CHAPTER ONE

INTRODUCTION

1.0 Background to the Study

The geodetic system of Nigeria (non-geocentric) was realized by the emplacement of physical (3D) control monuments, the primary triangulation and leveling network, and recently included the Zero Order GNSS Continuously Operating Reference Stations (CORS) at some locations.The geodetic system plays very important and critical roles in surveying, mapping, planning, engineering, environmental studies, management of natural and economic resources for sustainable development. CORS stations provide active geodetic controls network, which enable GNSS users to tie their positioning observations to the geodetic network without physically having to occupy a geodetic control point. Orthometric height (H) is an important factor in very many applications, ranging from topographical surveys and mapping, engineering constructions, planning of land resources among several others. Geodetic infrastructures in Nigeria are further divided into:-

a. Tidal Stations used to establish vertical datum e.g. Lagos Survey Datum (LSD).

b. Levelling Network (FBM, SBM) for connecting geodetic and conventional spirit leveling operations.

c. Gravity Survey Network used to provide computation of orthometric corrections and produce gravimetric geoid solutions and gravity data for other uses and users.

d. Continuously Operating Reference Stations (CORS) for global/international and zero order control networks recently established.

Geodetic Glossary (National Geodetic Survey 1986) defines height as “the distance, measured along a perpendicular between a point and reference surface (datum)”.Local geodetic datum have been developed in the past, in order to satisfy the surveying and mapping requirements of each country all over the earth and are used as computational reference surfaces e.g. Clarke 1880 for horizontal positions with Apapa or Yaba (MSL) Datum for Nigerian leveling network in Nigeria while NAVD88 is for vertical surface in USA. Featherstone and Kuhn (2006) observed that “most countries have adopted Mean Sea Level (MSL) as zero height surface for their national vertical datum” to provide critical information for a variety of applications. It is important to note that juxtaposing “geoid” for “mean sea level” and vice versa is not truly correct because the mean sea surface is not an equipotential surface and hence questionable as a reference/vertical datum in reality.The effect of climate change resulting in sea level rises coupled with geodynamical factors has rendered the mean sea level unreliable and unstable for vertical/reference surface.This is because, according to Hipkin (2002), mean sea level is constantly changing due to, for example, the changing amount of water in the oceans, plate tectonics changing the shape and volume of the ocean basins and the continents, and “thermal expansion of the oceans changing ocean density resulting in changing sea levels with little corresponding displacement of the equipotential surface”. Vertical reference datum forms the basis for all physical development requiring height information especially in transportation and fluid/water distribution. The difference between the normal to geoid and ellipsoid at a point is called deviation/deflection of the vertical which causes loop mis-closures for horizontal traverse surveys and also affects conventional leveling.They are mitigated by observation of azimuth on the 25th traverse leg in a traverse survey work and applying orthometric corrections in leveling differences before network adjustments.Such stations are called Laplace stations which are introduced to enforce parallelity of the axis of rotation of the reference ellipsoid and Earth’s rotation axis by observation of astronomical azimuth and longitude at the station. Solar observations are adequate for low order survey work while celestial bodies are necessary for high accuracy geodetic works.A physically meaningful vertical datum should guarantee height continuity across the adjoining boundaries and form part of solution to geoid problem nationally but msl though available worldwide, cannot guarantee this consistency/continuity.Development in space technology has however, provided a way out of the inconsistency arising from msl by geometric geoid development in the absence of national geoid model. Heiskanen and Moritz (1981) observe that geoid is of considerable importance for definition of a consistent height system. The geoid is taken as an equipotential continuous surface which is highly consistent for referencing heights and ensures continuity of heights across boundaries. Potential at a point is related to gravity by the following relationship given in Abeho *et al.* (2014):

$$\Delta w = \int_A^B g dx = \sum_i g_i \Delta h_i \tag{1.1}$$

Where W_A gravity potential at point A

g_i- is observed gravity values along leveling route

Δh_i -Incremental height differences obtained through leveling.

Global Navigation Satellite System (GNSS/GPS) is a space technique for determination of both horizontal and vertical **P**ositions as well as **V**elocity and **T**ime (**PVT**) information.Aves (2015) stated that the GNSS technology provides a good alternative to costly and strenuous process of spirit leveling provided an acceptable official geoid is available. Unlike the classical techniques, a global best-fit (mathematical model of the earth shape) ellipsoid, geocentric and coinciding with the mass center of the earth, WGS84 is used as a reference datum. The basic workings of the GNSS are that satellites in orbit transmit signals which are received and decoded by GNSS receivers to determine positions either in RTK or static post processing mode, the receivers may be single or dual or even triple frequency. Each satellite carries four atomic clocks so accurate that they might gain or lose an average of one second in 32000 years observed NASA SCIENCE (2002).

Multi - Networks as used in this study is the combined use of existing control stations of all orders ranging from primary (first order denoted by P), second order (XP or S) and tertiary (third order T).FCT development plans are divided into phases one to five, these physical developments require a lot of mapping, planning and civil engineering design and construction works. For the above to conform to FCT development master plan, survey controls are necessary and critical. These controls serve as basis for setting-out of plots, roads, drainages, reservoirs, dams and water distribution scheme i.e. design of various physical developments and also for scientific studies.

1.0.1 Modelling Concepts

This is a process of setting up a model written as equations, providing mathematical solutions and interpreting the results in physical terms of the behavior or reality of a phenomenon stated Kresyszig (2006) e.g. geoid undulation interpolation. To be acceptable, a model must by conception be simple to manipulate and consistent with observations/data with the intention to reproduce as accurately and realistically as possible natural phenomena. In the present study, two models were chosen to model the FCT geoid/orthometric height phenomena. Modelling involves several phases of planning which must be done sequentially to ensure a successful exercise.

The sequential procedures are presented for guidance. Firstly the type of model and the adequacy must be identified to model orthometric height processes. After this, the dataset available and to be acquired are identified as well as their quality with the mode of acquisitions. Then the equipment available, their quality and personnel on the ground for the exercise are also critical.

1.1 Nigerian Classical Datum

The first geodetic surveys of Nigeria were performed by the British Royal Engineers in 1910 – 1912 according to Mugnier (2009) going further to say that majority of the geodetic works were done beginning from 1928.

The Nigerian Geodetic network was established by classical methods of triangulation, trilateration, traversing for horizontal positions, while trigonometric and geodetic leveling was used for the height component of the coordinates system. The primary network enables collocation for various techniques like GPS, gravimetry to provide data for determination of shape and size of the earth as well as geoid etc.Since a datum must have origin and an orientation, the origin of the network was a point designated L40 and located at the northern end of the Minna Base of the triangulation system (in Niger State) with a computational reference surface as Clarke 1880 ellipsoid. The Geodetic height was derived from using leveling and trigonometric heighting and the parameters are given by Ono *et al.* (2004) as:

Ellipsoid	Clarke 1880
Semi-major axis (a)	6378249.145m
Flattening ($1/f$)	293.465
Latitude (φ)	09°38’ 09’’ N
Longitude (λ)	06°30’59’’E
Orthometric (H)	279.603m (above geoid)
$\alpha_{L40-L41}$	190°41’ 54’’.56

The inherent and significant deficiencies in the Nigerian primary triangulation network are well documented in Arinola (2006) who confirmed that errors propagate as survey works progress away from the origin and also as listed by Uzodinma (2005). And the one that relates to this research in the Uzodinma list is the absence of geoidal height (N) model.

Ezeigbo (1990) had earlier observed that effective utilization of data without a precise geoid in Nigeria has led to limitations and the only implication is that a local geoid model could be developed for GPS user community to enable conversion of ellipsoidal height to orthometric height in local applications within an area.

GNSS positioning methods naturally produce ellipsoidal heights (h), and these heights are best converted into elevations (H) using data from a geoid model.A first approximation to solving this problem could be to start from developing local geoid models of states using the availability of GNSS. Problem of circuit misclosures according to Ono (2002) makes the Nigerian Vertical system unreliable and unsuitable as a vertical control reference surface.All these existing controls or monuments are passive since they cannot communicate and are used as dead targets employed for survey observations. Active controls (CORS) realize their positions by continous transmissions of identity signals observed Ojinnaka (2007). A geodetic datum is defined by the size of reference ellipsoids and by its orientation and position with respect to each other i.e. the size is uniquely defined by two parameters, a and f . In conclusion, eight parameters uniquely define a geodetic datum via X, Y, Z of origin, α_X, α_Y, α_Z for rotation and a, f or (a,b) or (a,e) in geocentric Earth Centered Earth Fixed (ECEF) system.

The Nigerian Vertical monuments comprise primary conventional levels and trigonometrical heights. Primary levels include:-

Network in the southwest: A continuos line of levels from Lagos through Ibadan, Ilesha, Akure, Benin, Onitsha, Awka, Okigwe, Aba, and Portharcourt to Bende in Calabar Province and a further line from Ilesha to Oshogbo and along railway to Minna where it was connected to L41 the southern end/terminal of the Minna base. Total distance of double leveling is about 1800km.

Badejo *et al.* (2016) opined that defining and carrying mean sea level to the hinterland have resulted in uncoordinated or poor/inconsistent height system. The F.C.T. controls were established by classical methods by Geodata (trigonometrical stations), Development surveys, Seweje and Associates, Kaduna Polytechnic.

Trigonometrical heights have been determined by reciprocal trigonometric observations over all the primary chains. Kharaghani (1987) observed that reciprocal observations must be used to mathematically eliminate and check the effects of atmospheric refractions on heights which was actually the approach adopted for Nigeria. Adopting reciprocal procedures for height differences can yield centimeter order precision (about 8mm/km) for long lines according to Hasouna (2014).All optical/terrestrial measurements are affected by refraction and atmospheric errors, instrumental/personal errors, data processing and adjustments and hence the need for proper field techniques/processing to be designed to mitigate or eliminate them for reliable results.

Trigonometric heighting was done for triangulation stations that are usually located on top of hills while geodetic leveling was used for heighting Bench Marks (BM) that may range from Fundamental Bench Marks (FBM) to Standard Bench Marks (SBM) to other types including controls placed along roads/railways or not located on the hills. The vertical networks were connected to Tide gauges to define MSL as the zero surfaces. Featherstone (1995) maintained that MSL do not necessarily coincide with the same equipotential surface of the Earth's gravity field (geoid) at tide gauges possibly due to localized oceanographic phenomena among others. MSL at tide-gauge datum origin does not lie on a single equipotential surface observed and cannot lead to height consistency.

At this point, it is apt to state that Fotopoulos *et al.*(2003) specifically mentioned the Lagos tide gauge datum as suffering from assumptions that it is on a theoretical and approximate zero elevation surface without taking into consideration sea surface topography (SST) or river discharge corrections for tide gauge mean sea level determination.The implication is that national leveling networks based on MSL at a number of tide gauges lack consistency and is not compatible with space based positioning technology (that lacks local/national geoid model integrated). Heights can be obtained for all points within a geoidal surface unlike the current system that is datum dependent. MSL was possibly adopted because the ocean surface occupies 70% of the Earth's surface and available worldwide. The physical surface of the Earth as a computational surface is too complicated to contemplate its usage as a vertical reference surface.

Observations/measurements over MSL for a period of 18.67 years was accepted as long enough to average out the highs and lows of tides caused by changing effects of gravitational forces from sun and moon that produce tides.It takes the moon 18.67 (19 years) years for a complete circuit of the orbital plane around the ecliptic pole while one orbital pass of the moon takes one month to complete. National Geodetic Survey (1986) recommended 19 years as suitable period of mean sea level measurements at tide gauges.Varga (2016) reported that MSL in Croatia was defined by 18.6 years observation at tide gauges but without orthometric corrections due to lack of gravity data that was critical to to establishment of fundamental leveling networks. Hence, heights derived were not truly orthometric.

Non application of orthometric corrections was quoted to lead to a difference of 2 – 10 cm in Croatia. Canada is reported to determine MSL from measurements at five (5) tide gauges in the Coast over a 19 years period (Metonic cycle).Example of vertical Datums are: two in Argentina (mainland and Tierra del Fuego Island), Sweden RH70, Finland N60, Federal Republic of Germany FRGHN76 and Poland H60 observes Sjoberg (1993), Apapa and Yaba Datum in Nigeria among others.Level determination from different tide gauges generally results into statistically different height measurements. Level of inconsistencies arising from different tide gauges in Nigeria were not determined and hence not reflected in leveling adjustments to make heights consistent. In Canada for example, Vancouver water level was said to be higher than water level at Halifax by about 40 to 70cm and that it caused national scale tilt in published heights and had significant implications/inconsistency for border activities. Nigeria situation may not be too different from the Canadian or Argentinean experience (consider Yaba and Apapa datum) which may lead to inconsistency. There is no proof that the Nigerian leveling network complied with this time frame of 18.67 years. New Zealand was also reported by Amos (2007) as using observations of less than five years for vertical datum realization. Gulf of Mexico used modified five year epoch. Kumar and Burke (1998) stated that for vertical datum realization, a geoid model must be developed. The geoid is of considerable importance for the definition and realization of a consistent and homogeneous height system states Heiskanen and Moritz (1981).The geoid, according to Ahmed (2013) , is the only height which can be used for water flow contour elevations (that is the orthometric height). Rapp and Balasubramania (1992) gave ±2 m for global deviation of MSL from geoid based on two tide gauges. Featherstone (2000) suggested that this difference may be applied for unification of vertical datums.

Constant gravity potential of the geoid (W_0 = 62636850$m^2 s^{-2}$ related to GRS80 ellipsoid generated W_0) was changed by the IAG ICP1.2 Working Group at the IAG-IUGG in Perugia (2007) to 62636856 ± 0.5$m^2 s^{-2}$ as reported in Kasenda (2009).Kearsley $et\ al.$(2007) observed that this change of W_0 has led to both direct and indirect effects on the local height datum.

Kasenda (2009) suggested the following for determination of both effects as:

i. Direct effect $\Delta_{(\varphi,\lambda)}$is the seperation between the global physical and model reference surfaces at any point on the globe is computed from

$$\Delta_{(\varphi,\lambda)} = \Delta W_0 \,/\, \gamma_\varphi \qquad (1.2)$$

where ΔW_0 is the difference between W_0 values

γ_φ is computed normal gravity at the vertical controls

ii. Indirect effects are corrections to the local geoid resulting from change in W_0 computed from bias in height as:

$$B_{(\varphi,\lambda)} = (h - H)_{(\varphi,\lambda)} - (H)_{(\varphi,\lambda)} \quad (1.3)$$

Before the advent of space techniques, horizontal and vertical controls were independently done with the implication that the approach was cumbersome, labor intensive, slow, time consuming, prone to systematic and other errors, requires high number of personnel / logistics and costly to conduct over a large area.

1.1.1 Limitations of Classical Methods/Techniques

Geodetic networks were realized in the past as two dinstinct exercises namely (i) horizontal positions by angular and linear measurements between two physical points in triangulation and trilateration or in traversing by combination of angles and distances (the angles were not precisely measured due to atmospheric and systematic errors) and (ii) vertical positions done by spirit leveling and trigonometrical heighting. The 3D system decomposed into 2D + 1D cannot be entirely said to be consistent.

Whereas the 2D positions can be located widely apart, the vertical monuments are mostly located/sited along roads, railways, settlements and hence are rarely located in the hinterland. Interstation visibility is also a critical and limiting factor in the distances that can be covered. Different countries/continents are difficult to connect and hence the resort to adoption of different assumptions, conventions, projections and reference systems to suit their peculiar interest (be it economics, security, environmental and sociopolitical).

1.2 Space Geodetic System

Drawbacks of classical system made geodesists to start searching for alternative instrumentation and methods that can replace the classical approach without losing focus on accuracy level to be achieved within a highly reduced time frame and cost implication. Fortunately, Military exigencies led to development of space techniques of Global Navigation Satellite Systems, GNSS, and a generic name for all satellite global space systems ably and widely represented by the United States Global Positioning System (GPS) developed by Department of Defense (DOD) with the initial purpose of Military navigation. Surveyors and geomaticians took advantage of the 3D point positioning capabilities of GNSS to adapt to practical surveying needs, hence doing away with the classical 2D+1D system of old. GNSS has made it possible, very easy, and less cumbersome with reduced labor and costs of control establishment in a modern, seamless, interoperable and borderless way compatible with a worldwide projection system of Universal Transverse Mercator projection (UTM) complemented with fast computing facilities and scientific software.With GNSS, intercontinental survey works became feasible and practicable. Other modern techniques include but not limited to Very Long Base Interferometry (VLBI), Satellite Lunar Ranging (SLR) and others. GNSS can facilitate timely and accurate means of executing large engineering projects or applications, GIS operations, scientific or geo-scientific needs, global environmental data needs and mapping projects in 3D coordinates in a global reference system. Transformation formulae or software are integrated along with geoid model to transform from one system to the other is available.The geospatial data services which had earlier required enormous efforts and long period of time to acquire, collate/edit, process, analyse and interpret data can now be completed in a fraction of time and cost with the use of GPS when compared to conventional techniques.

That the GNSS results into 3D coordinates is not in doubt. Furthermore, that the 2D component of GPS observation is highly accurate is also not in contention. However,the height component which is referred to WGS84 ellipsoid differs from that produced from geodetic or trigonometric leveling referred to geoid (approximated by MSL) is obviously and highly significant due to the difference in reference surfaces used for computation as the "Zero" datum. Geodesy has always recorded height as reckoned from geoid. GPS has been designed to be geocentric i.e. mass center of the rigid earth coincides with the origin of the Earth Centered Earth Fixed (ECEF) system. Modern geocentric datum ensures uniformity by reducing effect of distortions to an acceptable level, removes the need for multiple coordinates system, overcome the limited territorial extent of the datum e.g. NTM (Nigerian Transverse Mercator) uses 4° width while UTM uses 6°, achieves compatibility with global datum as well as for survey systems observed Sergio (2003). Satellite techniques are used for geodetic connection over large distances because there is no line-of-sight constraint or limitations unlike the classical methods. At high altitude, satellites are regarded as "fixed". WGS84 ellipsoid is a mathematical surface used as reference for GNSS methods with the following parameters given by Leick (1995) as:-

Semi-major-axis, a= 6378137m

Inverse flattening f= 298.257223563

The geoid has been variously defined and in particular, Vanicek (2001) calls it an equipotential surface of the earth's gravity field. Komarov *et al.* (2007) see geoid as an equipotential surface of the gravity field suitably fitting the physical surface of the earth and determined in geodesy as the basic surface which orthometric heights are referred. Heiskanen and Moritz (1967) define geoid as a fundamental surface of physical earth that is closest to mean sea level (MSL): it is actually used as a mathematical model to represent the physical features of the Earth.Torge (2001) says the "geoid is a reference surface for vertical systems in geodetic measurements”. In summary, the geoid is used as a reference surface for orthometric height (H) required for engineering, environmental and geodetic/surveying applications.Fajemirokun (1988) defines geodesy as “that branch of applied mathematical physics, concerned with the determination of the size, shape and external gravity field of the earth, as well as the coordinates of points on the earth’s surface, in a three- dimensional, time-varying space, using appropriate observations and measurements”. Level surface/equipotential surfaces are surfaces where total potential is constant and can be mathematically represented as:-

$$W(x, y, z) = \text{constant} \tag{1.4}$$

This can be written by the expression as shown in Jensen (2011) as:

$$dW = \frac{\partial W}{\partial X}.dx + \frac{\partial W}{\partial Y}dy + \frac{\partial W}{\partial Z}dz \tag{1.5}$$

$$= \text{grad } W.\, dx \tag{1.6}$$

$$= g.dx \tag{1.7}$$

dW is the total change in potential which is zero when movement is along a level surface

i.e. $g.dx=0$

Therefore g is perpendicular or orthogonal to the equipotential surface.

The orthometric height (H) is measured along the curved line upwards from the geoid (W_0) i.e. opposite direction to the gravity vector.

$$g.dx = -gdH = dW$$

Jekeli (2000) gave the solution of g as:-

$$g = -\frac{dW}{dH} \tag{1.8}$$

It is important to note that the departure of MSL from equipotential surface due to various geodynamical effects, salinity, temperature variation, etc. is called Sea Surface Topography (SST) which may be as high as ± 2m.Hipkin (2000) also mentioned this phenomenon. Pan and Sjoberg (1998) suspects this difference of ± 2 m may be the same worldwide.The sea level was recorded at tidal gauges as a time series. A tidal gauge is an instrument which measures water levels for a certain time interval. These instruments account for natural phenomena, such as tidal waves and meteorological effects, with periods higher than the recording interval. The water level is continuously changing whether due to tidal influence, wind, currents, atmospheric pressure variations or temperature differences.

The focus of this research is centered on the height component of the 3D coordinates obtained from GNSS measurements which uses WGS84 ellipsoid for coordinate's computations. We recall that heights from GNSS are called ellipsoidal that is referenced to an ellipsoid surface. This height may be transformed to orthometric if by default a geoid model (usually a global one) has been integrated by the equipment manufacturers. Such models in most cases are global with the implication that such models may not fit the surface covering the study area and hence producing orthometric results, albeit an incorrect one, which may have serious consequences on construction or civil engineering works (in design and producing bill of quantities) and controls establishment where "height is a very important component of integrated geodetic datum" according to Odera and Fukuda (2015). Also Veronneau $et\ al.$ (2006) observed that a global geoid may not necessarily be needed for local applications for example within an area like FCT. The advantage of using geoid to define global mean sea level is the direct compatibility with global standard facilitating the geoid undulations with ellipsoidal heights obtained from GNSS. Moka (2011) says orthometric heights are usually presented in various countries as vertical datum for mapping. Kumar (2003) stated that a zero reference surface is equipotential and is time invariant with no slope and should not vary from place to place like MSL and other tidal surfaces and concluded that only geoid fits into the above description. Ayhan (1993) opines and correctly too “that orthometric heights (H) are the functional heights for mapping, engineering works, navigation and other geophysical applications” with geoid as the natural height datum.

Nwilo (2013) says heights modernization system implies geoid determination for the optimization of Global Navigation Satellite System (GNSS) in transformation of ellipsoidal height and scientific purposes. The International Earth Rotation Service (IERS) has advised that national agencies should make efforts to establish their precise national datums.When universal systems are used as a foundation, such as the International Terrestial Reference System (ITRS), compatibility with neighbouring countries’ systems is established at the same time and even globally. ITRF relates local measurements to a stable and accurate reference.

In the absence of this national vertical datum system (in a place like Nigeria/FCT for example), in a small area, a geoid model for the area may be developed for mapping, cadastral, engineering and construction applications. The Vertical Datum Unification project embarked upon by University of Lagos and funded by Shell Petroleum Development Company (SPDC) during the period 2002-2004 brought out the need for local geoid determination in Nigeria says Fajemirokun (2006).And we will concur by taking advantage of availability of modernized GNSS technology along with appropriate methodology. GNSS technology has made geoid development a possibility if points with orthometric (H) heights (FCT has most controls with orthometric height) are also heighted to obtain ellipsoidal (h) values for the same points or collocated. Recollect H is geoid related while his ellipsoid related and since geoid surface is different from ellipsoid, the geoid-ellipsoid separation (N) is called geoid undulation or height. The relationship between h, H and N is linear and is given by Kotsakis and Sideris (1999) as:

$$h = H + N + \xi \tag{1.9}$$

Where ξ = deflection of the vertical and curvature of the plumb line.

Seker and Yildirim (2002) says even if ξ=1", the error incurred is 8 x 10^{-2}mm i.e. 0.08mm which can be neglected and hence of no practical consequences.Sjoberg (2006) put the value at 1.5mm arising from the fact that H (in contrast to h) is curved along the plumbline. Furthermore, the difference between curved plumb line and straight ellipsoidal perpendicular (h) at h = 10000m and a case of deflection of vertical ξ=1" can be calculated from:

$$\Delta h = h \sin\xi \tan\xi \tag{1.10}$$

$$= 10000 \sin 1'' \tan 1'' < 1mm$$

which is negligible and Jekeli (2000) observed that "it considerably simplifies comparisons and conversions among different heights".This agrees with observations of Seker and Yildirim (2002). Also, determination of H from the relationship given by Nordin (2009) as:

$$H = h - N\cos\xi \tag{1.11}$$

For small $\xi = 30''$, cos ξ =1, hence H= h-N. The error due to small ξ is given by 0.0000011% which is highly negligible.

Therefore, with confidence, we can write without incurring errors that

$$h = H + N \tag{1.12}$$

Also, see Hofmann-Wellenhof (1997), Martenson (2002) andTorge (1980). The above can also be written as:

$$H = h - N \tag{1.13}$$

Since geoid is gravity related, geoid determination requires gravity measurements to cover area under investigation at acceptable distribution and density. This is the gravimetric approach. Kless and Prutkin (2010) have suggested however, that in the absence of high quality gravity data or sparse gravity data for gravimetric geoid, GPS leveling will produce a more acceptable result because systematic errors can be better controlled and lead to reduction in the long wavelength in leveling based heights. To buttress this, a study conducted in Ladak ,NW Himilaya by Banerjee *et al.* (1999), it was concluded that comparison of measured geoid with global geoid (OSU91,EGM 96) gravity model that GPS alone can be used for orthometric height determination.Various modes of using GPS are available ranging from static, fast static, real time kinematic (RTK) and so on.GPS can also be used as standalone or differential where two receivers or more are involved which Komarov *et al.* (2007) reported as providing ellipsoid heights with unprecedented accuracy up to one centimeter at regional and global scales which by implication means better accuracy at local scales.Bjelotomic and Basic (2016) confirmed that ellipsoidal heights with GNSS technology/CORS Networks will reasonably deliver 1 – 2 cm accuracy.However, surveying requires orthometric heights related to earth's gravity field like that produced by conventional spirit leveling. Conventional leveling realizes vertical datum by bench marks (BM) buried into the ground which is an unstable surface geodynamically to provide heights (H) at the benchmark. The geoid is realized relatively to WGS84 ellipsoid and represents a continuous surface known everywhere across the surface (under study).

Modeling of orthometric heights (H) from GPS observation is essentially to determine geoidal undulation (N) at several points to model geoid surface over the area. This is the interpretation of the relationship given by:

$$N = h - H \qquad (1.14)$$

which is the separation between geoid and ellipsoid. The Clarke 1880 ellipsoid is used in Nigeria. WGS84 is also an ellipsoid used worldwide as computational surface for GPS measurements. But the geoid as approximated by MSL is used in Nigeria and worldwide for orthometric height determination. However, the MSL has been noted to vary from country to country due to factors such as climate change, Sea Surface Topography (SST).For example, Andersen and Knudsen (2008) observed that global SST shows that MSL is approximately 10cm below the geoid at Cape Town, and 50cm above the geoid in Durban (60cm difference). Such figures are not available for Nigeria (between LSD and SBM Yaba) to determine accuracy of modeling SST and possible bias in geoid model. Hipkin (2000) stated that in fact SST "is notoriously difficult to quantify and model in coastal zone". Lamothe *et al*. (2013) observed that globally SSTvalues range roughly from -1.5m to + 1.5m in reference to the geoid. As recently as 2001, the IPCC (2001b) had predicted that global average temperature increases during the 20th century (between 1.4°C and 5.8°C by 2100) will lead to worldwide sea-level rise of up to 1m which will have an impact and several implications on coastal environments. Sahrum (2017) quoted AVISO (2013) which reported global mean sea level rise (GMSL) between 1993 and 2012 to be at the rate of 3.11 ± 0.6 mmper year.Global warming (temperature rise) leads to volume expansion of ocean water, melting of mountain of glacier, melting of the Arctic and Antarctic ice caps to contribute largely to sea level rises and lead to unreliability of MSL as a reference surface as problem in height datum definition and realisation.Ojinnaka (2006) stated that between 1972 and 1985, at Forcados, in Nigeria, a net rise of 0.04m was recorded while Lagos recorded an apparent fall of 0.02m. This shows how unreliable and unstable the MSL could be as a vertical reference surface.

Reports have it that from 2003 to 2006, melting ice sheets and glaciers (quoted to be about 150 billion tons) contribute to rising global sea level rise of about 0.3m per year from GRACE Mission measurements. Kumar (2016) emphatically confirmed that "MSL is not an equipotential surface because it has slope with respect to the geoid".Odera and Fukuda (2015) say the assumption of coincidence of geoid with approximate mean sea level is valid theoretically, as the difference between them is negligible, but practically impossible to achieve especially with the presence of Sea Surface Topography (SST).The above reveals that MSL is not a reliable and consistent surface for referencing heights. Pugh (1987) stated that non-linear response of waves on msl is a fundamental limitation to accuracy and consistency. Constancy of gravity is desirable for referencing heights.

From $$g=km_1m_2/r^2 \quad (1.15)$$

it is a fact that masses m_1 and m_2 and r (distance between m_1 and m_2) are constants (do not change) which means g = constant and it has been confirmed that the change of g is insignificant and therefore confirms the geoid as a constant and consistent surface to be relied upon to reference vertical heights as a vertical datum. Local geology, according to Rothstein (2016), has very small gravity differences, on the order of 0.01% or even less. At altitude of 5km (high mountain), gravity difference is less than 0.2%. The highest mountain in FCTis located at a primary (N series of control) N35 (H=940.960m) is less than 1km in height to imply that differences in gravity will be of no practical consequences to accept gravity as a constant surface to realize a vertical datum.FCT is geologically stable and not located on any known active zone and hence local changes in in geology may not have any significant effect on gravity.

Consider the following scenarios from the triplet h, H and N with a linear relationship given in equation (1.12) as: h=H+N, then for any method (gravimetric, geometric or astro-geodetic), we have:

(a) If N=0, then h = H, on a global basis, the error arising from this assumption can be up to ± 100m. If allowable standard deviation (**σ**) is 100m, then there is no difference between ellipsoidal (h) and orthometric height (H). For a scale of 1/100,000 and smaller, N of 50m may be negligible and h may be applied in place of H. This has serious consequences for aviation especially when the pilot believes the aircraft has more clearance than what is available in reality.

(b) Over a small area and assuming flat surface, ellipsoidal and geoid surfaces are parallel planes. The implication of this is that N is uniform or constant for all points on the surface which means the users must accept responsibility and bear the severe consequences that may arise from this assumption in the face of subsurface density varying realities.

(c) If the ellipsoidal and geoid planes are not parallel i.e. one is titled to the other, with three points of known N, linear interpolation can be used to compute the N for other points while points outside the plane defined by the three points are extrapolated but reasonable care must be exercised. A "best-fit" model is derived if many more points of known N are available. This yields better and accurate results of N and hence the orthometric heights.

(d) If the two surfaces are both curved and not concentric. Ellipsoid can be mathematically defined and easy for geodetic computations while the geoid, though a continuous surface but its irregularity makes the representation mathematically difficult and hence the separation between ellipsoid and geoid involves complex modeling mathematically.

1.2.1 Impacts of Space (GNSS) Methods

The problems associated with classical methods have been resolved by the development of space techniques of GNSS. The impacts include but are not limited to:-

a. Ability to acquire 3D positions devoid of the previous 2D+1D classical techniques

b. Possibility of relative postioning to mitigate common errors and improve on reliability of results.

c. Intervisibilty requirements between stations are not critical and in fact not a must.

d. Weather is not a serious and critical issue.

e. Globalization of networks within global reference systems are achieved while still retaining local referece systems where necessary and desirable.

f. Over large areas or long distances, space methods are more cost efficient and timely.

g. Consistency of height is enabled through globalization of height reference system.

h. Data exchange becomes seamless and compatible for interboundary projects.

1.3. Statement of the Problem

A critical characteristics of a reliable height system is consistency. The existing mean sea level (MSL) reference system in practice for centuries in the determination of orthometric heights is the conventional technique of spirit levelling which is tedious, requires time and labor intensive, costly, and prone to propagation of systematic error. Effects of sea surface topography (SST), temperature variation, salinity, geodynamical factors have rendered the use of MSL unsuitable and unreliable for achieving consistency. Ono (2002) observed that there is problem of circuit misclosures and concluded that Nigeria vertical system in its present status, is unsuitable and unreliable as vertical controls. Ojinaka (2006) reported MSL rise and fall at Forcados and Lagos respectively of 0.02m and 0.04m pointing to the unreliability and inconsistency arising from MSL reference surface. Amos (2007) stated that the difference in water levels between Vancouver and Halifax (40cm – 70cm) led to national scale tilt in published orthometric heights. Bello (1977) as well as Udoffia and Fajemirokun (1978) confirmed the lack of coincidence of Lagos Survey Datum and Standard Bench Mark with MSL. Fajemirokun (2006) declared that presently, Nigerian heights (including that of FCT Abuja) are strictly speaking not orthometric but heights closer to the geoid reference surface than the ellipsoid. Fotopoulos *et al.* (2003) specifically mentioned Lagos tide datum as suffering from assumptions of being on a theoretical zero elevation without consideration for Sea Surface Topography (that ranges globally from ± 1.5m) or river discharge corrections. Also Veronneau *et al.* (2006) observed that a global geoid may not necessarily be needed for local applications for example within an area like FCT. The advantage of using geoid to define global mean sea level is the direct compatibility with global standard facilitating the geoid undulations with ellipsoidal heights obtained from GNSS. However, continued use of global geoid models (EGM 96, EGM 2008) integrated by default for local applications has obvious implications for large scale base maps needed for design, mapping, planning and environmental studies and services as well as for physical developments/ constructions. In spite of this, the GNSS has created an opportunity for the determination of orthometric heights with poor quality especially in areas with poor quality gravity data. Ezeigbo (1990) opined that absence of Nigerian geoid model puts a limitation to the full utilization of data from GNSS conversion of ellipsoidal height to orthometric in local applications. In the absence of national geoid model, a local geoid could be developed for adoption by GNSS user community for orthometric height determination in geospatial data acquisition. Compatibility with global systems is also achieved by use of local geoid model in orthometric height determination. As surveyors/ geodesists, the critical role of orthometric heights in physical development projects cannot be overlooked especially in surveying, mapping, engineering and environmental projects. The efficiency of infrastructures should never be taken for granted considering human comfort, safety and huge investment costs involved. Nwilo (2013) says heights modernization system implies geoid determination to upgrade existing height to geoid related heights.

1.4. Aim and Objective of Study

1.4.1 Aim

The aim of this study is to model orthometric heights from Multi-Networks GNSS/ Precise leveling in FCT to enable GNSS users convert highly accurate ellipsoidal height (**h**) to geoid related orthometric heights (**H**) with a view to achieve consistency without precise leveling vertical datum or mean sea level. The specific objectives are:

i. To investigate the number and physical and physical status of the orthometric heights of existing controls in the FCT, with a view to establishing the stability of the stations and using check angles and distances to confirm stability by "in-situ" tests.

ii. To carry out GNSS observation so as to determine the ellipsoidal heights of the existing controls using relative technique.

iii. To determine geoidal heights (N) of the existing controls using the GNSS ellipsoidal heights (h) and the existing orthometric heights (H) from (N=h-H).

iv. To develop programs using Microsoft excel 2010 for interpolation of geoid heights from the polynomial equations (bicubic and multiquadratic) and hence model orthometric heights using GNSS ellipsoidal heights.

v. To compare the orthometric heights obtained using the two interpolation models with the existing orthometric heights using **t**-test statistics.

1.5 Study Area

Nigeria, officially called the Federal Republic of Nigeria, is a Federal Constituted Republic comprising thirty-six (36) states and the Federal Capital Territory, Abuja. The Country is located in West Africa and shares borders with the Republic of Benin in the West, Chad and Cameroon in the North- East and East respectively, and Niger Republic in the North. Nigeria falls between latitude 4° and 14°N and longitude 2° and 15° E with a total area of

923,768km^2. Its coast in the south lies on the Gulf of Guinea on the Atlantic Ocean with total length of border put at 853km. The highest point in Nigeria is on the Mambilla Plateau in Chappal Waddi in Taraba State at 2149 m above sea level. Major rivers are Rivers Niger and Benue that empties waters into the Atlantic Ocean.

The desirability or otherwise of Lagos remaining the Federal Capital of Federal Republic of Nigeria led to the establishment of the Aguda Panel that declared that the dual role of State Capital and Federal Capital played by Lagos, needed to be revised and reversed. This led to promulgation of Decree 6 of 1976 establishing Abuja as the new Federal Capital covering an area of about 8000km^2 carved out from Niger, Nasarawa, Kaduna and Kogi States citing factors such as centrality of location, accessibility from all parts of the country, favorable climatic conditions, availability of land for future expansion, physical planning convenience as highly favorable to Abuja. The development of Federal Capital Territory (F.C.T.) is handled by Federal Capital Development Authority (F.C.D.A). Such developments are in the area of mapping, planning, construction of houses and roads, water distribution network, recreation facilities among others. The FCT has six (6) Area Councils namely: Abuja Municipal Area Council (AMAC), Bwari Area Council (BAC), Kwali Area Council, Abaji Area Council (AAC), Gwagwalada Area Council (GAC) and Kuje Area Council (KAC) The FCT is bounded by the following States: Niger, Kaduna, Nasarawa, Kogi. Figure 1.1and Figure1.2 show the FCT and the 36 states of Nigeria.

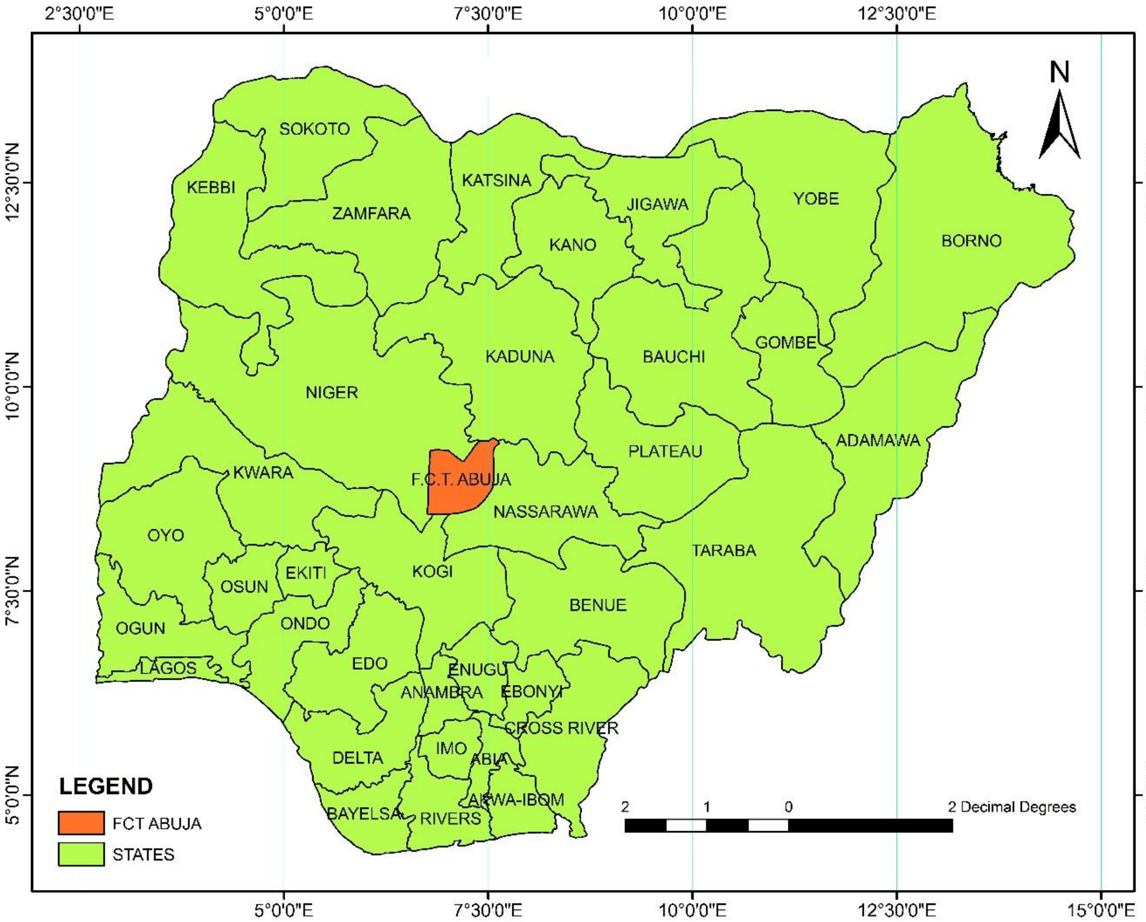


Figure 1.1: Map of Nigeria showing 36 States and FCT Abuja

Source: Arcinfo Shapfile 2010 (ESRI)

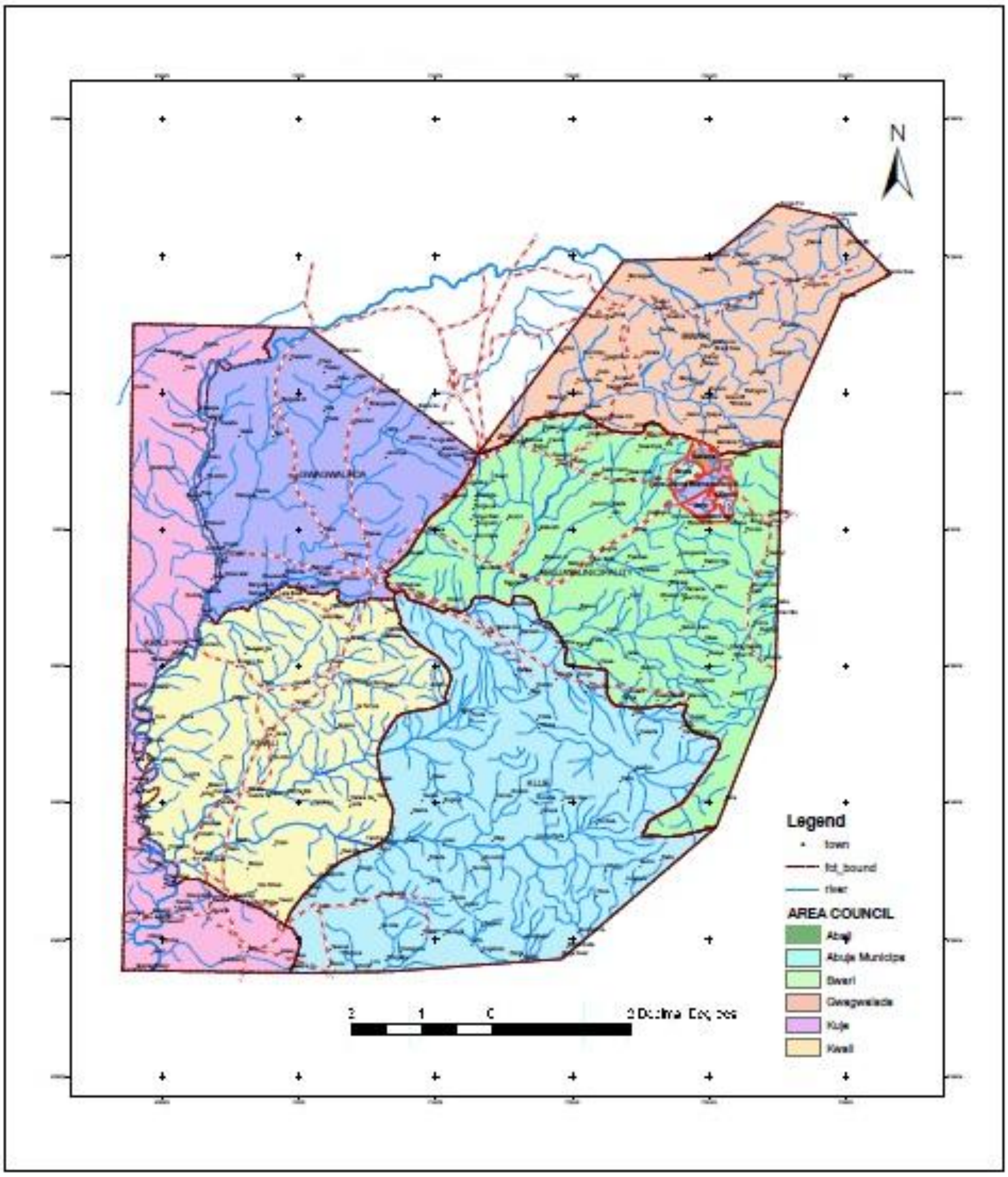


Figure 1.2: Map of FCT Six Area Councils

Source: Federal Capital Development Authority (Survey and Mappimg Department)

The study area is the Federal Capital Territory (FCT) which has an area of about 8000km^2 according to ABUJA Masterplan defined in the 1976 FCT decree with Abuja as the capital. FCT was created on 13th February, 1976 and is headed by a minister appointed by the President and Commander – in –Chief of the Nigerian Armed Forces. University of Abuja, Catholic University, African Business School, Abuja University of Technology Abaji (under construction), College of Education, Zuba are among higher institutions in the FCT. The

2006 population of FCT is put at 1,405,201 by the Nigerian Population Commission. The FCT is bounded to the north of River Niger and River Benue. To theWest is Niger state, Kaduna state to the Northeast, Nasarawa to the East and South and Kogi State to the Southwest.

FCT lies between latitude $8^{\circ}15'$ to $9^{\circ}12'$N and longitude $6^{\circ}\ 27'$ to $7^{\circ}\ 23'$ E. FCT is in the central region of Nigeria. Abuja city occupies 275.3km^2within the Federal Capital City (FCC) which has an area of 713km^2. FCT is situated within Savannah region with moderate climatic conditions. FCT has six area councils namely (i) Abuja Municipal Area Council (AMAC) (ii) Bwari Area Council(KAC) (iii) Kwali Area Council (iv) Kuje Area Council(KAC),(v)Gwagwalada Area Council (GAC) and (vi) Abaji Area Council (AAC) . Natural Resources found in the FCT include Tin, Marble, Mica, Clay and Tantalite.

For tourism, the popular Ushafa pottery village, Jabi Lake and Park, the National Church and Mosque in the Central Business District (CBD), Aso Rock which is the largest and highest mountain in FCT at primary station N35 (940.960m) elevation above mean sea level, Katampe Hill which is one of the highest rock outcrop and also acclaimed to be the geographical center of Nigeria, National Assembly Complex within the Three Arms Zone and theMillennium Park. The Abuja Carnival has been a yearly affair since 2005 among others.

