CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

Globalization has increased the pressure on organizations and companies to operate in the most efficient and economical way. This tendency promotes that companies concentrate more and more on their core businesses, outsource less profitable departments and services to reduce costs (Achermann, 2008). Competition in manufacturing is increasing exponentially as customers are becoming more exigent and demand becomes increasingly random, this is why the development of industrial strategies (maintenance and production) has become obligatory for manufacturing firms in order to reduce their costs (Mifdal, Hajej, Dellagi, and Rezg, 2013).

The importance of maintenance is ever increasing as a result of the widespread automation of manufacturing systems and the capital expenditure allocated to it, thus making maintenance of manufacturing equipment an investment opportunity to be maximised and not a cost centre (Horenbeek, Pintelon, and Muchiri, 2011). The economic downturn continuously drives manufacturing organisations to seek for more efficient strategies to manage assets maintenance.

According to Turuna Seecharan, Ashraf Labib (2016) the effective maintenance of assets is a vital strategic task given the increasing demand on sustained availability of those assets used for manufacturing. This is essential as sudden failures of manufacturing equipment can be prohibitively expensive because they result in immediate lost production outcome, inefficient quality characteristics and poor customer satisfaction. In the food manufacturing sector, asset maintenance is one of the most important essentials for an efficient manufacturing in the sector as this sector continuously face challenges that makes asset maintenance very critical due to the nature of manufacturing, as a result manufacturing companies in the sector must add or modify these assets to keep it running efficiently thus enhancing production. This is causing food manufacturers to invest more on manufacturing assets than any other manufacturing sector (Betts, 2018).

The main focus for food manufacturers is to improve efficiency and profitability through the reduction of total manufacturing costs by optimizing operation processes and maintenance activities achieved through continuously improved machine reliability and a hands-on maintenance culture. However, few manufacturers have the internal resources to implement such practical culture hence this research intends to contribute by developing an optimal maintenance method for improved cost and machine reliability in food manufacturing.

1.2 Statement of the Problem

In manufacturing, every sector faces its own individual problems or issues, but it can be said that the food manufacturing sector face more stringent issues than any other sector because their product is intended for human consumption, thus dealing with a wide range of regulations regarding food safety and maintaining complex machineries and equipment so that production is not affected to the detriment of the manufacturer. In other to stay competitive, production lines are sometimes run 24 hours a day. Hence when manufacturing equipment are not kept in good working condition, it can be prone to microbiological and physical contamination. Debris from worn or broken parts can contaminate the production line or even enter the product directly.

Furthermore, manufacturing equipment that do not meet operating parameter specifications can impact product quality as well as safety. Critical goals or targets can be missed, processes can be interrupted, and unwanted substances and chemicals can enter the production stream.

It is apparent that the maintenance team in any food manufacturing company has a lot to handle under such conditions, and to maintain these highly automated systems and keep equipment running optimally, food production and maintenance managers must stay on top of new techniques. There is a need to research, provide ideas and adopt newer and better maintenance strategies.

Trying to run such a sensitive system on reactive maintenance alone where components are left to fail before repairs are carried out is detrimental as downtime would be disproportionately high and the enterprise runs the risk of shortening the lifespan of their assets. Thus a proactive maintenance strategy is the most straightforward way to improve overall maintenance operations that will keep downtime and the associated stress of loss of revenue to the minimum.

1.3 Research Aim and Objectives

The aim of this study is to develop an optimal maintenance strategy for improving cost and machine reliability in food manufacturing. This will be pursed via the following specific objectives:

1. To collect, evaluate and categorise relevant data obtained from the case study for the optimal maintenance strategy.

2. To develop a multi-objective optimization model for an optimal maintenance strategy based on cost and reliability.

3. To validate the developed optimization model with an industrial application, adopting four solution techniques

4. To assess the potential contribution and economic implications the developed strategy can make to better justify the performance of maintenance operations.

5. To develop a generic user interface support system capable of providing an optimal maintenance schedule from one of the solution techniques presented in objective three.

1.4 Scope of Study

This study focused on maintenance and maintenance strategy, cost of maintenance and equipment reliability

The developed optimization model and programming execution codes were validated using the maintenance process at Tummy Tummy Foods Industry limited as an industrial applied case study.

1.5 Significance of the Study

An enhanced manufacturing system is key to sustaining the competitive nature of an economy. Thus it is crucial manufacturing systems in Nigeria continue to manufacture high quality products in an efficient and timely manner hence keeping the end user satisfied. Thus this study will aim at enhancing manufacturing in Nigeria in order to meet up with and also to able to compete favourably in the global market, giving insights on areas for further improvement. Hence this study will be significant in the sense that it will:

- Present new multi-objective maintenance optimization and cost forecasting models for use in manufacturing systems
- Provide bases for short and long term performance evaluation of maintenance management in manufacturing systems
- It will also contribute to the enhancement of knowledge in the academic world as well as in industrial practice.

CHAPTER TWO

LITERATURE REVIEW

2.1 Maintenance Concepts and Strategies

The efficient functioning of a manufacturing system depends on the smooth operations of complex systems that provide a variety of goods and services. According to Nicolai and Dekker (2008), each system built by humans is undependable in the sense that it depreciates with age and/or usage. A system failure occurs when it is no longer capable of delivering the designed outputs. Some failures can be disastrous in the sense that they can result in serious economic losses, affect humans and do serious damage to the environment. This depreciation and resultant failure can be controlled through several acts of maintenance concepts and strategies such as:

- Total Productive Maintenance
- Reliability Centred Maintenance
- Lean Maintenance
- Preventive maintenance
- Corrective maintenance

With effective implementation of such concepts and strategies, the likelihood of faults, failures and their resulting consequences can be reduced. According to Dekker and Schouten (1996) the principle idea of maintenance are defined as follows:

- Reducing breakdowns and emergency shutdowns.
- Maximizing production at lower cost, highest quality and within optimum safety standards.
- Optimizing the use of maintenance resources.
- Optimizing the utilization of resources to reduce downtime.

- Increasing reliability of the operating systems.
- Improving spares parts stock control.
- Optimizing capital equipment life.
- Improving equipment efficiency which reduces scrap rate.
- Identifying and implementing cost reductions.
- Optimizing the useful life period of the equipment.
- Minimising energy usage.

A study by Alsyouf (2009) opined that proper maintenance practices can contribute to overall business performance through their impact on the quality, efficiency and effectiveness of a company's operations which enhances the competitiveness, productivity advantages, value advantages and long-term profitability of the company as shown in figure 2.0.