The relief has lowest elevation in the extreme southwest where the flood plain of the Gurara River is at an elevation of about 70m above Mean Sea Level. The land rises irregularly eastwards, northwards and northwestwards. The highest part of the territory is in the northeast where there are many peaks over 760m above MSL. Hills occur either as clusters or form long ranges. It must be noted that the Federal Capital City (FCC) with five districts

(Maitama, Wuse, Central Area, Asokoro and Garki) was developed on the Gwagwa plains among several other plains.

The FCT has two main seasons, rainy season which starts from April to October and dry season starting from November to March. The undulating terrain and high altitude of the FCT territory provides regulating influence on its weather. The annual rainfall ranges between 1100mm to 1600mm.The soils of the FCT are generally shallow and sandy in nature which implies the soils are highly erodible but the location and development of Federal Capital City (FCC) on Gwagwa plains are on the deep and clayey part which are also fertile and therefore favorable for urban development.FCT falls within the guinea savannah zone of Nigeria. Patches of rain forest is however, found in the Gwagwa plains. The ecological problems are due to overstretching of social amenities and significant increase in population and are more pronounced in places like Karu, Nyanya, Gwagwalada among others. Soil erosion and degradation are physical signs of the FCT ecological problems. De-vegetation due to land clearing for various forms of development is also encountered.

1.5.1 Geological Setting of Federal Capital Territory (FCT)

The geology of FCT is said to be predominantly underlain by metamorphic and igneous rocks of Precambrian age and located within the basement complex in the Nigerian geological setting observes Kogbe (1989). For stability and geodynamics in Nigeria, two areas have been of great concern and they are (i) Ifewara –Zungeru fault system from the Southwest to the Northwest Nigeria where tectonic activities have been reported in the past and (ii) the Coastal region of Nigeria which has a sedimentary nature and hence generally unstable with possibility of subsidence arising from exploration of underground minerals (Nwilo,1995). Fortunately, FCT lies outside of these two zones to enable us to assume that geodetic infrastructures located within the FCT are geodynamically stable. Consideration for selection and location of controls includes ground stability geologically.

1.6 Justification and Significance of Study

For practical purposes in geospatial data acquisitions and applications, knowledge of geoid is essential in study of earth crust, geophysical and geoscientific studies, surveying, cartography, hazard and geohazard studies, engineering applications among several others.Within limited application areas, development of local geoid model is very important for GNSS users for consistency in orthometric height determination and compatibility with the space techniques of GNSS.

Provision of controls suitable for day to day surveying, mapping, and civil engineering construction activities is a sine qua non in Geodesy and cannot be wished away without facing dire consequences. The instrumentations and methodology needed to improve and increase productivity to meet timeline schedules must also be factored into a control scheme design without losing focus on minimum acceptable accuracy standard.Presently, there is a total absence of official geoid model for Nigeria, talk less of local geoid model for FCT and with the prevailing acceptance and widespread use of GNSS, the applications of GNSS require the development of F.C.T. geoid model to enable GNSS heights to be transformed to orthometric heights for use in geospatial services locally i.e. modeling orthometric heights. The geoid can be obtained everywhere as a continuous surface realizing a consistent reference system forming the basis of economic activities including mapping, environmental studies, etc. but it is not directly observable. The use of global geoid model as default in GNSS may introduce errors of fit. In actual fact, a research in Malaysia confirmed a difference of fit could be up to 2m. Also because of the lack of gravity data as input for the development of global geopotential models over most parts of Africa, necessary geoid models are unavailable to enable GNSS manufacturers integrate them for transforming GNSS height to orthometric: hence global models have been in use by default. Global geoid model is used to estimate geoid undulations as computed from gravity observations along with knowledge of topography and mass variations in the earth for global fit. National geoids are used for different areas of the world. Local geoids are used for different areas of a nation with large extent such as Nigeria for day to day mapping and engineering purposes.

Vanicek and Krawisky (1982) observe that global models are not as accurate as national geoids and their use should be avoided where better information is available. This necessitates the need for development of local geoid model for local applications in the interim.Note that global models are designed for global applications.Moka (2011) suggested that in the absence of official geoid model for Nigeria, EGM2008 should be adopted for orthometric height determination for regional applications. This suggestion was made without evaluation of the use of EGM2008 over Nigeria since global models are supposed to be used for global projects. Although Uzodinma and Oguntuase (2014) did a study in University of Nigeria, Enugu Campus whose results may not support Moka's suggestion. A contrary opinion was expressed by Kamguia *et al. (2007)*that GRACE gravimetric model for geoid solutions in Africa should be adopted instead of EGM96 because according to Merry (2003) ,EGM96 although is widely used but does not represent precisely the gravity data of Africa. Contrast this with suggestion by Moka (2011) that Nigeria should adopt EGM2008. GRACE has 1cm global accuracy according to Ihde (2010) with 100km resolution i.e. GRACE models may be very suitable for geoid solutions in Africa.

Gomaa (2010) observed that EGM 96 provides geoid undulation (N) values with uncertainty of ±37 cm (over thirty million surface gravity values were used in its development) while EGM2008 results in ±22cm (developed from over fifty-five million surface gravity values) as stated by Holmes *et al.(2008)*and may be in1', 2.'5, 5'or 10' (arcminutes grid). Global models are not accurate for engineering applications as well as cadastral surveying making development of local geoid models imperative.

The absence of a national geoid raises two posers and they are:-

(i) Non challant approach of doing nothing which has very serious implications for mapping/planning, cadastral, engineering and environmental and coastal studies.

(ii) Continued use of global geoid models (EGM 96, EGM 2008) integrated by GPS receivers' manufacturers for local applications with the obvious implications for large scale base maps needed for design, mapping, planning and environmental studies and services as well as for physical developments constructions and execution.

Local geoid model could be developed in the absence of gravity data for local use in Nigeria. Lack of gravity data coverage and official Nigerian geoid model requires use of GPS data (for ellipsoidal height h) in combination with available existing orthometric heights (H) to develop a local geoid model for FCT. Developed geoid model has the advantage of consistency and homogeneity in orthometric height since such heights are based on the International Terrestial Reference Frame (ITRF); compatibility is established at the same time which is quite different from reliance on MSL (which is not a level surface and hence unreliable). Ezeigbo and Adisa (1980) observed that the very poor gravity coverage of Nigeria poses a very serious challenge for the determination of the vertical reference datum (for orthometric height).

The definition of vertical datum based on geoid from this study will ensure homogeneity and consistency of heights across the whole of F.C.T. Abuja. Vanicek *et al. (2012)* supported this definition i.e. geoid based orthometric height. So, the modeling of orthometric height of F.C.T. will ease all environmental problems and engineering constructions in all ramifications. It will further provide research resources for further studies in the field. Odumosu *et al.(2015)*say orthometric height can be used to predict fluid flows efficiently. Dodo and Idowu (2010) listed geoid as one of the requirements of National Geospatial Data

Infrastructure (NGDI) as a reference surface for height. Geoid modeling is an indispensable part of geodetic infrastructure studies using GNSS.

Orthometric heights are used in G.I.S., civil engineering, cartography, hydraulic studies and projects. Satterfield (2010) define hydraulics to be that branch of engineering that focuses on the practical problems of collecting, storing, measuring, transporting, controlling and using water and other liquids. Hydrostatic pressures due to flow of liquids is believed to vary directly with elevation. Use of incorrect elevation leads to pressure drop that affects significantly the efficiency of water distribution as building floors rises up for example.

1.7 Scope of Study

The scope of study is as follows:-

i. Determination of ellipsoidal heights of the existing controls using GNSS observation in static mode.
ii. Computation of geoid heights of the existing controls using the GNSS geometric heights and the existing orthometric heights
iii. Development of programs for interpolation of geoid heights and computation of orthometric hieghts using two interpolation models (multiquadratic and bicubic models).
iv. Comparison of the geometric orthometric heights with the existing orthometric heights.
v. Computation of accuracy of each of the interpolation models.

1.8 Limitations of Study

The developed geoid model for GNSS leveling in modeling orthometric height

i. Is limited and valid over the Federal Capital Territory (FCT) and based on the existing orthometric height values in the official coordinate register of Surveying and Mapping, Dept.of F.C.D.A.

ii. Differential GNSS technique was adopted and limited to the FCT study area and accuracy of the technique.

iii. Comparisons of developed geoid models were limited to controls within the FCT geodetic network.

iv. Only one model of Dual frequency GPS was used throughout in this study i.e.Hi-Target V30 Pro.

1.9 Consistency and Compatibility

For this study, the issue of consistency and compatibility with space technique of GNSS is discussed below:

i) **Consistency: -** geodetic control monuments are physical marks assigned with numerical values in metric system and used to realize and make 2D planimetric and 1D orthometric data available. When a single datum is used for coordinate determination, then homogeneity or consistency is achieved but when in the case of vertical system where two different tidal gauges were used for MSL determination, then there is inconsistency and lack of homogeneity. Consistency and homogeneity is therefore achieved when heights are referred to a geoid equipotential surface.

ii) **Compatibility: -** in this study, compatibility refers to the ability of the local geoid model (N) combined with GNSS ellipsoidal height (h) to output orthometric height (H=h+N) directly by some conversion program integrated into the receiver or manipulated manually.

CHAPTER TWO

THEORETICAL FRAMEWORK

2.1 Geodesy

Geodesy can be defined as "as a global science of determining the geometry, gravity field, and rotation of the earth and their evolution in time" and geoid definition, realization and determination as of strategic interest to mankind. Geodesy is applied in surveying, positioning, bathmetry, mapping, navigation, etc. and can be further classified into i) geometrical geodesy which is concerned with describing geometry i.e. coordinate systems (N,E,H) either in Cartesian or curvilinear geographical coordinate systems, in either geocentric or geodetic systems with reference ellipsoids , ii) physical geodesy which concerns the earth's gravity field needed for height establishment and iii) satellite geodesy which involves using orbiting satellites to obtain data for geodetic purposes.Jekeli (2006) says the coordinate system becomes a reference frame when parameters are specified. Also Vanicek (2001) quoted Helmert (1880) as defining geodesy as the science of measuring and portraying the Earth's surface while Vanicek and Krakiwisky (1986) defines geodesy as the oldest geosciences believed to be a "discipline that deals with the measurement and representation of the earth, including the gravity field, in a 3D time varying space".

As a very important part of 3D geodetic networks, the geoid must be consistent and reliable to be relevant in National Geospatial Data Infrastructure (NGDI) and very germane to all physical developments and geo/scientific studies. A geocentric system, ECEF, places the origin of the coordinates at the center of mass of the earth.

The vertical datum differs from the traditional (uses MSL) to the space (uses the ellipsoid). The departure of MSL from the geoid (equipotential surface of gravity field) is $\pm$ 2m. Sea Surface Topography (SST) is the height of MSL above geoid. The knowledge of geoid is important to transformation of GNSS height to orthometric height. See figure 2.1for the three types of surfaces in geodesy.

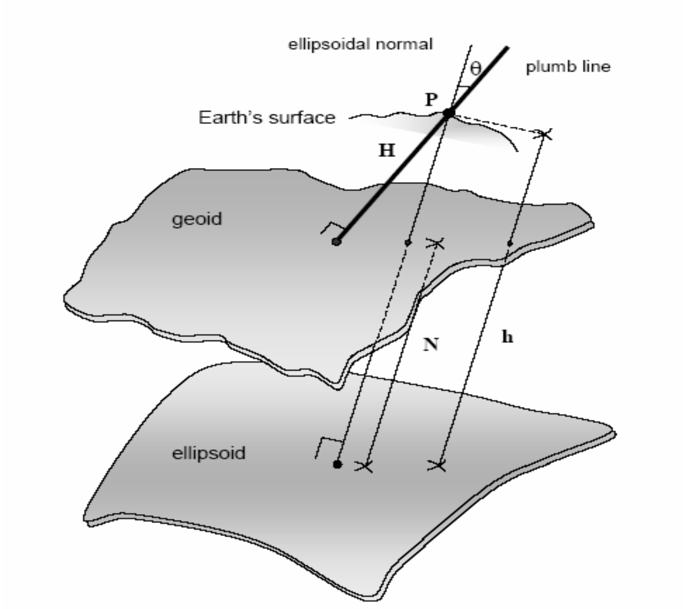


Figure 2.1: Surfaces in geodesy and relationship between ellipsoidal, geoid and orthometric heights.

Source: Fotopoulos, 2003

In geodesy, we define geodetic datum as determined by surveying process and for vertical; it is done conventionally by geodetic leveling from a datum origin point that is at a tide gauge. A datum is a level surface for which "Zero" value is attached. Datum can be local, astro-geodetic, gravimetric and global. The local datum has the advantage of better fit to the geoid and consistent with gravity field of the earth due to the fact that scientific, engineering and mapping needs require relationship with gravity via datum. Gravimetric datum is suitable for global application because it is usually consistent with the earth gravity field. Astro-geodetic datum requires measurement of the two components of deflection of vertical (ξ, η) and hence geoid based on this is systematically tilted and not suitable for global and scientific uses. All the various datums have relationship with gravity.

The Geodetic datum is resolved into horizontal (2D) for horizontal positions and vertical Datum (1D) for heights by adopting a single point as origin where the surface defined height is zero. From a point as origin, differential leveling is used to transfer height to other points without regard for absolute gravity potential because we are dealing with one point. However, where two points are used as different origin then transforming from one vertical to another requires gravity potential difference between the two origin points. The difference is not zero because MSL is not exactly a level surface. Knowledge of geodesy is necessary for transforming GNSS height to orthometric heights and vice versa. The geoid can't be seen or measured directly but can be modeled from gravity data or using geoid undulation.Isioye (2010) says that in Nigeria, like most African countries, the only geoid model available is a Global Geopotential Model (GGM) computed as a series of spherical harmonic expansions which is not accurate enough (at the one meter level) for mapping, surveying and civil engineering works. Hence, the need for development of a local geoid model becomes very critical and necessary for orthometric heights determination by geospatial data users.

2.1.1 Gravity

Gravity is important because geodetic instruments use gravity as reference/alignment to vertically (using plumb bob) define a horizontal plane of constant gravitational potential. This is a force which tries to pull two objects towards each other. It is the force that holds planet in their orbits around the sun and keeps the moon in orbit around the earth. Absolute gravity value are determined from relative gravimeter by calculating time difference between location and base location whereas in the static, gravimeter is fixed on a more stable earth surface with the major challenge being the irregular distribution of observation points. Airborne gravimetry is used to obtain information on regional gravity field for regional area.

Airborne method is faster, more efficient and points are more regularly distributed. Any methods used aims at gravity measurements with geodetic application such as geoids model development. This was also confirmed in Nwilo (2013) that "these data (gravity)were very useful in the determination of the geoid for the country" if available at specified intervals. Geoid model is needed for realization and definition of vertical datum especially when GNSS/GPS is to be used for orthometric height determination which Pinon (2016) called "physical height".

Satellite systems give global and uniformly distributed data for the earth's static gravity field and variations e.g. by CHAMP, GRACE, GOCE that has been dedicated for measuring gravity field for accurate geoid model. Pai *et al.* (2010) say gradiometer developed from gravimetry was used to measure gravity gradient models for regional or global usage. The geodetic boundary value problem can be solved by spherical harmonics expansion for global representation and by Stoke's integration for regional area (EGM 96) and EGM 2008 for global (whole earth).

Orthometric height cannot be divorced from gravity field, especially as it relates to equipotential surfaces of the earth. Level heights will be distorted due to undulation and divergence of surfaces if gravity is not taken into account.

Earth gravity field potential (W) can be resolved into potential V of the gravitation force and the potential of the centrifugal force ϕ, i. e.

$$W=V+\phi \tag{2.1}$$

Potential of gravitation force V is given by

$$V=G\iiint \frac{\rho}{l}\,dv \tag{2.2}$$

Potential of centrifugal force is given by:

$$\phi = \frac{1}{2} w^2 (x^2+y^2) \tag{2.3}$$

Where G is gravitational constant=$6.672 \times 10^{-11} m^2 s^{-2}$

dv=density of volume element

l= distance between dv and point (x,y,z)

w= angular velocity of earth of rotation

$\sqrt{x^2+y^2}$= Distance from axis of rotation

According to Hyo (2014), Moritz (1980b), Heiskanen and Moritz (1967), therefore, the gravity potential W satisfies the following relations:-

$$\Delta W = \begin{cases} 2\omega^2 & \text{outside topographical surface} \\ -4\pi G\rho + 2\omega^2 & \text{inside topographical surface} \end{cases} \tag{2.4}$$

Δ = Laplacian operator

$$g = gradW = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} \tag{2.5}$$

The Geoid is a special equipotential surface whereW (x, y,z)= W_o= Constant.

The distance of a point to the geoid along the plumbline is the orthometric height (H).

WGS84 reference ellipsoid has the following parameters:

a= 6378137m; b= 6356752m; ω=7292115x10^{-11}rads-^{1}

$u_{o=}$ 62636860.08497m^2/s^2 theoretical gravity potential of reference ellipsoid.

Heiskanen and Moritz (1967) gave the following:

$$e = \sqrt{\left(1 - \frac{b^2}{a^2}\right)} \tag{2.6}$$

$$\Delta u = \left. \begin{matrix} 2\omega^2 \text{outside reference ellipsoid surface} \\ -4\pi g G\rho_N \text{ inside reference ellipsoid} \end{matrix} \right\} \tag{2.7}$$

u=constant= normal level surfaces with the normal gravity direction/normal plumbline

ρ_N = Normal density

2.1.2 Orthometric Correction.

This is computed from acquired terrestrial gravity data and applied to successive height difference (dh) obtained from leveling and staff along levelling routes. The summation of height differences will normally arrive at zero if and only if this correction is applied to make them orthometric height differences as given by Moka (2011).

It is important to note that error analysis for a point in Swiss Alps with altitude of 2504m, an error in the mean value of gravity on the plumb line $\delta_{\bar{g}}$ of 1µGal corresponds to an error in height of 3mm as given in Holloway (1988). In Nigeria, a similar study shows that using computed normal gravity from International Gravity Formula in place of observed gravity results in height difference of 4.2mm in a relatively flat area consisting of Imo ,Cross-River and Rivers states as discussed in Okafor (1985).Odumosu *et al*. (2015) also reported 1mm difference in a study in Lagos state. This is insignificant for most surveying and mapping or low order applications.

It is important to note that Edan *et al*. (2014) observed that orthometric corrections are a function of gravity data which are insufficient and/or unevenly distributed in Nigeria and hence makes gravimetric method for geoid determination not adequate/sufficient for height conversion.

2.1.3 Geodetic Boundary Value Problems (GBVP)

The geodetic boundary value problem allows the determination of the geoid and the external gravity field and potential of the earth from the value of gravity scalar and the gravity potential given on it. However, in reality gravity observations are made on or above the earth's surface. Hence, the first task of solving geodetic boundary value problem involves the procedure of analytically predicting the gravity value on the geoid from observation made on or above the earth's topography observed Kebbie (2013).

These are problems dealing with the determination of the gravity potential in and outside the earth's surface from ground gravity data depending on the input data according to Hyo (2014) and namely:

(i) Third Geodetic Boundary Value Problem (3^{rd}GBVP) which has an input the orthometric Height (H) and the gravity g on the surface of the earth obtained via gravimetry and leveling, with the output being geoidal undulation (N) with the conditions that (a) center of reference ellipsoid coincides with the geocenter (b) mass of reference ellipsoid equals to mass of the earth, the constant equal zero. See Heiskanen and Moritz (1967) which leads to Stoke's problem.

(ii) Second GBVP which has as input h and gravity on the surface and GPS measurements with output as orthometric height (H) and geoidal undulation (N) and the external gravity potential when boundary surface is the geoid, thus becoming Hotine problem.

(iii) First GBVP which has as input data as N and topographic surface with output being gravity and external gravity potential and leads to inverse Stoke's/Hotine problem.

(iv) Downward continuation problems arising from acquisition of gravity data from mobile platform (e.g. aircraft). Such data must of necessity be brought down to earth surface as input data which leads to analytical downward continuation problems.

2.1.4 Global Navigation Satellite System (GNSS)

The GPS is the most widely known and used from the family of Global Navigation Satellite Systems (GNSS – a generic name to cover all global satellite systems). It was developed by United States Department of Defense (DOD) for Military navigation purposes. It consists of 24 (plus6 in reserve) satellites placed at about 20,200km above the earth with at least 4 satellites visible at least15° above the horizon in 6 orbital planes at about 55°inclinations to the equator. The adoption of 15° mask angle is to ensure that at least 4 satellites or more are used for fixes of positions. The design is that at any location on the earth at least four satellites will be visible 24hours a day throughout the year. Combination of multiple GNSS can significantly improve accuracy due to increased number of satellites which leads to strengthening of the orbit geometry resulting in increased accuracy, reduction in initialization times and increase in the overall satellite availability according to Pedro (2013). Schofield (2009) observed that more than the minimum of four satellites required for solution of position fixing are necessary for proper ambiguity resolution.

2.1.5. GPS Leveling and Second Geodetic Boundary Value Problem

Conventionally, controls establishment both horizontally and vertically may be either by triangulation, trilateration and traversing (if locations are in lowland). Spirit leveling, barometric as well as trigonometrical leveling is used for heights. This research is limited to orthometric height modeling and hence no further mention will be made of horizontal controls except as it is related to attaching height to a point. The GPS leads to savings in time and labor costs and no terrain constraint when deployed to making measurements with appropriate methodology and field planning.

The differential GPS (DGPS) technique has been tested and produces very satisfactory results in the horizontal components of the control establishment of the 3D data produced by GPS. The third component is ellipsoidal height (except if a geoid model has been integrated in the instrument by default, to produce orthometric height .GPS / leveling is used with leveling techniques. The fundamental relationship connecting the ellipsoidal height(h) and orthometric height (H) gives the geoid undulation (N) as stated in equation (1.14) is given by Ollikanen (1997) as: $N = h - H$.

Before the advent of GNSS, Stoke's formula was used to determine gravimetric geoid model by solving for N. In the second geodetic boundary value problem, the δg_p gravity disturbance is given by:

$$\delta g_p = g_p - \gamma_p \tag{2.8}$$

can be obtained from measurements of gravity and the h, on the topographic surface. General form of Stoke's Integral formula in spherical approximation is given by Heiskanen and Moritz (1967) as:

$$N=\frac{K\delta M}{RG} - \frac{\delta W}{G} + \frac{R}{4\pi G}\iint \Delta g S(\varphi) d\sigma \tag{2.9}$$

$$S(\varphi) = 1 + \frac{1}{\sin\frac{\varphi}{2}} - 6\sin\frac{\varphi}{2} - 5\cos\varphi - 3\cos\varphi \ln\left(\sin\frac{\varphi}{2} + \sin^2\frac{\varphi}{2}\right) \tag{2.10}$$

Where,

δg = gravity anomaly downward continiue to the reference sphere

K = gravitational constant

G = 9.80ms^{-2} mean value of surface gravity

δW = difference in gravity potential of the real earth on the geoid and normal potential of the model erath on the surface of this model

δM = difference between real earth and normal earth

R = Radius (=6371km) of the reference sphere that approximates the geoid

S(φ)= Stoke's function of spherical distance φ.

dσ = the integration unit

Assume mass of real earth is equal to mass of the normal earth and the potential generated by two masses to be equal, the first two terms in equation (2.9) become zero i.e.

$$N = R/4\pi G \iint \Delta g S(\varphi) d\sigma \tag{2.11}$$

Using Stoke’s formula requires no masses outside the geoid i.e. removing all residual masses to inside the geoid.The assumptions for solving Stoke’s boundary value problems fall flat with the reality of the existence of topography and atmosphere.High resolution and high accuracy geoid is necessary for orthometric modeling without using conventional process. Although geoid is a datum for orthometric and geodetic leveling and hence a geodetic boundary problem, Ezeigbo (1985) argued that “it does not belong to any of the traditional geodetic problems of potential theory, because the gravity potential W is not harmonic, and the “geoid surface is not known”going by Moritz (1965). The geoid is a closed and continuos level surface whose curvature displays discontinuities at abrupt density variation/changes. Since the geoid is not an analytic surface, it is avoided as a horizontal surface for position but highly suited for height referencing. An analytic surface is one that is defined; single valued and has a derivative at every point e.g. on the geoid surface, W_0=0. Surface of geoid is important to engineering, mapping and geosciences. Vergos (2008) observes that knowledge of geoid is important for sociopolitical and environmental issues like sea level change, urban and rural planning, engineering, ground subsidence and climate change among others.

2.2 Heights in geodesy

Moka (2011) says the “Height of a point P on the surface of the earth is the separation of the point from a specified reference surface, measured along a particular direction.” Surfaces that can be identified are (i) earth surface (ii) ellipsoid and (iii) Geoid as shown in figure 2.2. Height definition is a function of the reference surface.

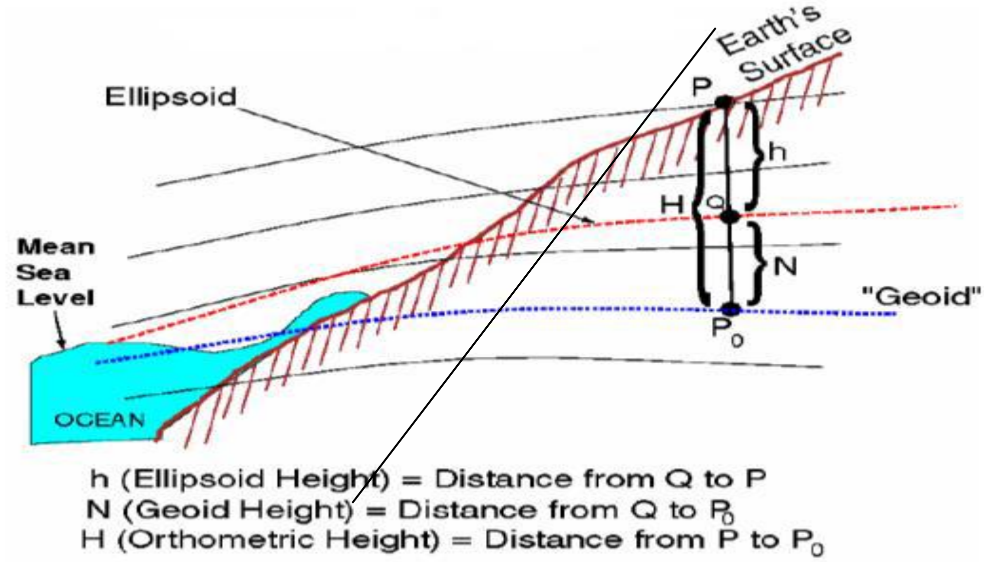


Figure 2.2: Relationship between the Geoid height, N, the ellipsoidal height, h and the Orthometric Height, H. N = h – H.

Source: Ono, M. N. (2009)

i. Earth Surface /topography: This is the physical surface for carrying out surveys as well as gravity measurements. It has variations noticed in hills, valleys, mountains, rivers, flats surfaces, among others. For geodetic, geophysical and geological applications, gravity measurements have to be reduced to the geoid to make them useful. Deflection or deviation of the vertical as a result of difference between normal to the ellipsoid and geoid are resolved into two i.e. ξ–deflections in meridian and η deflection in the vertical which must be applied to measurements.

ii. Ellipsoidal Surface: Irregularity in the surface of the earth makes computations mathematically complex and non-trivial. To avoid this complexity, a surface closely fitting the earth surface is generated by rotating an ellipse about its minor bi-axial axis to allow points to be projected through the normal into the ellipsoid. This projection can be done in either two ways given by Moka (2011) as the Pizettis double projection method from earth surface along plumbline to the geoid and then to the ellipsoid along the normal. Both projections are usually taken to be the same but Helmert is said to be suited for GPS application and hence accepted as better. Clarke 1880 is the ellipsoid used in Nigeria; WGS84 is also an ellipsoid like GRS80 amongst other, although it is worldwide and compatible with worldwide projection system, the Universal Transverse Mercator (UTM) zones. The difference between the World Geodetic System 1984 and GRS80 reference ellipsoid is at the 0.1 mm level (WGS84; NIMA, 2004).

iii. Geoid Surface: Is the equipotential surface of the earth's gravity field that approximates closely the Mean Sea Level (MSL) and suitable for vertical datum reference surface for orthometric heights. Orthometric heights are normally shown on topographic maps. For heights therefore, geoid determination is very important either at global, regional or local level. It is very close to the true figure of the earth and hence useful as a reference surface for height.

2.2.1. Vertical Datum

From figure 2.2, it is shown that three surfaces are available in geodesy but two (ellipsoid and geoid) are used when referencing heights. Vertical datums are used for specifying height of MSL as zero. Zero elevation varies from country to country In Nigeria, vertical datum is at the Lagos (Apapa) Survey Datum ((LSD) which was reported to be 0.067m below mean sea level obtained from a four year analysis (instead of recommended 18.67years) of observations at tide gauge at East Mole in Lagos but Fadahunsi (1985) mentioned another Datum Bench Mark recommended to be used for Geodetic leveling in Nigeria, the SBM Yaba with a height of 7.63237m. Lagos Survey Datum has been found not to be exactly coincident with the MSL from studies conducted by Bello (1977) and Udoffia and Fajemirokun (1978) among others. Swisstopo, Federal Office of Topography reported that the deviation of mean sea level from the geoid surface is about ± 5m in Switzerland.Ojinaka (2006) also stated that there is uncertainty attached to the value of the derived mean sea level height of SBM Yaba and this accounts for the difficulty in correlating geodetic level of

Nigeria level heights with neighbouring countries. Isioye (2011) states that geodetic leveling in Nigeria started in 1955 consisting of over 250 lines of 20000km covering fairly all part of the country especially South West and Northwestern parts of the country. Vertical Datum may be (i) Tidal (ii) Gravimetric and (iii) Geodetic. This is briefly described below:-

(i) Tidal Datum involves tide gauges attached at sea level, in Nigeria it is Apapa Tide Gauge of the Lagos Datum. Geodetic leveling and trigonometric leveling was used to transfer levels inward from the sea onto Bench Marks or trigonometric station (Primary, Secondary, and Tertiary). MSL concept is simple but fact be told MSL is not a level surface because according to Hipkin (2000) the Sea Surface Topography (SST) confirms that MSL at tide gauge origin may after all not lie on an equipotential surface which may hence be a drawback for vertical datum unification. Kumar and Burke (1998) suggested vertical datum unification by geoid modeling using GPS receivers on controls and computing offsets from $h\text{-}H\text{-}N=0$.Odera and Fukuda (2015) applied this in a study in Japan. Tidal datums are determined by means of water level observations over time, for example, MSL, Mean Low Water (MLW) and Mean High High Water (MHHW).

(ii) Gravimetric Datum is based on gravitational potential or geoid model which may be regional or global i.e. gravity field parameters.

(iii) Geodetic Vertical Datums are mostly determined through a process of surveying known as geodetic leveling, determining height difference between points and Bench Marks. A datum point in practice is usually chosen at a tide gauge to establish relationship between tidal and geodetic datum. SBM Yaba is an example in Nigeria; NAVD 88 is an example in North America.

2.2.2 A Vertical Datum Stability

The definition and realizations of datum must consider as very critical the ability to remain constant/invariant over time for it to be widely accepted and adopted for consistency. That is to say that a datum must be highly stable. The mean sea level used worldwide has been discovered to be unstable and hence unsuitable for height referencing. On the other hand the geoid is stable and can withstand change and is therefore a reliable, consistent and suitable surface for use as zero elevation for heights. Avstar *et al.* (2013) also restated that the geoid does not change from system to system.

Changes in geoid may arise from changes in the earth's gravity field that may be due to i) earth's diurnal rotation that produces centrifugal force component of gravity which decrease the length of the day by two milliseconds per century and seasonal variations of about the same order resulting into a diminishing centrifugal force; ii) the earth's mass and its spatial distribution; or iii) the spatial arrangement of objects massive enough and near enough that the gravitational fields have discernable effect on the geoid. Meyer *et al.*(2006) observed that all the above variations come to about 10^{12} radians per second that are far too small and insignificant to change the centrifugal force at discernable level within a geologic age (usually given in millions of years). This is therefore a guarantee that a geoid is a stable surface.

From equation (2.1), the gravity potential (W) is the sum of gravitational force (V) plus centrifugal force (ϕ) i.e. W =V + ϕ

If ϕ= constant, therefore W =V+ ϕ is also constant which makes geoid a stable surface for referencing height. It is only on a geoid surface that the gravity potential is constant.

2.2.3 Height Systems

The definition of a system of heights leads to different heights identifiable. From the onset, Moka (2011) says that height system misclosure which is path dependent can be avoided by using potential differences which depends on gravity values alone. The potential difference between a reference equipotential surface of fixed gravity W_0 and W_P of point P can be converted to height using

$$\int_o^P g\,dn = W_0 - W_P = C \qquad (2.12)$$

C	Geopotential number
g	Gravity value along leveling path
dn	Leveled height difference
W_0 & W_P	Potentials

Geopotential number C is assigned to a given geopotential surface expressed in geopotential units (1 gpu = 1meter *1kilogal) and used for determining the differences in surveyed geometric heights. Essentially it is the potential energy difference between two points.

The following heights are defined and listed in Moka (2011):

(i) Dynamic height (H^{dyn}): heights are defined with reference to normal gravity γat arbitrary latitude, usually 45°, of an international ellipsoid to divide the geopotential number C of the points given by:

$$H^{dyn} = C/\gamma_{45°} \qquad (2.13)$$

H^{dyn}has no geometrical meaning but mostly applied in hydraulic and fluid flow projects.

(ii) Normal Heights (H^N): mean value of normal gravity along curved normal gravity plumbline (γ) to the point is used to divide the geopotential number (C) given by:

$$H^N = C/\gamma \qquad (2.14)$$

$$\gamma = \frac{1}{HN}\int_0^{HN} \gamma(z)\,dz$$

where $\gamma(z)$ is actual gravity at the variable Point P of height z

$$\gamma = \gamma + 0.0424H \qquad (2.15)$$

where H is in km and γ is in gals.

(iii) Orthometric Heights (H_o): This is the geopotential number (C) divided by mean value of actual gravity (g) along earth's curved plumbline between the geoid and topographic surface.

H_o=C/g (2.16)

Where $g=\frac{1}{H_0}\int_0^{H_0} g(z)\,dz$ (2.17)

Since true value of g cannot be generally determined, approximations are necessary leading to several types of orthometric heights i.e. based on Pointcare-Prey relationship for mean gravity called:

(a) Helmert Orthometric Height (H)

$g^H = g^s + \frac{1}{2}FH_o - 2\pi G\rho H_O$

(2.18)

Where g^s=earth surface observed gravity

F=Linear vertical "free Air" gravity gradient

G=Universal gravitational constant

ρ= Constant topographic density

$2\pi G\rho H_O$is Bouguer plate gravity expression neglecting terrain effect but accounts for mass above geoid.

(b) Mader Orthometric Heights which is based on mean gravity as given by Mader (1954) and Krakwisky (1965) as:

(c) $g_m = g^s + \frac{1}{2}FH_o - 2\pi G\rho H_O + \frac{1}{2}(\delta g^T - \delta g_o^T)$ (2.19)

Where δg^T, δg_o^T are terrain corrections applied at the surface and geoid respectively.

$g_m = g^s + \frac{1}{2}FH_O - \frac{1}{2}(g^T - g_0^T)$ (2.20) without need for Bouguer plate correction. g^T and g^T_o are vertical components of gravity due to topographic mass at surface and geoid

(d) Neithammer Orthometric Heights are based in mean gravity equation given by Niethammer (1932) and Rapp (1961) as:-

$$g^N = g^s + \frac{1}{2}FH_O - 2\pi G\rho H_O + \delta g^T + \delta g^T \quad (2.21)$$

$$where\ \delta g^T = \frac{1}{H_0}\int_0^{H_0} \delta g^T dH$$

$g^N = g^s + \frac{1}{2}FHo-$ $g^T + g^T$ is the Niethammer mean gravity (2.22)

2.2.4 Other Methods of Obtaining Orthometric Heights

Geoid is the reference surface for orthometric height which Hiester *et al.* (1999) described as the distance from that point on the earth surface along the plumb line normal to the geoid. The plumb line is curved due to non-parallelism of equipotential surfaces. Geometric leveling, trigonometric leveling, gravimetric leveling is among the various methods used for heighting.

These are as follows:-

i. Geometric leveling which is the simplest and the most accurate technique to determine elevation differences by using a leveling instrument and a graduated vertically placed staves. For challenging terrain of rough, mountainous and large areas, this method is quite difficult to apply and prone to errors that propagate. Errors are mitigated by procedures designed for field works and careful selection of equipment and personnel. Due to effect of variation in geoid undulation of the earth, geometric leveling yields uncorrected results. Orthometric corrections computed from gravity data along the leveling routes must be applied for acceptable accurate results.Niemer (1986) observed that the speed of survey by this method is slow and hence leads to high survey cost of production. For long distances, considering geoid undulation variation of the earth, direct leveling without orthometric correction results into unacceptable results for some applications.

ii. Trigonometric leveling determines height differences by using vertically observed angles and slope distances. Kuntz and Scmitt (1986) listed three classifications as:

(a) Unidirectional

(b) Leap – frog and

(c) Reciprocal.

Reciprocal observations were done to reduce the effects of refraction on computed heights. EDM slant distances must also be corrected for atmospheric refraction using air pressure, temperature and water vapor to yield improved heights. Total Station is set-up at two stations to measure vertical angles at both stations to be used with slope distances measured to compute height differences. Reciprocal and geometric leveling are probably the most accurate leveling techniques among the various methods but they require more parties and equipment. For short distances, curvature and refraction effects can be neglected. Erengolu *et al.* (2012) observed that this method compares favorably with geometric leveling method.

iii. GPS Leveling.

This involves the application of GPS to obtain 3D coordinates of points on the earth surface in WGS84 reference ellipsoid. A geoid model is needed to transform the ellipsoidal height (h) to orthometric height (H). The geoid models are produced from geoid undulations (N). The relationship is given by equation (1.13) which is applied as: H =h-N to produce orthometric heights from G.N.S.S.

2.2.5 Mathematical Background of Differential Global Positioning System (DGNSS)

Differential GNSS technique gives the user position as a relative vector with reference to the known coordinated reference base stations .The method is used to compensate for natural errors (ionospheric or tropospheric) and induced errors (clock or orbit). This method basically requires the use of at least one known reference station to generate differential errors in GPS observations and apply this information to other user positions to obtain relative positions. Relative or Differential approach has been observed in several studies as a more realistic approach than absolute because it is based the difference in height over a baseline.Dey and Rao (2014) states that DGPS positioning are formulated to enable cancellation of common errors to two or multiple receivers relatively close to each other (about 150 km) to yield a final results that are more accurate .This is fundamental to modeling of orthometric height from GNSS/GPS. Souse-Silva (2007) identified errors removed by DGPS as ionospheric, tropospheric, signal noise, ephemeris data and clock drift .Hoffmann-Wellenhof *et al.* (2008) observed that multipath occurs when GNSS signals arrive at antenna via an indirect path as a result of deflections from nearby surfaces.Multipath is an uncorrelated error and hence not removed by differential technique. To avoid multipath, proper site selection for receiver antenna position must be located.

Let error at base reference station A be e_A from pseudorange ρ_A, receiver – satellite true range be d_A, receiver clock offset dT_A satellite clock offsets dt and velocity of light c, it was therefore formulated by Lapucha and Maynard (1992):

$$e_A = \rho_A - (c dT_A + cdt + d_A) \quad (2.23)$$

$$= \rho_A - c dT_A - cdt - d_A$$

Also at B, let e_B be pseudorange correction and for simultaneous observation observations at A and B we assume for short baseline $e_A = e_B$, then

$e_B = \rho_B$ – (cdT_B -+ cdt +d_B) (2.24)

But $e_A = e_B$

ρ_B - e_A –cdt =d_B - cdT_B

d_B is a function of E_B, N_B

cdT_B is receiver clock offset at B in meter

For post – processing, with the availability of raw data simultaneously obtained for both reference and user positions, we can write

$\rho_B - \rho_A$ = c($dT_B - dT_A$) + ($d_B - d_A$) (2.25)

But pseudo-range correction at A (e_A) can be decomposed in terms of

$e_A = d_d^A + d_{ion}^A + d_{tropo}^A$ + δt

d_d^A= orbit error

d_{ion}^A =ionospheric error distance dependent/spatially correlated

d_{tropo}^A =Tropospheric delay

δt =residual clock error which is negligible due to high stability of GPS satellite clock error

This can be similarly derived for station B, and then the residual differential error e^B is given by

$e^B = e_B$ - e_A=(d_d^B-d_d^A) + ($d_{ion}^B - d_{ion}^A$) + (d_{tropo}^B- d_{tropo}^A) + δ(t) – δ(t_0) (2.26)

Spatially correlated or position errors (dr) can be obtained from

dr =$\frac{e}{hd}$ (2.27)

Where e –specific error =2.6m (from IGS center)

d – Baseline

h – GPS height of 20200km

For a baseline distance of 100km,

$$dr = 1.2875x10^{-6}mm = 0mm.$$

The implication is that for baselines distances below 100km, the influence of spatially correlated errors on user position is negligible. This was also confirmed by Parker (2015).Therefore if known orthometric height is H_0, the elevation of other stations can be obtained from

$$\left.\begin{aligned} H &= H_0 + \Delta H \\ \text{But } \Delta H &= \Delta h - \Delta N \\ H &= H_0 + \Delta h - \Delta N \end{aligned}\right\} \qquad (2.28)$$

2.2.6 Surface Interpolation

The fields of geodesy, geophysics, geology among others use interpolation for generating values of unoccupied points and also in the construction of maps by using the appropriate software. Interpolation of geoidal heights (N) with available GPS positions has very important practical application/implication in geospatial data acquisition that is to use H= h-N conversion model for the direct transformation from ellipsoidal (h) to othometric height (H).

Problem formulation/Approximation

Let x, y, z be a sequence $\{(x_i, y_i, z_i) \in R^3, i=1, n\}$ of points in 3D space and D is a rectangular domain containing points $x,y=\{(x_i, y_i,) \in R^2, i=1, n\}$

Solving the interpolation or approximation problem will mean finding a continuous function of two independent variables f(x, y) for which

$$f(x, y) = z_i \text{ or } f(x_i, y_i) - z_i < \delta \tag{2.29}$$

Except for simple cases of plane or polynomial function of two independent variables of higher degree, simple analytic formula is usually not possible for interpolation. The procedure of gridding requires rectangular grid to cover the area whereby the z value at node are calculated or estimated using the surrounding x, y, z values if the densities are adequate to guarantee results.

2.2.7 Surface Algorithms

The following are commonly used techniques for solving interpolation/approximation problems in practical tasks and their applications. They are used in geoid determination/ modeling to produce orthometric heights; Triangulation with Linear Interpolation (TLI), Natural Neighbor (NN), Inverse Distance Weighting (IDW), Minimum Curvature (MC), Regression by plane with heights, Radial Basis Functions (RBF) and Kriging.

i. **Triangulation with Linear Interpolation (TLI):-** This is one of the first methods used before development of computers and based on division of surface into triangles. Each triangle defines a plane by its three vertices. It is a fast algorithm resulting in interpolative surface. The disadvantages are that domain is limited to the convex envelope of point's xyz, resulting surface is not smooth hence terrain changes need to be observed. It has these criteria to comply with: (i) circum-circle passing three points has no other data point within the circle (ii) No overlap of triangles (iii) No gap in triangulated surface. This method works best according to Ojigi (2011) when input data are evenly distributed over grid area.

ii. **Natural (Nearest) Neighbor (NN):-** This is based on partitioning of a plane with n points into n convex polygons such that each polygon contains exactly one point and every point in a given polygon is closer to its central point than any other. Nearest

Neighbors are used for computation. This method is fast and yields smooth surface. Resulting surface from this method is not acceptable in geology or hydrogeology. It founds application in GIS for digital modeling of terrain and fast interpolation of terrain data. See Ojigi (2011).

iii. **Inverse Distance to a Power (IDP):-** This is a weighted average interpolator which preserves sample data value and therefore can be either exact or a smoothing interpolator.

Computes the value of function f at an arbitrary point (x, y) as a weighted average of value z_i

$$f(x,y)=\sum_{i=1}^{n} z_i w_i \tag{2.30}$$

where,

$$w_i=\frac{h_i}{\sum_i^n h_i} \tag{2.31}$$

$$h_i=1/\sqrt{((x-x_i)^2+(y-y_i)^2+\delta^2)} \tag{2.32}$$

δ = smoothing parameter

It uses high computer time consumption for large n. “Bull’s eyes” effect are generated surrounding the position of point location and hence resulting function is not acceptable for most application.

iv. **Minimum Curvature Method: -** Developed in 1990 by W.H.F. Smith and P. Wessel to generate the smoothest possible surface with respect to input data as much/closely as possible but input data are not honored exactly, hence it is not an exact interpolator surface as observed by Ojigi (2011).In 1991, Smith and Wessel, developed a simple planar model using least squares regression of ax+ by+ c= z(x, y).

It has advantage of high speed of computation. It is also a suitable method for a large number of points.The disadvantages are that the algorithm is complicated as well as the computer implementation. This is a universal method suitable for smooth approximation and interpolation. See Ojigi (2011) for the details.

v. **Polynomial Regression: -**The method in based on regression by a plane f(x,y) =ax+by+c using weighted least squares fit. The weighting is assigned and computed as an inverse distance from point (x, y,) to (x_i,y_i).The algorithm is simple and has good extrapolation properties. For large n, the speed of computation is slow and the resulting interpolation functions approximate. This method has many choices or variants e.g. simple planar surface, bilinear saddle, quadratic surface, cubic surface, etc.

vi. **Radial Basis Function (RBF):-** This is a diverse group of data interpolation method. A multi quadratic method is considered best of this function. It has the advantage of easy implementation of smoothing and simple computer implementation. It is used for solving small problems because for large number of linear equations, the matrix required long computational time and possibility to propagation of rounding errors. It is a universal method suitable for use in any field.

vii. **Kriging: -** The most often used method for solving interpolation approximation problems because it is based on statistical formulation of the best linear unbiased estimate i.e. a variogram. It produces maps from irregularly spaced data. It can be exact or smoothing interpolator depending on user specification. It is very flexible and incorporates trends in an efficient and natural manner. See Ojigi (2005, 2007) for details. Chicaiza *et al*. (2017) also observed that using kriging method ensures unbiased distribution of errors especially in detailed engineering projects where elevation accuracy is very important.

viii. **Finite Element Method (FEM):-** Geoidal undulation can be computed by finite elements method using bilinear splines of the polynomial regression interpolation model according to Ezeigbo (1990). This may be employed for digital terrain modeling.For predicting geoidal undulation in a planar area; refer to Onwuzuligbo (2012).

2.3 Surface Fitting

Various techniques are available for modeling the surface of the earth. The choice of any model depends on the size of the study area as well as the nature of the points that will be used for ellipsoidal or orthomertric height data acquisition i.e. regular or irregular shape.

For small areas, plane or bilinear surface fitting is recommended and generally with kriging interpolation technique. In fact Romans (2004) suggested that for small areas, only model with four parameters or lower be adopted i.e. from a_0 …$a_3$at the plane or bilinear order.

When project area is less than 10km^2, assumption of flat surface is acceptable. For areas above 200km^2, the difference in chord and arc length is less than 8mm or one arcseconds and hence classified as a small areas believed to be reasonably and approximately representable by plane surface fitting according to Schofield and Breach (2007) but for other areas ,the geoid deviates from tangent plane at the point of contact by ammounts that are too significant to be neglected that therefore necessitates the development of a geoid reference surface rather than using the MSL or tangent plane i.e. geoid modeling for elevation determination. Lamothe $et\ al.$(2013) observed that geoid modeling could be adopted as practical alternative to conventional /spirit leveling for the realization of a vertical datum when GPS space technique is to be deployed.

For this research, the size of study area (F.C.T.) is put at 8000 km^2 which requires that higher order models be adopted. The controls to be used as observation points were established based on geodetic controls specifications that certainly were not of any regular pattern and hence Kriging interpolation technique is adopted for modeling. Ojigi (2011), Mohammed $et\ al.$(1996), Odera $et\ al.$ (2014) among several others suggested that Kriging and Radial Basis Function produce sufaces that are most similar to the original surface but Ojigi (2011) rated kriging as the best. Odumosu *et al.* (2014) also scored kriging very good in product appearance. Kriging will therefore be adopted in this research to process contours and DEM.

Generally, Kirici and Sisman (2017) observed that the orthogonal polynomials can be represented as follows:

$$N_{(x,y)} = \sum_{i=0}^{m} \sum_{i=0,j=k-i}^{k} a_{ij}\ x^i y^j \qquad (2.33)$$

Where a_{ij} shows polynomial coefficients, m shows the degree of polynomial and (x,y) or (e,n) shows the point plane coordinates

Two models were considered for F.C.T. surface fitting and they are:-

i) Multi – quadratic model(nine parameter) from Sanlioglu *et al.*(2009)

$$N = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 xy + a_6 x^2 y + a_7 xy^2 + a_8 x^2 y^2 \qquad (2.34)$$

Multiquadratic interpolation according to Yanalak and Baykal (2001) is an analytical method of representing irregular surfaces that involve the summation of quadratic surfaces. Kirici and Sisman (2017) quoted Teke and Yalcinkaya (2005) as stating the following advantages of multiquadratic method as:

a) Even if the reference points are not homogeneously distributed, the results of surface modeling are barely affected.

b) In case of an increase in the distance from reference points to the calculated point, the contribution to surface modeling decreases as much as the increase.

c) There aren't any overlay remains for behind the reference points.

This is particularly applicable to the present studies with reference to the lopsided distribution of controls selected (after reconnaissance surveys) for use in geometric geoid development.

ii) Bi- cubic model (third order polynomial)

$$N = a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy + a_6x^2y + a_7xy^2 + a_8x^3 + a_9y^3 \quad (2.35)$$

Multi-quadratic involves nine parameters ($a_0 \ldots a_8$) to be determined while bi-cubic requires determination of ten parameters ($a_0 \ldots a_9$).

Where x is easting (e) and y is northing (n) are Cartesian coordinates (WGS84)

N is geoidal undulation at the point of interest

$a_0,\ a_1,\ a_2, \ldots, a_n$ unknown parameters

Geoidal undulation of at least six points must be known within the study area to enable redundancies for robustness of least squres solution.

Odera (2014) suggested that for large areas, a bi-cubic polynomial should be adopted for surface fitting.For example in Turkey, a fifth degree polynomial was said by Erol and Celik (2004) to give the best solution over a study area of 72km * 72km (about 5000km^2).FCT study area is bigger than this and hence high degree polynomial may be a better surface representation but type of terrain becomes very critical.Manisa *et al.*(2016) also observed that a second degree polynomial was used for local geoid development in Botswana in an area of 100 km * 100 km delivering an accuracy of about ±20 cm. Some other higher order polynomial surfaces for geoid model development are given by Awange *et al.*(2010) as:

Quartic (15 parameters)

$$N= a_0 + a_1x + a_2y + a_3xy + a_4x^2 + a_5y^2 + a_6x^3 + a_7y^3 + a_8x^2y + a_9xy^2 + a_{10}x^4 + a_{11}y^4 + a_{12}x^3y + a_{13}x^2y^2 + a_{14}xy^3 \quad (2.36)$$

Quintic (21 parameters)

$$N= a_0 + a_1x + a_2y + a_3xy + a_4x^2 + a_5y^2 + a_6x^3 + a_7y^3 + a_8x^2y + a_9xy^2 + a_{10}x^4 + a_{11}y^4 + a_{12}x^3y + a_{13}x^2y^2 + a_{14}xy^3 + a_{15}x^5 +$$

$$a_{16}y^5 + a_{17}x^4y + a_{18}x^3y^2 + a_{19}x^2y^3 + a_{20}xy^4 \qquad (2.37)$$

Ning (2015), however, cautioned that using higher – order surface polynomial equations reduce "internal precision" and require more observations (which is costly to conduct) with a conclusion that gravimetric method used in flat and mountainous areas of Taiwan yielded precisions at the level of 19 cm and 22 cm respectively which are not adequate for high – accuracy surveying engineering applications.Surface models are required to account for datum inconsistencies and systematic distortions inherent among various height data.

Generally, polynomials have simplicity and flexibility for fitting data and guarantees good fit within the data range.

2.4. Geoid Surface Fitting Solutions (Local and Global)

It has been said that plane surface fitting methods would suffice for small areas. But the classification of a small area shows that FCT ($8000km^2$) is not small (hence plane surface cannot be assumed) but also not up to national or regional. Hence, the resort to using local surface fitting geoid solutions is imperative and instructive. For engineering and cadastral surveying applications, global geoids are not as accurate as needed. Local geoid developed from local field measurements are valid and good enough for projects of small or limited extent offering centimeter level of accuracy. But, for large areas (a country or continent) local geoid is not useful. Odera *et al.*(2014) suggested that for small areas like campuses of schools, local government areas (or headquarters) or area councils or of limited extent, a bi-quadratic or bilinear polynomial can be used since the change in slope is generally gentle and uniform but for large areas, a bi-cubic polynomial should be applied.

2.4.1. Local Geoid Fitting Solution

Fundamental theory is based on;

i. Adjusted GPS observations are of very high accuracy

ii. Orthometric heights of three or more controls to be collocated are known and accurate

iii. Area is small and geoid features do not vary rapidly.

Model for geoid surface fitting techniques are:

Model M1 $N_i = a_o + a_1 e_i + a_2 n_i$ (2.38)

M2 $N_i = a_o + a_1 e_i + a_2 n_i + a_3 e_i n_i$ (2.39)

M3 $N_i = a_o + a_1 e_i + a_2 n_i + a_3 \Delta h_i$ (2.40)

Where

h_o---- height at point selected as origin in central region of study area.

h_i---- ellipsoidal height of GPS point.

$\Delta h_i = h_i - h_o$

2.5. Current Research

It has been consistently asserted that the modeling of orthometric height is synonymous to geoid modeling or determination. The identified methods are: astrogeodetic methods, geometric methods, gravimetric methods and a combination of any of them. Recall also that ellipsoidal height (h) from GPS and orthometric height (H) are connected by the geoid undulation (N) i.e. by equation 1.13

$H = h - N$

This study used relative differential approach and GNSS dual frequency geodetic receivers. Primary control stations (4 no) were used as base or reference stations occupied in pairs i.e. (FCT260P- FCT 276P and FCT162P – FCT130P). All other controls were observed in rover static mode for two hours. Post- processing was done by online softwares and surface algorithm used to determine or derive a geoid model. Single site DGPS is said by Buhrke (1998) to work well over short distances because pseudo ranges are observed simultaneously by the reference and rover receivers through the same portion of the atmosphere but this study is improving on this by using GPS on the four primary base reference stations (FCT260P, FCT276P, FCT 130P, FCT162P) for observations and continuous data logging as well as ensuring triangular shape is formed with each rover station and the reference base station pair.

If gravimetric method were to be used (but not used in this study), then Stoke's formula which has some assumptions stated below are to be noted:

i. Gravity is known throughout the surface of the earth.

ii. There are no external masses outside the geoid

iii. The reference gravity (γ) must be that of the reference ellipsoids

iv. Reference ellipsoid must be geocentric.

When using gravimetric method, gravity observations are needed be it terrestrial, airborne, shipborne or satellite derived. Cost of doing terrestrial gravity survey is prohibitive and slow, airborne is fast but interpolation will introduce errors not to talk of satellite derived. The purpose for which the gravity information is needed will be a critical factor in the choice of acquisition method. In the case of Nigeria, due to sparseness/inadequacy of high quality gravity data, this gravimetric method may not be favored especially for GNSS applications in local areas for transformation of ellipsoidal height to orthometric. Marti (2001) observed that surface gravity can be interpolated with accuracy of 1 to 3mgal (1mgal=10^{-5} m/s) everywhere in Switzerland, which is sufficient for leveling and determination of orthometric heights. Nigeria gravity situation may not give such guarantee as in Switzerland and Marti (2001) again suggested Helmert approximation formula which is given by:

$$\bar{g} \simeq + 0.0424H \text{ mgal/m} \qquad (2.41)$$

The above relationship has been reported to give good results in flat and hilly regions but fails in mountainous area. The quoted value for gravity differences to orthometric height is less than 3cm for flatter area. A flat area is one where a slope of less 2^{o}-3^{o} variations in topography can usually be neglected as applicable or defined in Photogrammetry as observed in Avery and Berlin (1992).

Stokes formula is given by Hieskanen and Moritz (1967) in equation (2.11)

$$N=\frac{R}{4\pi G}\iiint_{\sigma} \Delta g S(\varphi) d\varphi$$

N- Geoid undulation

R= Mean radius of the earth

G= Mean gravity of the earth

S (φ) = Stoke's function is given

$$S(\varphi) = \frac{1}{\sin\frac{\varphi}{2}} - 6\sin\frac{\varphi}{2} + 1\text{-}5\cos \ln (\sin\frac{\varphi}{2} + \sin^2\frac{\varphi}{2})$$

Δg = (g-γ) = gravity anomaly

g = observed gravity

γ = normal (reference gravity)

Khairul (1994),Vergos and Sideris (2002) say gravimetric observations are the most precise method of obtaining accurate geoid height (undulation) and that Stoke's integral provides us with a local gravimetric geoid solution.Precise geoid requires sufficient and high quality gravity measurements/data which are still absent or unavailable in Nigeria. For small areas when no local gravimetric geoid solution is available, local geoid fitting surface is a viable alternative but Khairul (1994) says for large area (national, regional, continental), a regional or global geoid solution should be adopted.

2.5.1 Multi-Reference Station Technique

This approach improves the accuracy of observations and brings about improvements in ambiguity resolution (AR**)** as observed by Fotopoulos (2001).

In comparison to single baseline users, multi – reference stations lead to increase in reliability and availability of service. Note that in case one reference base station has issues, then the other reference stations will continue to provide corrections and hence assuring service availability but may have slight effect on accuracy which may not be as poor as if one single baseline DGPS were used.

In the case of single reference failure situation, single point positioning (SPP) results are presented. Multi-reference approach allows for quality of corrections from each reference base station and possible blunders detected and eliminated during post-processing to arrive at high quality final solution.

Multi-reference station approach also minimizes the spatially correlated errors to achieve highly accurate position fixes. When baseline distances increase to several hundreds of kilometers, the reliability of single baseline technique result depreciates and this further encourages use of multi-reference base station approach.

The limitation of this approach is high cost of geodetic GPS receivers needed but the advantages of efficiency and reliability of results are far more important especially as the geospatial products based on the data have far reaching impact on practical applications in surveying, mapping, engineering and environmental projects for sustainable and socioeconomic developments wider implications.

Fotopoulos and Cannon (2001) categorized multi-reference technique into:-

i) Partial Derivative Algorithm (PDA)

ii) Linear Interpolation Algorithm

iii) Condition Adjustment Algorithm

iv) Virtual Reference Station Technique.

Only the PDA is discussed because of the similarity in criteria with relative DGPS technique that both the master (base) receivers and rover receivers must use the same baseline and see the same sets of satellites.

Varner and Cannon (1997) gave the following as general form of PDA that is simply a truncated Taylor series expansion as:

$$g(P) = \propto + ß\Delta x + \gamma\Delta y + \delta\Delta z + \in \Delta z^2 + V_g \tag{2.42}$$

Where ß,γ,δ are spatially correlated errors computed from first order derivatives with respect to each of the horizontal axes x, y and z axis respectively. They are estimated from double difference carrier phase observations at known locations of the base reference stations.

$$\left.\begin{array}{l}
\propto = \text{constant} \\
ß = \frac{\partial g}{\partial x} \\
\gamma = \frac{\partial g}{\partial y} \\
\delta = \frac{\partial g}{\partial z} \\
\in = \frac{\partial^2 g}{2\partial z^2} \quad \text{for non-linear effects due to atmosphere} \\
\frac{\partial^2 g}{2\partial x^2} = \frac{\partial^2 g}{2\partial y^2} = \frac{\partial^2 g}{\partial x \partial y} = \frac{\partial^2 g}{\partial x \partial z} = 0 \\
\text{Assumptions are that the non-linear effects are insignificant.} \\
V_g \text{ is model prediction error at the rover stations.}
\end{array}\right\} \tag{2.43}$$

The truncated Taylor series expansion is about the base station and the secondary /observation rover stations used to compute ΔX, ΔY, ΔZ between baselines.

The PDA estimates field parameters for each satellite pair at base stations which are disseminated to rover stations. Varner (2000) observes that geometry of the network and number of reference stations determines the accuracy level.

For centimeter level positioning accuracy, Varner (2000) listed the following specifications:-

i) Carrier-phase GPS measurements was adopted.

ii) Use of relative mode was adopted.

iii) Known reference base stations were used for differential correction generation and

iv) For large scale applications, use of multiple differential reference stations is recommended.

2.6 Methods of Geoid Determination

Geiod determination can be accomplished by:

a. Astro-geodetic methods

b. Gravimetric methods and

c. Geometrical GPS/Leveling methods or

d. A combination of any of the above,

Each of this will be discussed briefly:

(a) Astro-Geodetic methods:-This method is based on dense availability of deviation of the vertical over an area. Astro-Geodetic method is a terrestrial method which has the ability of detecting short wavelength part of the geoid. The angular difference between normal to the ellipsoid and normal to the geoid at the same point on the earth surface is called the deviation of the vertical. See Figure 2.3

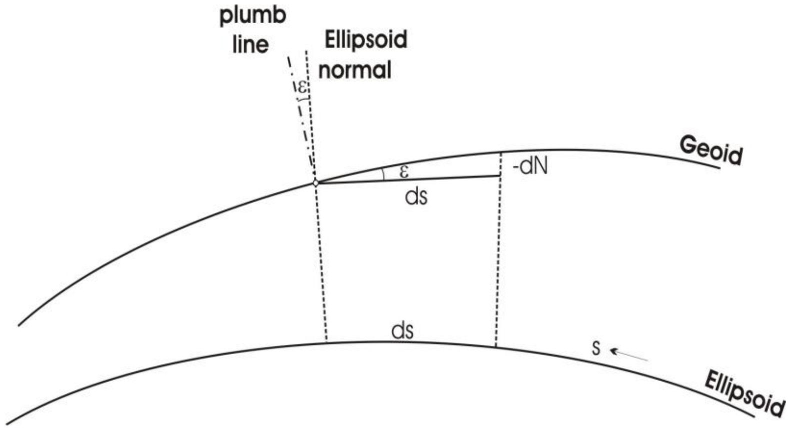


Figure 2.3: Geoid, Ellipsoid and Deviation of the vertical

It is important to note that the definition of a geoid as an equipotenial surface of earth's gravitation that is everywhere perpendicular to the direction of gravity as well as the axis of the observing theodolite is practical and hence, a physical reality.The deviation of the vertical ($\mathcal{E}$) is resolved into η along the meridian and ξ along the prime vertical where

$$\eta = (\lambda_A - \lambda_G)\cos\phi \qquad (2.44)$$

$$\xi = \phi_A - \phi_G \qquad (2.45)$$

Where, ϕ_A, ϕ_G are astronomical and geodetic latitude respectively

λ_A, λ_G Astronomical and geodetic longitude and hence

$$\mathcal{E} = \xi cos\alpha + \eta sin\alpha \qquad (2.46)$$

Where α- geodetic azimuth which is used to get rid of any accumulation of errors in the angles of the triangles and to control the survey e.g. at every 25th station of a traverse survey network (in Nigeria) astronomical observations to control bearing is introduced for rigid network.

From the relationship given by

$$dN = -\varepsilon\, ds \qquad (2.47)$$

Where dN = Change in undulation = N_B-N_A

$\mathcal{E}$= Deviation of the vertical

ds= Change in location or position

By integration of $-\varepsilon\, ds$, we get

$$N_B = N_A - \int_A^B \varepsilon\, ds \qquad (2.48)$$

It is known that N is a function of (ϕ, λ) or (N, E), therefore if N_A at point A is known and ε along AB is also computed from ξ and η, then N_B can be determined. Opaluwa (2006) opined that for accurate determination of geoid, very dense distribution of deviation of the vertical is required to be obtained from highly accurate instruments and tested techniques of processing. Laplace correction applied at Laplace stations are used to relate geodetic azimuth to an astronomical azimuth i.e. in broader thinking used to link physical world with a mathematical representation. This was applied to prevent datum rotation and ensure datum is rigidly fixed observed Onwuzuligbo (2012). In the USA for example, deviations/deflections of the vertical components are readily available from NGS website using "deflect" program (http:www.ngs.noaa.gov) unlike so many other countries including Nigeria. Hence, gravimetric geoid models are available for GNSS height transformation to orthometric.

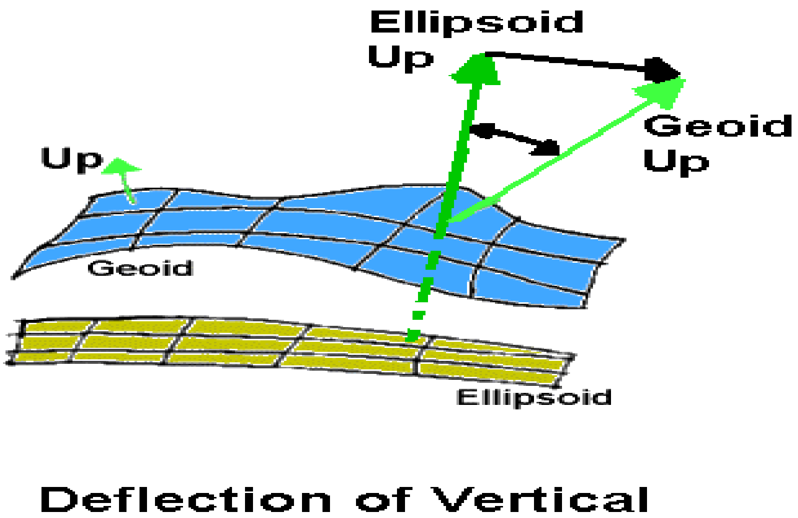


Figure 2.4: Deflection of the Vertical
Source: Clynch, J. R. (February 2006)

b. Gravimetric Method:- This method relies upon the solution of spherical geodetic boundary value problem says Novak *et al.*(2001) which requires the evaluation of Stoke's surface integral over the earth surface. In reality, gravimetric geoid is computed using combination of terrestrial and satellite- derived gravity data. This requires expensive and sensitive instruments and time consuming observation methods. They must cover a large area for reliable geoid modeling.

Terrestrial gravity data gives high frequency geoid while satellite-derived gravity data is for low- frequency. The technique is one of the classical means of geoid determination which requires entire earth's surface to be of sufficiently covered quantities (N,ξ, η) at any point P on the earth surface to be evaluated by Stoke's and Venning -Meinesz integrals. The most significant parameters, affecting accuracy of gravimetric geoid determination are the minimum spherical distance from the computation point, the Kernel (φ_0), the accuracy, size and distribution of gravity anomaly data as observed in Ezeigbo *et al.* (2007). The geoid is needed in geodesy for transforming GPS derived height to orthometric, for determination of topography of ocean surface, for mining and prospecting as well as in other applications.

Limitations of this approach is not knowing density distribution of anomaly but relying on assumptions. Gravimetric geoid solutions give a better geoid undulation estimate if a more detailed picture of local mass distribution and irregularities are known opined Vergo and Sideris (2002). Remove-Compute-Restore method is used but Fast Fourier Transforms (FFT) which is a method that handles large datasets efficiently in a shorter processing time especially with vast data coming from satellite space methods presently may be applied.

Heiskanen and Moritz (1967) gave the mathematical formulation for evaluation of

Gravimetric geoid determination a follows and is also found in Eziegbo *et al.* (2007):

$$\begin{matrix} N \\ \xi \\ \eta \end{matrix} = \frac{1}{4\pi\gamma}\begin{bmatrix} R\sum_{i=1}^{n}\Delta g_i \iint_{\sigma_i} s(\varphi)d\sigma \\ \sum_{i=1}^{n}\Delta\bar{g}_i \iint_{\sigma_i} \frac{ds(\varphi)}{d\varphi}\begin{Bmatrix} \sin \propto \\ \cos \propto \end{Bmatrix} d\sigma \end{bmatrix} \quad (2.49)$$

Where N, ξ, η- geoid undulation, deflection of vertical in the meridian and prime vertical respectively

R,γ – mean radius and mean normal gravity of the earth

Δg- mean gravity anomaly in the block m of the n-blocks in which gravity anomalies are available

α- Azimuth of integration point relative to computation point.

$$\tan\alpha = \frac{\cos\phi'\sin(\lambda'-\lambda)}{\sin\phi'\cos\phi' - \sin\phi\cos\phi'\cos(\lambda'-\lambda)} \quad (2.50)$$

Where ϕ, λ and ϕ', λ' are geographic coordinates of computation and integrating points respectively.

Heiskanen and Moritz (1967) say using dσ =$\cos\phi' d\phi' d\lambda$ (2.51)

The numerical solution is given by

$$\begin{bmatrix} N \\ \xi \\ \eta \end{bmatrix} = \frac{1}{4\pi\gamma}\begin{bmatrix} \sum_{i=1}^{n} \Delta\bar{g}\frac{\Delta\sigma}{m} s(\varphi_i) \\ \sum_{i=1}^{n} \Delta\bar{g}\frac{\Delta\sigma}{m}\sum_{j=1}^{m}\frac{ds(\varphi_j)}{d\varphi}\begin{Bmatrix} \sin\alpha_i \\ \cos\alpha_i \end{Bmatrix} \end{bmatrix} \quad (2.52)$$

Where m- number of subdivisions of $\Delta\sigma$

$\Delta\sigma$ - Area of each block in which Δg is defined and from (2.10)

$S(\varphi)=\frac{1}{\sin\frac{\varphi}{2}}$- 6sin $\frac{\varphi}{2}$+1-5cosφ-3cosφ ln (sin $\frac{\varphi}{2} + \sin^2\frac{\varphi}{2}$) is the Stoke's function

$$\cos\varphi = \sin\phi\sin\phi' + \cos\phi\cos\phi'\cos(\lambda'-\lambda) \quad (2.53)$$

is cosine of spherical distance between computation and integration point.

Using the Remove Compute Restore (RCR) method, compute geoidal quantities (N^c, ξ^c, η^c) at the computation point, compute gravity corresponding to the observed gravity among data. The residual anomaly ($\Delta g^b - \Delta g^c$) obtained from pair of anomalies Δg^b- and $\Delta g^c$are used in Stokes and Vening - Meinesz integrals to obtain residual geoidal quantities at the computation points. Desired gravimetric geoid is obtained by adding the residual geoidal quantities to the corresponding geodal quantities computed from geopotential coefficient i.e.

$$N^{-} = N^{c} + \Delta N \qquad (2.54)$$

Where,

N^{-} - estimated geoidal quantity from R-C-R procedure

N^{c}- Geoidal undulation computed with EGM96

ΔN = Residual geoid undulation computed from (Δg^{b}-Δg^{c})

The result shows the three most significant parameters that affect the accuracy of gravimetric geoid determination are by Ezeigbo *et al.* (2007) minimum spherical distance, the accuracy and the size and distribution of the observed gravity anomaly data. The fact is that an error of 1 mgal in observed gravity anomalies can lead to an error of as much as 1m in the computed geoidal undulation. The spherical distance (φ) from computation point should be carefully selected and it is suggested that for a good estimate φ should be between $0^{\circ}.005$ to $0^{\circ}.05$ i.e. terrestrial gravity data observed at 2km x 2km grid. This may not be feasible immediately and hence gravimetric approach may not meet the needs of our daily needed geoidal models.

Generally, most African countries lack terrestrial gravity data of high quality and distribution including Nigeria which then means gravimetric geoid cannot be reliable for orthometric heighting using GNSS and ellipsoidal height (h). In the absence of gravity data, Dumrongchai *et al.* (2012) stated that interpolation is then used to fill the voids which can result into significant errors for geoid computation. Merry (2003) reported that such generated gravity data led to a difference of up to 3m in Central Mozambique with the use of EGM 2008 and this may also be expected of majority of African countries whose gravity data were generated in the development of EGM 2008. Kenyon *et al.* (2007) stated that EGM 2008 accuracy worldwide is 15 cm but only achievable where high quality surface gravity data is available which is presently lacking in Nigeria and possibly was never a source of input data during the development of EGM2008.

(c) Geometric Approach:- This method involves the use of GNSS derived height in conjunction with existing orthometric heights of points within the study area to interpolate or determine geoid undulation at various points to enable GNSS instruments derive orthometric height automatically as an alternative to differential or spirit leveling. For this method, a geoid model must be developed to meet the purpose and accuracy needed.

The mathematical basis for this transformation is given by equation (1.13) as:

$$H = h - N$$

H=Orthometric height (geoid/MSL)

h= GPS Ellipsoidal height (ellipsoid)

N= Geoid- ellipsoid separation

The import of the above is that if h and H is known then the difference (h-H) will give the undulation (N). If the N is sufficiently known to a high accuracy from homogenous distribution of controls within the study area, then a model could be developed to produce orthometric heights automatically. For this method to be applicable, considerations must be given to the following factors as listed by Erol and Celik (2004):

(i) Homogenous distribution of observation points within the study area and the number of such stations so chosen to emphasize the changes of the geoid surface.

(ii) The accuracy of GPS derived heights and the existing orthometric heights

(iii) Characteristics of geoid surface in the area.

(iv) Depending on the size of study area, no single model might satisfy the different areas.

Table 2.1: Summary of various height systems

HeightSystems	Observation technique	Additional information	Quantities available for computation	Relative accuracy of orthometric height
Gravity field related	Geometric levelling	Surface gravity	Height increments potential differences	±1–2.5 cm per 10km
	Trigonometric levelling	Surface gravity	Height increments potential differences	±2–2.5 cm per 10km
Satellite based	GNSS/GPS	Geoid	Ellipsoidal height differences, geoid undulation	±2 cm per 10km

Source: Balasubraminia (1994)

2.7 Surfaces in Geodesy

Figure 3.4, shows the surfaces encountered in geodesy as:

i) Topographical is the physical earth surface

(i) The Ellipsoid

(ii) The Geoid

2.7.1 Topography or Physical Earth Surface

Topographical surface is the one that we inhabit with various terrain features like mountains, hill, and valley whose heights area referenced to either the geoid or the ellipsoid. Onuwuzuligbo (2012) quoting Iliffe (2003) as distinguishing between the geoid-ellipsoid separation either as geoid heights or geoid undulation (which refers to a possible wavy pattern or irregular shape of the geoid). Whether height or undulation, the triplet h, H and N are supposed to close at zero (h-H-N=0) like in conventional leveling where circuit closure is expected to be zero but a misclosure is always obtained due to several factors like:

(i) Random errors in h, conventional leveling H and N and datum inconsistencies among the heights which Featherstone and Whu (2006) attribute to slight difference in effects or movement or instability of observing and reference station.

(ii) Approximations made in data processing e.g. neglect of mean dynamic topography (Sea Surface Topography) or starting coordinates like in the Lagos Datum for vertical datum.

(iii) Effect of long wavelength geoid errors and atmospheric effects on GPS derived heights.

(iv) Effects of spatially variable gravity observations stated in Featherstone (1998).

Earth surface has abrupt height changes with undulations (complex and irregular) which makes it worthless as a surface for direct computation and because of this irregularity, the need for a mathematical model of the earth to do computations is needed, hence we have ellipsoidal (e.g. Clarke 1880 in Nigeria) and vertical datum (NAVD 88 in the US and Yaba datum in Nigeria).

2.7.2 The Ellipsoid

The ellipsoid is a mathematically defined regular surface with specific dimensions either a and f or a and e. Iliffe (2003) stated that the ellipsoid by design is meant to be a good approximation to the geoid,but with global difference of up to ±100m having a global root mean square error of around 30m. An ellipsoid is obtained by rotating an ellipse about its minor axis (a) and Newton during the development of geodesy postulated the earth to be flattened at the poles and bulges at the equator. This is a mathematical surface designed to fit the earth’s surface at various locations on the earth e.g. Clarke 1880 adopted in Nigeria and most African countries.

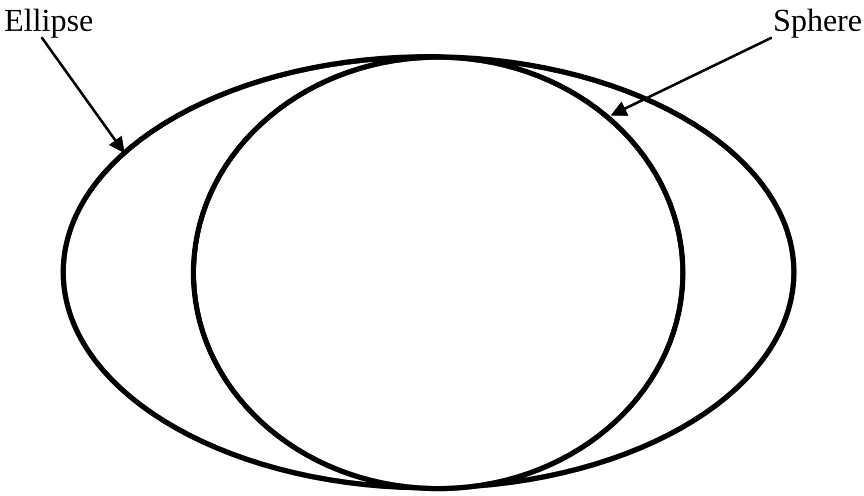


Figure 2.5: An ellipse flattened at the pole and bulging at the equator Source: Lowrie (2002**)**

Global geoid is designed to best fit the whole world. Local or regional geoids are available for each area or country and this leads to datum inconsistencies. For the whole world it is called global ellipsoid (WGS84). Only space methods have enabled worldwide survey connection with a global outlook based on WGS84 (best-fit the earth's ellipsoid). A best fit geoid in one area may poorly-fit in another area. Featherstone (2008) observed that GRS 80 differs from WGS84 by less than 0.1mm and hence the two ellipsoids are taken as the same.

2.7.3 Geoid

This has been defined as an equipotential surface which at every point is perpendicular to be direction of gravity. It shows mass distribution inside the earth. The shape of the earth does not allow mathematical representation hence an ellipsoid is also used to represent the geoid but differences (N) are noted to be about ±100 metres globally. The difference between the geoid and ellipsoid is termed geoid height/undulation (N) which is highly significant in GPS height transformation. Geodesy uses geoid (a) to interpolate geoid heights from GPS observations (b) to avoid drawbacks of MSL as a surface (effect of SST and others) and (c) for fluid/flow engineering projects e.g. water distribution and reservoir location. Majdanski (2009) stated that geoid undulations can be used to examine/explain variations of density at larger depth arising from activities within the earth crust and for lithospheric studies. Epuh *et al.* (2016) observed that variations in the height of the geoidal surface are related to the density variations/distributions within the earth surface and the geoid undulations help to understand the internal structure of the earth i.e. the geoid is inversely proportional to the distance from anomalous mass.

Okiwelu *et al.* (2011) found that the highest geoid undulation in Nigeria is centered over the North Central region with relatively low values (16 – 26m) confined to the sedimentary basins (Bornu basin and Benue trough). Lowrie (2007) shows that positive geoid undulations indicate the prescence of high density excess mass (as is the case in Nigeria) while negative undulations indicate regions of mass deficiency or low density mass deposits.

.

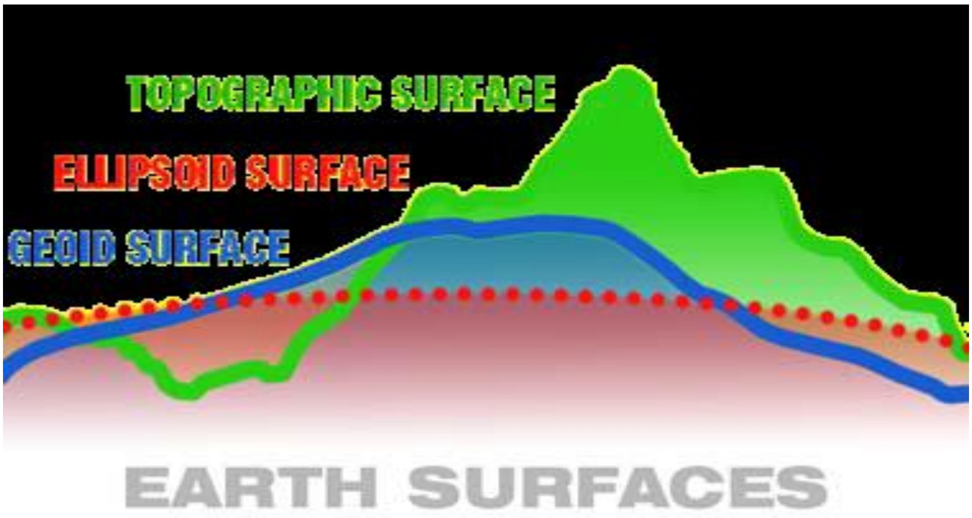


Figure 2.6: The Geoid, Ellipsoid and Earth's Surface
Source: Steve Kenyon, NGA

2.7.4 Geoid Surface Stability

A critical quality for a wide acceptability of a datum is stability. Change in geoid arises from changes in the earth's gravity field attributed to i) earth's diurnal rotation which produces centrifugal force component of gravity which is not constant and decreasing the length of the day by two milliseconds per century; ii) the earth's mass and its distribution; or iii) the spatial arrangement of objects massive enough and near enough that the gravitational fields have a discernable effect on the geoid. The summary is that despite these changes, Meyer *et al*.(2006) confirmed that all the above variations (about 10^{12} radians per second) are far too small to change the centrifugal force at discernable level in faster than a geologic time frame. A geological time frame is given in million of years .The geoid is therefore taken as a stable surface that will not change within a forseable time.

Mean sea level (msl) obviously does not fall into this stable (reference surface) datum category and hence cannot guarantee height consistency since the msl varies from place to place, has slope and hence unstable and unreliable.

2.7.5. Datum and Geoid Determination

Ono (2002) stated that Nigerian vertical control was designed to cover over 200 lines of geodetic leveling but has problems of circuit misclosures and therefore not suitable for vertical control reference system. In this present situation, inconsistency may be attributed to discrepancies between the two tidal gauges used as zero elevation,error propagation contribution from conventional leveling as well as lack of gravity/orthometric corrections and non-geocentricity of L40 origin. The suggestion is to use Nigerian vertical controls for providing heights for development and maintenance of infrastructure and engineering projects but with a caveat from Agajelu (1988) that they should not be relied upon when high precision projects are being contemplated especially for microgeodetic applications (deformation studies, crustal studies among others).

Gravity data corrections were not applied to leveled height difference to obtain the orthometric heights in Nigeria. Agajelu (1988) again emphasized the need for rapid gravity measurements for its importance and relevance to geoid determination for orthometric height using the relative measurement techniques while Ono (2002) says the relative approach can achieve 50μgal (which can translate to about 15cm) accuracy adequate for gravity measurements.

i) Datum ambiguity or bias

Vertical datum can be inconsistent due to measurement error or definitional issues. For a given area, datum ambiguity operates as a near constant. The ambiguity is evident in all three height components i.e. H, h, and N which can be written as:

$$(h + \delta h) = (H + \delta H) + (N + \delta N) \qquad (2.55)$$

where δ is the bias in each component.

From $h = H + N + (\delta H + \delta N - \delta h)$

$$= H + N + S \qquad (2.56)$$

Where $S = (\delta H + \delta N - \delta h)$ is a conversion value.

Milbert and Smith (1996) observed that it is possible to absorb the datum biases contained in S into the geoid model N to produce a new geoid model

$$N_{New} = N + S \qquad (2.57)$$

This is (S is very small compared to N) believed to support direct conversion between ellipsoid and orthometric vertical datum, even if they are not defined on a common reference. Kamaludin *et al.* (2005) observed that differential heighting techniques can be used to eliminate datum inconsistencies systematic error over short baselines (less than 150 km). Datum inconsistency are negligible for height conversions in less precise work e.g. for topographical and engineering constructions.

ii) Ellipsoidal height (h) bias

There is lack of agreement in the transformation parameters in Nigeria, and also arising from assumptions at L40 starting point of Nigerian Geodetic datum with WGS84 i.e. non-geocentricity. This according to Milbert and Smith (1996) may induce a systematic difference in ellipsoid heights which is put at a range of -28cm to -1.64m in the United States.The value range for Nigeria is not known yet or determined.

iii) Orthometric Height (H) Bias

Recollect the difference between point mean sea level and surface mean sea level earlier mentioned in this study. The Nigerian Levelling Network was realized or tied to Yaba Datum in Lagos i.e. a point mean sea level datum. This bias would have been minimized if surface mean sea level was adopted. The mentioning of Apapa datum also in Lagos did not link the two to suggest surface mean sea level was used. Zilkoski *et al.*(1992) observed that minimization of recompilation of national mapping products is a key factor in datum realization. It is not certain if this was considered in the planning of Nigerian Levelling datum realization.

iv) Geoid Height (N) Bias

Development of global geoid models depend on availability of gravity data. According to Schwarz *et al.* (1990), Remove-Compute-Restore (R-C-R) technique is used to obtain geoid. Fast Fourier Transform (FFT) may be used to evaluate Stoke's Integral.

2.7.6. Orthometric Height (H) From Ellipsoidal Height (h)

It is not trivial to derive H from h even though linear relationship H=h-N looks simple on the face value.The factors that will enable acceptable H are dependent on: errors in starting coordinate of h, H and developed N.

i) Ellispsoidal Height (h)

GPS observation is poorest in the vertical direction possibly because of the fact that satellites below the horizon cannot participate in computations for height, atmospheric effects etc. There are guidelines necessary for use of GPS to derive heights for example NOAA

Technical memorandum No.NGS-58 for geometric vertical control networks. The observations as well as the vector processing are identified as very important by Onwuzuligbo (2012) and are explained briefly in that study.

2.7.7. Observations

This is to determine the choice of GPS receivers. Dual - frequency GPS receivers are recommended, irrespective of baseline distance, geodetic quality antennas are also recommended. Higher order controls (primary and secondary) were used as base station in the survey process. Observation sessions used was 2 hours and in static mode at each rover control station. Epoch intervals for data collection were 5 seconds.

Vector processing: precise ephemeris was used in post-processing. Ambiguity resolutions (AR) for all integer fixing should be properly done. Atmospheric effects should be properly modeled. Data quality is determined from size of residuals. Least squares adjustments are used to produce final coordinates. Software vendors incorporate all the above processing requirements in their software generally.

2.7.8. Geoidal Height (N)

From (1.13), H=h-N, we see that H also depends on ellipsoidal height as well as N. The ellipsoidal height and geoid height must refer to the same reference ellipsoid, otherwise Moka (2011) lists procedure for transformation to bring them to the same ellipsoidal reference system. Kamaludin *et al. (2005)* has discussed how to resolve this issue in their studies. From the above, we can say that optimum value of orthometric heights is obtained when procedure, instrumentation and processing follow appropriate recommended guidelines.

2.7.9 Geoid Models

Global geoid model considers long and middle wave parts of the entire earth's gravity field. Care should always be exercised when using global models over small areas because the underlying assumptions might vary with size and purpose and in fact introduce avoidable errors and lead to erroneous results which may not be desirable.

The following is a list of geoids models among others:

2.7.10 Global Geoid Models

Among several global models are:

(i) EGM 2008 – Earth Geopotential Model 2008

(ii) EGM 96 – Earth Geopotential Model 1996

(iii) CHAMP- CHAllenging Mini- satellite Payload

(iv) GRACE- Gravity Recovery And Climate Experiment

(v) OSU – Ohio State University, among several others.

i) **EGM** 2008 as described by Pavlis *et al.* (2012) is the latest global geoid model published by the National Geospatial (NIMA) Intelligence Agency (NGA) to replace EGM 96 model which had been the default model (since 1996) and was developed from combination of both terrestrial and satellite data covering the entire Earth. Meyer-Gurr (2007) states that EGM 2008 was developed by Least Squares combination of ITG- GRACE 03S gravitational model based on 57 months of GRACE satellite to satellite tracking data as detailed in Pavlis (2012). Geoid undulation estimates can be obtained from Altrans software version3.002, retrieved and freely available and faster to compute for undulations from http://www.altrans.soft112,com/ .The model was computed from global 5 arc minute grid of gravity anomalies from land and sea based surfaces. It is complete to spherical harmonic degree and order 2159 translating to grid size of 6.5km. Global agreement with GPS Leveling is about 6cm. EGM 2008 is available from NGA website with software to convert spherical harmonics into grid of geoid undulations before they can be used for orthometric heighting over long distances (continental) extent. It uses the global datum to minimize the differences between mean sea level and the geoid at global scale. EGM 2008 has a better terrain representation than other Global Geopotential Models (GGM) over mountainous study area with height of over 900m. Height range within F.C.T. is between 70m to 960m above MSL. Pavlis *et al.*(2012) stated that the current high resolution EGM 2008 approximates the global geoid at an accuracy of ± 15 cm. This is certainly not acceptable for local projects in orthometric heighting. Odera and Fukuda (2015) consider a global geoid model of ± 1cm accuracy sufficient for recovery of orthometric heights and unification of vertical datums. Abeho *et al.*(2014) stated that a standard deviation of 26 cm was achieved using EGM 2008 due to inhomogeneous gravity data in Uganda used in the development of the global model resulting in a mean difference in height of about - 0.859m (about 1 m). For high accuracy surveying engineering applications (microgeodetic, deformation studies, etc), Ning (2015) opines that 22cm accuracy cannot be acceptable. Local projects hence require the development of local geoid for orthometric height determination using GNSS within the size of F.C.T area.

However, where high quality gravity data are available in adequate quantities, the difference between EGM 2008 and GPS/Leveling values is reported to be at centimeter level according to Rosa *et al.*(2016). Moka (2011) observed that for orthometric height determination, EGM have only served as reference with the implication that local geoid model is still desirable.

ii) **EGM 96.**This was developed jointly by the U.S. National Aeronautical and Space Agency (NASA) and National Imagery Mapping Agency (NIMA). It used harmonic coefficients in gravimetric geoid determination based on Remove-Compute-Restore technique to best represent the study area. EGM96 was widely used but Merry (2003) observed that this does not represent precisely the gravity data situation for Africa and suggested GRACE model in its place for Africa.EGM96 was used worldwide before release of EGM 2008. The expected accuracy is 1-2m at its maximum spatial resolution of 56km leading to error in height calculation if coverage of project is less than this resolution.

Global geoid models (EGM 96 and EGM 2008) as well as others need to be properly evaluated for geodetic and other geospatial data uses before being adopted for local applications. Abdulkadhum (2015) observed that global models may have truncation errors which could be of significant values in local applications. Nikolaos (2012) also confirmed that EGM96 and EGM2008 have accuracies varying from centimeters to even a meter for conversion of ellipsoidal to orthometric heights. With high quality gravity data of sufficient distribution, the difference in undulation between the EGM and GPS/Levelling data may be between ±5 cm to ±10 cm. Nigeria may not have met these gravity data requirements to benefit from such order of accuracies. Gravity anomalies can yield accurate geoid undulation only with high quality gravity data.

iii) **GRACE:** Is short for Gravity Recovery and Climate Experiment, launched by NASA as a twin-Satellite mission as stated by Tapley *et al.* (2004) which delivers static and time-variable external gravity field models. The satellites are in the same orbit around the Earth, one about 220km in front of the other at altitude 460km above the Earth's surface. They measure earth gravity field with a precision greater than any previous instrument. A global geoid of 1 cm accuracy with 199 km resolution is achieved as stated by Ihde *et al.* (2012).

iv) **GOCE**: this is short for **G**ravity Field and Steady-State **O**cean **C**irculation **E**xplorer and was launched in 2009 by European Space Agency at altitude 260km to collect gravity data to map surface of the geoid with further details stated in http:/geol-amu.org.It is a satellite gradiometry mission as described in Drinkwater *et al.* (2003) that delivers static external gravitational field models to higher degrees than from satellite tracking alone. The gravity data with sea–surface height are used to determine the direction and speed of ocean currents.

v) **OSU**-This is short for **O**hio **S**tate **U**niversity global geoid model. The OSU91A is a spherical harmonic model of the Earth's gravitation potential and it incorporates Geosat altimeter data, surface gravity data represented by a $1^{\circ} \times 1^{\circ}$grid of mean gravity anomalies.