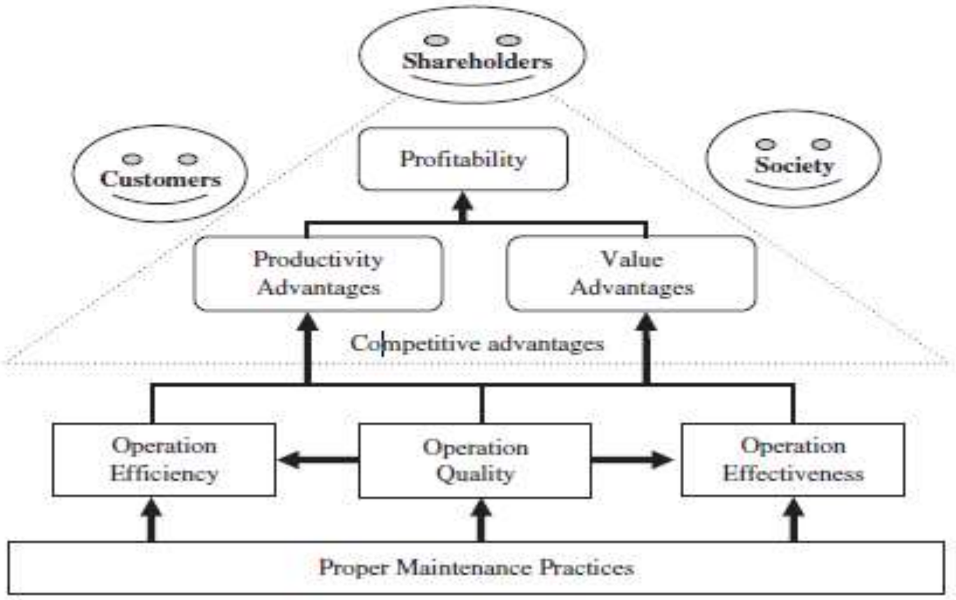


Fig 2.1: Impacts of effective maintenance   Source: Alsyouf (2009)

Thus the following section presents an in-depth understanding of such maintenance concepts and strategies through a review of existing literature.

2.1.1 Total Productive Maintenance

According to Wang and Lee (2001), manufacturing systems often operate at an inefficient capacity and with potential equipment breakdown thus leading to production wastes and losses, as a result productivity will be low and the cost of producing goods and services will be high. In order to combat these losses, the concept of total productive maintenance (hereinafter TPM) is one of the several methodologies used to eliminate losses in a manufacturing process. This is further supported by (Eti, Ogaji, and Probert, 2006).

A study by Brah and Chong (2004), further concluded that there is a positive correlation between implementing TPM and business performance thus necessitating the need for TPM to be an integrated effort of the entire manufacturing system.

Total productive maintenance a methodology developed by the Japanese in 1971 is a philosophy based on productivity maintenance and innovative in approach ensuring that there is no equipment and production breakdown, optimizes equipment effectiveness, eliminates defects in a production system and promotes autonomous maintenance through the establishment of a thorough system of preventive maintenance for equipment life span. According to Wakjira and Singh (2012), the objective of every TPM implementation in a manufacturing system is to advance productivity and quality along with better employee self-esteem and job satisfaction, ensuring joint responsibility between supervisors, operators and maintenance workers, and not simply to keep machines running smoothly, but also to extend and optimize their performance overall. Therefore according to Thomas (2000), TPM as a whole, places emphasis on:

- Maximizing overall equipment effectiveness.
- Establishing a planned system of Preventive Maintenance (PM) for the equipment's life span.

- Involving all employees from top management to shop floor workers.
- Empowering employees to initiate corrective activities.

TPM is successfully implemented through its unique eight pillar methodology as shown in the figure one, paving way for excellent planning, organizing, monitoring and controlling of manufacturing practices.

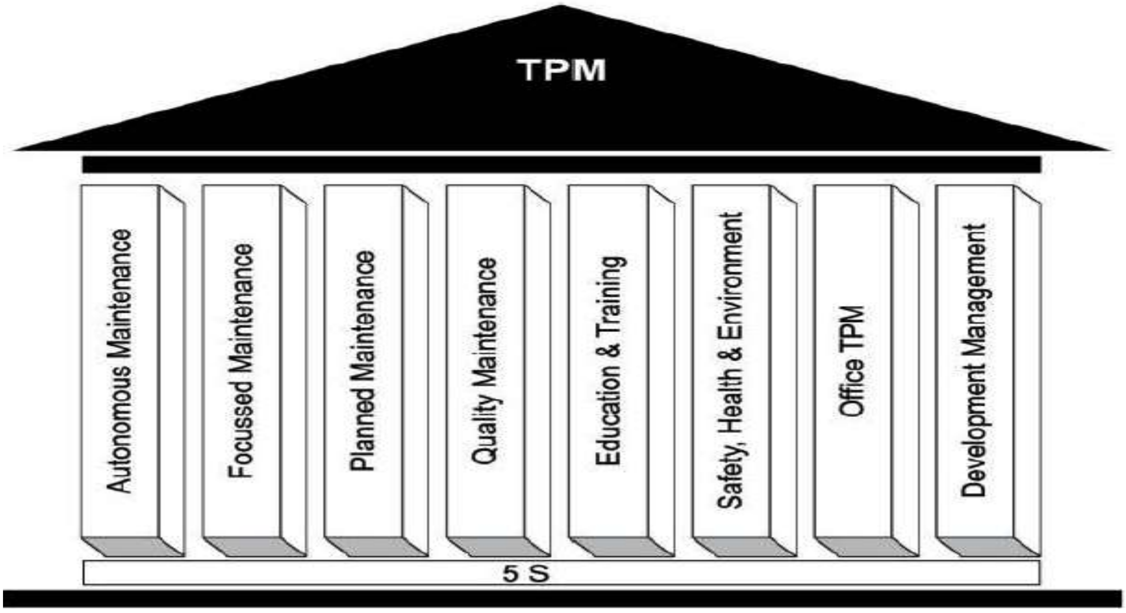


Figure 2.2: The eight pillars of Total Productive Maintenance Source: Ahuja and Kumar (2009)

Because it is the foundation on which TPM is built on, implementing TPM starts first with 5S. 5S according to A. K. Gupta and Garg (2012) is a methodical process of housekeeping to achieve a peaceful environment in the work place involving the employees with a commitment to sincerely implement and practice housekeeping. The philosophy starts with the cleaning and the arranging of the working environment and when implemented properly leads to reduction of defective products, lead time, unhappy customers, disheartened workers, and dwindling returns.

Table 2,1 and 2.2 outlines the key activities for 5S and TPM implementation in a working environment

Table 2.1: 5S activities Source: Wakjira and Singh (2012)

Japanese Term(English Term)	**Characteristics**
Seiri (Sort/Clear)	Sort out all unnecessary items from the working environment and get rid of them
Seiton (Set in order/Configure)	Arrange all necessary items in good order so that they can be easily picked up for use
Seisio (Shine/Clean and check)	Clean the workplace completely to make it free from dust, dirt and untidiness
Seiketsu (Standardize/Conformity)	Maintain a high standard of housekeeping and workplace organization
Shitsuke (Sustain/Custom and practice)	Train and motivate people to follow good housekeeping disciplines autonomously

Table 2.2: Description of the eight pillars of TPM Source: Jain, Bhatti, and Singh (2013)

Pillars	**Description**
1. Autonomous maintenance	Targeted towards developing operators that are able to take care of small maintenance tasks, thus freeing up the skilled maintenance people to spend time on more value added activity and technical repairs.