In Indonesia, Ramdani (2008) observed that the use of EGM96 yields accuracy value of 0.995m while EGM2008 produce 0.441m. Both accuracy levels of the global geoid models are definitely not appropriate for local applications/projects making the development of local geoid models for orthometric modeling imperative. Geoid models are very important and useful for economic activities like sea dredging, navigation, engineering, etc. Global geoid models (GGM) should be avoided when local projects are under consideration or be applied with cautions. In Central Mozambique, within South African sub region, a 3m difference has been reported using predicted anomalies in the absence of terrestrial gravity anomalies as stated in Barnejee *et al.* (1999). Stanway (2009) reported that EGM 2008 and EGM96 models used in the North of Papua New Guinea (PNG) gave an offset of 1.4 – 1.5 m from true mean sea level (MSL).Idrizi (2013) said though EGM96 is very suitable and useful for practical geospatial data acquisitions in the absence of national geoid, but it cannot be relied for practical geodetic issues. Alothman *et al.* (2013) reported that in Kingdom of Saudi Arabia (KSA) due to poor or lack of high quality terrestrial gravity data, some deficiencies have been noted with the use of EGM 2008 despite giving the highest accuracy of all the evaluated EGMs. These are pointers to the need for development of local geoid model in place of global geoids for GNSS users to determine orthometric heights for local applications in cadastral, mapping, engineering and environmental studies and generally for geospatial and geoscientific applications. Rapp (1997a) opined that modern geoid models (global) can provide geoid heights of any points on the earth surface with an accuracy ranging from 30cm to a few metres. For local applications, this is certainly not acceptable pointing to the important need for local geoid models for GPS users in local applications.It is most expedient to determine which of the GGM best – fits the project area before it is recommended for adoption and applications in geospatial data collections. The United States NOAA Technical Memorandum NOS NGS-58 gives the guidelines for use of GPS for establishing vertical controls.

2.7.11 Regional Geoid Models

These are models designed for regions to among others enable conversion of ellipsoid heights to orthometric height for regional applications. Examples: GEOID12B, GEOID 12 in U.S. In the New Zealand there is the NZGeoid 2009 which replaced NZ Geoid 05 with expected accuracy of 8cm based on comparisons with GPS/Leveling observation. The Australian Height datum (AHD) is also regional. South Korean KNGeoid 13 and KNGeoid 14 are regional models with accuracy of 5.2 cm and 3.9cm respectively.

Comparing global and regional geoid models in South Korea, Lee and Seo (2016) concluded that the regional model achieved decimeter accuracy and has advantages over global models for national applications with the implication that local geoid models will perform better than regional and global geoid models in local applications.Ning (2015) observed that polynomial techniques (quartic model) adopted in Taiwan for development of regional geoid undulation model achieved centimeter precision level.

2.7.12 Local Geoid Models

These are models developed for a state, local government areas or an area generally to be used to transform ellipsoidal heights to orthometric heights for the day to day engineering, planning and mapping needs in place of geometric leveling techniques when GPS method is adopted. The areas/size of project involved dictates the assumptions to be made about the surface of study area e.g. if a plane surface is to be assumed or a complex surface or if hybrid technique is so be adopted. The main fact to be stressed is that a local geoid model is limited in application to the study area and could be a better surface representation instead of global or regional model application. Local geoids are necessary in areas where the gravity point densities are spatially or poorly distributed or even lacking thereby creating voids. For engineering and cadastral applications, local geoid is adequate since they are developed from local field measurements to achieve centimeter accuracy over several kilometers with high resolution.

CHAPTER THREE

LITERATURE REVIEW

3.0 Brief Introduction

The present satellite age has put into considerations all perceived limitations and challenges of all the previous techniques by improving on radio transmitters aboard satellites orbiting the earth at finite distances (at high altitude) for wider coverage. Signals from several satellites orbiting the earth can cover the whole of the earth. Satellites act as reference points but are not fixed. They move at very high speeds with facilities to give instant locations in their orbits for 3D determination unlike fixed known land based transmitters systems for 2D determination. Accuracy of satellite positioning is dependent on accuracy of fixed reference points and also distance dependent. Orbit information is given as part of navigation message.

It must ab-initio be established and agreed that determination of orthometric heights (H) by space techniques of Global Navigation Satellite System (GNSS or as represented by GPS) is synonymous to determination of geoid which may be correctly termed geoid modeling. Orthometric heights determination has been conventionally done by methods of spirit or geodetic leveling (or trigonometric leveling) with mean sea level (MSL) as a reference surface. Spirit leveling yields millimeter accuracy, very expensive over large areas, time consuming and requires highly trained personnel. Over large area, time required for field work, data capture, the propagation of errors, the labor costs are clearly drawbacks of this method. For this method to produce orthometric heights in the true sense of it, a gravity dependent orthometric correction (OC) is expected to be applied. As noted by Dennis (2004), gravity measurements are essential for geoid determination and a high resolution geoid model is required to accurately determine orthometric heights (elevations) with GNSS. OC is applied to correctly compute orthometric heights from differential (spirit leveling) and trigonometric leveling. The need for applying OC to height differences arise due to non-parallelism of level surfaces. Leveling is path-dependent requiring gravity measurements along the leveling route to be applied before adjustments. Nigerian leveling network does not have a report or record of the application of this correction which possibly prompted Fajemirokun (2006) to suggest that the heights are not strictly orthometric but closer to heights above geoid rather than the ellipsoid. This is not semantics but clearly a statement of fact and clarity. With the possibility of developing a geoid model for local applications, the GNSS height (h) can be transformed into orthometric height (H) using the geoid undulation (N) and applying the relationship from equation (1.13).Kavzoglu and Saka (2005) reported that no orthometric corrections were applied to the leveling operations that produced orthometric heights in Istanbul, Turkey but that did not stop the development of geometric geoid model for the a study area of 10000 km^2.

The GNSS height is referenced to a mathematical ellipsoid surface designed to best-fit the earth surface as a whole and as much as possible whereas the orthometric height from differential leveling is referred to geoid (approximated by MSL). The difference between the two surfaces i.e. the geoid-ellipsoid separation is termed geoid height or undulation (N). The use of GNSS yields centimeter accuracy and is cheaper, quicker and requires fewer trained personnel and hence geoid information will enable delivery of orthometric heights over large areas and longer distances and remove the need for temporary bench marks (TBMs) or assumed heights in construction sites for topographical mapping, setting out of civil engineering works etc. A small area is one whose area is less than 200-250km^2 with the implication that a simple or plane surface will suffice to develop a geoid model. For area greater than 200-250km^2, a complex surface may be required to develop a geoid model for modeling of orthometric height from GNSS. FCT has a size of about 8000km^2.

3.1 Review of Some Previous Works on Geoid Modelling/ Orthometric Height Determination by GPS / GNSS.

The reviews of some literatures related to geoid modeling to enable orthometric height determination are arranged in the order listed below: Global, African and Nigerian and are presented in that same sequence.

3.1.1 Global

Jamil (2011) used GNSS techniques to upgrade reference systems, to enable orthometric heighting and improved vertical datum/ reference surface.

Dual frequency GNSS receivers with geodetic antenna were used in static mode for determination of ellipsoidal heights (h).Gravity data were obtained from airborne gravimetry for geoid undulation (N) computation achieving a quoted accuracy of 5cm. this is believed to be adequate for heighting requiring second order level of accuracy. This is obtained without input from spirit or trigonometrical leveling which is quite revealing and interesting. Processing was done with Trimble Geomatic Office (TGO).The various data sources were combined to develop a hybrid geoid model.

The method produced a geoid with 5cm quoted accuracy. This shows that with a precise geoid model (MYGE0ID) in combination with GNSS, the era for spirit leveling to obtain orthometric height is done away with for projects not requiring high precisions like deformation monitoring ,crustal movements etc. This accuracy level was achieved because gravity specifications in terms of distribution, gridding and density have been met substantially.

Saiful *et al.* (2014) developed a geoid model for determination of orthometric heights. Geoid undulation was obtained from gravity data complemented by global models (GOCE, GRACE), digital elevation models (DEM) and GNSS (GPS) Leveling. The KTH approach initiated in 1984 by L. E. Sjoberg was used to compute geoid undulation (N) without the additive corrections for the gravimetric model. Gravity data are obtained from surface gravity observations and outliers were removed by quality control cross validation procedures to ensure or clear data that will be used for geoid determination. An assumption of non-existent masses outside the geoid makes Stokes Integral without corrections less accurate and hence recommended for global applications only. The result show that the KTH approach without additive corrections obtained gravimetric geoid model of Peninsular Malaysia at an accuracy level put at ± 32.1cm. Need for an additive correction is very important for use in local applications.

The accuracy attained may be adequate for global applications and certainly better than EGM 96 (1-2m in accuracy) e.g. for tunneling works but for local applications, the additive corrections must be applied to improve the accuracy for both global and local applications.

.

McDonald (2004) compared the accuracy and reliability of several geoid models against empirically derived geoid undulations to determine suitability for GPS heighting i.e. if accuracy and precision of GPS plus geoid is equivalent to what conventional leveling gives for engineering applications.

GPS and Digital level (Zeiss DiNi 2i with accessories) were used for this study to obtain ellipsoidal height (h) and orthometric height (H) respectively. Gravimetric geoid was compared with developed geoid to determine which has the smallest variation and hence imply suitability for adoption with GPS for use.

For outlier's detection, where 3σ limit about the mean is exceeded, it probably means Ambiguity Resolution (AR) was not achieved by the GPS observations, baselines were short (Short baselines are not adequately covered by global geoids leading to erroneous results) or global geoid does not adequately define or reflect surface over study area. This may hence, affect the predicted undulations and subsequent orthometric height determined. The multipath effect and ambiguity resolution have significant impact and must be procedurally and by software removed/reduced to acceptable level.

Global geoids are not designed for projects that are less than their resolutions. For this type of project, it is reasonable to develop a local geoid model to correctly represent the surface within project area and avoid unnecessary introduction of errors.

Hyo (2014) determined gravimetric geoid model using heterogeneous data to develop a regional geoid model for orthometric heighting. Terrestrial gravity data, Airborne gravity data, Stokes integral, Shuttle Radar Topography Mission (SRTM), global geopotential models were all used in this approach. Validation of the developed models to establish accuracy and possibility for application were also stated.

The Remove-Compute-Restore approach was used to compute the geoid based on actual measurements involving free-air gravity anomalies. Low resolution of data affects the accuracy of geoid models. As expected, the regional geoid is affected by the accuracy of data from different sources. This model will perform quite well regionally but will not meet expectation for projects with lower resolution than the gravimetric resolution i.e. for local applications

The Stokes' method assumes gravity data are available covering the whole earth but in reality, it is not so with the expectation that truncation errors will be introduced.

Komarov *et al*.(2007)determined geoid by using GPS/ leveling technique in the study area.Precise leveling was performed to obtain orthometric heights on benchmarks using the optical level and invar staves. Double run leveling was adopted.

Dual frequency GPS receivers were used adopting differential GPS technique in static mode over the same benchmarks. Post processing of GPS baselines with Trimble Geomatic Office (TGO) with precise IGS ephemeris was used. The well-known relationship N = h – H is used to calculate the geoid undulation which is used with Linear interpolation method of Kriging to build up a map of local geoid undulation.

GPS heights can be transformed with the developed model to orthometric heights. Dual frequency GPS used in relative mode produce 2-5cm accuracy. Different accuracy levels were achieved for the dual and single frequency GPS equipment.

It was observed that the GPS network may not be homogeneous which may constitute a serious setback for achieving the accuracy desired. The levels were not corrected for gravity effects necessary before any adjustments, especially considering the length (355km) double run leveling. Definitely this will have a degrading effect on results.

The Use of differential GPS technique in relative mode contributes to attaining high accuracy in geoid modeling.

Pinon *et al.*(2015) developed a national geoid model for Argentina by gravimetric approach using Remove-Compute-Restore (RCR) technique incorporating Global Geopotential Model (GGM) with over 230000 land and marine gravity measurements with terrain corrections from SRTM, DTM model and 1000 locations of GPS/Leveling. Fast Fourier Transform (FFT) was used for gridding data to fit into solving the Stokes formula. GPS provides ellipsoidal height (h).Collocation was done using 1173 points. Use N = h – H for geoid undulation determination by trend surface approach.

The GEOAR fits the Argentinean vertical better than EGM 2008 and previous geoids possibly to support the need for local or regional model for applications that are not global.

Accuracy obtained is better than 10cm. The lack of homogeneous distribution of control points present gaps or blanks in gravity data computation Interpolation was used to fill the gaps in gravity data acquisition. Mountainous regions have unusual gravity variations which may negatively affect the computations. Lack of adequate gravity data to cover Argentina at such specified grid points/density led to a less accurate geoid.

Improvement can be made if more gravity distribution at homogeneous spacing is achieved, since it is a gravimetric geoid that was developed. This in reality may not be feasible considering costs i.e. interpolation cannot be totally ruled out but the consequences for each application should be evaluated for advice on when and where to adopt this approach.

GGM assumes gravity data results from uniformity gridding which may not be true and hence affects attainable accuracy.

Tranes *et al.*(2007) used differential (geodetic) leveling, static Global Positioning System observations and Real Time Kinematic (RTK) Global Positioning System (GPS) in the comparison of orthometric heights (H) using a derived geoid model.

Length of leveling route is 2.6km.GPS 3D observations and published orthometric height (H) were used to create local geoid models needed to convert ellipsoidal heights observed in static GPS and RTK mode to orthometric heights and results compared with differential method. No orthometric corrections were needed because of the size/length of study area and gentle topographic relief according to the researchers.

Differential leveling achieved 3mm accuracy without applying gravity correction since no significant gravity change can be expected to affect the leveling differences at this distance.

Differential leveling was done with TOPCON Electronic Digital Level DL-101 and invar staff. Equality of back sight and foresight distances were limited to within 30m, hence effects of collimation, curvature, refraction were eliminated. Double run leveling was done. Foot plates were used as turning points. Weighting was done by reciprocal length of each section to find a weighted mean of height differences between points of interest. Javad Legacy GPS and geodetic quality receivers mounted on fixed – height 2m range poles were used. The GPS network was connected to three CORS which are located at a range of 67-118km from study area. TOPCON (2000) GPS vector processing software PINNACLE was used to produce latitude north, longitude east and ellipsoid heights (h) for all survey points. Geoid undulation at the controls were computed from N = h-H.

Local geoid model were computed from four, low-order polynomial surface models. Planar and higher order models were computed using least-squares regression. Polynomial models are planar, bilinear and quadratic which when combined with GPS RTK derived heights (h) produce orthometric heights.

Network residuals were used to check quality of leveling heights from network adjustments.

The magnitude of largest residual was less than expected accuracy of GPS data, it was considered acceptable.

The local geoid equation was derived from the regression equations. It was found that the models used with static GPS derived ellipsoidal heights provided better orthometric heights than RTK derived ellipsoidal heights. Statistics was applied to understand differences between geoid models from two measuring methods i.e. whether they are significant or not based on F statistics. It was concluded that since residuals from static mode is smaller, it then implies that orthometric heights derived from static mode are better.

The size of study area or length of leveling route is short and could not allow gravity corrections which could have given a further insight into the analysis for categorical statements on the more acceptable geoid model for a particular study area.

The method used for development of geoid is adequate. The static method is highly favored over RTK due to long hours of observation. This static method is used to overcome errors associated with RTK approach. Higher order models for small area may not lead to significant increase in accuracy and may not even be realistic.

Featherstone *et al.* (1998) combined gravimetric and geometrical geoid model to accurately recover orthometric heights from GPS .Using H = h – N, the orthometric heights can be derived. For differential GPS method which is relative between base and rover stations, H = h – N is meaningless and not useful to surveying and geodesy. The change in orthometric height ΔH over the baseline A to B is determined by using the change in geoid -ellipsoid separation via

$H_A - H_B = h_A - h_B - (N_A - N_B)$

$\Delta H_{AB} = \Delta h_{AB} - \Delta N_{AB}$

Engelis *et al.* (1984, 1985) gave the following relationship:

$\Delta N_{AB} = \Delta h_{AB} - \Delta H_{AB}$

This has the implication that geoid undulation is essential to transform GPS heights (h) to orthometric height (H). That a geoid behaves well in one area is not a guarantee for the same behavior in other areas and this is one of the greatest drawback of geoid and use of GPS. This can be redressed by use of both gravimetry and interpolation between geometrically derived geoid undulations that surround the survey area.

Gravimetry requires homogenous coverage with terrestrial data which is logistically impossible to achieve. Gravisoft software is available for geoid determination from gravimetric geoid.

The geometric method uses GPS height to provide in conjunction with a geoid model orthometric heights by providing estimate of geoid at discrete points through a rearrangement of H = h – N to read N=h - H.

For a small area assumed as a flat surface, use linear interpolation to compute H at any point.

$h - H = N_o + N_1 e + N_2 n$

where N_o – bias

N_1, N_2 – tilt of geoid with respect to WGS84

Use least squares for an over determined equation. Multiple Regression equations are also applicable especially when multiple reference stations were used for observations.

Gravimetric geoid, Geometrical interpolation and combination of gravimetric and geometrical interpolation enables transformation of GPS derived heights to orthometric heights. The combined approach is said to be the most robust and when GPS orthometric height errors are minimized or mitigated, the GPS derived orthometric height is widely acceptable. Quality assurance is done to allow checks for accuracy.

Gravity data and geometric derived undulation are interpolated and definitely will generate errors that may reduce accuracy of orthometric height.

Bayoud and Sideris (2002)determined geoid from both ground and airborne gravity data and compare with GPS/Leveling data which indicated that normal free-air gradient gives a better fit than the Inverse Poisson Integrated technique.

The gravity anomalies (δg) using Stoke's integral and airborne gravity disturbances using Hotine's integral were solve as boundary value problems. It is discovered that Inverse Poisson integral is less practical for computation due to magnification of the errors in the data and due to rough field it generates when second Helmert condensation is used with consequences for the geoid.

The uses of the two methods were practically limited to large areas or global usage. Note that conducting gravity surveys is highly costly and hence the approach is restricted to global coverage.The practicability of the method due to errors generated may restrict the use to only large areas. Gravity grid spacing may not be feasible for terrestrial method and airborne method must be downward continued. Gravity observations are costly, however.

Yurt *et al.* (2005) aimed at the determination of geoid undulation for local geoid modeling by using GPS method within a small area. Spirit leveling was corrected by gravity reductions i.e. through gravimetric reductions. GPS leveling was carried on a network of 39 points within an area of 30km^2. The spirit leveling was GPS static observations were done for a least of 45 minutes using dual frequency receivers.

The orthometric heights have been determined at the level of ± 5.03mm with geoidal undulation at sub-centimeter level. Sub-centimeter level without leveling is implied for orthometric heights using GPS observation only. This accuracy level may be due to size of study area and possibly a fairly flat terrain. Hence effect of topography may be hidden to conceal real geoid model leading to faulty analysis and conclusions. Faulty model implies wrong orthometric height produced.

The combination of equipment and procedure to achieve this level of accuracy (5mm in orthometric height) is possible due to size of study area. This is a procedure to study, test and evaluate within a larger area of study by possibly using DGPS method with multiple reference stations.

Abdullah (2010) produced orthometric heights from GPS height using gravimetric geoid of the area.

GPS data along with known orthometric height of the same points are used with a gravimetric model of the area. Spirit leveling is used to provide orthometric heights. For small areas of gentle undulation, surface fitting solution may be used to determine local geoid surface, global geoid solution may be used for large area. The geoid undulation is the difference between ellipsoidal height and orthometric height i.e. N=h-H. For many controls available, a more complex surface can be modeled in place of plane surface.

For small area assumed to be a plane surface, simple polynomial model achieves accuracy of about 10cm which is adequate for most engineering, planning and mapping (large scale topographical surveying) applications. Size of study area was not stated to enable assessment or evaluation of model applied.

For topographic, engineering design and estimate of cost of construction, orthometric heights derived from this method is usually adequate.

Al-Bayari & Al – Zoubi (2007) produced gravimetric local geoid model from gravity only data. Using Gravisoft software, a local geoid model from gravity data was produced. OSU 91A and EGM 96 models were also used as Global Geopotential Models to compute geoidal undulation (N). GPS measurements were applied for validation to enable error margin to be determined and to determine ellipsoidal heights. Gravity only data achieved 40 cm accuracy while fitting to geoid developed by GPS leveling improves the accuracy to 10cm. it is significant to note that accuracy is better in flat areas when compared with mountainous areas. The accuracy level of 10cm is adequate for engineering, planning and mapping applications.

The standard error in flat areas was given as 0.2m and can be up to 1m in mountainous area due mainly to lack of gravity data.

Gravity data accuracy was not stated. Source and quality of gravity information was also not given i.e. whether it was terrestrial or airborne or from satellite The GPS leveling as well as technique adopted were not stated. The type of topography was stated as contributing to the differences in standard error over study area i.e. topographic effects which must be well provided for in order to avoid what is called border effects.

For planning and topographical mapping, this model may be useful for preliminary purposes and mapping where emphasis is not high accuracy e.g. for compensation and acquisition survey work and preliminary terrain studies before detailed requirements are listed. This method combined with GPS leveling will be adequate (10 cm accuracy) for derivation of orthometric heights of acceptable standard which has been demonstrated with the results from this study.

Soycan (2014) used polynomial models to determine local geoid by surface modeling techniques using kriging, polynomial regression, etc. to enable transformation of ellipsoidal height to orthometric heights for GPS users.

GPS RTK geometric leveling was made within study area for different number of control points distributed homogeneously to obtain the ellipsoidal heights. The different surface model was used to define the geoid model for the different number of controls.

The results show that the TIN works best for evenly gridded data. Nearest Neighbor results in high root mean square error. The evaluations show that polynomial regression, multi-quadratic and triangulation interpolation methods can be used for geiod surface yielding enough accuracy for determination of orthometric height from GPS.

This approach is solely associated with GPS method. May be some EGM models may be used for determination of some undulations and fitting to EGM be done to see the departure of the model. GPS model was not stated i.e. single frequency or dual frequency and in which mode i.e. RTK or static and the post processing software used and the baseline length were also not stated.

This approach is recommended where gravity data are not readily available considering the cost of conducting such operations. The methods of interpolation may also achieve the third order leveling accuracies adequate for survey services in physical development where speed is of importance using GPS.

Seker &Yildrimin (2002) obtained orthometric height from points that already has GPS ellipsoidal coordinates. Gravimetric geoid model was the method adopted, GPS campaigns was also done on about 600 points homogenously distributed within the study area. Gravisoft package interpolation (GEOIP) was used to obtain undulation while the GPS obtained height (h) and spirit leveled orthometric height (H) were used to obtain geoid undulation. The difference between the Gravisoft (N) and N_{GPS}/Leveling was computed.

Modeling the difference between gravimetric and GPS leveling geoid was successful leading to transformation of h_{GPS} to orthometric heights (H) within acceptable accuracy. This is believed to be adequate for engineering applications and alternative to spirit leveling. This method achieves time and financial savings.

The method gives different result for different areas i.e. from 12cm in the major part of the country to 20cm in the mountainous region where gravity data are insufficient.

This method may be deployed for use where mountains are not a feature of the terrain of the study area.

Kiamehr (2001) applied GPS/leveling as a replacement for 3rd order spirit leveling for engineering applications.

Some points within the Iranian first order leveling network were chosen for GPS observations in the different parts of the country. Global Geopotential Model (GGM) was used to compute geoid with improvement using Digital Terrain Model (DTM) on a one square km grid. Remove – Restore technique for transferring gravity anomalies to geoidal undulations using FFT approach. The gravity points involved were up to 12000.

GPS and orthometric height data were used to test the accuracy of the Iranian geoid. The results points to the need for improvement to enable application to surveying and geodesy projects. A value range of 0.90m to 1.92m is definitely not too good or accurate for transforming GPS heights to orthometric.

The inadequate gravity coverage and the mountainous topography in some parts of the study area have the tendency to degrade the accuracy achievable and further studies are needed to yield acceptable geoid model. The scale of the topographical map used for generating terrain data has a direct effect on the DTM accuracy obtained through interpolation and may also contribute to the poor geoid accuracy and hence poor orthometric heights produced.

For DTM generation to improve geoid computation, a small scale map should not be used. More gravity observations are needed and the GPS/Leveling points involved should be increased.

Featherstone *et al.* (1998) used combination of gravimetrically and geometrically derived geoid height to recover orthometric heights from GPS observations.

This involves use of terrestrial gravity measurements in conjunction with global geopotential model and digital terrain model. This is used generally for regional or nation-wide geoid model which is involving mathematically and computationally. For geometric input, GPS is used to provide ellipsoidal heights which when transformed through a geoid (N) model produces orthometric height (H) i.e. from H= h-N

For small areas, geoid was assumed to be a flat surface which may require the use of linear interpolation of a plane surface (model). If the separation is known over many points in study area, geoid can be modeled using low order polynomial surface or multiple regression equations.

The combination of gravimetric method and geometrical interpolation approach is said to be the most robust when compared with gravimetric geoid or geometrical geoid individually. This conclusion may not be true of all situations except all the input data are available to specifications i.e. gravity well distributed and of high quality, availability of orthometric heights also of high quality etc. to generate a geoid model for practical applications.

The conclusion arising from the results looks general. The gravity method applied was not stated as well as accuracy i.e. relative or absolute gravimetery. No effort was made to establish if there is a statistical difference between the undulation as obtained from gravity and geometric method.

The size of study area, the availability of data and type of data, the features for processing should guide the choice of method as well as the purpose i.e. whether engineering, mapping etc.

Abdullah (2010) determined orthometric heights (H) by using differential GPS survey of a baseline of a derived local geoid model and global geopotential model.

GPS baseline was measured in differential mode to produce ΔX, ΔY & ΔZ coordinate differences. X, Y, Z coordinates of the control points are transformed into φ, λ and h based on a reference ellipsoid. The H is determined from the relationship

$$H = h - N \text{ or}$$

$$H = h_{GPS} - N_{MODEL}$$

For the relative approach, orthometric height difference ΔH can be computed from $\Delta H = \Delta h_{GPS} - \Delta N_{MODEL}$

This is preferred for highest precision since ΔN_{MODEL} is more precise than geoid undulation (N) at either end of the points as well as Δh better than h at the end of the points of the baseline. A local geoid fitting surface is the surest alternative when gravimetric solution is unavailable. This can be developed from a minimum of three points whose GPS heights h as well as orthometric height H are also known or collocated. Least squares are used when more points are available to develop a complex surface than required i.e. provision of redundancy.

Global Geoid solution from global geopotential models from spherical harmonics are developed and used. These global models may be satellite only solution (GEM 9) or combined solution when surface gravity and altimetric data are added e.g. OSU91A

The N_{GM} is obtained from global geoid solutions and applied to determine H.

Model 1 $N_i = a_0 + a_1 x_i + a_2 y_i$

2 $N_i = a_0 + a_1 x_i + a_2 y_i + a_3 x_i y_i$

3 $N_i = a_0 + a_1 x_i + a_2 y_i + a_3 \Delta h_i$

Three different models were tested. Model 1 achieves 10.8cm accuracies, Model II achieves 3.5cm and Model III gives 6.6cm while OSU91A with correction for biases (e.g. geoid change) gives 10.4cm.

However, the GPS model was not stated i.e. whether single frequency or dual frequency. The mode of use also not stated either as static or RTK. The conventional leveling done was not corrected by gravity observation and loop closure was not disclosed and whether double run leveling was used or not.

The results achieved by Model II gave an acceptable accuracy for it to be used for engineering applications within the area by applying a simple polynomial model. The geopotential model is not useful for this study area. For the level of accuracy achieved; it is only reasonable that for certain application, GPS method and data may be used to replace conventional leveling at the third order level.

3.1.2 African

Gomma *et al*. (2014) employed a local geoid in the conversion of GPS derived heights to orthometric heights and evaluates the performance against some global models like OSU-91A, EGM 96 and EGM 2008.

This is a geometric method of GPS/Leveling. Factors that affect accuracy of geoid determination are as stated in Kayloop and Rabah (2008) and the interpolation algorithms given by Maher et al (2012). The polynomial regression model of degree m and order n was adopted for modeling the geoid undulation as a function of geocentric latitude (ϕ) and longitude (λ). Other interpolation options for computing geoid undulation are given as:- simple planar, Bilinear, Quartic and Cubic surface. For the polynomial regression model adopted, it was stated that the degree is very important so as to avoid losing the surface in reality and hence its suitability as a corrector surface if wrong degree was chosen. The degree depends on the number of points used for modeling and degree (m) of freedom. Schut (1976) and Yanak (1991) made reference to statistical tests found suitable for determination of most suitable coefficient. They asserted that the accuracy of the geoid model depends on the quality of the global geopotential models (EGM 2008, EGM 96, OSU-91A) in their study in Egypt. The OSU-91A to degree and order 360 has shortest wave length of 50km (0.5° latitude and longitude) and because of inadequate gravity data, fairly accurate representation of surface over Egypt cannot be realized using OSU - 91A. EGM 96 was collaboration between NASA, NIMA and Ohio State University (OSU). It was designed to be an improvement on OSU-91A.

EGM 2008 is the most recent EGM developed by least squares without incorporation of GPS/Leveling or deviation of the vertical according to the researchers. The degree an order is put at 2159 with coefficients extension to 2190. The resolution allows spatial extent of 9km with the implication of improvement by factor 6 in resolution (compare to 54km resolution for EGM 96) as well as 3 to 6m accuracy depending on geographic location and gravity data involved. Pavlis (2012) listed the wide range of areas of application of EGM 2008.The study area is $194km^2$ and 24 controls were used in the development of the model using GPS/Leveling as well as spirit leveling observations.

From the result, it was statistically observed that EGM 2008 gives the most precise geoid model when compared with EGM 96 and OSU-91A with the plot of N_{GGM} against N_{OBS} showing the nearness of EGM 2008 and N_{OBS}. GGM is global geoid model while OBS is observation.

It was also observed that increasing the number of observation points with adequate spatial distribution improves the precision of geoid model determination.

The size of study area and terrain may not necessarily reveal shortcomings expected. Stokes formula for computing N would have given other opinion/options if gravity data were available. Orthometric corrections were not applied or it was not stated for the spirit leveling operation. Type of level (precise or others), single or double run procedure was not mentioned as well as loop closure.

The type of GPS equipment used as well as the leveling instrument and accessories used were not stated. One other interpolation algorithm would have been applied as well as increasing the number of controls to study effect.

From the study, the results showed that the model adequately fits the surface over the zone under study.The EGM 2008 could be enriched if the GPS/Leveling data are incorporated to represent an update.The size of the study area should be increased to see the performance of the model within a larger study area.

Gledan and Algnin (2014) converted GPS based heights (h) into orthometric heights (H) for survey and mapping.

Optical level, automatic or digital levels were used to obtain difference in elevation between two points on the ground. For long distances, geiod undulation has a role to play in elevation differences. Geodetic leveling is one that takes geoid undulation into account. This results in orthometric height (H). GPS was used to derive ellipsoidal height (h) from the three dimensional positions (N.E h). The previous geoid was based on Doppler positions at 19 points and also from points with weighting applied according to inverse distances between stations for adjustments.

Automatic level was used for leveling from known benchmark to close on another known benchmark. Orthometric heights (H) were obtained. GPS was also used along the same line to derive ellipsoidal height of same points. Using N = h – H, geoid undulation were computed for study area. The orthometric heights of any point can then be computed from H = h+N whose ellipsoidal height h is known from GPS.

The value of geoid-ellipsoid separation (N) is very important in GPS derived orthometric heights. The accuracy (of 0.216m i.e. 22 cm) of heights obtained is adequate for contouring at map scales of 1:5000 and smaller but not for flow / fluid projects because small scale plan cannot be used for engineering designs

The 22cm accuracy has limited the use of the model developed to low order survey mapping and physical planning works. Leveling never took gravity into account for orthometric corrections; level equipment model was not indicated. Relative DGPS was not adopted although with reference to one base reference station used. Multiple base station was also not used which limited the accuracy achievable from use of redundant observations.

For engineering and design works, the results from this model are adequate. Topographical mapping for planning purposes can be done with minimum delay.

Odera and Fukuda (2015) produced orthometric heights from ellipsoidal height obtained from GNSS by fitting geoid model to a Local Vertical Datum (LVD). Leveling was normally used to produce the orthometric heights (H) while GNSS gives the ellipsoidal heights (h). The method warps the gravimetric model to fit into the local vertical datum (LVD). The offset of existing local vertical datum O_{LVD} = H – h – N where N is gravimetric geoid undulation obtained from precise geoid model.

The offsets are interpolated using kriging. That means there is N and offsets for every grid point to recover H = h – N – O_{LVD}. The recovered orthometric height compares favorably with established orthometric heights. The standard deviation between the differences of established and converted is given as ±3.3cm for offsets and ± 4.0cm for planer fitting methodrespectively.

Gravity data density was not stated. It is said that the surface realized after fitting is not an equipotential surface, hence physical applications are limited. That is to say heights from such a surface cannot be referred to as orthometric but closer to vertical reference surface than the ellipsoid.

The accuracy achieved is adequate for local applications but the aim is defeated if after the study, an equipotential surface is not realizable. Orthometric height produced may not be reliable for some other applications in general. Probably, assumptions of a planar surface are a fallacy and the local vertical datum could be varying for different parts of the area.

Sabah (2007) determined orthometric height by using GPS data and EGM 96 at the test point. Handheld Global Positioning System (GPS) is used to measure ellipsoidal height and the global geoid model (EGM 96) used to compute the undulation (N). An Etrex Vista Garmin GPS model (±4m accuracy) was used for each location/position measurement. EGM 96 geoid correction calculator was used for geoid undulation computation. The geoidal map of the area may be computed all over the site making surveyors need for orthometric height easier than before. This is a desirable development for construction site applications and hence avoiding establishment of temporary benchmarks.

The accuracy of handheld GPS used is too poor or low. The accuracy of the EGM 96 is also poor at 1m. The orthometric heights of the existing were not given to enable comparison with those obtained from the GPS study.

For a small area within the University, this exercise may be adequate but generally the sources of input data is too low to give accurate information needed but the heights are a fair reflection of the local variations in the topographic gradients or surfaces.

Odera *et al*. (2014) determined a local geoid model for application of GPS in heighting.GPS leveling data from dual frequency GPS and spirit leveling using a level instrumentwas used to provide height data for interpolation of local geoid models. Surfer software was used to generate/interpolate data for points not occupied. The geoid is used with the ellipsoidal height (h) to produce orthometric heights (H).
Accuracy of ±1cm in the study area using bi-quadratic polynomial interpolation method was achieved. This accuracy is sufficient for most engineering, planning and mapping projects of limited extent and relatively flat terrain or slope variation.

The spirit leveled differences was not corrected for gravity and the circuit closure was not stated. Type of leveling instrument was not stated as well as whether single or double run leveling was done. The study area size was not indicated.

Orthometric heights, obtained using this approach will be tremendously useful for engineering construction and topographical mapping for designs and preparation of bill of quantities as well as planning of physical developments and environmental studies.

3.1.3 Nigerian

Olaleye *et al*.(2013) determined geoid model using "sat-level" in which the ellipsoidal height from any satellite-based system is combined with orthometric height from geodetic leveling to model the geoid over the area, and use it as alternative to traditional/conventional methods for orthometric heighting i.e. conventional spirit leveling.

GNSS receiver was used to obtain ellipsoidal height (h) of points of observation within study area. Geodetic level was used for leveling over the same points to obtain orthometric heights (H) from reduced level over the same points.

Least squares adjustment was used to obtain satlevel geoidal coefficients. Surfer software was used to plot the undulations to show the geoidal surface from the 3D coordinates (East, North, and Undulations). Aleem (2013) developed “orthometric Height on fly” to compute the undulation which had been done with Microsoft Excel earlier. The same results were achieved after comparison. This acts as a check on the computations of undulations.

Geoid undulation computed from $N = h - H$ and “satlevel” were put in tabular form and the differences are computed (as residuals). They are close. Coefficients obtained from least squares adjustment are used to develop a model of the undulation surface which can be used to compute orthometric height of points within the study area. Differential GPS (DGPS) method was adopted with spirit leveling to give h and H respectively. The GPS is a dual frequency model.

Orthometric corrections were not applied to elevation differences between control points before adjustments. Leveling routes were not stated for circuit closure. Stability of the observation stations was not mentioned either in the form of date stations was established orconstructed especially with the type of terrain within study area.

The process of removing outliers is adequate. But the optimum geoid for Nigeria is not yet officially given and may not have been integrated in the GNSS receivers used in the country, hence users are compelled to use whatever model the manufactures integrated as default. The need for local geoid model for transformation to orthometric height is important as a possible replacement for conventional leveling methods of height determination.

Onwuzuligbo (2012) realized a local geoid model of NnmadiAzikwe University, Awka for processing orthometric heights within study area.

GPS/Leveling was done using DGPS Promark III and leveling (differential) was done with a leveling instrument. From N = h – H, interpolation was done to enable unoccupied points have the geoid undulation applied to ellipsoidal height(h) at that point to determine the orthometric height(H).

Ono (2011) was referred to highlighting the differences between global and local ellipsoids. One of the assumptions made at starting point of geodetic datum is that there is no undulation at that point i.e. N =0 with the implication that deviation of the vertical is also zero and that normal to the ellipsoid and geoid coincides.Polynomial regression methods were reviewed and used to compute the differences and compared with observed .Surfer was used to plot the profile.Primary and secondary data sources were listed as well as the instruments used for data capture as well as processing methods with software used.

The results show the heights from GPS/leveling and spirit leveling which was used to compute the undulations (from N = h – H). Geoidal map was produced with surfer software where the height value is replaced with the undulation values (may be termed NAUGEOID). This will enable use of GPS for orthometric height determination instead of spirit leveling for day to day application for engineering and survey works with the study area. The era of creating Temporary Bench Mark (TBM) in construction sites within NAU may be over with the development of this model i.e. no assumed height values anymore and the real surface is physically depicted and related to works being done.

As admitted by the researcher in the report, geodetic level and accessories with geodetic techniques were not employed. Since no gravity data was collected over the study area which is of small size, no orthometric corrections were also applied. The topography/slope and geology of the study area was not mentioned as well as the size/area.

For the size of the study area, the method used is adequate for applications in engineering and civil works and planning purposes. The topography would have assisted in the distribution of observation points.

Okiwelu *et al*. (2011) determined the Nigerian geoid undulation model using Spherical harmonic analysis to approximate the shape of the geoid (a surface of constant gravitational and centrifugal potential). The relationships for gravitational pull in Cartesian and polar coordinates were given as well as Laplace equation for a spheroidal earth. The geoid undulation as a function of the disturbing potential (T) was also stated. The relationship using GPS height (h) and orthometric height (H) through geoidal undulation (N) is well known.The gravity anomaly (Δg) is related to wavelength (λ) of the geoid undulation and degree (m) of the spherical harmonics. Geoid undulation for Nigeria was computed using a program (hsynth WGS84f) and minimum curvature program and surfer used for plotting.

EGM 2008 was very suitable with reference to WGS84 (this is the reference ellipsoid for GNSS/GPS) for geoid modeling because the extension to degree 2190 enables wavelengths from short (tens of kilometers) to long wavelengths (thousands of kilometers) to be obtained.

Nigerian geoidal undulation is vital to studying regional problems and to understand the relationship between geoid undulation and topographic features on the earth. The geoid undulation (N) peaks at Jos plateau and decreases towards the ocean.Nigeria has positive undulation i.e. $N=h - H>o$ and has overall good correlation with topography. This highlights the need for terrain description in works concerning geoid modeling especially for deciding on interpolation techniques to be adopted.

GPS/Leveling data would have impacted positively on the Nigerian geoid but for the large size of the country and the needed logistics and political will to embark on such projects.Since the author suggests that the geoid would be vital for regional problems, it is assumed that it may not be adequate for applications whose resolutions are not up to the geoid resolution i.e. day to day surveying, mapping and engineering activities.No validation was stated except the correlation with topography.

Reference to WGS84 ellipsoid is important since GPS uses this ellipsoid for referencing height. This is a national / regional issue and hence this approach may be better.The GPS/leveling data would have been integrated to assist the geodesists in defining a vertical datum for orthometric heights.

Nwilo *et al*. (2014) used interpolation method for geoid modeling over a small area of less than 10 square km. The local geoid model is to serve as alternative to conventional leveling operations and to determine orthometric height from GPS in the absence of an official geoid model for Nigeria and lack of quality gravity data over the country for gravmetric geoid development. A review of existing methods of geoid modeling was done and emphasis was placed on the interpolation technique. Model was formulated from the relationship between ellipsoidal height (h) and orthometric (H) i.e. N=h-H as a function of the coordinates of observed points i.e. $N_i = h_i - H_i = a_0 + f_i$ (e,n) which can be written as

$$N = a_0 + e_i x_i + n_i x_2$$

Spirit leveling was used to to obtain the orthometric height while Promark 2 was used for ellipsoidal heighting determination in rapid static mode for 5 minutes.

Emelife (2012) used gravity data to develop a local geoid model for Awka and environs. The gravity observations were used to determine the geoid undulation (N) at several points to compute the gravity anomalies (δg) which was used with Stoke’s formula to provide geiod undulation (N). N computed from EGM84, EGM96, and EGM2008 was also used for comparisons with the observed terrestrial gravity. Gravity anomaly obtained from Nigerian Geological Survey Agency (NGSA) was used to plot map of study area as well as anomaly from terrestrial observations. Residuals were computed for each of the method.

It was recommended that to avoid generating parallel geospatial database infrastructure,geoid determination should be standardized.

Aina (2014) aimed at geoid model determination for transforming ellipsoidal height (h) using the space-based technique of GNSS within the Universitycampus as study area.GPS was used to determine **h** over points that have earlier been spirit leveled for orthometric height (**H**) determination. The difference between the two heights is called geoid undulation (N) which is given by N=h-H. Least squares technique was used to produce the most probable values of the coefficients of the curve fitting surface to derive a geoid model for the campus. Inverse Distance Weighting (IDW) and Kriging was the interpolation technique used. The studyrevealed the feasibility of adopting this method for orthometric heighting as an alternative to conventional leveling at the third order level of accuracy.

3.2 Identified Gaps from Reviewed Literatures

i. Real Time Kinematic (RTK) approaches limited to few minutes of observations were adopted in majority of the studies as in Nwilo (2014) and Olaleye (2013) and Tranes *et al.*(2007).

ii. Conventional one base reference station was used in almost all the related works consulted. This was well elaborated in Odera *et al.*(2014), Onwuzuligbo (2012), Nwilo (2014) , Aina (2014), Seker and Yildrim (2002), Gledan and Algnin (2014) , Tranes *et al.*(2007),Jamil (2011) . Parker (2015) says multiple base reference station observations can reduce the effect of random errors (which are errors that vary in sign and magnitude according to normal distribution of errors).

iii. Most of the related studies consulted were carried out within small areas and hence require simple surface for modeling as advocated by Romans (2004). The change of slope and geoid in small area is generally uniform and gentle. This was reinforced by Onwuzuligbo (2012), Aina (2014) and Sabah (2007)

iv. Global Geoid Models (GGMs) like EGM96 or EGM2008 are available but Odera *et al.*(2014) say in many cases they are too generalized and not accurate enough for orthometric height determination. Evaluation of the accuracy of the global models should be considered before its recommendation for use in the absence of an approved national geoid model for any country as alternative. The attempt by Uzodinma *et al.* (2014) to evaluate EGM2008 was quite instructive in the face of the proposalby Moka (2011) on adopting EGM 2008 as Nigerian national geoid model. Gomma (2008) observed that GGMs when used in a spherical harmonic expansion produces quasigeoid not geoid solution since the processing yields height anomalies not geoid undulations. This agrees with the works of Okiwelu *et al.*(2011), Gomaa *et al.*(2014), and Featherstone *et al.*(1998).

v. The network geometry was not discussed as important to choice of interpolation techniques as noted in almost all the entire studies consulted. For irregular networks, kriging and Radial Basis Function (RBF) are usually adopted. The DGPS relative technique was adopted in most of the studies based on single reference station e.g. in Aina (2014) and Nwilo (2014). This yielded low accuracy which may be due to geometrical instability of the field data capture technique.

vi. None of the related works were based on stability of triangles in the field technique for data acquisitions to ensure stability of the results.

3.3 General Comments on the Previous Works.

3.3.1 Global geoids: EGM 2008, EGM96 etc. are developed from global cooperation by having various data as input into the processing of the geoid model. The resolutions are different and hence found application globally especially as default geoid models integrated into the GPS to enable derivation of orthometric heights. Evaluation is needed to evaluate fit to each area of the earth by comparison with existing orthometric heights obtained from differential/spirit leveling with geodetic levels, invar staves or others. Sophisticated equipment, procedure and processing are used for this. Ezeigbo *et al.*(2006) reported an accuracy of 1 m for the gravimetric geoid solution of Nigeria.

3.3.2 Regional Models: Are those developed by each country for their territorial needs and applications. Their resolutions are however lower than that of global geoid and the methods of developing regional geoids are less sophisticated compared to global. They are designed to be the official surface for converting GPS derived ellipsoidal height to orthometric heights in a region. It may be improved by fitting it to GPS/ leveling geoid obtained from geometrical interpolation (taking cognizance of topographical features and masses).