2. Focused maintenance	Through which focused maintenance activities maximizes overall effectiveness of equipment and processes by elimination of wastes/losses and continuous improvement.
3. Planned maintenance	Establishes and maintains optimal conditions through planned maintenance, achieved through daily, weekly and monthly assessments to monitor defects and implement improvement programmes.
4. Quality maintenance	Ensures customer satisfaction through zero defects by placing emphasis on eliminating non conformance cost.
5. Education & Training	Aims at upgrading the skills and morale of the operators and workers with the goal to create experts in the working environment.
6. Safety, Health & Environment	Aims to create a safe working environment with the goal of achieving zero accidents etc.
7. Office TPM	Follows the first four pillars of TPM to improve productivity and efficiency of

	organizational activities through the automation of essential processes
8. Development Management	Aims to reduce overall the cost of maintenance in the working environment, reducing Mean Time to Repair (MTTR) and improving Mean Time Between Failure (MTBF)

According to F. Wang and Lee (2001), the basic goal of TPM is to increase the productivity of any manufacturing system through autonomous maintenance by operators, maximise output to improve manufacturing efficiency by reducing the six big losses ((1) reduced yield—from start up to stable production, (2) process defects, (3) reduced speed, (4) idling and minor stoppages, (5) set-up and adjustment, and (6) equipment failure.).

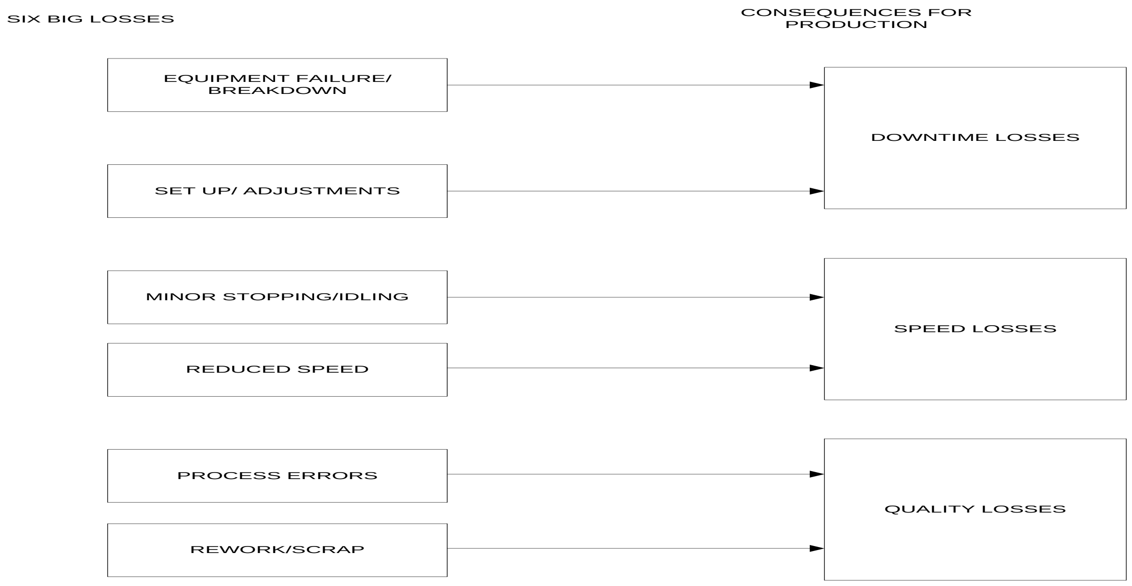


Fig 2.3 Six Big Losses Source: Rodrigues and Hatakeyama (2006)**;** Mwanza and Mbohwa, (2015)

In evaluating manufacturing performance capability through the implementation of TPM, overall equipment effectiveness (OEE) is used as a key performance indicator. It is a function of the equipment availability rate, quality rate and performance rate in a manufacturing system and is represented as:

$$\text{OEE} = \text{Availability x Performance Rate x Quality Rate} \qquad (2.0)$$

Where availability accounts for losses as a result of equipment failure, setup and adjustment and is calculated as the ratio of operating time to loading time and is calculated as follows:

$$\text{Availability} = \frac{\text{Plannedruntime} - \text{Planneddowntime}}{\text{Plannedruntime}} \times 100 \qquad (2.1)$$

And performance rate accounting for losses due to idle time and minor stoppages and is calculated as ratio of net operating time to operating time and is calculated as follows:

$$\text{Performance rate} = \frac{\text{Total Actual amount of product}}{\text{Target amount of product}} \times 100 \qquad (2.2)$$

Quality rate factors in the defects in process and reduced yield and is defined as ratio of valuable operating time to net operating time and is calculated as follows:

$$\text{Quality rate} = \frac{\text{Processed Quantity} - \text{defective quantity}}{\text{Processed quantity}} \times 100 \qquad (2.3)$$

Generally 85% OEE is considered to be the world class goal and serves as a benchmark for any manufacturing firm with the intent of implementing TPM.

Table 2.3: World class goals for OEE Source: Mwanza and Mbohwa (2015)

OEE Factor	**WORLD CLASS RATE (%)**
Availability	>90.0%
Performance Rate	>95%
Quality Rate	>99%
OEE	85%

A study by Mckone, Schroeder, and Cua (2001) investigated the impact of total productive maintenance practices on manufacturing performance in a manufacturing system, stating that TPM has a positive and significant relationship with low manufacturing cost (as measured by higher inventory turns), high levels of quality (as measured by higher levels of conformance to specifications), and strong delivery performance (as measured by higher percentage of on-time deliveries and by faster speeds of delivery). Several other studies by (Ahuja & Kumar, 2009), (Wakjira & Singh, 2012), (Albert & Chan Tsang, 2009), (Ahmed, Hassan, & Taha, 2005), (Ahuja & Khamba, 2007), (Brah & Chong, 2004) and (Perera, 2013) also highlighted the effects of TPM on manufacturing system performance stating that it significantly improve cost effectiveness, product quality, on-time delivery and volume flexibility, thus reducing the cost of rework and repairs due to very limited products rejected as a result of equipment failure. These findings also suggested that effective TPM implementation can significantly contribute towards the realisation of strategic performance improvements through effective maintenance for competing in a highly dynamic global marketplace.