3.3.3 Local Geoid Models: Are developed for transformation of GPS derived heights to orthometric heights for low order work especially to replace conventional leveling and hence fasten the ease of obtaining orthometric heights or elevation needed for engineering, planning, surveying and mapping needs over areas not up to regional in extent like a state or an area council or local government area. The GPS/Leveling method is mostly used but its accuracy may not allow its use for precision projects like deformation monitoring, crustal motion studies i.e. micro geodetic studies. Ono (2009) stated that the approach to geoid modeling depends on data availability and accuracy requirements either for i)gravimetric method ii) geometric method and iii) astrogeodetic method.

It is important to note that Okafor (1985) computed the difference between orthometric corrections and normal corrections obtained from measured gravity and normal gravity and gave a value of 4.2mm. The implication of this is that in the absence of observed gravity, normal gravity may be useful in computing gravimetric geoid adequate for geoid determination for GPS height transformation purposes. International gravity formula must however, be used. However, the conditions for terrestrial gravity observations given by Angus-Leppan (1982) may be difficult to meet since the controls to be used for observations may not be considered in the geodetic network design for both horizontal and vertical datum e.g. gridding intervals of 2-3km.

From a perusal of previous works on geoid determination which is a critical factor in the use of GPS to produce orthometric height, it will be apparent from all the researches that for small areas, interpolation of geoid by kriging, plane, bi-quadratic, minimum curvature, etc. are used. With large areas and availability of controls (N, E, H) in the F.C.T.Abuja,bi-cubic and multiquadratic polynomial interpolation models was adopted.

3.4 Comparison of Datum

Orthometric height has been referenced to mean sea level but with observed shortcomings, modern times pointed to the need for developing an alternative reference surface which is referred to as ageoid. The geoid model is necessary for issues relating to engineering and environmental problems like flooding; water retention etc.since the shape of the surface of terrain has implications on velocity of water movement during rainfall especially in drainage designs and engineering activities. A geoid model provides an accurate and consistent surface for referencing heights called orthometric heights which are consistent with direction of flow of fluids. Table3.2 gives comparison between Conventional and Modern Datum highlighting the need for modernization of height systems.

Table 3.1: Comparison between Conventional and Modern Datum.

Conventional	Modern
Origin is a fixed point on earth surface with geodetic coordinates ($\phi,\lambda,h=H$) e.g.L40 in Nigeria	Origin is earth centered i.e. geocentric with coordinates (0,0,0) at earth center of mass
Uses a best- fit ellipsoid for a region or state e.g. Clarke 1880 for Nigeria	Best- fit ellipsoid for the whole earth (geocentric reference system)e.g. GRS 80, WGS84
Vertical datum is also at points with assumed MSL elevation e.g., Apapa and Yaba Tidal datum in Lagos Nigeria	Vertical datum is not fixed to a point but referred to equipotential (constant) surface equivalent to mean sea level called geoid
Land masses are considered homogeneous throughout the earth and assume datum to be fixed and immovable	Reference frames (ITRF,etc.) takes actual movement of earth land masses into consideration

Source: Sergio (2003) modified

The conventional way of determination of orthometric (H) heights by classical leveling has limitations both in techniques, cost implication, time needed for field work and personnel and training involved.Leveling achieves highly accurate results when orthometric corrections from observed gravity are applied while trigonometric method is restricted to controls located on hill tops or high grounds but limited by refraction and curvature which are procedurally mitigated but does not attain classical leveling accuracy. The sea surface has been discovered not to be a level or equipotential surface after all due to salinity, sea surface temperature variations and geodynamical factors as well as sea level rise. This has the implication of leading to inconsistencies in vertical reference definition for heights. The Yaba or Apapa Datum mean sea level are point mean sea level definition of vertical datum and there is no record of using the two or more points mean sea level to produce surface mean sea level. Fortunately, development of GNSS opens a new vista in height determination since it delivers three dimensional 3D coordinates i.e. North (N),East(E) and Height(h) or latitude, longitude and ellipsoidal height h above a global best–fit ellipsoid WGS84 as opposed to mean sea level required for the much needed orthometric height H.

CHAPTER FOUR

METHODOLOGY

4.1 Introduction

Reconnaissance was carried out by physical visitation and evaluation of all selected controls that met the selection criteria. Interstation travelling distances were also very critical in view of observation duration. During the recce exercise, some controls could not be located because of difficulty in accessing them in the thick forest generally in the south eastern part of FCT and the advice from security agencies was to avoid that area as much as practicable. South Eastern part of FCT from observation of distribution of controls revealed that for some unstated reasons, have highly limited number of controls. Visual observations of the satellite imagery of FCT (fig. 4.1) showed thick forest occupation of this area and hence a possible reason for the limited controls within this part of FCT.

Some of the controls were found to have been removed e.g. FCT160P, FCT 2905S, FCT2910S, FCT2896S, FCT11791T, FCT11792T among several others. FCT 3636S, though on the ground physically, was found within a fenced plot and hence rendered inaccessible and useless. FCT 3634S located near GSS Kuje could not be found at all like many others. Resulting from this exercise, the following multi- network of controls (Table 4.1) were eventually selected and found to be fit for use as base and rover observation stations.

Table 4.1: Multi- Network of controls in FCT UTM ZONE 32 with coordinates

	FCDA COORDINATE REGISTER VALUE			REMARKS
CONTROL POINTS	EASTINGS(m)	NORTHINGS(m)	ORTHOMETRIC HEIGHTS(m)	
FCT10774T	327004.820	995328.184	430.653	ROVERSTATION
FCT12087T	332582.788	994837.597	474.612	„
FCT14384T	342631.664	994893.886	413.667	„
FCC11S	331888.114	998442.043	485.447	„
FCT9P	329821.512	1007612.091	497.253	„
FCT12P	333743.992	1008308.730	735.707	„
FCT19P	337452.408	996344.691	635.644	„
FCT24P	322719.776	1001884.850	453.804	„
FCT35P	322183.380	992926.363	427.171	„
FCT53P	308943.361	993406.773	351.943	„
FCT57P	303234.270	992916.402	323.844	„
FCT66P	299148.035	998114.283	297.111	„
FCT103P	340639.766	998375.578	532.558	„
FCT260P	255881.175	993666.807	201.944	BASE STATION
FCT276P	351983.716	1025998.314	625.572	„
FCT2107S	308926.908	989748.256	316.092	ROVERSTATION
FCT2168S	310554.927	1009739.930	431.087	„
FCT 2583S	294859.311	992582.162	225.618	„
FCT4154S	329953.882	1003831.280	476.981	„
FCT4159S	326124.422	1003742.860	452.230	„
FCT4652S	329441.767	997474.808	462.711	„
FCT3473S	329962.784	988829.321	501.459	„
FCT4028S	330164.634	1001388.240	449.592	„
FCT162P	270791.291	934625.533	189.696	BASE STATION
FCT130P	330982.584	952889.869	695.608	,,
FCT2327S	282526.612	973821.470	183.287	ROVERSTATION
FCT2652S	271370.273	945385.429	138.952	,,
FCT2656S	272644.591	941062.460	204.724	,,
FCT83P	332954.205	987231.606	568.752	,,
XP382	284074.729	983364.863	274.586	,,

Source: Surveying and Mapping Dept., FCDA.

Table 4.1 shows the multi controls selected for this study and the various capacities they will functionor used with their registered coordinate values and orthometric heights referenced to mean sea level.

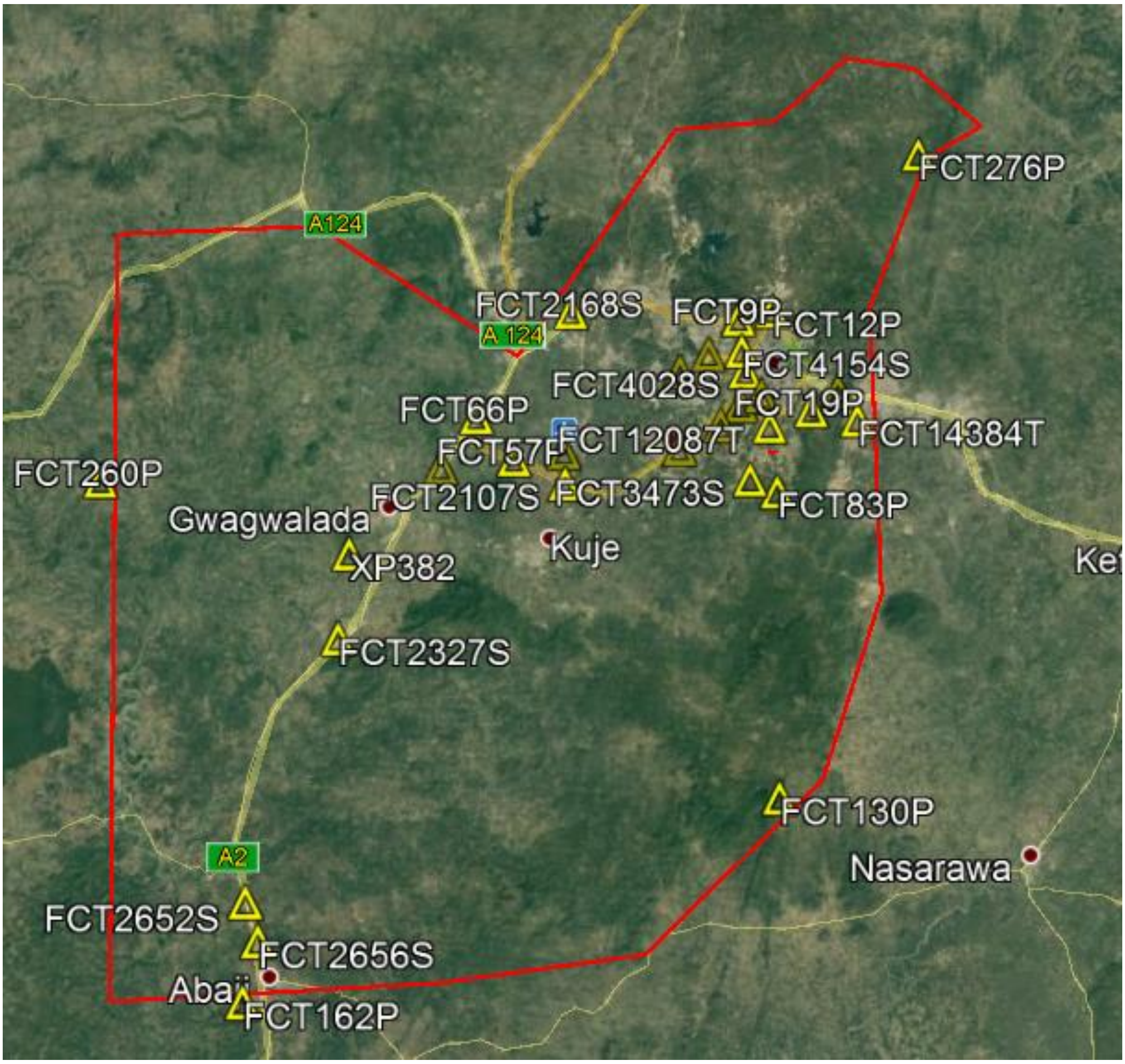


Figure 4.1 Google Earth Satellite Imagery of Multi Networks shewing general distribution of the Ground Control Points.

Data Source: Survey and Mapping Departmenr,FCDA Abuja.

4.1.1 Flowchart of Methodology

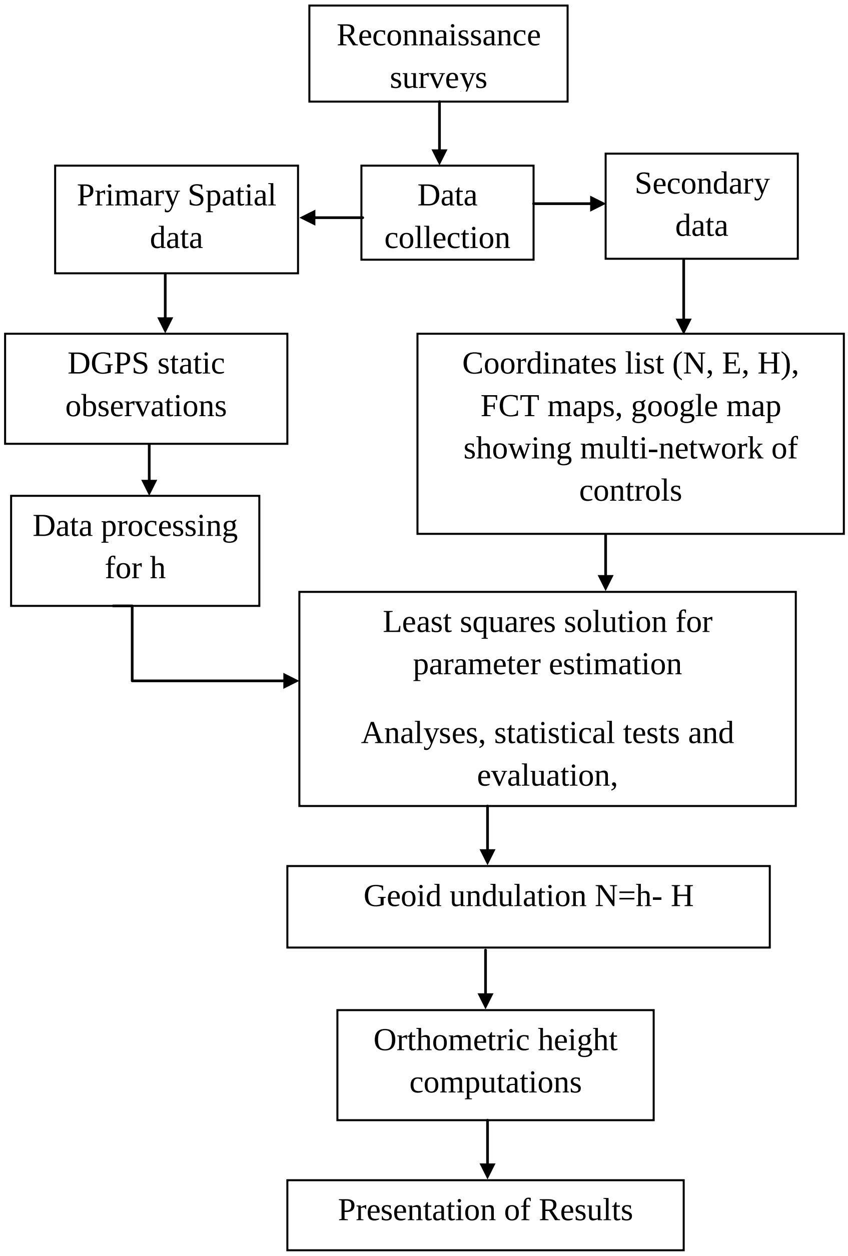


Figure 4.2: Flowchart of Methodology

4.2 Steps Taken To Fulfill Objective 1 (Physical Status)

Multi-Network of controls is very important to geospatial data acquisitions for sustainable developments in all stages and hence all efforts must be made to show their reliability and accessibility for various applications. The present investigation of physical status is to confirm the existence of these controls, their accessibility, usability and the stability for this study. Check computation of distances and angles were done also for the "in- situ" tests. The controls documented in table 4.1 were all visited. Observations revealed that primary stations visited were all intact except for FCT 160P that had been removed. Some secondary stations were found to have been removed due to road and building construction activities while most tertiary stations located on the ground were not traceable at all possibly due to farming and physical development activities. The number of controls sighted is adequate for this study although the distribution may not be homogeneous.This in essence requires that a polynomial model that can take this lopsidedness into consideration must be exploited.Table 4.2 details the physical status of controls used for geometric geoid modelling in the F.C.T.

Table 4.2 Physical Status of Controls

S/N	CONTROL ID	YEAR OF ESTABLISHMENT	STATUS (2017)	REMARKS/ USED AS
1	FCT260P	1980	FIXED/STABLE	PRIMARY/BASE STATION
2	FCT162P	1980	FIXED/STABLE	PRIMARY/BASE STATION
3	FCT130P	1980	FIXED/STABLE	PRIMARY/BASE STATION
4	FCT276P	1980	FIXED/STABLE	PRIMARY/BASE STATION
5	FCT35P	1980	FIXED/STABLE	PRIMARY/ROVER STATION
6	FCT53P	1980	FIXED/STABLE	PRIMARY/ROVER STATION
7	FCT57P	1980	FIXED/STABLE	PRIMARY/ROVER STATION
8	FCT83P	1980	FIXED/STABLE	PRIMARY/ROVER STATION
9	FCT9P	1980	FIXED/STABLE	PRIMARY/ROVER STATION
10	FCT12P	1980	FIXED/STABLE	PRIMARY/ROVER STATION
11	FCT66P	1980	FIXED/STABLE	PRIMARY/ROVER STATION
12	FCT24P	1980	FIXED/STABLE	PRIMARY/ROVER STATION
13	FCT4159S	1980	FIXED/STABLE	SECONDARY/ROVER STATION
14	FCT2168S	1980	FIXED/STABLE	SECONDARY/ROVER STATION
15	FCC11S	1980	FIXED/STABLE	SECONDARY/ROVER STATION
16	FCT4652S	1980	FIXED/STABLE	SECONDARY/ROVER STATION
17	FCT3473S	1980	FIXED/STABLE	SECONDARY/ROVER STATION
18	FCT2652S	1980	FIXED/STABLE	SECONDARY/ROVER STATION
19	FCT3424S	1980	FIXED/STABLE	SECONDARY/ROVER STATION
20	FCT3401S	1980	FIXED/STABLE	SECONDARY/ROVER STATION
21	FCT2107S	1980	FIXED/STABLE	SECONDARY/ROVER STATION
22	XP382	1980	FIXED/STABLE	SECONDARY/ROVER STATION
23	FCT2022S	1980	FIXED/STABLE	SECONDARY/ROVER STATION
24	FCT4028S	1980	FIXED/STABLE	SECONDARY/ROVER STATION
25	FCT2656S	1980	FIXED/STABLE	SECONDARY/ROVER STATION
26	FCT2337S	1980	FIXED/STABLE	SECONDARY/ROVER STATION
27	FCT12028T	1980	FIXED/STABLE	TERTIARY/ROVER STATION
28	FCT14384T	1980	FIXED/STABLE	TERTIARY/ROVER STATION
29	FCT2910S	1980	REMOVED	SECONDARY STATION
30	FCT2896S	1980	REMOVED	SECONDARY STATION
31	FCT11791T	1980	REMOVED	TERTIARY STATION
32	FCT11792T	1980	REMOVED	TERTIARY STATION
33	FCT3636S	1980	INACCESSIBLE	SECONDARY STATION
34	FCT3634S	1980	REMOVED	SECONDARY STATION
35	FCT2413S	1980	REMOVED	SECONDARY STATION
36	FCT160P	1980	REMOVED	PRIMARY STATION
37	FCT2778S	1980	FIXED/STABLE	SECONDARY STATION
38	FCT10774T	1980	FIXED/STABLE	TERTIARY STATION
39	FCT160P	1980	FIXED/STABLE	PRIMARY

Source: Surveying and Mapping, FCDA/remarks after field recconnaissance.

4.2.1 Computation of Check Angles and Distances for Objective I

The base reference stations were used for check of stability of controls (“in-situ” check). Base station coordinates are given in Table 4.3

Table 4.3: Base Station Coordinates in UTM ZONE 32 Coordinate System.

Point id	Coordinates in FCDA register		GNSS processed coordinates	
	N(m)	E(m)	N(m)	E(m)
FCT260P	993666.807	255881.175	993666.814	255881.173
FCT276P	1025998.314	351983.716	1025998..904	351983.716
FCT162P	934625.533	270791.291	934625.533	270791.291
FCT130P	952889.869	330982.584	952889.869	330982.584

Source: Surveying and Mapping Dept., FCDA/ GNSS Observed Coordinates.

Using check angles and distances from existing coordinates (established in 1980) and GNSS determined coordinates (obtained in 2017) to compute angles and distances, we have from existing coordinates:-

Angle at base station FCT260P = 94° 25’ 26”.71

Distance (m) FCT276 P - FCT260 P = 60.895 km

From GNSS observations in 2017, we have

Angle at base station FCT260 P = 94° 25’ 17”.4

Distance (m) FCT276 P - FCT260 P = 60.895 km

Table 4.4: Summary of Check Computations for Stability

	Existing (1980)	GNSS (2017)	Differences
Angle at FCT260P	94° 25' 26".71	94° 25' 17".4	00° 00' 09".31 not significant
DistanceFCT276 P - FCT260 P	60.895 km	60.895 km	0.000 m insignificant
Comments :Controls are in Stable and reliable condition for use in this study			

From S=dϴ where d =60.895 km and ϴ=4.365079365 x 10^{-5} in radians, then S=0.003m i.e. 3mm due to 9" difference in angles.

From the differences in the Table 4.4 above, and considering the 37 years interval (1980 to 2017), the differences are not significant and it can be concluded that the controls are "in-situ" i.e firmly fixed and that no movement is suspected in their physical status.

4.3 **Polygonal Bases for Adopted Field Procedure**

A polygon is defined as a closed plane figure with three or more straight sides, flat and can be drawn on paper. It has two dimensions (length and width). The name of a polygon is determined by the number of sides e.g. triangle has three sides; quadrilateral has four sides, etc.

4.3.1 **Stability of Shapes Computed from Grashof's Formular**

Use of two base stations with each rover position forms a triangular shape (a 2D polygon) to become the basis of the adopted field method. Use of one base station has been the conventional method of terrestrial observations in GNSS measurements by relative technique and may result into poor position computation due to poor geometry.

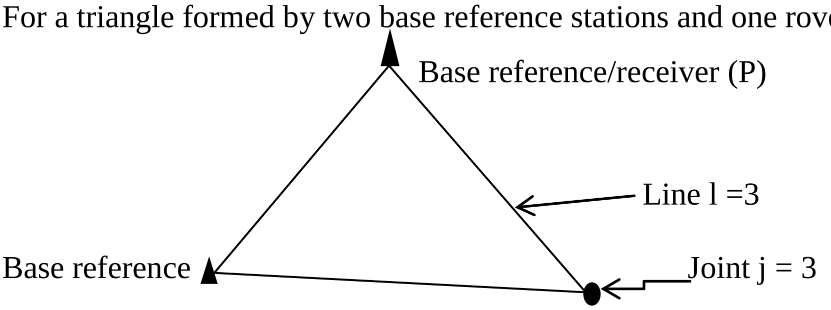
For triangles, the side physical structures as an example, diagonal bracing is used to create triangles in squares and quadrilaterals, etc. For planar shapes, the degree of freedom (n) is given by Grashof's law as quoted in Quora http://www.quora.com by:

$$n = 3\,(l - 1) - 2j - h \qquad (4.1)$$

Where l is no of links, h is no of higher pairs.

j is no of joints

For a triangle formed by two base reference stations and one rover station, see Figure4.3.



From figure 4.3: A triangle

for a triangle, l = 3 , j= 3 , h = 0

n = 3(3-1)-2*3 – 0

=6 – 6 – 0 = 0

The implication of the above n = 0 is that the triangular shape is geometrically stable and believed to be the strongest perfect frame in physical structures.

Assuming for a line in fig. 4.4, we have l=1, j=2, h=0 therefore n= - 4 to show that line adopted for field observation may actually not guarantee stability of results.

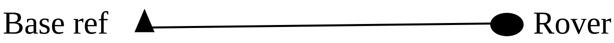


Figure 4.4: A line

For a pentagon in figure 4.5, we have l=5, j=5, h=0 we have n= 2 to indicate and imply deformed and unstable shape that may affect the resulting data.

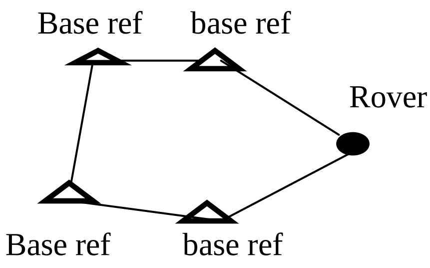


Figure 4.5: A Pentagon (5- sided polygon)

When degree of freedom (n) is greater than one e.g. in quadrilaterals and other polygons, they become deformed shapes and hence unstable.

4.3.2 Least Squares Technique

In surveying, measurements often satisfy established numerical relationships or geometric constraints. And that the number of equations in a parametric least squares adjustment is always equal to the number of unknown variables. Often, the system of equations becomesquite large because for robustness, more measurements than needed are required for redundancies. Matrix algebra provides at least two important advantages:

(i) It enables reducing complicated systems of equations to simple expressions that can be visualized and manipulated more easily.

(ii) It provides a systematic, mathematical method for solving problems that is well adapted to computers.

The least squares principle is based on minimization of sum of the squares of weighted residuals. In this study, the weight is assumed to be unity (i.e. w =1) since by design same set of equipment, personnel, time of observations durations and technique were used. Least Squares may be represented generally by

$$\sum wv^2 = w_1 v_1{}^2 + w_2 v_2{}^2 + w_3 v_3{}^2 + \ldots w_n v_n{}^2 \quad (4.2)$$

From the online processing software, it was decided that the online processed data would be averaged and used for ellipsoidal height in the geoid development. The OPUS, CSRS-PPP and MagicGNSS were the three online post-processing softwares used. GAPS and AUSPOS did not process all the points observed and hence omitted.

4.3.3 Mathematical Model

The mathematical model is a set of one or more equations that properly represents reality e.g. a polynomial equationto represent a geoid surface for modelling of geoid undulation (N) and by implication orthometric heights. A model may be conditional or parametric. While a conditional model enforces geometric conditions on measurements and their residuals, the parametric expresses equations in terms of observations and are more commonly used (i.e. as observation equations). Conditional models are not commonly used because of the difficulty to express all conditions in a complicated measurement network.

4.3.4 Outliers

Outliers are data points that are statistically inconsistent with the rest of the data.The existence of gross errors in geodetic computations cannot be denied from the point of view of statistics.An outlier is also viewed as observation that lies outside the overall pattern of a distribution and observations outside $\pm 3\sigma$ is considered an outlier i.e. if the deviation from the mean is three times greater than the standard deviation. Soycan (2013) also gave ±3σ as threshold for outlier detectionor data rejection. Olliver and Clendining (1979) also suggested that an observation whose residual is greater than three times the standard devaiation (σ) may reasonably be suspected of some other form of error and when the number of observations is small there are good grounds for rejection. Das $et\ al.$ (2017) gave ±3σ (±2.83σ) as criterion to identify points suspected to contain gross errors/ mistake and hence eliminated from futher participation in processing of information.

Table 4.5: Outlier Computation

CONTROL POINTS	Undulation N_{obs}=h-H (m)	N_{obs}-Mean N
FCC11S	23.949	-0.321
FCT260P	22.787	-1.483
FCT103P	24.278	0.008
FCT12P	24.485	0.215
FCT19P	24.18	-0.09
FCT2107S	26.041	1.771
FCT2168S	24.187	-0.083
FCT24P	24.183	-0.087
FCT276P	24.276	0.006
FCT4154S	24.251	-0.019
FCT4159S	24.323	0.053
FCT66P	24.004	-0.266
FCT9P	24.440	0.170
FCT35P	24.128	-0.142
FCT57P	23.951	-0.319
FCT4028S	24.35	0.08
FCT53P	24.012	-0.258
FCT4652S	24.402	0.132
FCT162P	25.395	1.125
FCT130P	23.775	-0.495
FCT2327S	24.195	-0.075
FCT2652S	24.789	0.519
FCT2656S	24.505	0.235
FCT83P	24.067	-0.203
XP382	23.804	-0.466
$mean$N	24.270	**σ** =0.575m

From Table 4.5, outlier is computed for the geoid undulation. The observed geoid undulation was computed as the difference between ellipsoidal height and existing orthometric height of each control point N_{obs}= **(h-H)**.For the outlier, the ±3σ criterion was applied. The standard deviation **(σ)** is computed as 0.575m and ±3σ = ± 1.725m .The range of the observed geoid undulations in the FCT is 22.546m to 25.994m. From Table 4.5,it is FCT2107S with **(N_{obs}-Mean N) =** 1.771m that lies outside ±3σ range of ± 1.725m and hence it was removed from further participation in the geometric geoid development process.

4.3.5 Observation Equations

These are written for the parametric model as one equation for each observation and generally as given in Ono $et\ al.$ (2014),

$$AX = L \tag{4.3}$$

Where A is a design matrix

X is the coefficients

L is measurements

When the number of measurements (m) is greater than the number of unknowns (n), then we have redundant observations (m>n) requiring least squares adjustments solutions. The (m-n) term is called the degrees of freedom (d.o.f). When there are redundant equations, the system is said to be over determined: A is not square, but $A^T A$ is according to Mikkhail & Ackerman (2000) and we have:

$$A^T AX = A^T L \tag{4.4}$$

$$\text{Let } N = A^T A \tag{4.5}$$

Let n= $A^T L$

Then X = N^{-1}n

NX = $A^T L$

Solution is given by X=$(A^T A)^{-1}(A^T L)$ for unit weight (4.6)

Unit weight is due to equal reliability of observations.

Where A = design matrix; X = vector of unknowns parameters/coefficients, P= Weighted Matrix and L = geoid undulations (N=h- H).

Standard deviation of observations (σ) is given as:

$$\sigma = \sqrt{\frac{v^2}{(n-1)}} \tag{4.7}$$

and the results are shown in table 4.10

4.3.6 Centroid Computation

Centroid coordinates are used in conjunction with coordinates of the different controls to estimate coefficient parameters of the various models. This is very importantant for transformation of GPS height coordinates to orthometric heights i.e between two datums. Ziggah *et al.* (2013) listed several centroid computational methodologies as:

i) Arithmetic mean centroid

ii) Harmonic mean centroid

iii) The median centroid

iv) Root mean square centroid.

This study adopted the arithmetic mean centroid because it works well for data that are simply added together and divided by number of data in the computation of the geometric geoid (N) determination and are given by

$$\left.\begin{aligned} x &= \frac{\sum_{k-1}^{n} x_i}{n} \\ y &= \frac{\sum_{k=1}^{n} y_i}{n} \end{aligned}\right\} \qquad (4.8)$$

This may be used for normalizing coordinates in geometric geoid modeling.

4.3.7 Least Squares Solution

To use least squares equations, first solve for the values of the polynomial constants ($a_{0,}$ $a_{1,,\ldots,}a_n$). This was accomplished by using the known/observed geoidal undulation (N) values along with the known northing (y or n) and easting (x or e) coordinates of the geoid model observation/control points. The known/observed geoidal undulation for each sample point and the difference in northing and easting coordinates between each observation point and the central (centroid) sample point are substituted into the polynomial equations (multi-quadratic and bi-cubic), hence creating a system of twenty-four observation equations. The system of equations expressed in matrix notation would take the form shown in equation 5.7 and equation 5.8

The solution to the vector of unknown polynomial constants is then be expressed as X=$(A^T\mathrm{A})^{-1}(A^T\mathrm{L}$) and the values are shown 5.6.2 for both models: multiquadratic (A) and bicubic (B) respectively.

4.3.8 Steps To Compute and Solve the Least Squares.

Microsoft Excel (2010) spreadsheet was used to compute the values needed to populate the design matrix A and results are shown in appendix 3 and appendix 4 for both multiquadratc and bicubic models. The model equations are a function of poitions x (e) and y (n) of each points observed. The positions are the results of the the coordinate differences from the computed centroid normalised positions shown on the sheets.

The L matrix contains the observed geoid undulations and the X contains the coeffient values to be determined from equation X =$(A^T\mathrm{A})^{-1}(A^T\mathrm{L}$).

To solve the above equation, the online matrix calculator was used and the results are shown in 5.6.2 for both models.

4.3.9 Data Processing

The computation of x, y, x^2, y^2, xy, x^2y, xy^2, x^2y^2, x^3 and y^3 was carried out in excel spreadsheet (Appendix 3 and 4). Model A is multiquadratic and B is bicubic.

The constants a_o, a_1, a_2, a_3, a_4, a_5, a_6, a_7 and a_8 for model A were determined with least squares method using online matrix calculator (Huobi.pro). Also the constants a_o, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 and a_9 for model B were computed using least squares method and the same online matrix calculator. Appendices 5 and 6 respectively show the computed constants and the least squares models

Excel programs were developed for interpolation of geoid heights, N and orthometric heights of various points in the study area with the models (model A and model B) using the computed constants, a_o, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 and a_9 and the evaluated values of x, y, x^2,

y^2, xy, x^2y,xy^2, x^2y^2, x^3 and y^3. The excel program for model A and model B were respectively developed using equations (2.34) and (2.35).The developed programs are respectively shown in appendices 7 and 8.

4.4 Steps Taken To Fulfill Objective II (GNSS Observations for Ellipsoidal Height h)

To fulfill objective ii, the following processes were used to acquire data in static mode for 2 hours and post processed to determine the ellipsoidal heights of the occupied controls. Below is a breief description of the processes:-

4.4.1 GPS Observations /Data Acqusition for Primary Data

4.4.2 Instrumentation

Dual frequency Hi-Target V30 Pro DGPS geodetic receiver models and accessories were adopted and used throughout the field observations for raw data. All the necessary logistics for achieving results were identified including safety and security of the field parties, vehicles, commuincations, equipment and others. Geodetic receivers were used to mitigate for errors especially the ionospheric error.

4.4.3 Static GPS Observations

To avoid or minimize the magnification of distance dependent errors and obtain centimeter – level accuracy, the study wass designed to ensure that rover-base station distances are less than base-base reference station distances generally and from GPS Guidebook (2004) “for utmost accuracy, a minimum of 45 minutes data is required”. This study was based on 2 hours data which was also a condition recommended for online post-processing. Note that the base reference stations (primary controls) were chosen to form perimeter of study area to ensure that rover observation stations are all within the perimeter and remove edge effects.

Primary data were acquired using Hi-Target V30 Pro DGPS model to obtain highly accurate 3D coordinates (N, E, h). The geodetic receivers were mounted on tripods set up directly over the base and rover observation stations. All settings were done including control point identifier, mask angles, height of instrument, etc. The rover control positions were designed and chosen to reflect changes in topography over the study area. Manufacturer of Hi-Target V30 Pro geodetic receivers quoted 2 cm accuracy as achievable for ellipsoidal height if used for data capture.

Four primary controls (FCT 260P, FCT276P, FCT162P& FCT130P) were used as base reference stations for continuous data logging while for rover stations a minimum of 2 hours static mode of observations was used. During the first session of observations on 20th of May 2017, the base reference stations used were FCT 260P and FCT 276P. The second session on 10th of June 2017 was based on FCT 162P and FCT 130P. The base receivers were powered by car batteries that ensured constant power supply throughout the duration of observations. For rover positions, the duly charged internal battery of each receiver was adequate and okay. Height of instrument was measured by using the steel tape in the instrument accessories to measure from the top of control mark to the antenna mark on each receiver and recorded as well as input into the data logger unit.

The rover - base reference stations distances are the recommended means of computing duration of observations using NGS guidelines outlined in Zilkoski *et al.* (1977) but in this study, it was decided to adopt 2 hours as minimum duration. Manilowski and Kwiecien (2016) concluded from studies that at least two-hour measurement session allow for determination of the horizontal distance or relative height with accuracy of 1-2 centimeter and as claimed by the manufacturer. Grinter and Janssen (2012) suggested that from studies where less than 4 hours observations dataset will be used, then differential technique should be adopted.Rover station FCT 2778S was the only station whose occupation time was 25 minutes (due to challenges linked to transportation on the way to the location) and therefore eliminated from post- processing.

High accuracies of coordinates require long period of observations necessary for proper ambiguity resolutions, hence the two hours static duration adopted for this study and all the receivers in use continue to generate and update coordinates of one another to yield results that are highly accurate and reliable for further uses.

4. 4.4. Secondary Data

Secondary data are data not directly acquired from field observations. These are i) list of coordinates obtained from Survey and Mapping Department of F.C.D.A. ii) map of Nigeria showing the 36 states and F.C.T. and iii) map of F.C.T. showing the six area councils,and iv) goggle map showing the multi-network of controls used within the study area by goggle earth.

4.5 Instrument Selection (Software and Hardware)

The following instruments were deployed for the field observations for raw data capture:-

a. (i) Hi-Target V30 Pro model GNSS receivers (Dual frequency) and accessories.DGPS is important for the cancellation of ionospheric error which becomes significant on baselines longer than 10 km.

(ii) Handheld GPS of various models e.g. Garmin, Etrex, etc for navigating to observation points (controls) for reconnaissance and during field work.

(iii) Bipod, tripod, etc for holding receivers.

(iii) Cutlasses, Handsets for communications between the base reference stations and the rover stations teams. The communication is very important for alerting and giving proper directives to the various rover positions of the readiness of the base reference receivers for start and close of observations.

b. **Hardware**

(i) HP Laptops models and desktops (Quilink, HP).

(ii) AO HP Plotter

(iii) A3 HP Deskjet printer 1515 series and accessories

(iv) 32GBHP, 2GB Transcend flash drives for storage of data, drawings, information.

c. **Software**

(i) Hi-Target Geomatic Office (HGO), CHC, 5 online (AUSPOS, GAPS, CSSR-PPP, magic GNSS, OPUS) post-processing software

(ii) Surfer Software

(iii)Autocad software (2007, 2009, 2010)

(iv) Autocad Land Development (2007)

(v) Microsoft software (MsWord2007, Ms Excel 2010, Ms PowerPoint, etc)

(vi) Least Squares software (online matrix calculator Huobi.pro)

4.6 Study Area Muti- Networks of Controls

Some controls within the F.C.T., Abuja was selected to serve as both the base reference stations and observation stations as shown in Figure 4.7. The selection was to ensure even distribution (as much as practicable) and location of the controls for geoid modeling. Their status was determined during reconnaissance surveys to confirm existence and physical state of the pillars with pictorial evidence as attached in appendix 1.

The topography of the study area is uneven with orthometric height ranging from the flood plain of the Gurara River at an elevation of about 70m above Mean Sea Level in Abaji area to around N35 with height of about 940.96m and hence surface model will show variations as indicated by the various orders of the multi-networks selected as base and observation stations i.e.the land rises irregularly eastwards, northwards and northwestwards of the FCT.

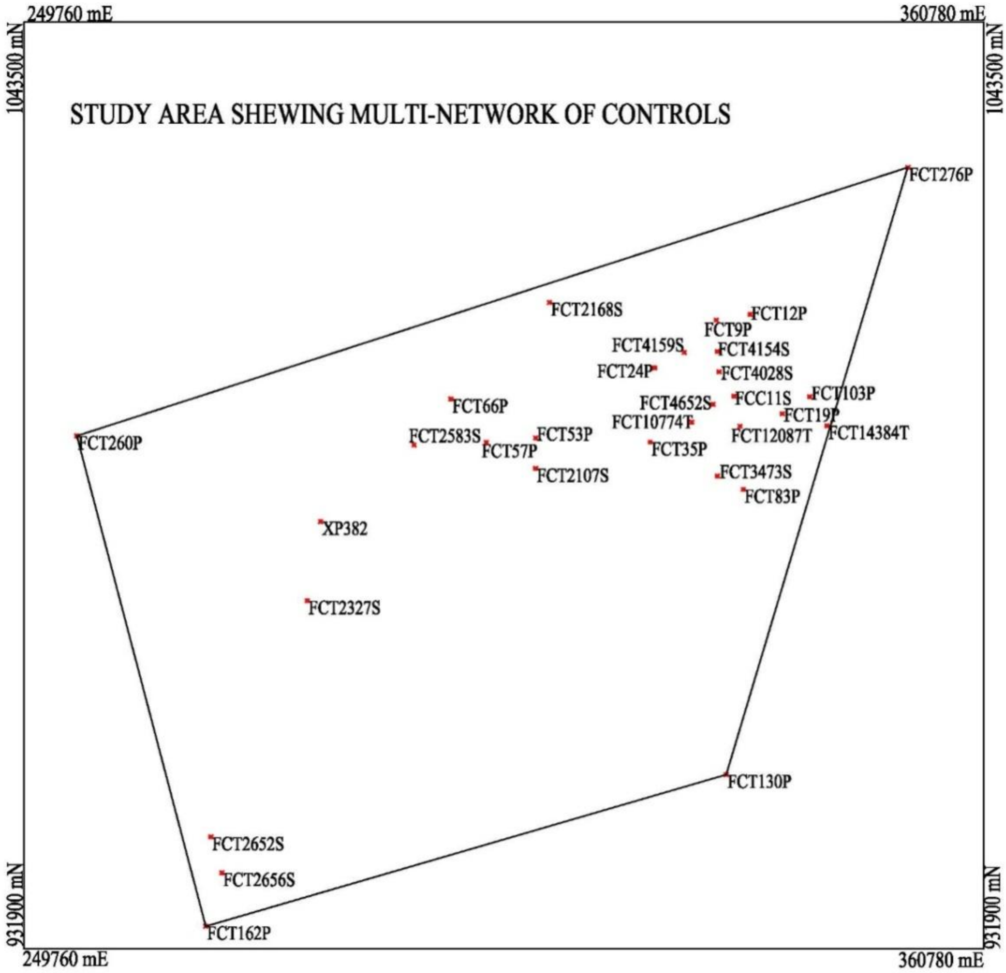


Figure 4.6: Plan of some selected Multi - Networks controls

4.6.1 Geometry of Study Area Base Reference Stations

The base reference stations are made up of primary controls and were chosen to sorroud the FCT. FCT 260P is located on the ground while the remaining three are located on hill tops. A sketch of the geometry is shown in figure 4.7.

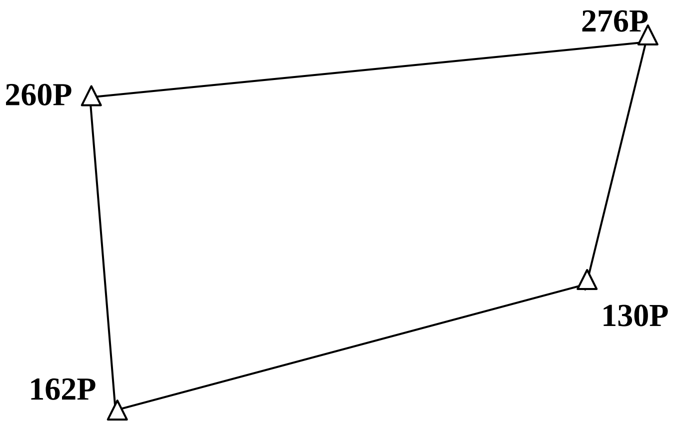


Figure 4.7: Sketch of Primary Base Reference Station controls

The primary controls are located around the perimeter of the study area and the observation points are distributed within the area of study but according to criteria that meets triangulation/control network scheme including strength analysis. The geodetic controls support the production of spatially accurate data for surveying and mapping arising from the permanency/stability of the controls. The geodetic controls ensure accurate representation of curved features on the earth surface on a flat paper as medium for map production in a chosen recognized projection system that manages distortions.

4.6.2 DGPS Observations Program

Schofield and Brench (2007) gave the five principles of surveying expertise as: working from whole to the part, economy of accuracy, independent control/check, data consistency and data safety /security.Based on these principles, the observations commenced by setting up dual frequency receiver V30 Pro Hi – Target DGPS on each end of the baselines (Primary controls) for continuous data logging. The baselines are arranged in such a way as to avoid extrapolation of positions within the F.C.T.i.e. observation positions are to be obtained by interpolation only. It is worthy to note that the primary triangulation stations forming the perimeter of the study area are of the highest quality in instrumentation, observations, processing and stability due to the very stringent conditions and specifications of primary triangulation/controls establishment and position determination.

Static GPS consists of the combination of antenna, receivers and static mode of observation was adopted for a 2 hour session timespent on each of the rover stations. This duration of observation is generally accepted as adequate for resolving ambiguities and produce accurate results. Various studies have indicated that there are virtually no significant improvements in accuracy after 2 hours of observations. Barnes *et al.* (2003) listed constellation of satellites, multipath and tropospheric effects as the dominant parameters that affect the accuracy of GPS baselines but Eckl *et al.* (2001) had earlier pointed out that duration of observations is the most dominant of factors determining accuracy in baseline length.

After the completion of the observations, the data were downloaded and then converted to rinex data for post - processing.

4.6.3 Satellite Geometry

GPS surveys involvedconsideration of factors such as:-

i) Number of satellites available

ii) Elevation or mask angle (usually set at 15°).The reason for this recommendation is to reduce the effects of systematic errors in GPS surveying namely tropospheric delay, ionospheric delay and multipath.

iii) Positional Dilution Of Precision (PDOP)

iv) Obstructions to satellite visibility

v) Vertical Dilution of Precision (VDOP) which is very critical for vertical GPS surveys.

4.6.4. Vertical Dilution of Precision (VDOP)

This is used to describe weakness in geometry of satellites in the vertical direction because satellites are usually situated above the antenna hence affecting the ellipsoidal height. This implies importance must be attached to times and areas in which GPS vertical surveys will take place to achieve a VDOP as low as possible. Caltrans Surveys Manual (2012) recommended a maximum VDOP of 4 (or 6 maximum) and minimum number of satellites observed simultaneously as 5 with 3 receivers using fixed antenna height tripods with dual frequency receivers. Precise ephemeris is applied to the computation. For this research where dual base stations were used on known primary control stations at any given time, the expectation of high accuracy and reliability is most desirable and expected.

4.6.5 Position Dilution of Precision (PDOP)

DOP is a measure that allows describing the influence of satellite geometry in accuracy of obtained measurements. Lower PDOP implies better satellite geometry and hence better position reliability. In general, Shruthi and Bindu (2016) have a list of DOP ranges and the interpretations.

DOP comprise Horizontal Position Dilution of Precision (PDOP), VerticalDilution of Precision VDOP, and TimeDilution of Precision (TDOP). These are measures of geometrical strength of a position determined by GPS. In the FieldGenius Technical Notes-GPS Terminology, it was stated that a DOP of between 4 and 6 can be considered as the threshold for good and poor geometry. Lower PDOP values denote better satellite geometry and thus a better reliability in positioning. A DOP value of 2.8 was achieved.

4.7 Processing Of GPS Observations

GPS data was acquired in Static mode and hence require post processing by either offline or online software.