2.1.2 Reliability Centered Maintenance (RCM)

A maintenance methodology initially used in the aviation industry during the 1960s to reduce maintenance costs and to increase safety and reliability, is an approach that helps in deciding what maintenance tasks must be performed at any given point of time Sainz and Sebastián (2013) (Shafeek, 2014). This maintenance methodology is reliability based driven in the sense that the target is reducing the need for maintenance of equipment and manufacturing assets by improving the reliability of the equipment and manufacturing assets. It is also a method of preserving a system's or asset's function by selecting and applying effective preventive maintenance (PM) (Misra, 2007).

According to Misra (2007) The main features of RCM are:

- A focus on the preservation of assets function
- Classification of specific failure modes to define loss of this function
- Prioritization the failure modes, as not all functions or functional failures have the same importance
- Identification of effective and applicable preventive maintenance tasks that will prevent, and discover or detect the onset of appropriate failure modes based on cost-effective options.

An implementation of RCM when properly conducted according to (Robin P. Nicolai , Rommert Dekker, 2008) should address the following questions:

1. What are the system functions and the associated performance standards?
2. How can the system fail to fulfil these functions?
3. What can cause a functional failure?
4. What happens when a failure occurs?
5. What might the consequence be when the failure occurs?
6. What can be done to detect and prevent the failure?
7. What should be done when a suitable preventive task cannot be found?

With the questions identified and addressed, the process followed while implementing RCM methodology is as follows:

1. The objectives of maintenance with respect to a particular asset are defined by the functions of the asset and its associated desired performance standards.
2. Functional failures are identified.
3. Failure modes which are likely to cause loss of each function are also identified.

4. Failure effects are assessed.

5. Failure consequences are quantified to identify the criticality of failure in terms of the following categories: hidden failure, safety and environmental, operational and non-operational.

6. Functions, functional failures, failure modes, and criticality are analyzed to identify opportunities for improving performance and/or safety.

7. Preventive tasks are established. These may be one of three main types: scheduled on condition tasks which employ condition based or predictive maintenance, scheduled restoration, and scheduled discard tasks.

Even though the main objective of using RCM is to reduce the total costs associated with system failure and downtime in manufacturing, Robin P. Nicolai and Rommert Dekker,(2008) suggests that evaluating the returns from an RCM program solely by measuring its impact on costs may hide many other less tangible benefits such as:

- Increased reliability leading to fewer equipment failures and, therefore, greater availability for patients and lower maintenance costs.
- Reduction in total of total maintenance cost as failures are prevented and preventive maintenance tasks are replaced by condition monitoring.
- Increasing Efficiency and Productivity as a result of the RCM approach to maintenance that ensures that the proper type of maintenance is performed on equipment as needed.
- Reducing lifecycle costs including acquisition phase and operation phase since decisions made early in the acquisition cycle profoundly affect the life-cycle cost. Savings of 30–50 % in the annual operations and maintenance costs are often obtained overtime through the implementation of a balanced RCM program.

- Improving maintenance sustainability as RCM planning involves decisions made at all phases of equipment life cycle.
- Optimizing spare parts inventory.
- Identifying component failure significance and hidden failure modes as well as previously unknown failure scenarios.
- Providing training opportunities for system engineers and operations personnel.
- Identifying areas for potential design enhancement.
- Providing detailed review and improvement where necessary.

RCM functions by finding an equilibrium point between high maintenance costs and cost of preventive maintenance policies while, taking into consideration the potential shortening of useful life of the piece of equipment (Demoly & Kiritsis, 2012).

2.1.3 Lean Maintenance

Lean maintenance is a methodology that makes use of the ideas and concepts from lean manufacturing thus combining it with TPM and RCM methodologies. It is also a planning and scheduling approach of implementing lean in maintenance. In adopting lean maintenance, the goal is to attain the highest maximum obtainable in preventive maintenance through the elimination of all types of wastes in the maintenance process (Frediksson & Larsson, 2012). Wastes exists in maintenance as it does in manufacturing, Davies and Greenough (2000) provided a useful comparison between the types of wastes found in maintenance and in manufacturing as showing in table 2.4

Table 2.4: wastes as applied in manufacturing and maintenance Davies and Greenough (2000)

S/N	Waste in Manufacturing	Waste in Maintenance
1	Transportation	Centralised maintenance
2	Inventory	Excessive stock
3	Motion	Double handling
4	Waiting time	Waiting for resources
5	Over production	Too much preventive maintenance
6	Over processing	Non-standard preventive maintenance
7	Defects	Poor maintenance
8	Human potential	Lack of training
9	Inappropriate systems	Poor information keeping
10	Energy	Inappropriate energy management
11	Wasted materials	Too much preventive maintenance
12	Waste in service and office	Poor service operations
13	Customer time	Production inconvenience
14	Defecting customers	Poor maintenance

From a maintenance point of view, improved efficiency and profitability can be obtained while increasing value through the elimination of wastes in maintenance. According to Smith and Hawkins (2004), applying lean maintenance will prove successful in following areas, improved inventory control as a result of efficient planning and scheduling, increased accuracy in maintenance budgeting, also a factor due to improved equipment reliability; and also, reduced maintenance costs. The maintenance function according to Davies and Greenough (2000), is expected to add value through its activities, requiring greater management integration within the manufacturing organisation.

Overall lean maintenance implementation in the maintenance of manufacturing assets and equipments can be summarised as followed:

Table 2.5: Lean Maintenance Implementation Source: Sima Ghayebloo (2010); Smith and Hawkins (2004)

LEAN MAINTENANCE					
Planning And Scheduling	DOCUMENTATION	WORK ORDER SYSTEM	CMMS	PREDICTIVE MAINTENANCE	ROOT CAUSE FAILURE ANALYSIS
TOTAL PRODUCTIVE MAINTENANCE (TPM)					

2.1.4 Preventive maintenance

Introduced in the 1950's because of the need to prevent failure, and another alternative to corrective maintenance according to Basri, Hamimi, Razak, Ab-samat, and Kamaruddin (2017), preventive maintenance is aimed at reducing the probability of failure occurring during the operation of an equipment due to planned maintenance tasks carried out over a specific period of time. These tasks are planned to change components before they fail and are scheduled during machine stoppages or shutdowns. If not done or implemented properly, preventive maintenance can become very expensive for the manufacturing organisation.

According to Brammer (2018), the importance of preventive maintenance in a manufacturing organisation is as follows:

- Aims at reducing equipment downtime and improving system reliability thereby eliminating premature replacement of machinery and equipment.
- Aims at reducing environmental and workplace hazards and injuries thus improving safety and quality conditions for everyone.
- Aims at saving maintenance and replacement cost.

Preventive maintenance can be performed either as periodic (time based) maintenance or predictive (condition based) maintenance.