4.7.1 Post – Processing For Static Observations

Every static observation is post-processed after downloading using appropriate software. Post processing will yield centimeter level accuracy. Due to long period of raw data acquisitions

(2 hours in this study), post – processing was not vulnerable to poor satellite geometry or visibility, multipath and unreliable data link from reference base stations and it is a way to significantly increase/improve the accuracy of DGPS observations. Better positional accuracy were produced by taking advantage of both IGS stations network and IGS product range and works with data collected anywhere on the earth.

4.7.2 On-Line Post-Processing

For this research, five free on-line post processing software was used. They are given as:

i) OPUS …On-line Positioning User Service that is operated by the National Geodetic Survey (NGS).It is the most common in the U.S. and requires 2 hours static observations data. Only GPS observations are used for position solution. It does not work well in Africa because of lack of stations in Africa or Europe.

ii) AUSPOS…AUStralian On-line GPS POsitioning Service. It uses Bernese GNSS software for processing baselines, IGS orbits and IGS network stations for solutions everywhere on the earth. Quality of computed coordinates depends on proximity to the IGS stations, quality of GPS orbits and quantity of submitted data. Where positions could not be processed, then significant issue of geometry may not be ruled out since geometry is the most important phase in GPS campaign. It is operated by National Mapping Division, Geoscience Australia. It does not process GLONASS data. Produces ITRF 2008 coordinates after processing.

iii) GAPS…GNSS Analysis for Positioning Software developed by University of New Brunswick (UNB). It processes only GPS data. GAPS can be used for estimating ionospheric delays, satellite clock errors, code multipath, etc.

iv) CSRS-PPP …Canadian Spatial Reference Service Precise Point Positioning operated by the Canadian Geodetic Division of Natural Resource Canada (NRCan). GNSS data are submitted over internet for processing.

v) MagicGNSS which presently supports only dual frequency PPP data.

Other on-line post-processing software is available but the above five were used in position determination in this research. The success rate of each of the post-processing software is shown in Table 4.6.The common features of all GPS processing software services is the use of RINEX file format as input file standard from the user end.

Online post-processing software was adopted for the fact that i) it does not matter which computer you use, ii) you can work from anywhere, iii) no need for installing customized(offline) software on the computer and iv) it is fast.

Table 4.6. Statistics of Online Post-Processing Software

Online Software type	No processed out of 30	No not processed	Success rate in%
MagicGNSS	30	0	100 accepted
GAPS	21	9	70
CSRS-PPP	30	0	100 accepted
AUSPOS	24	6	80
OPUS	28	2	93.3 accepted

4.7.3 Baseline Processing: General Overview of the GNSS Observations

Figures 4.8 and 4.9 show the processing of the observations using two base reference stations in the field data capture.

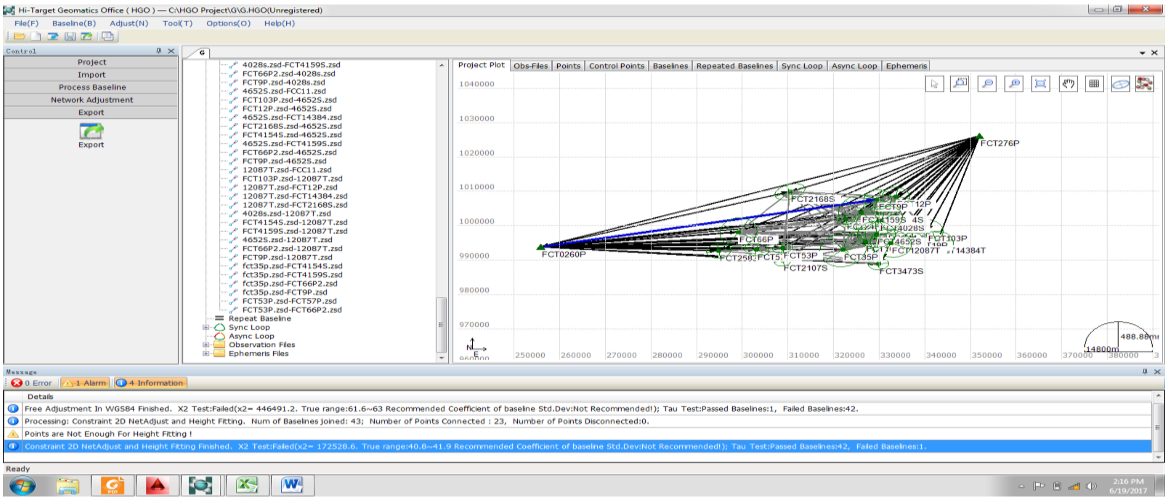


Figure 4.8: GNSS observation with FCT 260Pand FCT 276P as Base Stations

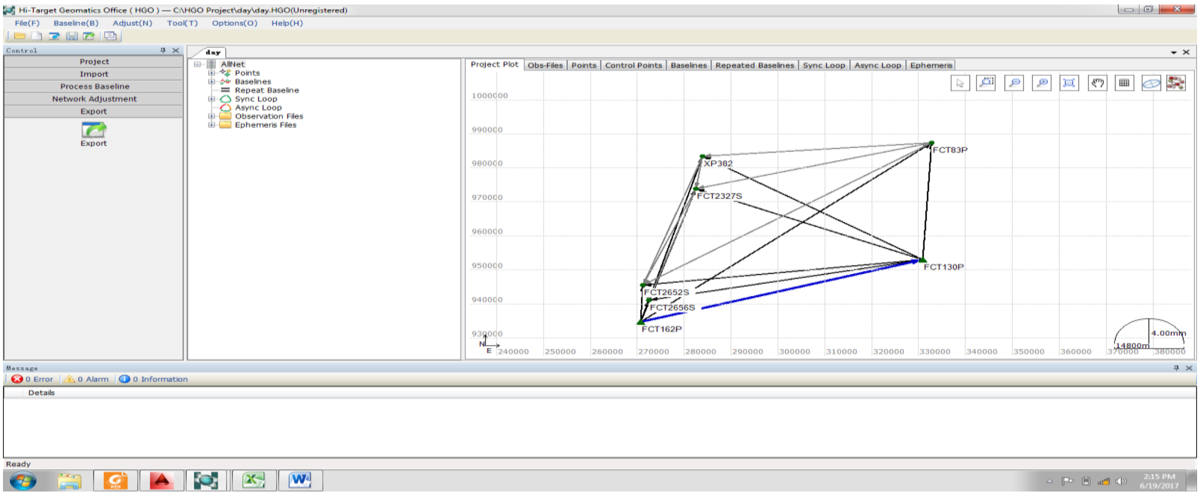


Figure 4.9: GNSS observation with FCT 162P and FCT 130P as Base Stations

The results of the post processing for ellipsoidal height determination are given in Table 4.7

Independent baselines are needed for computing number of whole baselines vectors from number of GPS receivers (n_r) and is given by (n_r) (n_r - 1)/2. The independent baseline vector is given by (n_r - 1). For this study, the independent baseline for 23 GPS receivers is 23-1=22. Only independent baselines should participate in network adjustments observed Wei (2011).

4.8 Fixed Baseline Information

This analysis is possible when GPS campaigns are based on fixed controls whose GPS WGS 84 coordinates are known as was the case in the study. This is done to verify the accuracy of

GPS measurement process and the controls held fixed and are always given in the post-processing reports. The loop closure analysis for example, are made by the baselines to enable large erroneous/blunders in GPS baseline measurements be exposed. None occurred in this campaign.

4.9 Baseline Network Adjustment

In GPS campaign, when one control is used for baseline adjustment, a minimally constrained adjustment is done to expose mistakes such as antenna height. On the earth's surface loss of navigation solution does not occur as long as the antenna has an open view of the sky. Where two or more controls are all used in baseline processing, scaling problems are exposed. This study used two primary controls (FCT260P-FCT276P) and (FCT160P-FCT130P) as fixed baselines, hence both minimally and fully constrained baselines were done for input data into the geometric geoid modelling of FCT. Uzodinma *et al*. (2013) may be consulted for details on fixed baseline information and baseline adjustments. Baseline vectors affect the quality of efficiency and precision of resuls in post procession of network opined Wei (2011).

Chang and Lin (1999) reported from studies using one and multiple base reference stations, that results obtained from the latter are more reliable and consistent achieving over 60% improvement in values both in horizontal and vertical components using DGPS. Differential GPS (DGPS) was developed as an augmentation system to meet the needs of positioning and distance-measuring applications that requires higher accuracies than stand-alone Precise Positioning Service (PPS) or Standard Positioning service (SPS) GPS could deliver observed Sabatini and Palmerini (2008).

Table 4.7 shows the ellipsoidal heights determined from online post-processing softwares to do geometric geoid modelling.

Table 4.7: Ellipsoidal Heights from 5 Online Post-Processing Software

CONTROL POINTS	OPUS	AUSPOS	CSRS-PPP	GAPS	MagicGNSS
PID	**ELIP HEIGTH(m)**	**ELIP HEIGTH(m)**	**ELIP HEIGTH(m)**	**ELIP HEIGTH(m)**	**ELIP HEIGTH(m)**
FCT10774T	454.816	454.618	454.857	0.000	454.813
FCC11S	509.413	509.365	509.410	509.379	509.365
FCT260P	224.737	224.720	224.753	224.721	224.731
FCT103P	556.836	556.812	556.851	556.821	556.821
FCT12P	760.201	760.176	760.185	760.178	760.189
FCT14384T	437.969	437.878	437.841	437.803	437.844
FCT19P	659.837	659.838	659.817	659.813	659.817
FCT2107S	342.112	342.113	342.063	342.057	342.133
FCT2168S	455.252	455.241	455.290	455.290	455.28
FCT24P	477.973	478.099	478.013	477.980	477.974
FCT276P	649.841	649.809	649.851	649.819	649.851
FCT3473S	0.000	525.490	525.486	525.411	525.523
FCT4154S	501.178	501.230	501.247	501.282	501.27
FCT4159S	476.589	476.586	476.442	0.000	476.627
FCT66P	321.096	321.029	321.126	321.123	321.122
FCT9P	521.648	521.653	521.720	521.694	521.712
FCT12087T	498.816	498.845	498.772	498.768	498.748
FCT35P	451.315	451.203	451.276	451.265	451.306
FCT57P	347.771	347.765	347.845	347.773	347.768
FCT4028S	473.905	474.521	473.994	0.000	473.926
FCT53P	375.938	375.903	375.991	375.946	375.936
FCT4652S	487.076	486.935	486.992	487.247	487.27
FCT162P	215.006	215.034	215.073	0.000	215.193
FCT130P	719.357	0.000	719.411	719.368	719.381
FCT2327S	207.433	0.000	207.446	0.000	207.561
FCT2652S	163.774	0.000	163.774	0.000	163.674
FCT2656S	229.230	0.000	229.244	0.000	229.212
FCT83P	592.759	0.000	592.876	592.769	592.822
XP382	298.410	0.000	298.432	0.000	298.329

GAPS and AUSPOS online software did not process for all controls and hence were omitted from ellipsoidal height determination. The accepted three online software ellipsoidal height are shown in Table 4.8. And the average ellipsoidal height was computed and used for geoid undulation computations subsequently.

Table 4.8: Ellipsoidal Heights from the Three Accepted Online Post Processing Software

	COORDINATE REGISTER VALUE			**OPUS**	**CSRS-PPP**	**MagisGNSS**
CONTROL POINTS	**EASTINGSx(m)**	**NORTHINGSy(m)**	**HEIGHTS, H (m)**	**HEIGHT, h(m)**	**HEIGHT, h(m)**	**HEIGHT, h(m)**
FCC11S	331888.114	998442.043	485.447	509.413	509.410	509.365
FCT260P	255881.175	993666.807	201.944	224.737	224.753	224.731
FCT103P	340639.766	998375.578	532.558	556.836	556.851	556.821
FCT12P	333743.992	1008308.730	735.707	760.201	760.185	760.189
FCT19P	337452.408	996344.691	635.644	659.837	659.817	659.817
FCT2107S	308926.908	989748.256	316.092	342.112	342.063	342.133
FCT2168S	310554.927	1009739.930	431.087	455.252	455.290	455.28
FCT24P	322719.776	1001884.850	453.804	477.973	478.013	477.974
FCT276P	351983.716	1025998.314	625.572	649.841	649.851	649.851
FCT4154S	329953.882	1003831.280	476.981	501.178	501.247	501.27
FCT4159S	326124.422	1003742.860	452.230	476.589	476.442	476.627
FCT66P	299148.035	998114.283	297.111	321.096	321.126	321.122
FCT9P	329821.512	1007612.091	497.253	521.648	521.720	521.712
FCT35P	322183.380	992926.363	427.171	451.315	451.276	451.306
FCT57P	303234.270	992916.402	323.844	347.771	347.845	347.768
FCT4028S	330164.634	1001388.240	449.592	473.905	473.994	473.926
FCT53P	308943.361	993406.773	351.943	375.938	375.991	375.936
FCT4652S	329441.767	997474.808	462.711	487.076	486.992	487.27
FCT162P	270791.291	934625.533	189.696	215.006	215.073	215.193
FCT130P	330982.584	952889.869	695.608	719.357	719.411	719.381
FCT2327S	282526.612	973821.470	183.287	207.433	207.446	207.561
FCT2652S	271370.273	945385.429	138.952	163.774	163.774	163.674
FCT2656S	272644.591	941062.460	204.724	229.230	229.244	229.212
FCT83P	332954.205	987231.606	568.752	592.759	592.876	592.822
XP382	284074.729	983364.863	274.586	298.410	298.432	298.329

This was used to fulfill objective two i.e.determination of ellipsoidal heights of various controls used for geometric geiod development.

4.10 Steps Taken To Fulfill Objective III (Determination of Geoidal Undulations N) From N= h-H

After post-processing of GPS observations, the average ellipsoidal heights (**h**) were computed and combined with the existing orthometric height (**H**) of each control point to determine the geoid undulation from N= h- H. See the results in Table 4.9

Table 4.9: Showing Average Ellipsoidal Heights and Computed Geoid Undulation.

	COORDINATE REGISTER VALUE			**post processing**	**UndulationN**
CONTROL POINTS	**EASTINGS(m) (e)x**	**NORTHINGS(m) (n)y**	**ORTHO HEIGHTS H(m)**	**AVERAGE h (m)**	**N=h-H (m)**
FCC11S	331888.114	998442.043	485.447	509.396	23.949
FCT260P	255881.175	993666.807	201.944	224.74	22.787
FCT103P	340639.766	998375.578	532.558	556.836	24.278
FCT12P	333743.992	1008308.730	735.707	760.192	24.485
FCT19P	337452.408	996344.691	635.644	659.824	24.18
FCT2107S	308926.908	989748.256	316.092	342.103	26.041
FCT2168S	310554.927	1009739.930	431.087	455.274	24.187
FCT24P	322719.776	1001884.850	453.804	477.987	24.183
FCT276P	351983.716	1025998.314	625.572	649.848	24.276
FCT4154S	329953.882	1003831.280	476.981	501.232	24.251
FCT4159S	326124.422	1003742.860	452.230	476.553	24.323
FCT66P	299148.035	998114.283	297.111	321.115	24.004
FCT9P	329821.512	1007612.091	497.253	521.693	24.440
FCT35P	322183.380	992926.363	427.171	451.299	24.128
FCT57P	303234.270	992916.402	323.844	347.795	23.951
FCT4028S	330164.634	1001388.240	449.592	473.942	24.35
FCT53P	308943.361	993406.773	351.943	375.955	24.012
FCT4652S	329441.767	997474.808	462.711	487.113	24.402
FCT162P	270791.291	934625.533	189.696	215.091	25.395
FCT130P	330982.584	952889.869	695.608	719.383	23.775
FCT2327S	282526.612	973821.470	183.287	207.482	24.195
FCT2652S	271370.273	945385.429	138.952	163.741	24.789
FCT2656S	272644.591	941062.460	204.724	229.229	24.505
FCT83P	332954.205	987231.606	568.752	592.819	24.067
XP382	284074.729	983364.863	274.586	298.390	23.804

4.11 Steps Taken to Fulfill Objective IV (To develop Microsoft excel program for interpolation of geoid heights and model orthometric heights)

Development of program is very important in facilitating ellipsoidal height conversion to orthometric. This was achieved by adopting the Microsoft Excel 2010 spreadsheet.

4.11.1 Spreadsheet

This is a computer software application used for manipulation and keeping track of data organized in rows and columns adaptable to various applications. It can be used for graphical display of data and computations. The electronic spreadsheet program uses mathematical formulas for automatic calculation of data input.

4.11.2 Microsoft Excel Software

This is an example of spreadsheet package where data of the same type can be manipulated. The worksheet cosists of rows and columns labelled A, B, C …Z and rows 1, 2, 3… with 256 columns and 16,384 rows. After column Z, we have column AA … AZ, BA …BZ and so on.The intersection of a column and a row is called a cell. Formulas always start wiyh equal sign (=) and are designed to manipulate data in cells that has values.

The Microsoft Excel 2010 was designed for the geometric geoid model. The polynomial equation was designed to accept each term as input in the different columns. The constant term a_0 has the same value for its column (for each model surface) whereas the remaining columns contain data values depending on positions (e, n) and the coefficient of each of the terms. The developed program computes the geoid undulation (N) and orthometric heights (H).

4.11.3 Data Processing

The model equations are written in terms of positions x, y.

The computation of x, y, x^2, y^2, xy, x^2y, xy^2, x^2y^2, x^3 and y^3 was carried out in excel spreadsheet (Appendix 3 and 4) to populate the A matrix for each model. x is the easting coordinates and y is similarly the northing coordinates.

Models A are B represents the multiquadratic and bicubic respectively.

The constants a_o, a_1, a_2, a_3, a_4, a_5, a_6, a_7 and a_8 for model A were determined with least squares method using online matrix calculator (Huobi.pro). Also the constants a_o, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 and $a_9$for model B were computed using least squares method and the same online matrix calculator. Appendices 5and 6 respectively show the computed constants and the least squares models

Excel programs were developed for interpolation of geoid heights, N in the study area with the models (model A and model B) using the computed constants, a_o, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 and a_9 and the evaluated values of x, y, x^2, y^2, xy, x^2y,xy^2, x^2y^2, x^3 and y^3. The excel program for model A and model B were respectively developed using equations (2.34) and (2.35).

The developed programs were used to achieve objective IV of this research respectively and shown in appendices 7 and 8 i.e. interpolation of geoid heights (N) and modelling of orthometric heights (H).

4.12 Steps Taken to Fulfill Objective V (To compare modelled orthometric heights using their standard deviations)

The developed program was used to model orthometric heights from the two models (multiquadratic and bicubic) and the results are in appendices 7 and 8.

The differences between existing and the developed orthometric heights are shown in Table 4.9 and were used to compute the standard deviation (σ) for each of the models.

Table 4.10: Comparison between the Accuracy of Multiquadratic and Bicubic Models

DIFF B/W EXISTING AND MULTIQUADRATIC MODEL ORTHOMETRIC HEIGHTS (A)	DIFF B/W EXISTING AND BICUBIC MODEL ORTHOMETRIC HEIGHTS (B)	A^2	B^2
0.285841605	0.298482551	0.0817054233	0.0890918334
0.01911206	0.012296746	0.0003652708	0.0001512100
0.122675646	0.151970694	0.0150493142	0.0230950917
0.119230612	0.205745694	0.0142159389	0.0423312904
0.059281596	0.060457902	0.0035143076	0.0036551580
0.000472599	0.009592057	0.0000002233	0.0000920076
0.002586221	0.119698876	0.0000066885	0.0143278209
0.008313477	0.066451743	0.0000691139	0.0044158342
0.08526864	0.069715646	0.0072707410	0.0048602712
0.039248531	0.001560435	0.0015404472	0.0000024350
0.185538642	0.252681189	0.0344245876	0.0638477831
0.081473227	0.140784399	0.0066378867	0.0198202470
0.080509242	0.105236756	0.0064817381	0.0110747748
0.096904403	0.075769114	0.0093904633	0.0057409586
0.049988498	0.043316085	0.0024988499	0.0018762832
0.000778872	0.068313577	0.0000006066	0.0046667448
0.204730669	0.165304325	0.0419146470	0.0273255200
0.001755336	0.092805825	0.0000030812	0.0086129211
0.02890435	0.011997291	0.0008354615	0.0001439350
0.066385996	0.003844076	0.0044071005	0.0000147769
0.007916644	0.139298723	0.0000626733	0.0194041341
0.009012951	0.221467692	0.0000812333	0.0490479387
0.157723116	0.120082336	0.0248765813	0.0144197674
0.186634409	0.184536082	0.0348324025	0.0340535656
STANDARD DEVIATIONσ (SQRT OF AVERAGE OF A^2 or B^2) =		**0.109959231m**	**0.135719119m**

From Table 4.10 above, the standard deviation (σ) of 0.109959231m and 0.135719119m for both multiquadratic and bicubic modelsrespectively implies that a multiquadratic model achieves 11cm accuracy as against 14cm from bicubic model though the two models can be applied throughout the FCT. Standard deviation is one of the indicators of how the model fits the FCT surface. The smaller the σ, the better the geoid model. This means the multiquadratic geoid model of 11cm accuracy is better.

Table 4.10A: Accuracy of the Two Models (Multiquadratic and Bicubic Models) using Densly Distributed Points in Particular Part of the Study Area.

CONTROL POINTS	DIFF B/W EXISTING AND MULTIQUADRATIC MODEL ORTHOMETRIC HEIGHTS (A)	DIFF B/W EXISTING AND BICUBIC MODEL ORTHOMETRIC HEIGHTS (B)	A^2	B^2
FCC11S	0.285841605	0.298482551	0.081705423	0.089091833
FCT103P	0.122675646	0.151970694	0.015049314	0.023095092
FCT12P	0.119230612	0.205745694	0.014215939	0.042331290
FCT2168S	0.000472599	0.009592057	0.000000223	0.000092008
FCT24P	0.002586221	0.119698876	0.000006689	0.014327821
FCT66P	0.185538642	0.252681189	0.034424588	0.063847783
FCT9P	0.081473227	0.140784399	0.006637887	0.019820247
FCT35P	0.080509242	0.105236756	0.006481738	0.011074775
FCT57P	0.096904403	0.075769114	0.009390463	0.005740959
FCT4028S	0.049988498	0.043316085	0.002498850	0.001876283
FCT53P	0.000778872	0.068313577	0.000000607	0.004666745
FCT2327S	0.066385996	0.003844076	0.004407101	0.000014777
FCT2652S	0.007916644	0.139298723	0.000062673	0.019404134
XP382	0.186634409	0.184536082	0.034832403	0.034053566
STANDARD DEVIATIONσ (SQRT OF AVERAGE OF A2 or B2) =			**0.122391029**	**0.153398946**

Also, table 4.10A shows the computed accuracy/standard deviation for multiquadratic and bicubic models using densely distributed points within the study area. This was done to compare the accuracy of the models using the total number of points (24 points) distributed within the entire study area with those using densely distributed points (14 points) in a particular part of the study area. From table 4.10A, it can be seen that the accuracy of the models, multiquadratic and bicubic are repectively 0.122391029m and 0.153398946m.

Comparing that accuracy of the models using tables 4.10 and 4.10A, it can be seen that the accuracy of the models are better when the total number of points distributed within the entire study area was used than when a limited number of points within a particular part of the study area was used. This implies that the models can be applied across the entire study area with high accuracy/reliability irrespective of spatial distribution of the points.

4.12.1 Correlation Coefficient (R) and Coefficient of Detemination (R^2)

Correlation coefficient (r) between orthometric heights based on MSL and modelled geoid can be determined from R= $\frac{n(\sum x)-(\sum x)(\sum y)}{\sqrt{((n\sum x^2-(\sum x^2-(n\sum y^2-(\sum y)^2))}}$ (4.9)

Where x= $H_{multiquadratic}$

Y = H_{MSL}

n=24 no of stations in this study

R = correlation coefficient to estimate quality of fit of the model to the existing msl H

For existing and multiquadratic modelled orthometric, R = 0.995 (1) and R^2 = 99% which confirms the multiquadratic as an acceptable and perfect model since the closer R^2 is to 100%, the better the model and the adequacy of statistical prediction. Also, 99% of variation in modelled orthometric height can be explained by the surface polynomial adopted.

4.12.2 Hypothesis testing for comparison of orthometric height

The null hypothesis is given by H_0 while the alternative is H_1 and is formulated as follows:

H_0 : R = 0 no relationship between $H_{multiquadratic}$ and H_{MSL}

H_1 : R≠ 0 there is relationship between $H_{multiquadratic}$ and H_{MSL}

Significance level α = 0.05 i.e. 95%

Decision rule: reject H_0 if |computed t| > $t_{20,0.05}$ from t- distribution table

Scenario 1: $H_{multiquadratic}$ **and** H_{MSL}

The statistical significance of the relationship can be computed by t-test statistics formular given by janda.org/c/10/lectures/topic/L as:

t is computed from $t = R\sqrt{(n-2)/(1-R^2)}$ (4.10)

In the case of $H_{multiquadratic}$ *and* H_{MSL} the computed **t** = 0 while table **t** =1.717. From the decision rule above, we reject H_0 i.e .the existing heights do not have any correlation with the modeled heights. Then H_1 is accepted and further, it may be an indication of coincidence of the two surfaces but referenced to different reference datum, the geoid and the mean sea level. Height values based on the geoid (multiquadratic or bicubic models) are the desired orthometric heights, however.

It has to be emphatically stated here that the use of sophisticated techniques cannot serve as replacement for using high quality coordinates as starting coordinates in data acquisition and processing. This research was based on primary controls that were established by classical methods to the highest standard of constructions (of pillars on stable ground geologically), high quality and rugged instrumentations with self checking tested field methodology for observations ,processing and analysis to obtain highly reliable coordinates.

4.13 Evaluation of Surface Fitting Techniques

Alevzakou and Lambrou (2011) stressed the need to determine if a surface of higher degree is necessary in geometric geoid modelling by using the relationship given in equation 4.11 as

$(r_1\, \sigma_2 / r_2\, \sigma_1) \leq F_{1,}r_2$ (4.11)

Where r_1 , r_2 degrees of freedom of the smaller degree surface and the greater surface respectively

σ_1 ,σ_2 standard deviation of the two surfaces respectively

$F_{1,}r_2$ F distribution for one degree difference between the tested surfaces

σ_1 = 0.109959231m σ_2=0.135719119m

r_1 = 16 (multiquadratic surface) , r_2 = 15 (bicubic surface)

The decision rule is if (r_1 σ_2/ r_2 σ_1) ≤ $F_{1,}r_2$, then no higher surface is needed for geometric geoid modelling of FCT.

Computed (r_1 σ_2/ r_2 σ_1) =1.31655213437

$F_{1,}r_2$ From F Tables = 4.531 (http://www.stat.ucla.edu/-dinov)

From the relationship (r_1 σ_2/ r_2 σ_1) and F distribution $F_{1,}r_2$, since 1.13655213437<4.531, no higher degree surface is needed for geometric geoid model in the FCT. This is an indication that either multiquadratic or bicubic model can be used to model orthometric height although the multiquadratic model performed better and could be taken as the optimum.

4.14 Bias and Skill Computations

The skill parameter can be seen as a measure of the model predictive capacity in relation to the observations. This skill parameter ranges from negative values to one, corresponding value of one implies a total agreement between observations and the model results.

Table 4.11: Orthometric Heights and Differences for Accuracy (**σ**) Estimates

PID	H_Multi quadratic (m)	H_Bicubic (m)	H_EXIST (m)	X =H_Multiq -H_EXIST	Y =H_Bicubic -H_EXIST	X^2 multiquadratic	Y^2 bicubic
FCC11S	485.161	485.149	485.447	-0.286	-0.208	0.081796	0.088804
FCT260P	201.963	201.956	201.944	0.019	0.012	0.000361	0.000144
FCT103P	532.681	532.71	532.558	0.123	0.152	0.015129	0.023104
FCT12P	735.826	735.913	735.707	0.119	0.206	0.014161	0.042436
FCT19P	635.703	635.704	635.644	0.059	0.06	0.003481	0.0036
FCT2168S	431.087	431.097	431.087	0	0.01	0	0.0001
FCT24P	453.807	453.684	453.804	0.003	-0.12	0.000009	0.0144
FCT276P	625.58	625.506	625.572	0.008	-0.066	0.000064	0.004356
FCT4154S	476.896	476.911	476.981	-0.085	-0.07	0.007225	0.0049
FCT4159S	452.269	452.228	452.23	0.039	-0.002	0.001521	0.000004
FCT66P	296.925	296.858	297.111	-0.186	-0.253	0.034596	0.064009
FCT9P	497.334	497.394	497.253	0.081	0.141	0.006561	0.019881
FCT35P	427.252	427.276	427.171	0.081	0.105	0.006561	0.011025
FCT57P	323.747	323.768	323.844	-0.097	-0.076	0.009409	0.005776
FCT4028S	449.642	449.635	449.592	0.05	-0.317	0.0025	0.100489
FCT53P	351.944	352.011	351.943	0.001	0.068	0.000001	0.004624
FCT4652S	462.916	462.876	462.711	0.205	0.165	0.042025	0.027225
FCT162P	189.694	189.789	189.696	0.268	0.093	0.071824	0.008649
FCT130P	695.579	695.596	695.608	-0.029	0.348	0.000841	0.121104
FCT2327S	183.221	183.283	183.287	-0.066	-0.004	0.004356	0.000016
FCT2652S	138.96	139.091	138.952	0.008	0.139	0.000064	0.019321
FCT2656S	204.715	204.503	204.724	-0.009	-0.221	0.000081	0.048841
FCT83P	568.91	568.872	568.752	0.158	0.12	0.024964	0.0144
XP382	274.399	274.401	274.586	-0.187	-0.185	0.034969	0.034225
					SUM	0.362499	0.661433
					STD DEV σ	0.126m	0.170m

Using Table 4.11, bias and skill was calculated by relationship given by Sutherland *et al.* (2004). Bias and skill can be defined, for the purpose of the present thesis, as presented in expressions (4.12) and (4.13), respectively follows:-

$$\text{Bias} = \frac{1}{N}\sum_{i=1}^{N}(H_{MODEL} - H_{EXISTING}) \quad (4.12)$$

$$\text{Skill}=1-\frac{\sum_{i=1}^{N}(H_{MODEL} - H_{EXISTING})^2}{\sum_{i=1}^{N}(H_{EXISTING})^2} \quad (4.13)$$

From calculations based on Table 4.11, we have the following:-

N= 24,

For Multiquadratic model,

$\sum_{i=1}^{N}(H_{MODEL} - H_{EXISTING})$ =0.196

$\sum_{i=1}^{N}|H_{MODEL} - H_{EXISTING}|$ =0.196

Bias= 0.008166667

Skill = 1- 1.510E-10

= 1

for Bicubic model

$\sum_{i=1}^{N}(H_{MODEL} - H_{EXISTING})$ = -0.305

$\sum_{i=1}^{N}(H_{MODEL} - H_{EXISTING})$ = 0.305

bias = -0.012708333

skill = 1 – 2.704E-10

= 1

The bias values computed as zero simply imply that the data used and equipment used has not shown any bias whatsoever in this study of modelling orthometric height or in geoid interpolation.

The skills parameter are computed for both models indicated that there was total agreement between the observations and the results from the models. This also suggests that the selection and combination of equipments, personel, field techniques and processing methods adopted yielded high quality data to produce the FCT geoid surface information as much as possible. Orthometric heights from the surface are hence based on geoid and compatible with GNSS technique and the adopted dual base reference station technique.

4.15 Diagnostic Test for Multiquadratic and Bicubic Models

In this research, two models were developed for orthometric height modelling and there is the need to know which is better of the two.

i) One method is to use the standard deviation (**σ**) of each of the models for comparison. Multiqudratic model has **σ = 11 cm** and bicubic model has **σ = 14 cm** to imply that the multquadratic model is surely the better.

ii) Adopting diagnostic tests as stated in Sinha and Prasad (1979) to compute a value

$$B=N \sum_{1}^{N} a_i^{\,2} \qquad (4.14)$$

where N= 24 is the number of controls.B is compared with (1.98 / $\sqrt{N}$) at 95% confidence limit.The decision rule is if 1.98 / $\sqrt{N}$< N$\sum_{1}^{N} a_i^{\,2}$, the the model is valid.

1.98 / $\sqrt{N}$= 0.404

N $\sum_{1}^{N} a_i^{\,2}$= 581.397 for multiquadratic model

N $\sum_{1}^{N} a_i^{\,2}$ = 564.020 for bicubic model

Since 0.404 < 581.397 and 0.404 < 564.020, the models are valid at 95% confidence limits for modelling orthometric heights from GNSS techniques.

Using the Chi squares (χ^2) test at the various degees of freedom (d.o.f), we have for multiquadratic d.o.f = 15, at 95% χ^2 =24.996

Bicubic d.o.f = 14 at 95% χ^2 =23.685

Since 0.404 < 24.996 or 23.685,the models are satisfactory at 95% confidence limits for modelling orthometric heights from GNSS techniques as confirmed by the diagnostic tests.

CHAPTER FIVE

RESULTS, PRESENTATION, ANALYSIS AND DISCUSSION

5.0 Brief Introduction

At the completion of fieldwork/observations, the data acquired are processed into usable format to produce information and the results are presented and discussed in order to link the findings with the set out research objectives. This chapter discusses how the observations were post-processed and results presented. From the determined/observed geoid undulation (N= h – H), the least squares technique was used to obtain the polynomial model parameters ($a_0, a_1, a_2 \ldots, a_n$) from online matrix solver. Microsoft Excel 2010 spreadsheet was used to generate the matrix elements from the positions (e, n) and the model parameters determined and given in table 4.4 and table 4.5 for each model. The geoid model equation is then written in terms of the eastings (e or x), northings (n or y), and model parameters. The followings are the sub-headings under which this chapter will be highlighted:

5.1 Results

The sample post-processing reports for observations are given appendix 9(online post processing reports) attached as an example.

.5.1.1 Model Parameters

Solution of the least squares problem led to the determination of the cofficients of the polynomial surface model equations given below:

i) MULTI – QUADRATIC MODEL A CONSTANTS (9 coefficients)

$$X = \begin{pmatrix} a_o \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{pmatrix} = \begin{pmatrix} 24.2248901210000000000 \\ -0.00002409340580587179 \\ -0.00008013620770038382 \\ 0.00000000009699046795 \\ 0.00000000370280953876 \\ 0.00000001166702184889 \\ -0.00000000000021600943 \\ -0.00000000000045716237 \\ 0.00000000000000000886 \end{pmatrix}$$

ii) BICUBIC MODEL B CONSTANTS (10 coefficients)

$$X = \begin{pmatrix} a_o \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{pmatrix} = \begin{pmatrix} 23.50081592604167925515 \\ 0.00004395285221636346 \\ 0.00009105227330487502 \\ -0.00000000156204910634 \\ -0.00000000352164634358 \\ -0.00000000178532065159 \\ 0.00000000000004116279 \\ 0.00000000000002433215 \\ 0.00000000000000729517 \\ 0.00000000000002928003 \end{pmatrix}$$

Finally, once the polynomial constants have been solved, they were substituted back into the original polynomial equations, and they yield the practical interpolation equation for determining an accurate geoidal undulation for various points of interest within the defined geoid model limiting area (FCT).

Interpolation equations developed then require only the difference in northing (y) and difference in easting (x) coordinates between a point of interest and the central sample point (computed average coordinates) in order to solve the point of interest's geoidal undulation. The geographic location and size of a project area may point to the need for different polynomial interpolation surfaces used for different regions of the project in situations where a single surface model may not cover all.

Microsoft Excel (2010) spreadsheet was used to compute the values needed to populate the design matrix A and results are shown in appendix 3 and appendix 4 for both multiquadratc and bicubic models. The model equations are a function of poitions x (e) and y (n) of each points observed. The positions are the results of the the coordinate differences from the computed average positions in the centroid and shown on the sheets.

The L matrix contains the observed geoid undulations and the X matrix contains the coeffient values to be determined from equation X =$(A^T \mathrm{A})^{-1}(A^T \mathrm{L})$.

To solve the above equation, the online matrix calculator was used and the results are shown in 5.7 and 5.8. See appendix 8 for both models.

These coefficients are used to write the geoid models equations used to copmpute the geoid undulation for any point within the study area. The A matrices for both models are shown below:

Multiquadratic model is given by equation 2.34

N = $a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy + a_6x^2y + a_7xy^2 + a_8x^2y^2$

$$AX = L$$

$$\begin{pmatrix}
1 & x_1 & y_1 & x_1^2 & y_1^2 & x_1y_1 & x_1{}^2y_1 & x_1y_1^2 & x_1{}^2y_1^2 \\
1 & x_2 & y_2 & x_2^2 & y_2^2 & x_2y_2 & x_2{}^2y_2 & x_2y_2^2 & x_2{}^2y_2^2 \\
1 & x_3 & y_3 & x_3^2 & y_3^2 & x_3y_3 & x_3{}^2y_3 & x_3y_3^2 & x_3{}^2y_3^2 \\
1 & x_4 & y_4 & x_4^2 & y_4^2 & x_4y_4 & x_4{}^2y_4 & x_4y_4^2 & x_4{}^2y_4^2 \\
1 & x_5 & y_5 & x_5^2 & y_5^2 & x_5y_5 & x_5{}^2y_5 & x_5y_5^2 & x_5{}^2y_5^2 \\
1 & x_6 & y_6 & x_6^2 & y_6^2 & x_6y_6 & x_6{}^2y_6 & x_6y_6^2 & x_6{}^2y_6^2 \\
1 & x_7 & y_7 & x_7^2 & y_7^2 & x_7y_7 & x_7{}^2y_7 & x_7y_7^2 & x_7{}^2y_7^2 \\
1 & x_8 & y_8 & x_1^2 & y_8^2 & x_8y_8 & x_8{}^2y_8 & x_8y_8^2 & x_8{}^2y_8^2 \\
1 & x_9 & y_9 & x_9^2 & y_9^2 & x_9y_9 & x_9{}^2y_9 & x_9y_9^2 & x_9{}^2y_9^2 \\
1 & x_{10} & y_{10} & x_{10}^2 & y_{10}^2 & x_{10}y_{10} & x_{10}{}^2y_{10} & x_{10}y_{10}^2 & x_{10}{}^2y_{10}^2 \\
1 & x_{11} & y_{11} & x_{11}^2 & y_{11}^2 & x_{11}y_{11} & x_{11}{}^2y_{11} & x_{11}y_{11}^2 & x_{11}{}^2y_{11}^2 \\
1 & x_{12} & y_{12} & x_{12}^2 & y_{12}^2 & x_{12}y_{12} & x_{12}{}^2y_{12} & x_{12}y_{12}^2 & x_{12}{}^2y_{12}^2 \\
1 & x_{13} & y_{13} & x_{13}^2 & y_{13}^2 & x_{13}y_{13} & x_{13}{}^2y_{13} & x_{13}y_{13}^2 & x_{13}{}^2y_{13}^2 \\
1 & x_{14} & y_{14} & x_{14}^2 & y_{14}^2 & x_{14}y_{14} & x_{14}{}^2y_{14} & x_{14}y_{14}^2 & x_{14}{}^2y_{14}^2 \\
1 & x_{15} & y_{15} & x_{15}^2 & y_{15}^2 & x_{15}y_{15} & x_{15}{}^2y_{15} & x_{15}y_{15}^2 & x_{15}{}^2y_{15}^2 \\
1 & x_{16} & y_{16} & x_{16}^2 & y_{16}^2 & x_{16}y_{16} & x_{16}{}^2y_{16} & x_{16}y_{16}^2 & x_{16}{}^2y_{16}^2 \\
1 & x_{17} & y_{17} & x_{17}^2 & y_{17}^2 & x_{17}y_{17} & x_{17}{}^2y_{17} & x_{17}y_{17}^2 & x_{17}{}^2y_{17}^2 \\
1 & x_{18} & y_{18} & x_{18}^2 & y_{18}^2 & x_{18}y_{18} & x_{18}{}^2y_{18} & x_{18}y_{18}^2 & x_{18}{}^2y_{18}^2 \\
1 & x_{19} & y_{19} & x_{19}^2 & y_{19}^2 & x_{19}y_{19} & x_{19}{}^2y_{19} & x_{19}y_{19}^2 & x_{19}{}^2y_{19}^2 \\
1 & x_{20} & y_{20} & x_{20}^2 & y_{20}^2 & x_{20}y_{20} & x_{20}{}^2y_{20} & x_{20}y_{20}^2 & x_{20}{}^2y_{20}^2 \\
1 & x_{21} & y_{21} & x_{21}^2 & y_{21}^2 & x_{21}y_{21} & x_{21}{}^2y_{21} & x_{21}y_{21}^2 & x_{21}{}^2y_{21}^2 \\
1 & x_{22} & y_{22} & x_{22}^2 & y_{22}^2 & x_{22}y_{22} & x_{22}{}^2y_{22} & x_{22}y_{22}^2 & x^{22\,2}y_{22}^2 \\
1 & x_{23} & y_{23} & x_{23}^2 & y_{23}^2 & x_{23}y_{23} & x_{23}{}^2y_{23} & x_{23}y_{23}^2 & x_{23}{}^2y_{23}^2 \\
1 & x_{24} & y_{24} & x_{24}^2 & y_{24}^2 & x_{24}y_{24_1} & x_{24}{}^2y_{24} & x_{24}y_{24}^2 & x_{24}{}^2y_{24}^2
\end{pmatrix}
\begin{pmatrix} a_o \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{pmatrix}
=
\begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \\ N_8 \\ N_9 \\ N_{10} \\ N_{11} \\ N_{12} \\ N_{13} \\ N_{14} \\ N_{15} \\ N_{16} \\ N_{17} \\ N_{18} \\ N_{19} \\ N_{20} \\ N_{21} \\ N_{22} \\ N_{23} \\ N_{24} \end{pmatrix} \quad (5.7)$$

Bicubic model is given by equation 2.35

$$N = a_o + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 xy + a_6 x^2 y + a_7 xy^2 + a_8 x^3 + a_9 y^3$$

$$AX = L$$

$$\begin{pmatrix}
1 & x_1 & y_1 & x_1^2 & y_1^2 & x_1 y_1 & x_1{}^2 y_1 & x_1 y_1^2 & x_1^3 & y_1^3 \\
1 & x_2 & y_2 & x_2^2 & y_2^2 & x_2 y_2 & x_2{}^2 y_2 & x_2 y_2^2 & x_2^3 & y_2^3 \\
1 & x_3 & y_3 & x_3^2 & y_3^2 & x_3 y_3 & x_3{}^2 y_3 & x_3 y_3^2 & x_3^3 & y_3^3 \\
1 & x_4 & y_4 & x_4^2 & y_4^2 & x_4 y_4 & x_4{}^2 y_4 & x_4 y_4^2 & x_4^3 & y_4^3 \\
1 & x_5 & y_5 & x_5^2 & y_5^2 & x_5 y_5 & x_5{}^2 y_5 & x_5 y_5^2 & x_5^3 & y_5^3 \\
1 & x_6 & y_6 & x_6^2 & y_6^2 & x_6 y_6 & x_6{}^2 y_6 & x_6 y_6^2 & x_6^3 & y_6^3 \\
1 & x_7 & y_7 & x_7^2 & y_7^2 & x_7 y_7 & x_7{}^2 y_7 & x_7 y_7^2 & x_7^3 & y_7^3 \\
1 & x_8 & y_8 & x_1^2 & y_8^2 & x_8 y_8 & x_8{}^2 y_8 & x_8 y_8^2 & x_8^3 & y_8^3 \\
1 & x_9 & y_9 & x_9^2 & y_9^2 & x_9 y_9 & x_9{}^2 y_9 & x_9 y_9^2 & x_9^3 & y_9^3 \\
1 & x_{10} & y_{10} & x_{10}^2 & y_{10}^2 & x_{10} y_{10} & x_{10}{}^2 y_{10} & x_{10} y_{10}^2 & x_{10}^3 & y_{10}^3 \\
1 & x_{11} & y_{11} & x_{11}^2 & y_{11}^2 & x_{11} y_{11} & x_{11}{}^2 y_{11} & x_{11} y_{11}^2 & x_{11}^3 & y_{11}^3 \\
1 & x_{12} & y_{12} & x_{12}^2 & y_{12}^2 & x_{12} y_{12} & x_{12}{}^2 y_{12} & x_{12} y_{12}^2 & x_{12}^3 & y_{12}^3 \\
1 & x_{13} & y_{13} & x_{13}^2 & y_{13}^2 & x_{13} y_{13} & x_{13}{}^2 y_{13} & x_{13} y_{13}^2 & x_{13}^3 & y_{13}^3 \\
1 & x_{14} & y_{14} & x_{14}^2 & y_{14}^2 & x_{14} y_{14} & x_{14}{}^2 y_{14} & x_{14} y_{14}^2 & x_{14}^3 & y_{14}^3 \\
1 & x_{15} & y_{15} & x_{15}^2 & y_{15}^2 & x_{15} y_{15} & x_{15}{}^2 y_{15} & x_{15} y_{15}^2 & x_{15}^3 & y_{15}^3 \\
1 & x_{16} & y_{16} & x_{16}^2 & y_{16}^2 & x_{16} y_{16} & x_{16}{}^2 y_{16} & x_{16} y_{16}^2 & x_{16}^3 & y_{16}^3 \\
1 & x_{17} & y_{17} & x_{17}^2 & y_{17}^2 & x_{17} y_{17} & x_{17}{}^2 y_{17} & x_{17} y_{17}^2 & x_{17}^3 & y_{17}^3 \\
1 & x_{18} & y_{18} & x_{18}^2 & y_{18}^2 & x_{18} y_{18} & x_{18}{}^2 y_{18} & x_{18} y_{18}^2 & x_{18}^3 & y_{18}^3 \\
1 & x_{19} & y_{19} & x_{19}^2 & y_{19}^2 & x_{19} y_{19} & x_{19}{}^2 y_{19} & x_{19} y_{19}^2 & x_{19}^3 & y_{19}^3 \\
1 & x_{20} & y_{20} & x_{20}^2 & y_{20}^2 & x_{20} y_{20} & x_{20}{}^2 y_{20} & x_{20} y_{20}^2 & x_{20}^3 & y_{20}^3 \\
1 & x_{21} & y_{21} & x_{21}^2 & y_{21}^2 & x_{21} y_{21} & x_{21}{}^2 y_{21} & x_{21} y_{21}^2 & x_{21}^3 & y_{21}^3 \\
1 & x_{22} & y_{22} & x_{22}^2 & y_{22}^2 & x_{22} y_{22} & x_{22}{}^2 y_{22} & x_{22} y_{22}^2 & x_{22}^3 & y_{22}^3 \\
1 & x_{23} & y_{23} & x_{23}^2 & y_{23}^2 & x_{23} y_{23} & x_{23}{}^2 y_{23} & x_{23} y_{23}^2 & x_{23}^3 & y_{23}^3 \\
1 & x_{24} & y_{24} & x_{24}^2 & y_{24}^2 & x_{24} y_{24_1} & x_{24}{}^2 y_{24} & x_{24} y_{24}^2 & x_{24}^3 & y_{24}^3
\end{pmatrix}
\begin{pmatrix} a_o \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{pmatrix}
=
\begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \\ N_8 \\ N_9 \\ N_{10} \\ N_{11} \\ N_{12} \\ N_{13} \\ N_{14} \\ N_{15} \\ N_{16} \\ N_{17} \\ N_{18} \\ N_{19} \\ N_{20} \\ N_{21} \\ N_{22} \\ N_{23} \\ N_{24} \end{pmatrix} \qquad (5.8)$$

5.2 Geometric Geoid Program

After the computation of the model coefficients, the excel spreadsheet was used to develop a program to compute the geoid undulation (N) at various points of interest within the FCT. The position of points determined from GPS observations (e,n) as well as the ellipsoidal height (h) is entered appropriately to compute the orthometric heights of those points. Appendix 3 and 4 show the results for geometric geoid models (multiquadratic and bicubic), the input-output window and processing window respectively.