2.1.4.1 Periodic Maintenance

Periodic maintenance can be described as preventive maintenance tasks carried out according to a predetermined schedule to maintain the condition or operational status of manufacturing equipments. It is more cost-effective than suffering downtime waiting for a repair to be effected after a failure has occurred (CoJ, 2015).

CoJ (2015), identified some examples of the tasks likely to be required in periodic maintenance as follows:

- Checking high speed shaft alignment
- Checking brake adjustment, pad wears
- Checking performance of yaw drive and brake
- Bearing greasing
- Checking security of cable terminations
- Pitch calibration checks (for pitch regulated machines)
- Oil filter replacement

2.1.4.2 Predictive maintenance

Predictive maintenance can be described as a maintenance process that helps to determine the condition or state of operating equipment in order to predict when maintenance should be performed. The aim is to predict when failure might occur, and to prevent the occurrence of the failure by performing maintenance. This ensures that maintenance is planned before failure occurs. It allows the frequency of maintenance to be as low as possible to prevent unplanned reactive maintenance, without incurring costs associated with doing too much preventive maintenance (Flix, 2018). When predictive maintenance is implemented effectively and efficiently, maintenance is only performed when it is required. This ensures reductions in cost in the following:

- Reduction in the time spent on equipment maintenance
- Reduction on production hours lost to maintenance,
- Decrease in the cost of spare parts and supplies.

Examples of predictive maintenance according to Szwedo (2012), includes the following but not limited to:

- Vibration Analysis
- Infrared Thermograph
- Oil Analysis
- Visual Inspections

The key benefits of predictive maintenance according to Flix (2018), Misra (2007) and Szwedo (2012) are as follows:

- Provides increased operational life
- Results in decrease of downtime

- Allows for scheduled downtime
- Allows for money to be budgeted for repairs
- Lowers need for extensive parts inventory

It is important to note that predictive maintenance implementation is expensive, therefore is rarely used for less important parts of a piece of equipment (Liu, Wang, and Golnaraghi, 2010).

2.1.5 Corrective maintenance

Corrective maintenance consists of the actions taken to restore a failed piece of equipment or system to an operational state (Basri et al., 2017; Misra, 2007). The major aim is to maximise the effectiveness of all critical systems, minimise breakdowns, minimise unnecessary repairs, and reduce the deviations from optimum operating conditions (Olumuyiwa, 2014). It is a simple and straight forward maintenance strategy focusing on the principle of "fix it when it is broken" (Fredíksson and Larsson, 2012), involving the repair or replacement of the component or part that caused the overall system failure. It is important to note that this maintenance strategy leads to high levels of parts, components and systems breakdown and high repair and replacement costs, due to sudden failures that can occur (Basri et al., 2017). That is to say that it is the most expensive type of maintenance strategy and usually equipment service levels are generally below acceptable levels and the quality of product is usually affected (Olumuyiwa, 2014).

Corrective maintenance is carried out in three basic steps according to Misra (2007), in the following order:

- Diagnosis of the fault:
- Repair or replacement of faulty components:
- Verification of the repair action

This type of methodology is difficult to predict as equipment failure behaviour is stochastic and breakdowns are unanticipated (Krajewski and Sheu, 1994). Examples of such actions include:

- Replacement of a failed light bulb
- Repair of a ruptured pipeline
- Repair of a stalled motor.

Corrective maintenance and preventive maintenance identified from literature as the two basic strategy of maintenance with the aim of reducing unplanned downtime and increase available productive time, but with different methods or ways of achieving the aims and objective of maintenance as summarised in table 2.6.

Table 2.6 : Maintenance Strategies Source: (Basri et al., 2017; Prajapati and Bechtel, 2012)

Features	**Maintenance Strategy**	
	Corrective Maintenance	**Preventive Maintenance**
Approach to Maintenance	Reactive	Proactive
Maintenance Strategy	Fixing after failure	Periodic maintenance Predictive maintenance
Downtime	Highest	Less
Good for Failures	Random age-based	Age- based
Extensive (manpower)	Maximum	Little less
Required Schedule	Not applicable	Based on the

		standard useful life of component or history of failures
Action	Inspect, repair or replace after failure	Inspect, repair or replace at predetermined intervals, forecasted by design and updated through experience

2.2 Economic Implications on maintenance in manufacturing

According to Lazim, Taib, Lamsali, and Najib (2016), problems in maintenance can dampen manufacturing performance as equipment faces the probability of unplanned stoppages, breakdowns, failures and so forth occurring. This may increase the cost of maintenance and lead to the inability to deliver the product to the customer on time and also affect the quality of product produced.

Manufacturing organisations as a result of the growing importance of maintenance and maintenance management are making efforts to increase profitability by increasing labour productivity, while at the same time maintaining a high level of quality, service and timelines. Many studies (Al-Najjar and Alsyouf, 2003, 2004; Al-Najjar, 2007; Alsyouf, 2009), have discussed the economic implications of maintenance as it applies to the manufacturing industries showing how and effective maintenance policy affects productivity and profitability of a manufacturing process. In which general, improvements in the performance of a maintenance policy aims to reduce production cost, increase company's profit and competitiveness through enhancement of process availability, performance efficiency and quality rate (Al-Najjar, 2007).

A study by Al-najjar and Gomiscek (2015) describes this relationships as illustrated in the figure below:

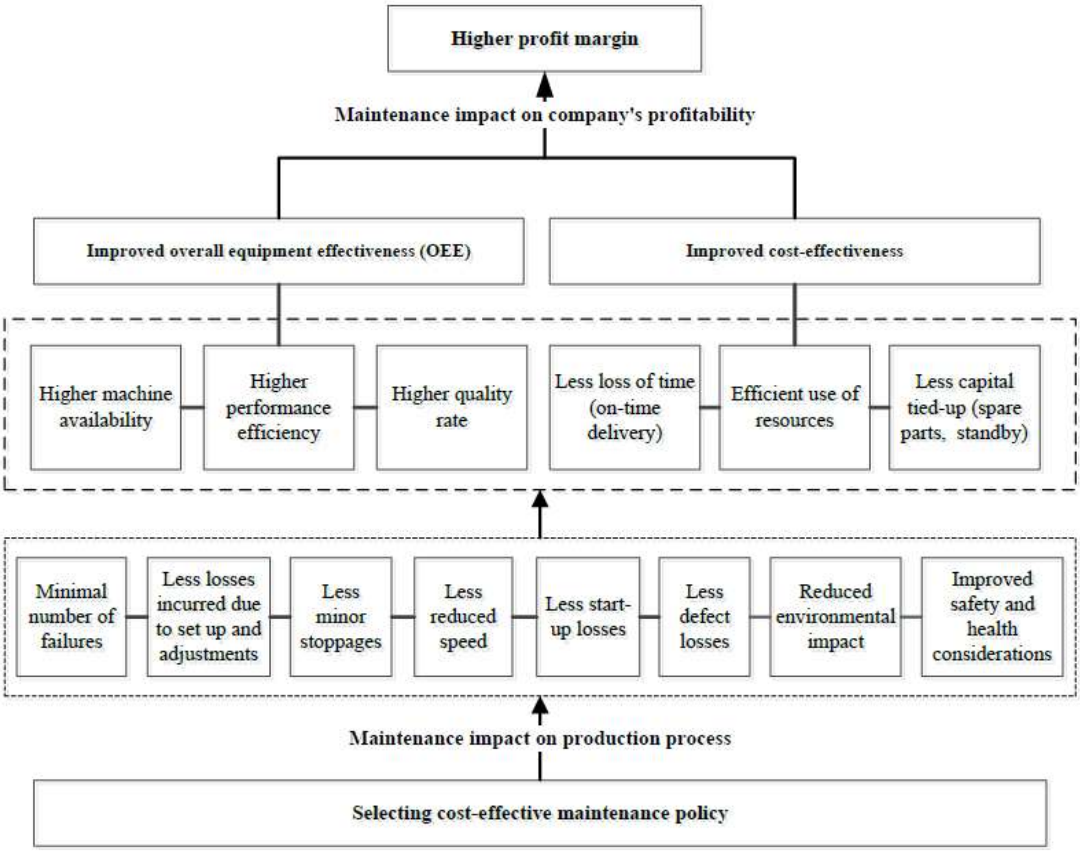
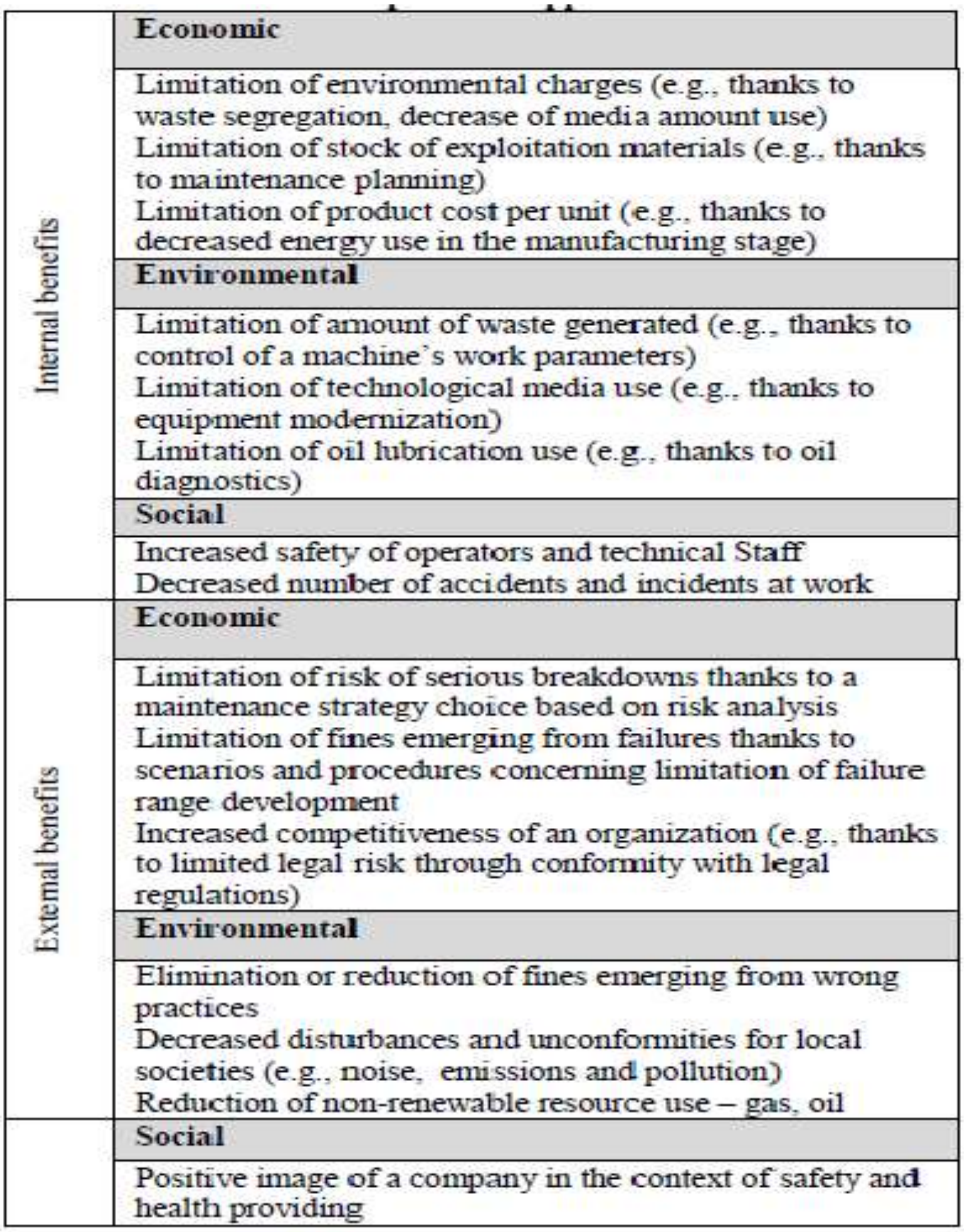


Fig 2.4: Relationship between maintenance and company's profits margins Source: Al-najjar and Gomiscek (2015)

Another study by Jasiulewicz (2013), identified the internal and external benefits gain by a manufacturing organisation through sustainable maintenance policies as outlined in figure 2.4



Internal benefits	**Economic**
	Limitation of environmental charges (e.g., thanks to waste segregation, decrease of media amount use) Limitation of stock of exploitation materials (e.g., thanks to maintenance planning) Limitation of product cost per unit (e.g., thanks to decreased energy use in the manufacturing stage)
	Environmental
	Limitation of amount of waste generated (e.g., thanks to control of a machine's work parameters) Limitation of technological media use (e.g., thanks to equipment modernization) Limitation of oil lubrication use (e.g., thanks to oil diagnostics)
	Social
	Increased safety of operators and technical Staff Decreased number of accidents and incidents at work
External benefits	**Economic**
	Limitation of risk of serious breakdowns thanks to a maintenance strategy choice based on risk analysis Limitation of fines emerging from failures thanks to scenarios and procedures concerning limitation of failure range development Increased competitiveness of an organization (e.g., thanks to limited legal risk through conformity with legal regulations)
	Environmental
	Elimination or reduction of fines emerging from wrong practices Decreased disturbances and unconformities for local societies (e.g., noise, emissions and pollution) Reduction of non-renewable resource use – gas, oil
	Social
	Positive image of a company in the context of safety and health providing

Figure 2.5: Maintenance Internal and External Benefits Source**:** Jasiulewicz (2013)

2.3 Maintenance Optimization

Maintenance has become a frequent practice of manufacturing industries, academic researchers and practitioners have worked on various ways to efficiently implement and manage maintenance (Olumuyiwa, 2014). Implementing an effective maintenance strategy or policy can be formulated as an optimization problem with the aim finding an optimum balance between maintenance cost and maintenance objectives while considering all possible constraints. Maintenance optimization is the problem of determining cost-optimal maintenance decisions for an object (system or structure or one of its components) to ensure a safe and economic operation (Mazzuchi, Noortwijk, & Jan Kallen, 2008). Thus it is performed by minimizing or maximizing one or two objective function presented in table 2.7 with consideration on various maintenance criteria presented in table 2.8.