5.3 Products from Study

After the post-processing of the raw data, position coordinates are determined and geoid development by solving for the model equation parameters was embarked upon. Developing programin Microsoft excel 2010 for geometric geoid undulation determination and orthometric height computation was carried out,then products in form of i) digital elevation models (DEM),ii) geoidal maps (from geoid undulations computed for multiquadratic,bicubic) ,iii) contour maps for models using orthometric heights (computed from multiquadratic , bicubic , and existing msl heights).

5.3.1 Digital Elevation Models

This is a 3D raster representation of terrain's continuos surface created from the elevation data for existing msl heights, models (multiquadratic and bicubic) orthometric height. This was used to quantify the land surface characteristics over an area. DEMs are very important for hydrologic studies, civil engineering, and urban planning among others and for 3D visualisations. The accuracy of DEM is critical to spatial analysis and applications but previous studies by Shi *et al*. (2005) confirmed that accuracy of generated DEM is the same for both multiquadratic and bicubic models and more accurate than linear/bilinear interpolation models. The resulting maps are shown in figure 5.1 to figure 5.3

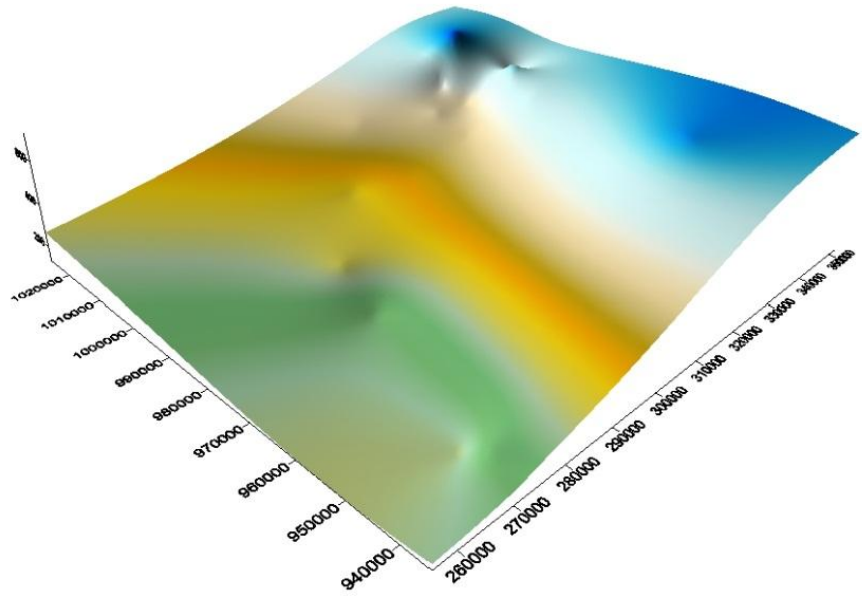


Figure 5.1: FCT DEM from Multiquadratic Model Orthometric Height

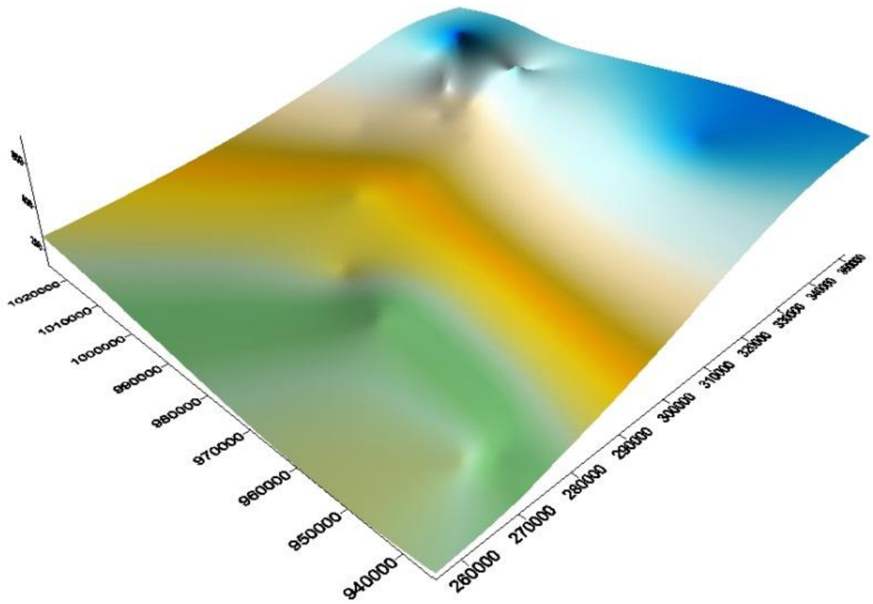


Figure 5.2: FCT DEM from bicubic model orthometric height

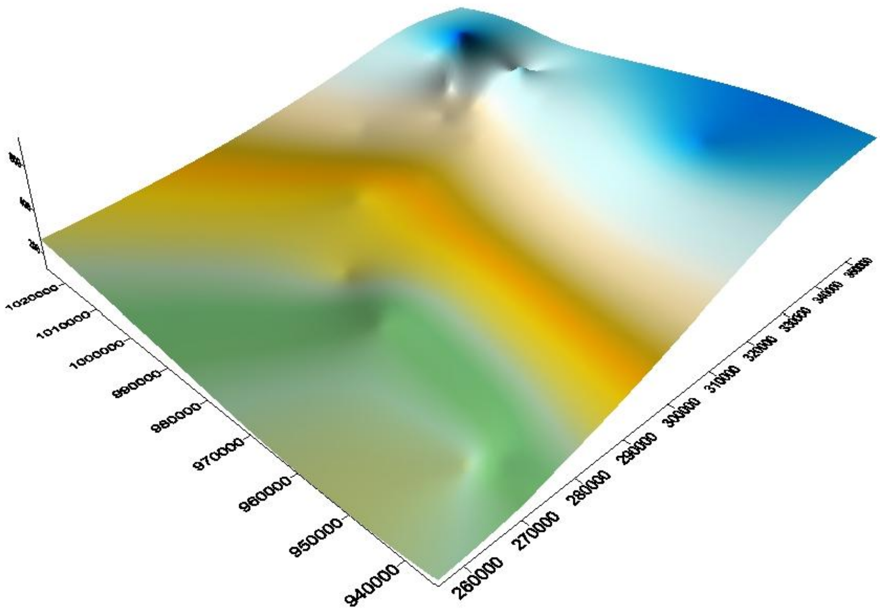


Figure 5.3: FCT DEM from Existing MSL Orthometric Height

5.3.2 Geoidal Maps

This is done by replacing the height (H) value of each point by the geoid undulations (N) and using kriging interpolation software to plot surface. It has been said that kriging produces surfaces very close to original surfaces. The results are shown in figure 5.4 to figure 5.5.

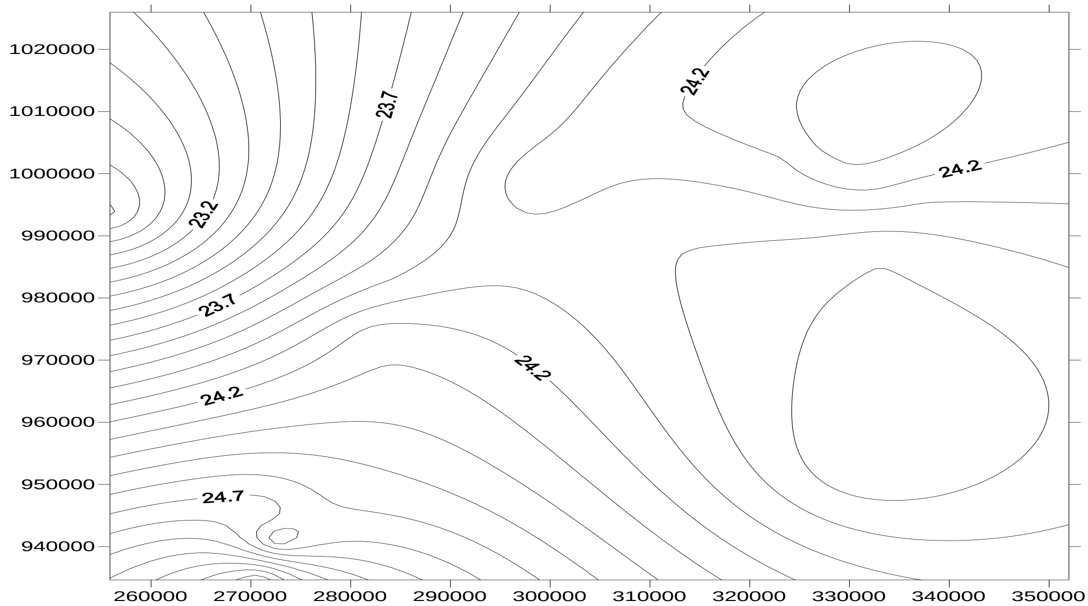


Figure 5.4**:** Multiquadratic Geoid Height Map

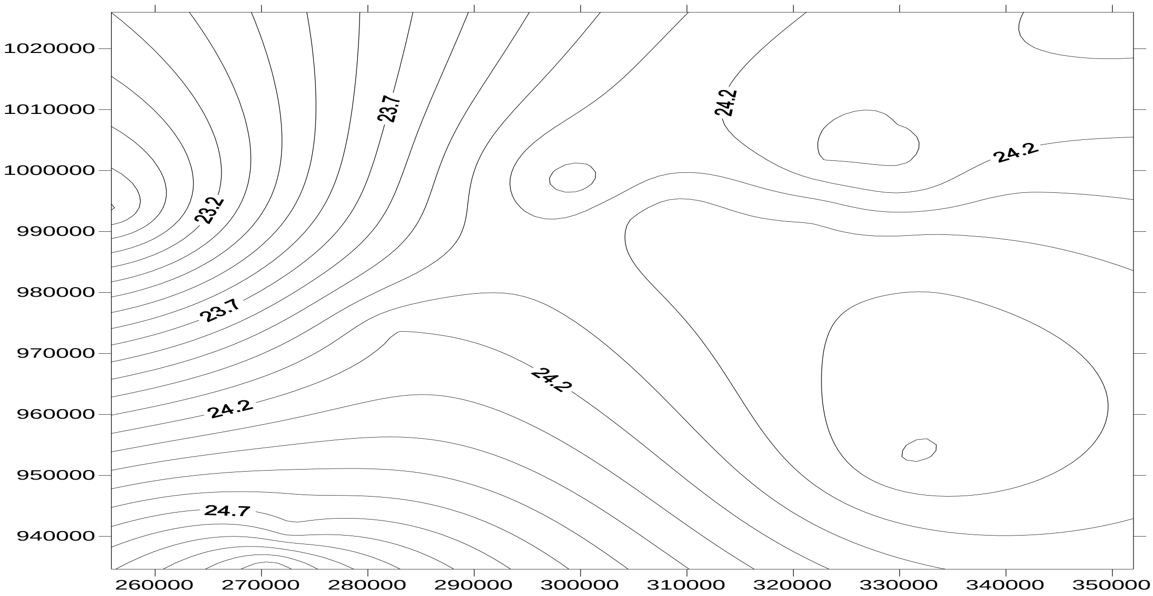


Figure 5.5: Bicubic Geoid Height Map.

The plotting of geoid heights from both multiquadratic and bicubic geoid models resulted into identical geoid surfaces.

5.3.3 Application areas of the geoid model

a) For GIS applications in landuse/landcover change studies in FCT as well as in precision farming for investigating development scenarios.

b) They are used for transforming GPS ellipsoidal heights (h) to orthometric height for practical surveying and engineering applications

c) The geoid is important in National Geodetic Data Infrastructure (NGDI).

d) The map can also be used to interpolate for geoid heights at any point of interest in FCT.

e) This is useful where conventional methods of spirit levelling is costly, tedious, time consuming and costly.

f) For production of large scale maps for different applications in GIS, mapping, cadastral, engineering and environmental studies.

The importance of the generated geoid models in orthometric height are:-

a) Consistency and compatibility with GNSS technique is achieved with these models for orthometric height determination.
b) Orthometric heights can be generated for all points of interest within the FCT.
c) The models can be used to generate topographical maps at 1m contour interval for civilian, surveying and mapping as well as engineering/environmental applications.
d) Geoid model defines a vertical datum continuos surface for orthometric heighting in FCT.

The above discussion of application and importance areas generally refers to both models and their products.

5.3.4 Contour Maps

The orthometric heights computed from the two models, and the existing msl orthometric heights are used to produce contour maps by kriging interpolation software. The results are shown in figure 5.6 to figure 5.8.

a) Contour plans of developed models and existing orthometric height.

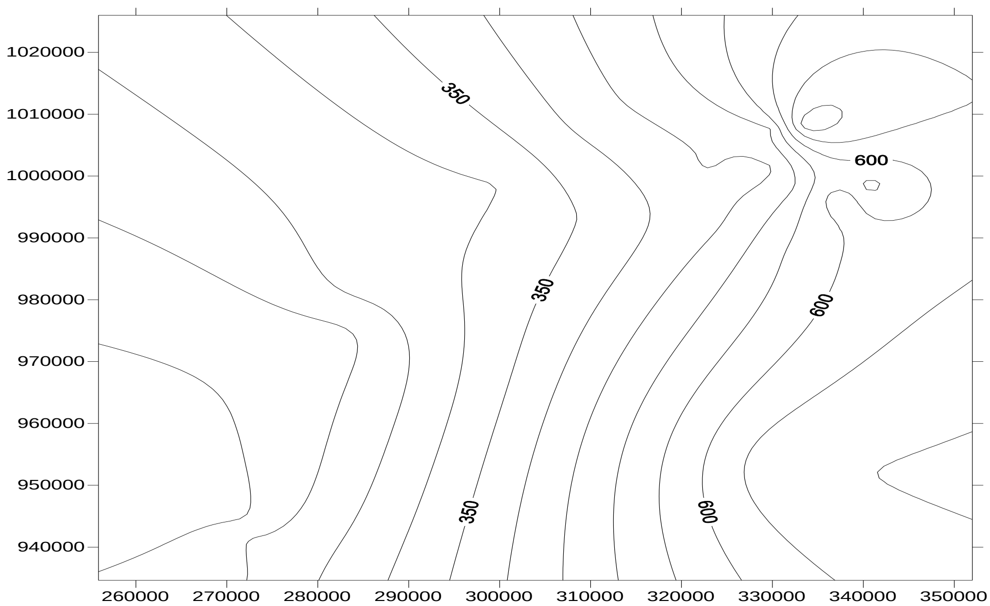


Figure 5.6 :Multiquadratic Orthometric Height Map

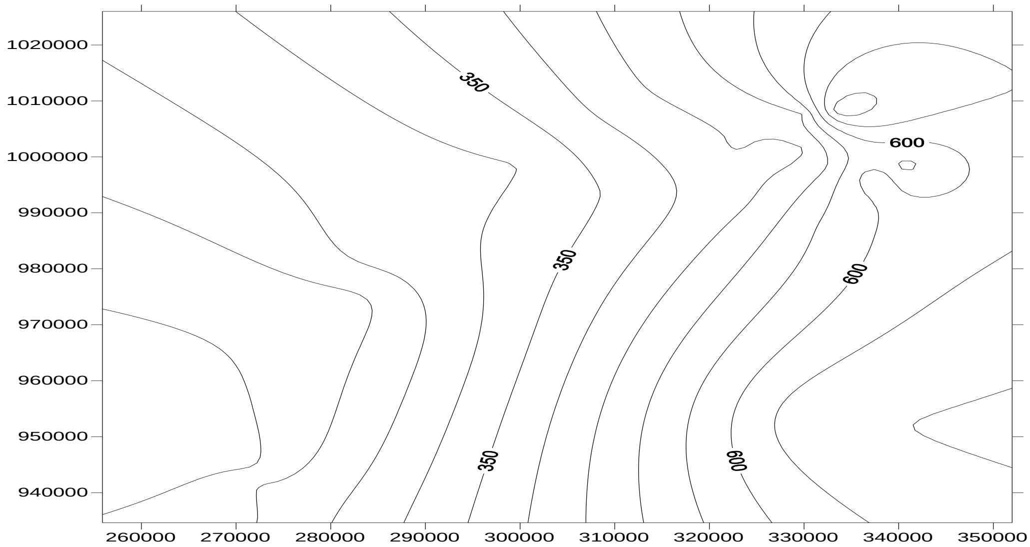


Figure 5.7**:** Bicubic Orthometric Height Map

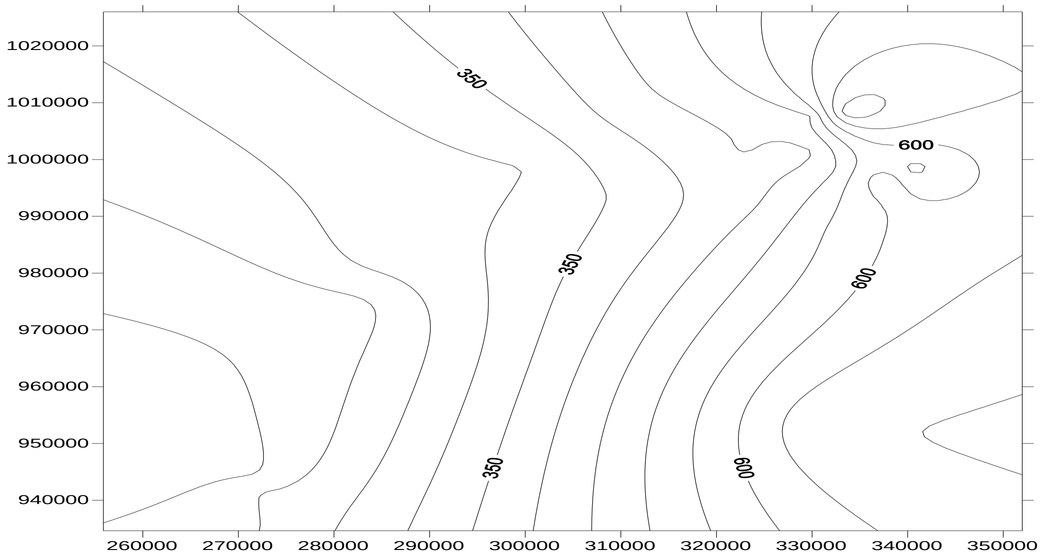


Figure 5.8**:** Existing Orthometric Height Map

Contour maps of existing orthometric heights and from the two developed models were plotted and compared (Figure 5.6 to 5.8). it can be seen that the contour maps resulted into identical shapes which is also an indication of the agreement or fit of the the developed models with the terrain of study area.

5.3.5 Plot of geoid undulation against controls

To show the differences between the multi-qudratic and bi-cubic models ,a plot of the geoid undulation values are plotted against the controls and shown in figure 5.9.

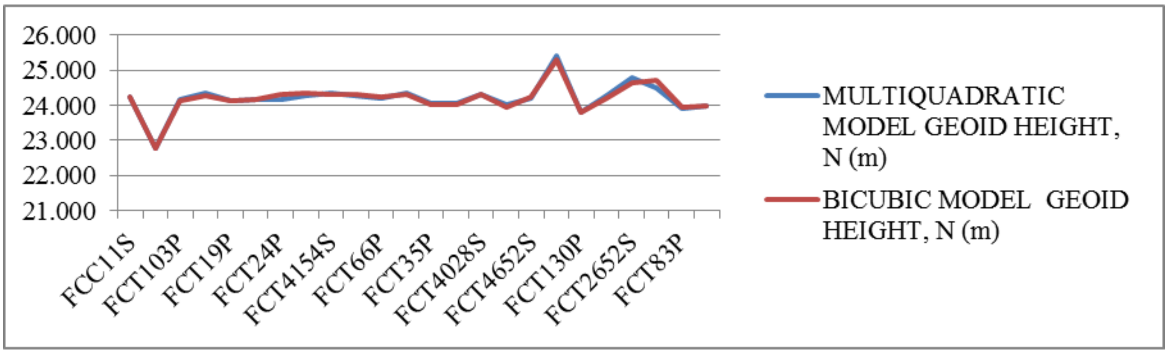


Figure 5.9: geoid undulation models from both multiquadratic and bicubic models.

It can be stated that the two surfaces are nearly coincident from a visual inspection of figure 5.9. It clearly implies and confirms the interchangeability and acceptability of the two models for orthometric height determination within the FCT.

5.3.6 (ASPRS 1993) Speifications for Topographical Survey

This is to use the standard deviation of the models to check against the specifications given by American Society of Photogrammetry and Remote Sensing (ASPRS 1993) in Table 5.1 as a guide to the expected application area of each model.

Table 5.1: ASPRS topographic elevation accuracy requirement for well-defined points

Contour Interval (m)	Class I (m) High Accuracy/ Standard Deviation Accuracy	Class II (m) Lower than Class 1 Accuracy Standard Deviation	Class III (m) Lower than Class 11 Accuracy Standard Deviation
0.5	0.08	0.16	0.25
1.0	0.17	0.33	0.5
2.0	0.33	0.67	1.0
4.0	0.67	1.33	2.0
5.0	0.83	1.67	2.5

Source: American Society of Photogrammetry and Remote Sensing (ASPRS 1993)

From Table 5.1, it is seen that multi-quadratic and bi-cubic models with σ=11cm and 14cm respectively, checked against the specification above, can be used to produce topographical plan of 1m contour interval for accurate survey, engineering and environmental studies and preparation of engineering drawing plans but inadequate for survey microgeodetic applications where a high accuracy is required.

5.4 Discussion of Results

The results will be discussed and analyzed to discover if aim of the research has been substantially realized or fulfilled.

5.4.1 Documentation of differences between models and existing height

Table 5.2 contains the differences between existing orthometric heights and developed model (multiquadratic and bicubic) orthometric heights of the controls.

The orthometric heights of controls from the two developed models (program) were compared with existing controls as shown in Table 5.2. This shows the fit of the two surfaces to the study area.

The standard deviations (**σ**) of the two surfaces were also determined as shown in Table 5.6. The σ for mutiquadratic = 0.126m and bicubic is = 0.170m which also shows fit of models to the terrain of the study area.

In comparing the two accuracy values, the multiquadratic model fits the study area better than bicubic model but both can be applied in the study area for orthometric height determination to accuracy of 13cm.

5.4.2 Acceptability and validity of results

From the results obtained in this study, and considering the the standard deviation of the models, the evaluation of the two surfaces, the check of standard deviation against ASPRS (1993) topographical specification,bias , skillsand diagnostic tests and the hypothesis test have all combined together to confirm the acceptability of the procedure ,processing and results. The details are given below:-

For standard deviation (**σ**), from Table 5.2

σ= 11cm for multiquadratic model

σ=14cm for bicubic model

a) For **bias and skill** computed values values

Multiquadratic bias = 0.008166667 implies unbias and skill = 1 implies high predictive ability

Bicubic bias= -0.012708333 implies unbias and skill =1 implies high predictive ability

b) Validation

i) For validity, the diagnostic tests show that the models are valid at 95% confidence limit since the values computed meet the conditions $1.98/\sqrt{N} < N\sum_{1}^{N} a_i^{2}$

0.404 is less than 581.397 for multiquadratic model

0.404 is less than 564.020 for bicubic model

Chi (χ^2) squares test at 95% confidence limits are confirmed by

0.404 is less than 24.996 for multiquadratic model

0.404 is less than 23.685 for bicubic model

ii) Internal validation.

Furthermore, internal validation of geometric geoid models was done to determine how closely the models predict orthometric height of controls used in this study. The differences between the existing orthometric heights and the modelled orthometric heights are shown in Table 5.2.

From table 4.10 and 5.2, the smaller the the standard deviation (**σ)** the better the developed models perform. The multi quadratic model with **σ =11cm** is better than **σ =14cm** for bicubic**.**

Multiquadratic model takes good care of lack of homogeneous distribution of selected controls in geoid modelling observed Doganalp and Sevi (2015) and as well has the capacity to generate precise geoid model. Thus multiquadratic model is very appropriate to non homogeneous distribution of controls encountered in this study for geoid modelling.

Table 5.2: Comparison between Multiquadratic Model, Bicubic Model and Existing Orthometric Heights

STATION	MULTIQUADRATIC MODEL ORTHOMETRIC HEIGHT, H (m)	BICUBIC MODEL ORTHOMETRIC HEIGHT, H (m)	EXISTING ORTHOMETRIC HEIGHT, H (m)	DIFF B/W EXISTING AND MULTIQUADRATIC MODEL ORTHOMETRIC HEIGHTS (A)	DIFF B/W EXISTING AND BICUBIC MODEL ORTHOMETRIC HEIGHTS (B)
FCC11S	485.161	485.149	485.447	0.285841605	0.298482551
FCT260P	201.963	201.956	201.944	0.01911206	0.012296746
FCT103P	532.681	532.710	532.558	0.122675646	0.151970694
FCT12P	735.826	735.913	735.707	0.119230612	0.205745694
FCT19P	635.703	635.704	635.644	0.059281596	0.060457902
FCT2168S	431.087	431.097	431.087	0.000472599	0.009592057
FCT24P	453.807	453.684	453.804	0.002586221	0.119698876
FCT276P	625.580	625.506	625.572	0.008313477	0.066451743
FCT4154S	476.896	476.911	476.981	0.08526864	0.069715646
FCT4159S	452.269	452.228	452.230	0.039248531	0.001560435
FCT66P	296.925	296.858	297.111	0.185538642	0.252681189
FCT9P	497.334	497.394	497.253	0.081473227	.0.140784399
FCT35P	427.252	427.276	427.171	0.080509242	0.105236756
FCT57P	323.747	323.768	323.844	0.096904403	0.075769114
FCT4028S	449.642	449.635	449.592	0.049988498	0.043316085
FCT53P	351.944	352.011	351.943	0.000778872	0.068313577
FCT4652S	462.916	462.876	462.711	0.204730669	0.165304325
FCT162P	189.694	189.789	189.696	0.001755336	0.092805825
FCT130P	695.579	695.596	695.608	0.02890435	0.011997291
FCT2327S	183.221	183.283	183.287	0.066385996	0.003844076
FCT2652S	138.960	139.091	138.952	0.007916644	0.139298723
FCT2656S	204.715	204.503	204.724	0.009012951	0.221467692
FCT83P	568.910	568.872	568.752	0.157723116	0.120082336
XP382	274.399	274.401	274.586	0.186634409	0.184536082

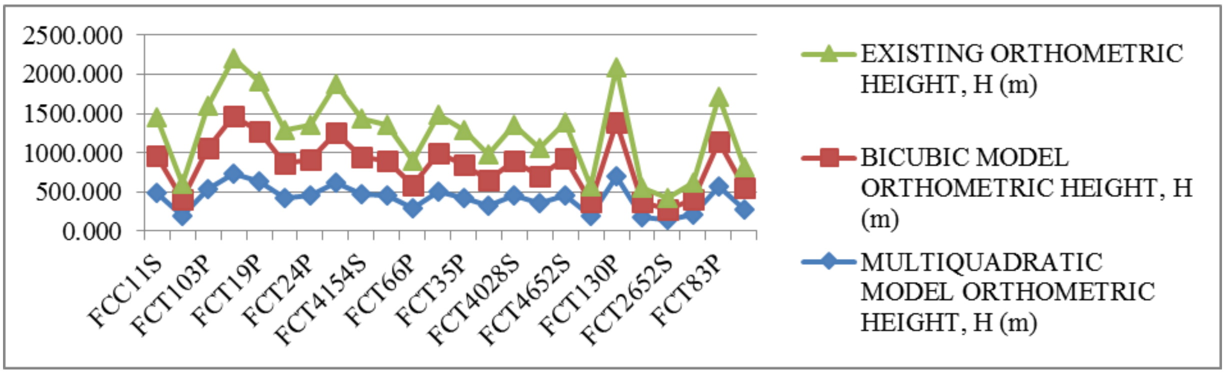


Figure 5.10: Plot showing similarity of the three surfaces

Figure 5.10 shows the similarity of surfaces (multiquadratic, bicubic and existing orthometric heights) and visually confirming the status of multiquadratic model as most suitable and adequate for modelling orthometric heights in FCT by GNSS technique.

d) **Coefficient of correlation (R) and coefficient of Determination (R^2)**

Values of **R** = 0.995m and R^2=99% respectively indicate the multi-quadratic model has a high predictive ability at 95% confidence limits.

As a summary therefore, the geometric geoid model approach using differential GPS relative technique had been adopted by several researchers both locally and internationally with acceptable results for various purposes over varying sizes of study areas by polynomial interpolation models. Abdullah (2010) reported 3.5cm to 10.8cm in their investigation from various models in Malaysia. Odera *et al*. (2014) also used biquadratic model in Kenya and reported 1cm accuracy over the study area. Aina (2014) also adopted the polynomial model for geoid modelling of Federal University of Technology of Minna, and Nwilo *et al.* (2014) also adopted polynomial interpolation for geoid modelling in Lagos over a small study area. Common to all the above studies was the use of one base reference station and mostly by RTK. This study however, adopted dual base refence stations on primary controls and static 2 hours observations and online post-processing with two polynomial models (multiquadratic and bicubic) for geoid modelling to produce orthometric height of F.C.T. Abuja with accuracy of 11cm adequate for engineering/environmental studies , mapping and day to day geospatial data acquisitions replacing or complementing the levelling technique.

Martensson (2002) recommended the use of network that resembles a triangulation network in GPS campaigns where the aim is to obtain surface cover for geometric geoid modeling to ensure that no deterioration of results are experienced and hence it can be stated that the results from this study is highly stable and consistent since the FCT trangulation network was used for all measurements.

CHAPTER SIX

CONTRIBUTION TO KNOWLEDGE

6.1 Introduction

For this research and from a review of existing knowledge of previous works on modeling of orthometric heights, the following outlines our contributions to knowledge.

6.2 Contibutions to Knowledge

i) An approach for modelling of orthometric height from Multi-Networks of GNSS/Precise levelling was developed for F.C.T., Abuja, Nigeria.

ii) The adopted (dual) base reference stations methodology used along with the rover control stations to form triangular shape/geometry has been proved to be most stable 2D polygon shape from Grashof's law and believed to imply stability of data/results from this adopted field procedure is a contribution to the methodology of field procedures.

iii) The accuracy value indicator (σ= 11cm) imply that the multquadratic model works better than bicubic model (σ= 15cm) especially when there is lopsidedness or lack of homogeneous distribution of controls involved in the development of the geoid models.This has also confirmed previous studies that came to the same conclusions.

iv) From the statistical indicators computed, no higher polynomial surface than multiquadratic model is needed for modeling geoid surface in FCT. Both multiquadratic and bicubic surfaces can be reliably and interchangeably be used for modelling of orthometric heights for geospatial data acquisitions by GNSS user community and is also a contribution to knowledge.

v) $N = a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy + a_6x^2y + a_7xy^2 + a_8x^2y^2$ is the model developed for geometric geoid of FCT to model orthometric heights and provide reference materials for feature researches and researchers and is also a contribution to knowledge.

CHAPTER SEVEN

CONCLUSION AND RECCOMENDATION

7.1 Introduction

This section is where the findings of the research with conclusions and recommendatios arising are clearly stated. Limitations are also highlighted with suggestions for further studies.

7.2 Summary of Research Findings

Following are our findings:

Polynomial surfaces can be used to model geoid undulation over a defined area if high quality orthometric heights are available within the territory can be collocated to obtain GNSS ellipsoidal height in the absence of gravity data.

Statistical evaluation revealed that no higher surface than multiquadratic is necessary for geoid modeling in the FCT.

Dual base reference stations were actually adopted for data acquisition instead of conventional single base reference station. This enabled stability of results by exploiting Grasshof's law of stability of polygons.The a_0 term of the coefficient of the geoid model equations should be close to the observed geoid undulations.

Conversion programme was developed in Microsoft excel format to interpolate the geoid undulation (N) and orthometric heights (H) from both multiquadratic and bicubic models just by copying the positions (e,n,h) from post processed results and pasting into the appropriate column of the geometric geoid program.

Centroid position $(X_0, Y_0)$of the study area was used to determine the absolute $(X-X_0)$ and $(Y-Y_0)$ as normalized coordinates of each point of interest which was later substituted into X, Y in the determination ($a_{0,}$ a_1,…,a_9) of the coefficients of the polynomial surfaces especially when the significant figures are high/large(20 decimal places were used in this calculation).

Diagnostic tests show that the models are satisfactory and valid at 95% confidence limits for modelling of orthomertric height within the FCT.

7.3 Conclusion

The research is on the modeling of othometric heights using GNSS leveling in FCT which in every marterial fact tantamonuts to geoid modeling. From the study, the following can be concluded:

i) Practically, this research leads to consistency of orthometric heights at the centimeter level over FCT for geospatial data acquisitions if the developed conversion geometric geoid model programis adopted by surveyors and engineers in geospatial applications inmapping, engineering, hydraulics, surveying and environmental applications.

ii) From the centimter level of accuracy of the geometric geoid, geomaticians and other users' of height data may sieze the opportunity of the developed models to acqiure height related to geoid instead of GNSS global geoid model presently used to acquire orthometric heights for their various applicationsguaranteeingglobal continuity and compatibility at borders.

iii) The feasibility of developing geoid model for GPSusers community by GNSS/Levelling in FCT has been demonstrated as an alternative approach to conventional spirit leveling in orthometric height determination

iv) The use of dual–based reference stations taking advantage of stability of triangles technique was adopted/exploited against the conventional single base reference approach which has no mechanism in place to ensure reliable results are obtained.The stability of the adopted dual base reference (primary) stations and one rover station was computed to be 0 and hence geometrical stability and quality of results is not in doubt.GNSS surveys based on primary controls in static mode will ensure reliability.

v) Geoid modelling could be taken as feasible alternative to conventional levelling.Geoid-based datum would be compatible with Global Navigation Satellite Systems across all locationswithin FCT.

vi) Both multquadratic and bicubic models can be used to determine orthometric heights though the multiquadratic has a better accuracy and is the optimummodel in this study.

vii) Third order leveling accuracy in Nigeria is given by 24mm√ K km.For a distance of 100km, accuracy=24cm whereas the model will give 11cm.This means GNSS/Levelling can actually replace the conventional leveling within the FCT over large areas or long distances and provide results within the shortest time possible.

viii) Propagation of errors, lack of accessibility to hinterlands, cost of conducting operations for orthometric height determination using the conventional approach can practically lead to the adoptionof this model to minimize the ever present economic reality and lack of political to fund surveying projects.

ix) FCT geoid model has realized an accurate and homogeneous vertical reference surface across the study area and hence compatible with GPS method.

x) With the development of the FCT geometric geoid model, a user has the choice of continue with global geoid models (with low reliability in orthometric heights) for height determination for local applications or resort to the local geoid for accurate and practical solutions in geospatial data acquisitions using GPS.

xi) The level of accuracy derived from these models can lead to production of large scale contour maps at 1m contour intervals for various applications based on GNSS technology.

7.4 Recommendations

From the study, the followings can be recommended:

i. The use of EGM2008 with a standard deviation of about 1m in orthometric height acquisition within FCT has limitations in applications (not adequate enough for production of base topographical map for engineering and allied projects) and the developed multiquadratic geoid model is recommended for orthometric height determination at the centimeter level (σ=11cm) and generating contours at 1m contour intervals.

ii. Height modernization can be undertaken using this technique in the absence of a national geoid model at local government areas and state levels.

iii. This approach could also be replicated in other states and all data collected throughout the country combined to support development of a national geometric geoid model for GNSS user community in the country prior to production of national geoid model.

iv. This model should be used for applications in the production of topographical base maps for engineering and planning designs, for water distribution projects, cadastral and other mapping and environmental applications.

7.5 Limitations

The developed geoid models are limited to the FCT only.

Though millimeter accuracy may not be achieved with these developed models (especially for microgeodetic applications), it has created an alternative to conventional leveling for orthometric height determination for daily needs in surveying, engineering and environmental projects.

7.6 Suggestion for Further Studies

Further studies should be to integrate gravity model (data) observed on at least four base reference stations used in this study to improve the accuracy of the local geoid models.

More controls (including benchmarks) can be collocated and other polynomial surfaces tested/evaluated.

Combined online and offline post processing ellipsoidal height should be used for geoid model development.

Controls located on the ground (e.g. FCT260P) should be used for base reference stations.

Evaluation of water distribution network design in an existing estate based on existing elevation map may be compared with the base map resulting from these models to determine efficiency of water flows.

REFERENCES

Abdallah, A. (2009). Determination of gravimetric geoid model of Sudan using the KTH Method, M.Sc. Thesis, Royal Institute of Technology (KTH), Sweden.

Abdallah, A.K. (2010). Height determination using GPS data, local geoid and global geopotential models, Universiti Teknologi Malaysia Institutional Repository, EPrints3

Abdulkhadum, A.J. (2015). A comparison of national orthometric heights with DGPS/Leveling and ITRF 00 datum with WGS 84 (EGM2008) geoid, *International Journal of Advancements in Research and Technology, Vol.4*, Issue 11, November 2015.

Abeho, D. R., Hipkin R. & Bedada, B.T. (2014). Evaluation of EGM 2008 by means of GPS leveling in Uganda, *South African Journal of Geomatics, Vol. 3, No. 3* November 2014.

Adam, J. M. (2004). Which geoid model should be used for GPS heighting on the Toowoomba bypass project? Unpublished Dissertation, Faculty of Engineering and Surveying, University of Southern Queensland.

Ahmed, E., M. (2013). GPS ellipsoid height calibration to determine the approximate mean sea level (orthometric) height ,*International Journal of Advanced Research in Engineering and Applied Science, Vol.2 ,No. 8*, August 2013,pp.1- 11.

Aina,V.O.K. (2014).Geoid modeling using Global Positioning System/Leveling method of Federal University of Technology Minna,Gidan Kwano Campus, UnpublishedM.Sc thesis, Department of Surveying and Geoinformatics, Federal University of Technology, Minna, Niger State.

Alevizakolu, E &Lambrou, E. (2011). Fast and convenient determination of geoid undulation (N)in urban area, FIG working week 2011,Marrakech,Morroco,18-22 May 2011.

Angus-Leppan, P.V. (1982). The required frequency of gravity measrements for level reduction, Aust.J.Geod.photogramm.Surv. 36, 65-82.

Amos, M.J. (2007). Geoid modeling in New Zealand to unifymultiple local vertical datum, PhD. Thesis, Curtin University of Technology, Australia.

Amos, M. J& Featherstone, W, E. (2003). Preparations for a new gravity geoid of New Zealand and some Preliminary Results ,New Zealand Surveyor, No. 293 ,June 2003.

ASPRS (1993). American Society of Photogrammetry and Remote Sensing (ASPRS 1993).

Arinola, L. (2006). Error propagation in the Nigerian geodetic network: Imperative of GPS observations to strengthen network, XXIII International FIG Congress, Munich, Germany, October 8-13, 2006.

Avery, E.T& Berlin, G.L. (1992).*Fundamentals of Remote Sensing and Airphoto Interpretation*, 5th ed., Prentice Hall, Inc.

Avstar, N.B., Kutoglu, S.H., & Erol, B. (2013). GNSS/Levelling data in assessing the fit of the recent global geoid in Turkey, *International Symposium on Global*

Navigation Satellite Systems(ISGNSS), YTU Davutpasa Congress Center, 22-25 october 2013.

Awange, J.L., Grafarend, E.W., Palanez, B& Zaletnyik, P. (2010).*Algebraic Geodesy and Geoinformatics,* Springer Science and Business Media, 2010.

Ayhan, M, E. (1993). Geoid determination in Turkey (TG-91), jounal of Geodesy, 67 (1):10-22. March 1993, DOI: 10.1007/BF00807293.

Badejo, O. T., Aleem, K.F.A. & Olaleye, J.B. (2016).Replacing orthometric heights with ellipsoidal heights in engineering surveys, *Nigerian Journal of Technology, Vol.35, No.4, October 2016, pp.761 – 768, Faculty of Engineering, University of Nigeria, Nsukka.*

Balasubraminia, N. (1994). Definition and realization of a global vertical datum, Scientific Report No.1, Department of Geodetic Science, the Ohio State University, Columbus, OH 43210.

Bello, S.I.T. (1977). "Establishment of height datum for Nigeria", B.Sc project Report, Dept.of Surveying, University of Lagos, Lagos, June.

Bjelotomic, O & Basic, T. (2016).Initial consideration on modernisation of the Croatian height reference system, *SIG- International Symposium on Engineering Geodesy, 20 – 22 May 2016, Varazdin, Croatia.*

Chicaiza, E.G., Leiva, C.A., Arranz, J.J., & Buenano, X.E. (2017).Spatial uncertainity of a geoid undulation model in Guayaquil,Ecuador, De Gruyter Open, Open Geosci.2017; 9:255-265.Downloaded on 22ndJuly 2017.DOI 10.1515/geo-2017.0021

Clynch, J. R (2006): Earth Coordinates

Dawod, G. (2008). Towards the redefinition of the Egyptian geoid: performance analysis of recent global geoid and digital terrain models, *Spatial Science, Vol. 54, No. 1, June 2008.*

Dodo, J.D & Idowu, T.O. (2010). A framework for the implementation of the Global Navigation Satellite System (GNSS) in Nigeria, *the Proceedings of the Nigerian Union of Planetary and Radio Sciences (NUPRS) Conference 2010.*

Doganalp, S. & Selvi, H.Z. (2015). Local geoid determination in strip area projects by using polynomials, least-square collocation and radial basis functions. Measurement, 73,429=438. DOI: 10.1016/J.

Dumrongchai, F., Whichienchareon, C., & Promtong, C (2012). Local geoid modeling for Thailand, *International Journal of Geoinformatics. Vol.8, No. 4, December 2012.*

Eckl, M.C., Snay, R.A., Soler, T., Cline, M.W. & Mader, G.L. (2001): Accuracy of GPS – derived relative positions as a function of interstation distance and observing-session duration. *Journal of Geodesy, 75,633-640.*

Edan, J.D., Idowu, T.O., Abubakar, T. and Aliyu, M.R. (2014). Determination of orthometric heights from GPS and Levelling Data , *International Journal of Electronics,Communication & Instrumentation Engineering Research and*

Development (IJECIERD) ISSN(P):2249-684X; ISSN(E):2249-7951 Vol. 4,Issue 1,Feb 2014,123-134 .

Epuh, E.E., Olaleye, J.B. & Omogunloye, (2016). Gongola basin geoid determination using isostatic models and seismic reflection data and geophysical interpretation, Department of Surveying and Geoinformatics, University of Lagos, Nigeria.

Erol, B & Celik, R, N. (2010). Modeling local GPS/Leveling geoid with the assessment of inverse distance weighting and geostatistical kriging methods, Commission IV, WG IV/1.

Ezeigbo, C. U. & Adisa, A.A. (1980). "Gravity values and the Nation needs", The Map Maker, *Journal of Nigerian Inststution of Surveyors, Vol. 6, No.2, pp.45 – 55.*

Ezeigbo, C. U. (1990): Nigerian Geodetic Networks – The control Question, The Map Maker, *Journal of Nigerian Institution of Surveyors*, Vol.10, No.1, pp.82 – 86.

Ezeigbo, C. U. (1985). Definition of Nigerian geodetic datum, *Proceedings of International Symposium on the Definition of Nigerian Geodetic Datum, Nigerian Association of Geodesy*, pp.110-143.

Ezeigbo, C.U, Fajemirokun, F.A, & Nwilo, P.C. (2006). "Determination of an optimum geoid for Nigeria", A research Report Submitted to National Space Research and Development Agency (NASRDA), Federal Ministry of Science and Technology, Abuja.

Ezeigbo, C.U., Fajemirokun, F. A., Nwilo, P. C., Ayodele, E. G & Oyewusi, A.M. (2007). Analysis of some factors that affect the accuracy of a gravimetric geoid determination in Nigeria, Department of Surveying &Geoinformatics, Faculty of Engineering, University of Lagos.

Fadahunsi, O. (1985). Status of geodetic computations at Federal Surveys , Definition of the Nigerian Geodetic Datum , *Proceedings of International Symposium on Definition of Nigerian Geodetic Datum,* May 15 – 17 , 1985.

Featherstone, W.E. & Kuhn, J. (1995).Quasigeoid modelling in New Zealand to unify multiple local vertical vertical datums,https://www.linz.govt.nz

Featherstone, W.E. (1995).On the use of Australian geodetic datums in gravity field determination, *Geomatics Research Australasia, 62:17 – 36.*

Featherstone, W.E (2000). "Comparison of different satellite altimeter- derived gravity anomally gridswith shipborne gravity data around Australia, Gravity and Geoid, pp. 326 – 331

Featherstone, W.E (2008). GNSS-based heighting in Australia: current, emerging and future issues, *Journal of Spatial Sciences Special issues on GNSS (in press)*, pp.115-133.

FieldGenius Technical Notes-GPS Terminology

Fotopoulos, G., Featherstone, W.E., & Sideris, M.G. (2003). Fitting a gravimetric geoid model to the Australian height datum via GPS data, Ziti publishing, Thessaloniki, Greece, pp. 173-178.

Fotopoulos, G., Kostakis, C. & Sideris, M.G. (1999). Evaluation of geoid models and their use in combined GPS/Leveling/Geoid Height network adjustments, Technical Reports Nr. 1999.4, TheUniversity of Calgary, Canada.

F tables: http://www.stat.ucla.edu/~dinov downloaded on25/1/18 by 12.17 am

GPS Guidebook (2004). Standards and guidelines for land surveying using gobal positioning system methods, Natural Resources Canada.

Grinter, T & Janssen, V. (2012). Post-processed precise point poisoning, a viable alternative.

Hassouna, R.M.K. (2014). Orthometric Correction for Trigonometric Leveled Heights, Journal Geomatics, Indian Society of Geomatics, Vol. 8, No.1, April 2014.

Heiskanen, W.A and Moritz, H. (1967). Physical Geodesy, New York. W.H. Freeman & Co., 364p.

Heiskanen, W.A. and Moritz, H (2006). Physical Geodesy. San Francisco, WH Freeman& Co.

Hipkin, R. (2002b). Defining the geoid by W=Uo, Theory and practice of a modern height system in gravity and geoid 2002, 3rd meeting of International Gravity and Geoid Commission. Thessaloniki, Greece. pp. 367-377.

Hipkin, R.G. (2000). Modeling the geoid and sea surface topography in coastal areas, Physics and Chemistry of the earth, Vol. 25, No. 1, pp. 9-16, doi:10.1016/ 51464-1895(00)00003-x.

Hoffman – Wellenhof, B., Lichtenegger, H. and Wasle, E. (2008). GNSS-Global Navigation Satellite Systems, Springer Wien, New York.

Hyo Jin Yang (2013). Geoid Determination based on a combination of terrestrial and airborne gravity data in South Korea, Published Ph.D. Dissertation, The Ohio State University.

Idrizi, B. (2013). Developing Model for Utilization of Global Earth Gravimetrical Models in Macedonia Territory, FIG working Week 2013.

Ihde, J., Liesbach, G., Sacher, M & Sanchez, L. (2010). Realization of a Global Unified Height System and its Advances for Hydrographic Survey and Coastal Mapping, HYDRO 2010, Rostock- Warnemunde, 02 – 05, November 2010, International Association of Geodesy.

Iliffe, J.C. (2000). Datums and Map Projections for Remote Sensing, GIS and Surveying, Caithness, Scotland, Whittle Publishing.

IPCC (2001b). "Changes in Sea Level" in Climatic Change 2001: The Scientific Basis, Contribution of Working Group 1 to the Third Assesment Report of the Intergovernmental Panel on Climatic Change, Cambridge University Press.

ISO (2003). International Organisation for Standardisation (19111) – Geographic Information Spatial referencing by Coordinates-ISO Copyright Office, Geneva ,Switzerland,pp. 1-43.

Isioye, O. A. & Fajemirokun, F.A. (2010). Current Trend in GNSS Based Heighting: Conditions and Limitations in Nigeria, Asian Journal of Engineering, Science & Technology Vol. 1, and Issue 2.

Jekeli, C. (2006). Geometric Reference Systems in Geodesy, Division of Geodesy and Geospatial Science, School of Earth Sciences, Ohio State University.

Jensen, T. (2011). Notes on physical Geodesy

Kamaludin, M.O., Sharum, S. & Azhair, M. (2005). Enhancement of Height System for Malaysia Using Space Technology, Faculty of Geoinformation Science and Engineering, University Teknologi, Malaysia.

Kamaluddin, T., Saiful, A., Jasmee, J, Mohd, Z. & Azhari, M. (2014). Absolute and Relative GPS Orthometric Heights using Regional Fitted Geoid, FIG Congress, Kuala Lumpur, Malaysia, 16-21 June 2014.

Kamguia,J.,Tabod,C.T.,Noujou,R.,Tadjou,J.W.,Mangelle-Dicom.E.,& Kaude,H.L. (2007).The Local Geoid Model of Cameroon:CGM05,Nordic Journal of Surveying and Real Estate Research 4:2(2007) 7-23.

Kasenda, A. (2009). Precision Geoid for Modernisation of Height Systems in Indonesia,Ph.D Thesis,Schol of Surveying and Spatial Information Systems, The University of New South Wales,Sydney, 2052 , Australia.

Kavzoglu, T. &Saka, H.M.(2005). Modelling GPS/LevellinG Geoid Undullation using Artificial Neural Networks, *Journal of Geodesy*, Vol. 78 (9), pp.520- 527.

Kearsley, A.H., Govind, R & Lutton, G (2007). The Distortion of the Australian Height Datum with respect to a global Geoid Presented at theIAG General Assembly at IUGG XXIV 2007, Pergia, Italy, July 2 – 13, 2007.

Kebie, B. D. (2013).A Thesis submitted to School of Graduate studies, in partial fulfillment of the requirement of degree of Master of Science in Civil Engineering, Department under Geodesy, Addis Ababa University Institute of Technology.

Kenyon, S., Factor, J., Pavlis, N. & Holmes, S (2007). Towards the next earth gravitational model, Paper presented at the society of Exploration Geophysicists 77th Annual Meeting, San Antonio, Texas, USA, September 23 – 28, 2007.

Khairul, A.A. (1994). Height Determination Using GPS Data, Local Geoid and Global Geopotential Models (GGM).

Kharaghani, G.A. (1987). Propagation of Refraction Errors in Trigonometric Height Traversing and Geodetic Leveling, Technical Report No. 132, Department of Surveying Engineering, University of New Brunswick, Fredericton, Canada, November 1987.

Kirici, U. & Sisiman, Y. (2017). The Comparison of the Adjustment Methods in Geoid Determination Method, FIG Working Week 2017

Klees, R. & Prutkin, I. (2010). The Combination of GNSS Leveling and Gravimetric Heights in the presence of noise, Journal of Geodesy, pp. 731-749.

Kogbe, C.A. (1989). An overview of the geology and hydrogeology of Nigeria. DOI:10.1201/9780203889497.ch11.

Komarov, R, V, Kascheev, R, A, & Zagretdinov, R, V. (2007). Geoid Determination by GPS /Leveling Method in the Republic of Tartasan, International Journal of Science, Georesources, 2(10)2007.

Kotsakis, C. & Sideris, M.G. (1999). On the adjustment of combined GPS/Levelling geoid networks.J.Geodesy, 73(8), 412-421.

Kumar, A. & Burke, M.K. (1998). Surface positioning for DP operations, Dynamic Positioning Conference.

Kumar, P, Kumar S.C, Shukla, G & Kumar, S. (2013). Assessment of Positional Accuracy of DGPS: A case study of Indian School of Mines Dhanbad, Jkarhand, India International Journal of Advances in Earth Sciences, volume 2, issue 1, 2013, pp. 1-7.

Kuntz, E. & Schmitt, G. (1986). Precise height determination by simultaneous zenith distances, the symposium on Height Determination and Recent Vertical Crustal Movements in Western Europe, Hannover, Germany, September 15-19.

Kreyszig, E (2006).Advanced engineering mathematics, 9th edition, Wiley International Edition.

Lamothe, P., Veronneau, M., Goadsby, M& Berg, R. (2013). Canada's New Vertical Datum, Ontario Proffessional Surveyor, Fall 2013, pp.24 – 28.

Lapucha, D & Maynard, K.L. (1992). Investigation of the Real-Time Accuracy of the DGPS Method, John E. Chance and Associates, Inc., Lafayette, LA 70506.

Leick, A (1995).*GPS Satellite Surveying*, Second Edition, John Wiley and Sons, Inc., New York, NY.

Manisa, M., Sas, R.K, Segobye, M. & Maphale, L. (2016). Developing Local Geoid Model to assess the Accuracyof Orthometric Heights in Botswana, *Spatial Inform. Res. Korean Spatial Inform.* Soc., Vol.24, Issue 5, 2016, pp.607 – 616, http://dx.doi: 10.1007/S41324-016-0057-3.

Madjanski, M, Kozlovskaya, E., Swieczak, M& Grad, M. (2009). Interpretation of Geoid Anomalies in the Contact Zone between the East European and the Paleazoic Platform. Estimation of Effects of density inhomogeneities in the crust on geoid undulations, Geophys J Int (2009) 177(2): 321 – 333.https://doi.org/10.1111/j.1365-246X.2008.

Martensson. S. G. (2002). Height Determination by GPS – Accuracy with respect to Different Geoid Models in Sweden, *FIG XXII International Congress, Washington*, 19 – 26 April 2002.

Merry, C. (2003). The African Geoid Project and its relevance to the Unification of African Vertical Reference Frames, in Proceedings of the 2nd FIG Regional Conference, Markerch, Morocco, December.

Merry, C. (2009).EGM 2008 Evaluation for Africa, Newton's Bulletin, Issue No.4, April 2009.

Meyer, T.H, Roman, D.R&Zilkoski, D.B (2006). "What does Height really means? Part 1V: GPS Orthometric Heighting", Department of Natural Resources and the Environment Articles, University of Connecticut.http://digitalcommons.uconn.edu/nrme_articles/5

Milbert, D.G & Smith, D.A. (1996). Converting GPS height into NAVD88 Elevation with GEOID 96 geoid height model, National Geodetic System, NOAA.

Moka, E.C. (2011).Requirements for the determination of orthomatric heights from GPS – determined heights, contemporary issues in surveying and Geoinformatics, BPrint Lagos

Moritz, H. (1965). The Boundary Value Problem of Physical Geodesy, Publication of Isostatic Institute, I.A.G. 83 Helsinki.

Moritz, H. (1967).*Physical Geodesy*, San Francisco, Freeman, pp.178-188.

Moritz, H (1980b).*Advanced Physical Geodesy*. Whichman Verlag, Karlsruhe

NASA SCIENCE (2002). Tick-Tock atomic clock. http://sciencenasa.gov

National Geodetic Survey (1986). Geodetic glossary.

NIMA (2004). Department of defense World Geodetic System 1984: Its definition and relationships with local geodetic systems, third edition with revisions, Technical Report TR8350.2, National Imagery and Mapping Agency, Maryland, USA. http://earthinfo. nga.mil/GandG/publications/tr8350.2/tr8350_2.html

Ning, Fang-Shii (2015). Using surface and buffer analysis to estimate regional geoid undulation, *Geod.Vol.21.No. 3, Curtiba*, July/Sept 2015.

Nordin, S.B (2009). Height modernisation using fitted geoid models and MyRTKNET, M.Sc Thesis, Faculty of Geoinformation Science and Engineering, Universiti Teknologi, Malaysia.

Novak, P, Vanicek, P, Veronneau, M, Holmes, S &Featherstone, W. (2012). On the accuracy of modified Stokes integration in high-frequency gravimetric geoid determination, department of geomatics engineering, The University of Calgary, Canada

Nwilo, P.C. (1995). Sea level variations and the impacts along the coastal areas of Nigeria, PhD thesis, university of Salford, Salford, United Kingdom.

Nwilo, P.C (2013): Technological advancement in surveying and mapping: The Nigerian Adaptation, *FIG Working Week*, Abuja Nigeria 2013.

Odera, P.A& Fukuda, Y (2015). Recovery of orthometric heights from ellipsoidal heights using offsets method, *Earth Planets and Space 67(1),* 1, 2015.

Odera, P.A., Musyoka, S.M& Gachari, M.K (2014). Practical application of the geometric geoid for heighting over Japan, Nairobi County and its environs, *JAGST Vol. 16 (2)2014*.

Odumosu, J.O., Ajayi, O.G., Idowu F.F & Adesina, E. A. (2015). Evaluation of the various orthometric height systems and the Nigerian scenario – A case study of Lagos State.

Odumosu, O.J, Idowu, F.F. & Okorocha, C.V. (2014). Selecting optimal data – fitting model for surveying and geodetic applications, FIG Congress 2014. pp. 1-12.

Ojigi,M.L (2011).Determination of suitable terrain surface modelling algoritms for hydraulic deasign of storm sewer,Contemporary issues in Surveying and Geoinformatics,pp.103 – 133, Bprint,Lagos.

Ojinaka, O.C. (2006). The Nigeria vertical datum and the influence of degenerate amphidrom, geodesy and national development, *the Proceedings of Nigerian Association of Geodesy Conference*, 23-25 August 2006.

Ojinaka, O.C. (2007). *Principles of hydrographic surveying from sextant to satellite*, El-Demak, Enugu, pp.11-17.

Okafor, U.Z. (1985). Nigerian geodetic heights and orthometric correction, Proceedings of *International Symposium on the Definition of Nigerian Geodetic Datum, May 15-17, 1985*, pp.144-170.

Okiwelu, A., Okwueze, E. & Ude, I.A. (2011). Determination of Nigerian geoid undulations from spherical harmonic analysis,DOI:.5539/apr.v3n1p8.

Ollikanen, M (1997) .Determination of orthometric heights using GPS leveling data , FIG Working Week 2009, Eilat, Israel, 3-8 May 2009.

Olliver, J.G, & Clendinuing, J. (1979).*Principles of Surveying*, Vol.2, Van Nostrand Reinhold Company, New York.

Ono, M.N. (2002) .On problems of coordinates, coordinate systems and transformation parameters in local map production, Updates and Revisions in Nigeria, FIG Working Week 2009, Eilat, Israel.

Ono, M.N., Igbokwe, J.I., Ojiakor, J. C. & Emengini, J.E (2004). Three Dimensional Integration Approach for Adjusting National Geodetic Network for Satellite Mapping, Nigerian Institution of Surveyors, Garden City Benin.

Ono, M.N. & Dodo, J.D. (2005). In support of handheld GPS for third order Control Densification and monitoring surveys, program for the *2nd International Conference of the Nigerian Union of Planetary and Radio Sciences*, Port Harcourt.

Ono, M.N. & Igbokwe, J.I. (2009).GIS-Based map production and revision: Which coordinates, coordinate system and transformation parameters? *Nigerian Journal of Surveying and Geoinformatics, Vol.2 Number 1, 2009.*

Onwuzuligbo, C. (2012). (2009).Realisation of geoid over Nnamdi Azikwe university campus, unpublished M, Sc thesis, dept. of surveying and Geoinformatics,Nnamdi Azikwe university, Awka, Anambra state ,Nigeria.

Opaluwa, Y.D. (2006). “Geoid definition and determination using classical methods “, A Seminar Paper on SVY 804 (Advanced Physical Geodesy), Department of Surveying and Geoinformatics, University of Lagos.

Pan, M & Sjoberg, L.E. (1993). Baltic Sea level project with GPS, Bullentin Geodesique, 67.pp 51-59.

Parker, A. (2015). Development of best practice for GNSS surveys in South Africa, *Geomatics Indaba Proceedings 2015*-Stream 1.

Pedro, E.F.N. (2013). Positioning with combined GPS and GLONASS observations, Master of Science Degree, Technico, Lisbon.

Pugh, D.T. (1987).*Tides, Surges, Mean Sea Level. A Handbook for Engineers and Scietists*. John Wiley and Sons, Chichester, England, 472pp

Quora, http://www.quora .comon Polygons

Rapp, R.H. & Balasubramaia, N. (1992). A conceptual formulation of a world height system, the Ohio State University, Columbus, U.S.A

Roman, D. (2007). Datum, heights and geodesy, central chapter of the professional land surveyors of Colorado 2007 annual meetings.

Rosa, R., Jana, S.K., Das, R.K., & Pal, D.K. (2016). Evaluation of orthometric heights from GPS survey using a geoid model - A case study for Madang, Papua New Guinea, *International Journal of Advancements in Research and Technology*, Vol.5, Issue 5, May 2016.

Rothstein, D. (2016), Does gravity varies across the surface of the Earth?http://curios.astro.cornell.edu/a accessed on 15/10/2016 at 11.57pm

Sahrum, S. (2017). ONGD14/OMAN DATUM, national survey authority, ministry of defence, Sultanate of Oman, 2017.

Sanlioglu, I., Maras, S.S. & Uysal, F. (2009). Determination of orthometric heights with Real-Time Kinematic surveying, Konya Sample, *FIG Working Week*, 2009.

Schofield, W. (2009).*Engineering surveying: Theory and Examination Problems for Students*, 5th Edition Kingston University.

Schwarz, K.P, Sideris M.G & Forsberg, R. (1990). The use of FFT Techniques in physical geodesy. Geophysical Journal International, 100, pp. 485 – 514.

Seker, D.Z. & Yildrim, A. (2002). Orthometric height derivation from GPS observations, *FIG XXII International Congress, Washington D.C.USA*, April 19-26 2002.

Sergio, C.A. (2003). Technical aspects in the upgrading and implementation of PRS92, Policy Forum and Mid –Year Assembly of Geodetic Engineers, GEP-NCR, Queen City, Philippines.

Shruthi, H.O.& Bindu, A.T. (2016). Different GNSS (Global Navigation Satellite System) receiver's combination and its spatial information analytics, Volume 2 issue: 04, April

2016, IJMTST.

Shi, W.Z., Li, Q.Q. and Zhu, C.Q. (2005). Estimating the propagation error of DEM from higher –order interpolation algoriths. International journal of remote sensing, vol. 000, No. 000,0month 2005, 153-168.

Sinha, N. K. & Prisad, T. (1979). Some stochastic modelling techniques and their applications, appl. Math.modelling.vol.3.February 1979.

Sjoberg, L.E. (2006). A new technique to determine geoid and orthometric heights from satellite positioning and geopotential Numbers, *J. Geod (2006) 80*, dox – 312. DOI: 10. 1007/500190-006-0057-5

Sousa-Silva, T. (2007). Error analysis in GNSS receivers. Instituto Superior Technico Lisbon, October 2007.

Soycan, M. (2013). Analysis of geostatistical surface model for GPS height transformation: A case study of Izmir Territory of Turkey, *Geodeski vestnik 57/4* (2013)

Stanway, R. (2009). Using GNSS to establish a height datum in a Project, 43rd Association of Surveyors PNG Congress, Lae, 12th – 15th August 2009.

Suffolk County (2008). Global Positioning System (GPS) data collection guidelines, draft, Apri 2008.New York.

Torge, W. (1980). *Geodesy*: An Introduction. New York, de Grunter, pp.134- 144.

Torge, W. (2001).*Geodesy*, 3rd Edition, de Gruyter, Berlin.Trimble Business Center Tutorial, Processing GNSS Baselines.

Udoffia, I.M& Fajemirokun, F. (1978). “On a height datum for Nigeria”, *Proceedings of the International Symposium on Geodetic Measurements and Computations*, Ahmadu Bello University Zaria, Kaduna, March.

United States NOAA Technical Memorandum NOS NGS -58.

Uzodinma.V.N. (2005).VLBI,SLR and GPS data in the Nigerian primary triangulation network – What Benefits to Future Research and National Economy ? “*In Proceedings of the 1st International Workshop on Geodesy and Geodynamics”*, Toro.Nigeria.

Uzodinma.V.N, Oguntuase, J.O., Alohan, N.O & Dimgba, C.N. (2013).*Practical GNSS Surveying.* Enugu: Professor’s Press Limited.

Vanicek, P. (2001).*Geodesy, an Online Tutorial*, University of New Brunswick, Geodesy Group, Academic Press.

Vanicek, P., Kingdon, R and Santos, M. (2012). Geoid vs. Quasi-geoid in Contribution to Geophysics and Geodesy, Vol. 42/1, 2012(101-117).

Vanicek, P & Krakiwsky, E. J. (1986).*Geodesy*: The Concepts, 2nd edition, North Holland, Amsterdam.

Vanicek, P. & Krakiwisky, E. (1982).*Geodesy*, the Concepts, North-Holland publishing company, the Netherlands, pp. 98-99.

Varga, M., Bjelotomic, O& Basic, T. (2016).Initial considerations on modernisation of the Croatian height reference system,*SIG,International Symposium on Engineering Geodesy*,20 – 22 May 2016,Varazdin,Croatia.

Varner, C.C. (2000). DGPS carrier phase networks and partial derivative algorithm, PhD Thesis, Dept. of Geomatics Engineering, The University of Calgary, Alberta, Canada. February 2000

Verna,C & Cannon, M.E. (1997). “The Application of multiple reference stations to the determination of multipath and spatially decorrelating errors “, *Pro. Of ION NTM-97 (Santa Monica), the Institute of Navigation*, VA, pp. 323-333

Vergos, G. S. & Sideris M.G (2002). Evaluationof geoid models and validation of of geoid and GPS / leveling undulations in Canada, GIG working week 2005,Cairo, Egypt.

Vergos, G. S. & Tziavos I.N. (2008). Monitoring systems with the recent and forthcoming gravity-field dedicated satellite missions, *International Conference on Planet Earth, June 1-6*, Greece.

Veronneau, M. (2006).Demysitifying the vertical datum in Canada, Natural Resources Canada, pp 24.

Veronneau, M., Duval, R., Huang, J. (2006). A gravimetric geoid model as a vertical datum in Canada. *Geomatica, Vol.60.No.2, 2006*, pp.160 to 172

Yanalak, M& Baykal, O. (2001). Transformation of ellipsid heights to local levelling heights, *J. Surv Eng 127, pp. 1-14*

Ziggah, Y.Y., Youjian, H., Odutola, S.A., & Nguyen, T.T (2013).Accuracy assessment of centroid computation in precise GPS coordinates transformation- a case study, Ghana. *European scientific journal, vol.9, no. 15, pp.200-220, May 2013*

Zilkoski, D.B, Richard J.H & Young, G.M. (1992). Results of the general adjustment of the North American Vertical Datum of 1988. Surv.and Land Info.Sys.52 (3). 133 – 149.

APPENDIX 1: PICTURES OF STATIC OBSERVATIONS AT BASE/ROVER CONTROLS STATIONS


Surveyor at FCT 260P Base Station   Surveyor at a Rover Station


FCT 2107S Rover Observation Station   Rover Observatio Station

-

FCC011SXP382

FCT 2652S Rover Observation Station

XP382 FCT130P BASE

FCT 036P

FCT130P BASE

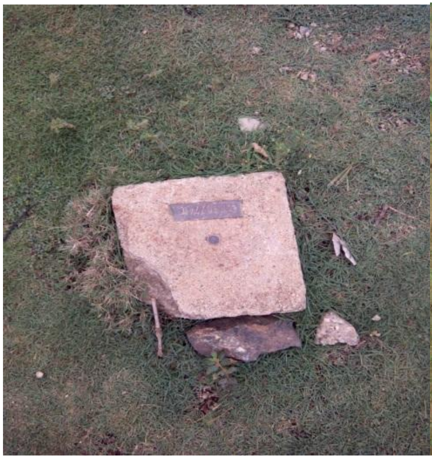
APPENDIX TWO

PHYSICAL STATUS OF SOME EXISTING CONTROLS USED WITHIN STUDY AREA OF FCT ABUJA

FCT2168S

FCT260P

XP382



FCT 10774T **FCT 2652S**

FCT 10774T **FCT 130P (BASE CONTROL)**

\

APPENDIX THREE

PROGRAM FOR GEOMETRIC GEOID INTERPOLATION (MULTIQUADRATIC MODEL) FOR POPULATING **A** MATRIX

AVERAGE EAST	312884.309		MULTIQUADRATIC MODEL (PROCESSING WINDOW)													
AVERAGE NOR	989273.136															
STATION	**EASTING (x)**	**NORTHING (y)**	**ELLIPSOIDAL HEIGHT, h (m)**	**CHANGE IN E**	**CHANGE IN N**	a_0	a_1x	a_2y	a_3x^2	a_4y^2	a_5xy	a_6x^2y	a_7xy^2	$a_8x^2y^2$	**GEOID HEIGHT, N (m)**	**ORTHOMETRIC HEIGHT, H (m)**
FCC11S	331888.114	998442.043	509.396	19003.805	9168.906625	24.2248901210	-0.4578663797	-0.7347614057	0.0350275833	0.3112909349	2.0329098533	-0.715272257	-0.730375397	0.268998551	24.235	485.161
FCT260P	255881.175	993666.807	224.74	57003.134	4393.670625	24.2248901210	-1.3733996457	-0.3520921018	0.3151566865	0.0714803001	2.9220405821	-3.083881477	-0.503065117	0.555758593	22.777	201.963
FCT103P	340639.766	998375.578	556.836	27755.457	9102.441625	24.2248901210	-0.6687234828	-0.7294351526	0.0747180986	0.3067942239	2.9475846906	-1.514702599	-1.051319353	0.565517806	24.155	532.681
FCT12P	333743.992	1008308.73	760.192	20859.683	19035.59362	24.2248901210	-0.5025808015	-1.5254402844	0.0422031097	1.3417271983	4.6326995475	-1.789182056	-3.45550101	1.396953563	24.366	735.826
FCT19P	337452.408	996344.691	659.824	24568.099	7071.554625	24.2248901210	-0.5919291731	-0.5666875702	0.0585426197	0.1851659701	2.0269659848	-0.921999551	-0.56165784	0.267427843	24.121	635.703
FCT2168S	310554.927	1009739.93	455.274	2329.382	20466.79363	24.2248901210	-0.0561227518	-1.6401312249	0.0005262724	1.5510685595	0.5562251008	-0.023988553	-0.446078055	0.020137932	24.187	431.087
FCT24P	322719.776	1001884.85	477.987	9835.467	12611.71363	24.2248901210	-0.2369698917	-1.0106549025	0.0093825093	0.5889515582	1.4472017744	-0.263534065	-0.715177185	0.136323859	24.180	453.807
FCT276P	351983.716	1025998.314	649.848	39099.407	36725.17763	24.2248901210	-0.9420378736	-2.9430164620	0.1482754977	4.9941224184	16.7530576936	-12.12765829	-24.1084036	18.26845702	24.268	625.580
FCT4154S	329953.882	1003831.28	501.232	17069.573	14558.14363	24.2248901210	-0.4112641432	-1.1666344213	0.0282601431	0.7847717719	2.8992699999	-0.91627115	-1.653884304	0.547130623	24.336	476.896
FCT4159S	326124.422	1003742.86	476.553	13240.113	14469.72363	24.2248901210	-0.3189994094	-1.1595487778	0.0170024858	0.7752679779	2.2351770557	-0.547918941	-1.267309327	0.325190284	24.284	452.269
FCT66P	299148.035	998114.283	321.115	13736.274	8841.146625	24.2248901210	-0.3309536298	-0.7084959623	0.0183006688	0.2894333425	1.4168946400	-0.360345589	-0.490858838	0.130673888	24.190	296.925
FCT9P	329821.512	1007612.091	521.693	16937.203	18338.95463	24.2248901210	-0.4080748991	-1.4696142768	0.0278235427	1.2453187463	3.6239005716	-1.13639852	-2.604121923	0.854803411	24.359	497.334
FCT35P	322183.38	992926.363	451.299	9299.071	3653.226625	24.2248901210	-0.2240462852	-0.2927557276	0.0083870293	0.0494179359	0.3963475494	-0.068238336	-0.056736593	0.010225064	24.047	427.252
FCT57P	303234.27	992916.402	347.795	9650.039	3643.265625	24.2248901210	-0.2325023117	-0.2919574908	0.0090320683	0.0491488144	0.4101851440	-0.073286116	-0.058557325	0.010951499	24.048	323.747
FCT4028S	330164.634	1001388.24	473.942	17280.325	12115.10362	24.2248901210	-0.4163418767	-0.9708584604	0.0289622871	0.5434825947	2.4425251503	-0.781454406	-1.159515722	0.388321814	24.300	449.642
FCT53P	308943.361	993406.773	375.955	3940.948	4133.636625	24.2248901210	-0.0949508654	-0.3312539631	0.0015063660	0.0632697279	0.1900610130	-0.013867765	-0.030784762	0.002351255	24.011	351.944
FCT4652S	329441.767	997474.808	487.113	16557.458	8201.671625	24.2248901210	-0.3989255487	-0.6572508608	0.0265898793	0.2490784350	1.5843679331	-0.485693621	-0.509177126	0.163390118	24.197	462.916
FCT162P	270791.291	934625.533	215.091	42093.018	54647.60337	24.2248901210	-1.0141641703	-4.3792516944	0.1718498629	11.0579243479	26.8374469583	-20.91529365	-57.46756341	46.88091697	25.397	189.694
FCT130P	330982.584	952889.869	719.383	18098.275	36383.26738	24.2248901210	-0.4360490779	-2.9156170712	0.0317689900	4.9015650409	7.6824348530	-2.574238391	-10.95244418	3.841594065	23.804	695.579
FCT2327S	282526.612	973821.47	207.482	30357.697	15451.66638	24.2248901210	-0.7314203192	-1.2382379459	0.0893854242	0.8840605655	5.4727317224	-3.07599542	-3.313522668	1.949494517	24.261	183.221
FCT2652S	271370.273	945385.429	163.741	41514.036	43887.70737	24.2248901210	-1.0002145220	-3.5169944337	0.1671548473	7.1320957162	21.2567990001	-16.33824958	-36.55537344	29.41097564	24.781	138.960
FCT2656S	272644.591	941062.46	229.229	40239.718	48210.67638	24.2248901210	-0.9695118613	-3.8634207754	0.1570503531	8.6063265959	22.6338361108	-16.86264891	-42.75745582	33.34494714	24.514	204.715
FCT83P	332954.205	987231.606	592.819	20069.896	2041.530375	24.2248901210	-0.4835521428	-0.1636005022	0.0390678299	0.0154327409	0.4780364073	-0.177631012	-0.038240828	0.01487427	23.909	568.910
XP382	284074.729	983364.863	298.39	28809.580	5908.273375	24.2248901210	-0.6941209081	-0.4734666223	0.0805013042	0.1292565433	1.9859006766	-1.059271175	-0.459757233	0.256701701	23.991	274.399

APPENDIX FOUR

PROGRAM FOR GEOMETRIC GEOID INTERPOLATION (BICUBIC MODEL)FOR POPULATING **A** MATRIX

AVERAGE EASTING	*312884.309*	**BICUBIC MODEL (PROCESSING WINDOW)**
AVERAGE NORTHING	*989273.136*	

STATION	EASTING (x)	NORTHING (y)	ELLIPSOIDAL HEIGHT, h (m)	CHANGE IN E	CHANGE IN N	a_0	a_1x	a_2y	a_3x^2	a_4y^2	a_5xy	a_6x^2y	a_7xy^2	a_8x^3	a_8y^3	GEOID HEIGHT, N (m)	ORTHOMETRIC HEIGHT, H (m)
FCC11S	331888.114	998442.043	509.396	19003.805	9168.906625	23.5008159260416	0.8352714217	0.8348497919	-0.5641255918	-0.2960607536	-0.3110816103	0.136302391	0.038873724	0.0500676371	0.0225696159	24.247	485.149
FCT260P	255881.175	993666.807	224.74	57003.134	4393.670625	23.5008159260416	2.5054503356	0.4000536986	-5.0756556890	-0.0679830639	-0.4471389069	0.587664926	0.026775292	1.3512372941	0.0024834419	22.784	201.956
FCT103P	340639.766	998375.578	556.836	27755.457	9102.441625	23.5008159260416	1.2199314887	0.8287980026	-1.2033485524	-0.2917840481	-0.4510477385	0.288641959	0.055955743	0.1559841794	0.0220823467	24.126	532.710
FCT12P	333743.992	1008308.73	760.192	20859.683	19035.59362	23.5008159260416	0.9168425532	1.7332340733	-0.6796887487	-1.2760820217	-0.7089087757	0.340946806	0.183916644	0.0662153248	0.2019625251	24.279	735.913
FCT19P	337452.408	996344.691	659.824	24568.099	7071.554625	23.5008159260416	1.0798380136	0.6438811244	-0.9428395260	-0.1761065631	-0.3101720627	0.175696375	0.029893849	0.1081807689	0.0103541921	24.120	635.704
FCT2168S	310554.927	1009739.93	455.274	2329.382	20466.79363	23.5008159260416	0.1023829938	1.8635480868	-0.0084757123	-1.4751811736	-0.0851151367	0.004571262	0.023742195	0.0000922057	0.2510272964	24.177	431.097
FCT24P	322719.776	1001884.85	477.987	9835.467	12611.71363	23.5008159260416	0.4322968165	1.1483251958	-0.1511070168	-0.5601365881	-0.2214549050	0.050219092	0.03806481	0.0069409728	0.0587345735	24.303	453.684
FCT276P	351983.716	1025998.314	649.848	39099.407	36725.17763	23.5008159260416	1.7185304466	3.3439109103	-2.3880038280	-4.7497806112	-2.5636002285	2.311048418	1.283153057	0.4360596688	1.4503179840	24.342	625.506
FCT4154S	329953.882	1003831.28	501.232	17069.573	14558.14363	23.5008159260416	0.7502564085	1.3255520722	-0.4551347384	-0.7463761265	-0.4436544880	0.17460477	0.088026845	0.0362830151	0.0903419616	24.321	476.911
FCT4159S	326124.422	1003742.86	476.553	13240.113	14469.72363	23.5008159260416	0.5819407190	1.3175012301	-0.2738281231	-0.7373373140	-0.3420331092	0.104411517	0.06745166	0.0169320861	0.0887058426	24.325	452.228
FCT66P	299148.035	998114.283	321.115	13736.274	8841.146625	23.5008159260416	0.6037484321	0.8050064988	-0.2947355953	-0.2752725631	-0.2168172216	0.068667511	0.026125621	0.0189078556	0.0202347245	24.257	296.858
FCT9P	329821.512	1007612.091	521.693	16937.203	18338.95463	23.5008159260416	0.7444383694	1.6698035086	-0.4481032105	-1.1843904375	-0.5545395058	0.216552276	0.138602583	0.0354454485	0.1805906434	24.299	497.394
FCT35P	322183.38	992926.363	451.299	9299.071	3653.226625	23.5008159260416	0.4087206824	0.3326345891	-0.1350746300	-0.0470001202	-0.0606502220	0.013003508	0.003019766	0.0058661623	0.0014275830	24.023	427.276
FCT57P	303234.27	992916.402	347.795	9650.039	3643.265625	23.5008159260416	0.4241467490	0.3317276174	-0.1454631012	-0.0467441657	-0.0627676898	0.013965413	0.003116673	0.0065557541	0.0014159373	24.027	323.768
FCT4028S	330164.634	1001388.24	473.942	17280.325	12115.10362	23.5008159260416	0.7595195600	1.1031077264	-0.4664428955	-0.5168922335	-0.3737621005	0.148914071	0.06171442	0.0376435971	0.0520658445	24.307	449.635
FCT53P	308943.361	993406.773	375.955	3940.948	4133.636625	23.5008159260416	0.1732159160	0.3763770117	-0.0242602989	-0.0601742011	-0.0290836733	0.002642643	0.001638498	0.0004465166	0.0020680851	23.944	352.011
FCT4652S	329441.767	997474.808	487.113	16557.458	8201.671625	23.5008159260416	0.7277474936	0.7467808464	-0.4282348364	-0.2368920547	-0.2424444582	0.092553851	0.027100599	0.0331143613	0.0161539468	24.237	462.876
FCT162P	270791.291	934625.533	215.091	42093.018	54647.60337	23.5008159260416	1.8501082105	4.9757885180	-2.7676732613	-10.5169057278	-4.1067419699	3.985621555	3.058671196	0.5440835803	4.7784261477	25.302	189.789
FCT130P	330982.584	952889.869	719.383	18098.275	36383.26738	23.5008159260416	0.7954707955	3.3127792048	-0.5116453561	-4.6617516844	-1.1755878900	0.490547261	0.582936244	0.0432460999	1.4101866903	23.787	695.596
FCT2327S	282526.612	973821.47	207.482	30357.697	15451.66638	23.5008159260416	1.3343073809	1.4069093498	-1.4395684959	-0.8408071292	-0.8374528728	0.586162157	0.176359945	0.2040994777	0.1080183365	24.199	183.283
FCT2652S	271370.273	945385.429	163.741	41514.036	43887.70737	23.5008159260416	1.8246603002	3.9960755266	-2.6920591820	-6.7831516956	-3.2527754497	3.113419337	1.945634392	0.5219396589	2.4751424641	24.650	139.091
FCT2656S	272644.591	941062.46	229.229	40239.718	48210.67638	23.5008159260416	1.7686503895	4.3896916815	-2.5293244673	-8.1852545401	-3.4634935596	3.213348954	2.275735925	0.4753354562	3.2809619275	24.726	204.503
FCT83P	332954.205	987231.606	592.819	20069.896	2041.530375	23.5008159260416	0.8821291619	0.1858859817	-0.6291944975	-0.0146776806	-0.0731504819	0.033849393	0.002035342	0.0589753825	0.0002491375	23.947	568.872
XP382	284074.729	983364.863	298.39	28809.580	5908.273375	23.5008159260416	1.2662632231	0.5379617221	-1.2964881278	-0.1229325539	-0.3038881332	0.201854877	0.02447026	0.1744400526	0.0060388364	23.989	274.401

APPENDIX FIVE

COMPUTED COEFFICIENT CONSTANTS (MULTIQUADRATIC MODEL)

MULTI – QUADRATIC MODELCONSTANTS (9 coefficients)

$$X = \begin{pmatrix} a_o \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{pmatrix} = \begin{pmatrix} 24.2248901210000000000 \\ -0.00002409340580587179 \\ -0.00008013620770038382 \\ 0.00000000009699046795 \\ 0.00000000370280953876 \\ 0.00000001166702184889 \\ -0.00000000000021600943 \\ -0.00000000000045716237 \\ 0.00000000000000000886 \end{pmatrix}$$

APPENDIX SIX

COMPUTED COEFFICIENT CONSTANTS (BICUBIC MODEL)

BICUBIC MODEL CONSTANTS (10 coefficients)

$$X = \begin{pmatrix} a_o \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{pmatrix} = \begin{pmatrix} 23.50081592604167925515 \\ 0.00004395285221636346 \\ 0.00009105227330487502 \\ -0.00000000156204910634 \\ -0.00000000352164634358 \\ -0.00000000178532065159 \\ 0.00000000000004116279 \\ 0.00000000000002433215 \\ 0.00000000000000729517 \\ 0.00000000000002928003 \end{pmatrix}$$

APPENDIX SEVEN

GEOMETRIC GEOID MODEL AND ORTHOMETRIC HEIGHTS DETERMINATION OF MULTIQUADRATIC MODEL(INPUT-OUTPUT AND PROCESSINGWINDOW)

GEOMETRIC GEOID MODEL AND ORTHOMETRIC HEIGHTS DETERMINATION OF FCT (MULTIQUADRATIC)

INPUT AND OUTPUT WINDOW

STATION	EASTING (x)	NORTHING (y)	ELLIPSOIDAL HEIGHT, h (m)	GEOID HEIGHT, N (m)	ORTHOMETRIC HEIGHT, H (m)
FCC11S	331888.114	998442.043	509.396	24.235	485.161
FCT260P	255881.175	993666.807	224.74	22.777	201.963
FCT103P	340639.766	998375.578	556.836	24.155	532.681
FCT12P	333743.992	1008308.730	760.192	24.366	735.826
FCT19P	337452.408	996344.691	659.824	24.121	635.703
FCT2168S	310554.927	1009739.930	455.274	24.187	431.087
FCT24P	322719.776	1001884.850	477.987	24.180	453.807
FCT276P	351983.716	1025998.314	649.848	24.268	625.580
FCT4154S	329953.882	1003831.280	501.232	24.336	476.896
FCT4159S	326124.422	1003742.860	476.553	24.284	452.269
FCT66P	299148.035	998114.283	321.115	24.190	296.925
FCT9P	329821.512	1007612.091	521.693	24.359	497.334
FCT35P	322183.380	992926.363	451.299	24.047	427.252
FCT57P	303234.270	992916.402	347.795	24.048	323.747
FCT4028S	330164.634	1001388.240	473.942	24.300	449.642
FCT53P	308943.361	993406.773	375.955	24.011	351.944
FCT4652S	329441.767	997474.808	487.113	24.197	462.916
FCT162P	270791.291	934625.533	215.091	25.397	189.694
FCT130P	330982.584	952889.869	719.383	23.804	695.579
FCT2327S	282526.612	973821.470	207.482	24.261	183.221
FCT2652S	271370.273	945385.429	163.741	24.781	138.960
FCT2656S	272644.591	941062.460	229.229	24.514	204.715
FCT83P	332954.205	987231.606	592.819	23.909	568.910
XP382	284074.729	983364.863	298.390	23.991	274.399

APPENDIX EIGHT

GEOMETRIC GEOID MODEL AND ORTHOMETRIC HEIGHTS DETERMINATION OF BICUBIC MODEL(INPUT-OUTPUT AND PROCESSINGWINDOW)

GEOMETRIC GEOID MODEL AND ORTHOMETRIC HEIGHTS DETERMINATION OF FCT (BICUBIC MODEL)

INPUT AND OUTPUT WINDOW

STATION	EASTING (x)	NORTHING (y)	ELLIPSOIDAL HEIGHT, h (m)	GEOID HEIGHT, N (m)	ORTHOMETRIC HEIGHT, H (m)
FCC11S	331888.114	998442.043	509.396	24.247	485.149
FCT260P	255881.175	993666.807	224.74	22.784	201.956
FCT103P	340639.766	998375.578	556.836	24.126	532.710
FCT12P	333743.992	1008308.730	760.192	24.279	735.913
FCT19P	337452.408	996344.691	659.824	24.120	635.704
FCT2168S	310554.927	1009739.930	455.274	24.177	431.097
FCT24P	322719.776	1001884.850	477.987	24.303	453.684
FCT276P	351983.716	1025998.314	649.848	24.342	625.506
FCT4154S	329953.882	1003831.280	501.232	24.321	476.911
FCT4159S	326124.422	1003742.860	476.553	24.325	452.228
FCT66P	299148.035	998114.283	321.115	24.257	296.858
FCT9P	329821.512	1007612.091	521.693	24.299	497.394
FCT35P	322183.380	992926.363	451.299	24.023	427.276
FCT57P	303234.270	992916.402	347.795	24.027	323.768
FCT4028S	330164.634	1001388.240	473.942	24.307	449.635
FCT53P	308943.361	993406.773	375.955	23.944	352.011
FCT4652S	329441.767	997474.808	487.113	24.237	462.876
FCT162P	270791.291	934625.533	215.091	25.302	189.789
FCT130P	330982.584	952889.869	719.383	23.787	695.596
FCT2327S	282526.612	973821.470	207.482	24.199	183.283
FCT2652S	271370.273	945385.429	163.741	24.650	139.091
FCT2656S	272644.591	941062.460	229.229	24.726	204.503
FCT83P	332954.205	987231.606	592.819	23.947	568.872
XP382	284074.729	983364.863	298.390	23.989	274.401

APPENDIX NINE

ONLINE POST PROCESSING (SAMPLECSRS-PPP REPORTS)

CSRS-PPP (V 1.05 11216)

FCT9P

Data Start Data EndDuration of Observations

2017-05-20 09:09:20.000 2017-05-20 11:44:30.000 2h 35m 10.00s

Apri / Aposteriori Phase Std Apri / Aposteriori Code Std

0.015m / 0.008m 2.0m / 0.704m

Observations Frequency Mode

Phase and Code L1 and L2 Static

Elevation Cut-Off Rejected Epochs Observation & Estimation Steps

10.000 degrees 0.00 % 5.00 sec / 5.00 sec

Antenna Model APC to ARP ARP to Marker

V30 Ant. not in PPP (0 m) 0.077 m

(APC = antenna phase center; ARP = antenna reference point)

Estimated Position for FCT9P.17o

Latitude (+n) Longitude (+e) Ell. Height

ITRF14 (2017) 9º 06’ 47.5112’’ 7º 27’ 02.5899’’ 521.720 m

Sigmas(95%) 0.010 m 0.025 m 0.047 m

Apriori 9º 06’ 47.491’’ 7º 27’ 02.643’’ 522.273 m

Estimated - Apriori 0.631 m -1.608 m -0.553 m

95% Error Ellipse (cm)

semi-major: 3.101cm

semi-minor: 1.201cm

semi-major azimuth: 93º 14’ 34.81’’

UTM (North) Zone 32

1007731.667m (N) 329745.821m (E)

Scale Factors

0.99995871 (point)

0.99987683 (combined)

(Coordinates from RINEX file used as apriori position)

19:29:31 UTC 2017/07/06 / FCT9P.17o 1 IGS Final

Estimated Parameters & Observations Statistics

IGS Final 2 19:29:31 UTC 2017/07/06 / FCT9P.17o

19:29:31 UTC 2017/07/06 / FCT9P.17o 3 IGS Final

IGS Final 4 19:29:31 UTC 2017/07/06 / FCT9P.17o

19:29:31 UTC 2017/07/06 / FCT9P.17o 5 IGS Final

IGS Final 6 19:29:31 UTC 2017/07/06 / FCT9P.17o

~~~ Disclaimer ~~~

Natural Resources Canada does not assume any liability deemed to have been caused directly or indirectly by any content of its PPP-On-Line positioning service.

If you have any questions, please feel free to contact:

EMail: nrcan.geodeticinformationservices.rncan@canada.ca

Phone: 343-292-6617

19:29:31 UTC 2017/07/06 / FCT9P.17o 7 IGS Final
~~~