Table 2.7: Objectives for Maintenance Optimization Source: (Hilber, 2008)

Objective Function	Description
Availability	Maximize availability under given constraints (e.g. cost constraints).
Minimal Cost	Minimize cost given constraints (on availability and/or maintenance requirements).
Minimal Total Cost	Minimize total cost (of interruptions and maintenance).

Table 2.8: Generic list of Optimization Criteria Source: (Horenbeek, 2013)

Maintenance Optimization Criteria	
Maintenance Cost	Reliability
Maintenance Quality	Maintainability
Personnel Management	Environmental impact
Inventory of spare parts	Safety/risk
Overall equipment effectiveness	Logistics
Number of maintenance interventions	Output quantity

Capital replacement decisions	Output quality
Life-cycle optimization	
Availability	

According to Dekker (1996), maintenance optimization process involves four aspects:

1. Description of a technical system, its function and importance.
2. Modelling of the deterioration of the system in time and possible sequences for the system.
3. A description of the available information about the system and the actions open to management
4. Objective function and an optimization technique which helps in finding the optimal balance

Horenbeek et al. (2011) suggested a generic maintenance optimization classification framework with the aim of collecting factors that have an impact on the optimization method such as optimization objectives and parameters. It provides a general overview of all possible maintenance optimization models making it possible to select the appropriate model based on the user experience. This classification framework is shown in figure 2.5.

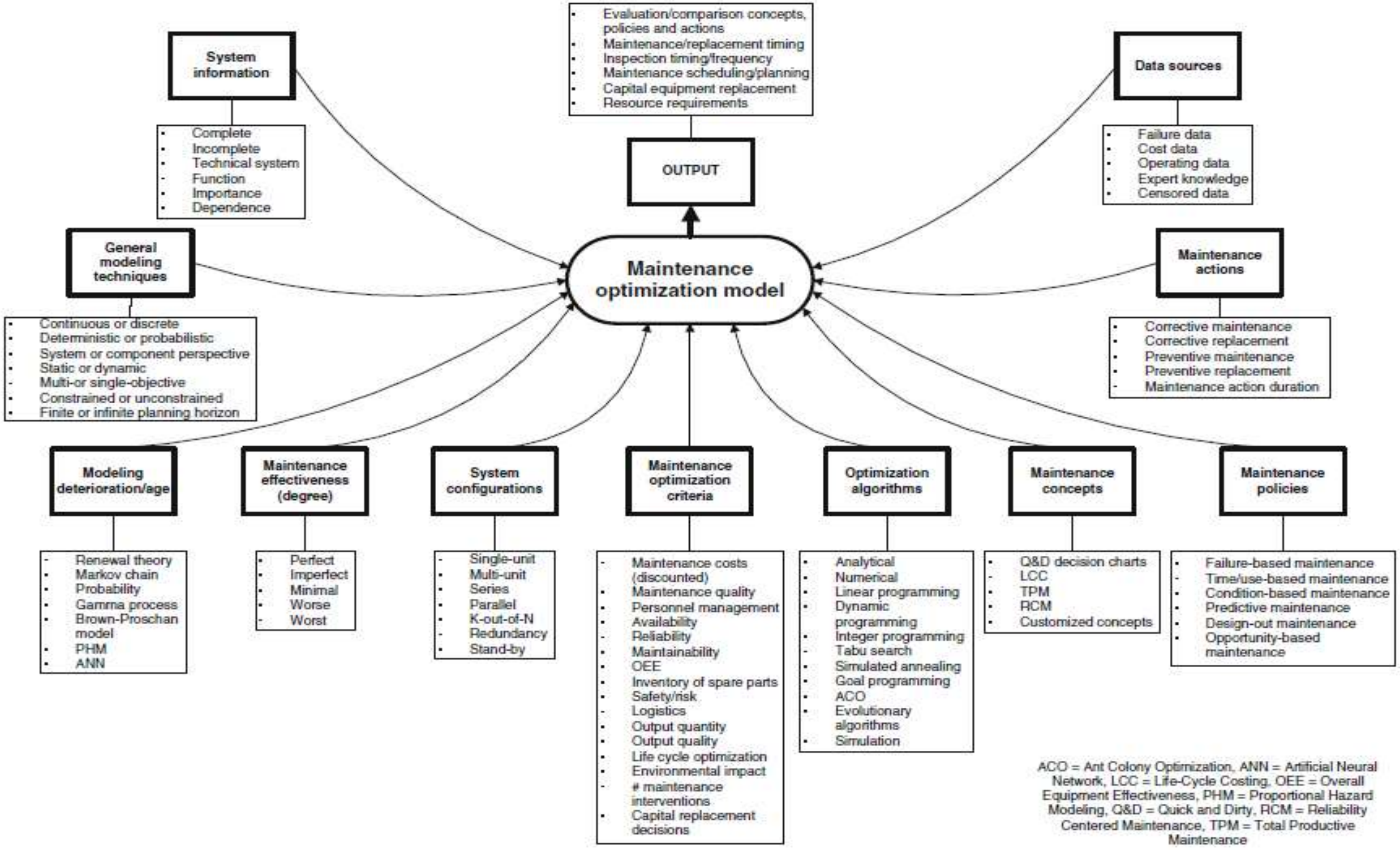


Fig 2.6: Maintenance Optimization Classification Framework Source: Horenbeek et al., 2011

2.4 Maintenance Optimization Methods

The optimization of maintenance for manufacturing components can be achieved through various means of quantitative methods and techniques. These techniques vary with the kind of problem being addressed which eventually leads to the model and method applied for optimization (Olumuyiwa, 2014).

2.4.1 Exact Optimization Methods

Exact optimization methods are methods designed to find the real optimal solution of a problem. These methods are efficient at finding the optimal solution for small problems. The computation time and effort increases significantly as the problem becomes larger and more complex. This method includes analytical and numerical methods, linear and nonlinear programming, mixed integer and dynamic programming (Rothlauf, 2011).

2.4.2 Analytical and Numerical Methods

This method makes use of mathematical analytic functions to provide solutions to equations describing changes in a system. Analytical methods have been used as an optimization approach for maintenance optimization (Olumuyiwa, 2014). A study by Oke, (2005) presented an analytical model to measure profitability of a maintenance system, using a case study for application. The study used simulation experiments and demonstrated that maintenance profitability can be realised through the use of differential calculus.

2.4.3 Linear & Nonlinear Programming Methods

Linear programming is an optimization method applied when solving problems with objective functions and constraints appearing as linear functions of the decision variables, in which the constraint equation can be in either equality or inequality forms

(Olumuyiwa, 2014). Nonlinear programming is an optimization method used to solve problems that have objective functions and constraints that are not stated as explicit functions of the design variables.

2.4.4 Integer Programming Methods

This method can either be a mixed-integer linear program, which is a mathematical program that involves the minimization or maximization of a linear function subject to linear constraints, where the decision variables assume only integer values or a mixed integer non-linear program which is a mathematical program with continuous and discrete variables and nonlinearities in the objective function and constraints (Olumuyiwa, 2014).

Vassiliadis and Pistikopoulos (2001) presented an optimization framework using mixed integer nonlinear optimization model. The objective was to identify the number of Preventive Maintenance and Corrective Maintenance actions required over a given time horizon of interest as well as the time instants and sequence of these maintenance actions on the various components of the process system, so that the system efficiency is maximized. Mixed integer linear program was applied by Matsuoka and Muraki (2007) to evaluate the balance between labour cost, material cost and opportunity cost and this was solved to get an optimal solution in the area of short-term maintenance scheduling of utility systems.

2.4.5 Dynamic Programming Method

Maintenance decision making involves several decisions to be taken at different times, and the mathematical technique used to optimize such a sequence of interrelated decisions over a period of time is the dynamic programming method (Sharma, 2010).

A study by Zhou, Xi, and Lee (2009) developed an opportunistic preventive maintenance scheduling algorithm based on dynamic programming and integrated imperfect effect into maintenance actions. Simulation was used to optimize the maintenance practice by maximizing the short term cumulative opportunistic maintenance cost savings for the whole system under study.

2.4.6 Simulated Annealing Method

This method is an approach to finding the approximate solution of difficult combinatorial optimization problems. It is based on randomness for global optimization problems, imitates the annealing process in the material processing when a metal cools and freezes into a crystalline state with the minimum energy and larger crystal size so as to reduce the defects in metallic structures (Olumuyiwa, 2014). Simulated annealing has being applied by several studies to optimize maintenance, Safaei, Banjevic, and Jardine (2008) proposed a multi-objective simulated annealing (MOSA) algorithm to solve a real maintenance workforce scheduling problem (MWSP) with the aim of simultaneously minimizing the workforce cost and the flow time of the work requests. Manuela, Fata, and Passannanti (2017) developed a model combining Simulated Annealing-based algorithm with Monte Carlo simulation for the joint optimization of age replacement Preventive Maintenance (PM) and inventory control policies with the aim of minimizing the total expected cost per unit time. The model was formulated with reference to a continuous production system characterized by a random deteriorating behaviour so that the presence of a buffer is considered to ensure a continuous products supply during interruptions of service caused by breakdowns or planned maintenance actions on the production system. Doostparast, Kolahan, and Doostparast (2015) in their study applied simulated annealing in order to find the optimal frequency and types of maintenance actions required to achieve a certain level of system availability with minimum total cost.

2.4.7 Genetic Algorithm

Genetic Algorithm is a search heuristic that mimics the process of evaluation modelled after evolutionary mechanisms (Lynch, 2006; Malhotra, Singh, and Singh, 2011). It is a popular optimization method for non-linear systems with a large number of variables (Lynch, 2006). The main advantage of this optimization algorithm is that it is capable of exploring a larger area of the solution space with a smaller number of objective function evaluations (M Ali Ilgin & Tunali, 2007; Mehmet Ali Ilgin, 2006).

Using genetic algorithm as a solution to an optimization problem consists of two main steps: (i) Defining a data structure, which consists of possible solution and (ii) Defining an objective function, which evaluates the possible solutions to select the optimum (Lapa, Pereira, and de Barros 2006).

The following outlines the procedure for carrying out genetic algorithm (Yang, 2010):

- Encoding the objective functions.
- Defining a fitness function or selection criterion.
- Initializing a population of individuals'
- Evaluating the fitness of all individuals in the population.
- Creating a new population by performing crossover, mutation fitness proportionate reproduction etc.
- Evolving the population until certain stopping criteria are met.
- Decoding the result to obtain the solution to the problem

Several studies have been carried out implementing genetic algorithm in the optimization of maintenance. M Ali Ilgin and Tunali (2007) adopted a simulation optimization approach using genetic algorithms for the joint optimization of preventive maintenance (PM) and spare provisioning policies of an automotive manufacturing system. In the study a factorial experiment was carried out to identify the best values for the genetic algorithm parameters, including the probabilities of crossover and mutation, the population size, and the number of generations. The computational experiments showed that the parameter settings given by the approach achieves a significant cost reduction while increasing the throughput of the manufacturing system. Tsai, Wang, and Teng (2001) incorporated genetic algorithm in planning periodical preventive maintenance in a manufacturing system based on maximizing the unit cost life of a system.

Samhouri, Al-Ghandoor, Fouad, and Ali (2009) presented genetic algorithm method on how to decide whether a particular item requires opportunistic maintenance or not, and if so how cost effective this opportunity-based maintenance will be when compared to a probable future grounding. Marseguerra, Zio, and Podofillini (2002) used a Genetic Algorithm (GA) to determine the optimal degradation level beyond which preventive maintenance has to be performed in a continuously monitored multi-component system. Lapa et al., (2006) applied GAs to present a model to optimize preventive maintenance policies based on the cost-reliability model. Saranga (2004) applied genetic algorithm to decide whether opportunistic maintenance is cost effective during analysis of manufacturing components that need opportunistic maintenance.

2.4.8 Simulation Optimization Method

Simulation optimization can be defined as the process of finding the best input variable values from among all possibilities without explicitly evaluating each possibility (Amaran, Sahinidis, Sharda, and Bury 2015; Carson and Maria, 1997)**.** The objective is to minimize the resources spent while maximizing several objectives. According to Amaran et al.,(2015) simulation optimization methods are most commonly applied to as either discrete-event simulations, or systems of stochastic nonlinear and/or differential equations.

Discrete event simulation (DES) is widely applied to the area of maintenance, mainly due to its ability to model stochastic changes in flexible systems Gopalakrishnan, Skoogh, and Laroque (2014), while Stochastic differential equations may be used to model phenomena ranging from financial risk to the control of nonlinear systems to the electrophoretic separation of DNA molecules (Carson and Maria, 1997).

A common simulation model comprises *n* input variables (X_1, X_2, X_3......X_n) and m output variables (Y_1, Y_2, Y_3 ......Y_m) as illustrated in figure 2.6

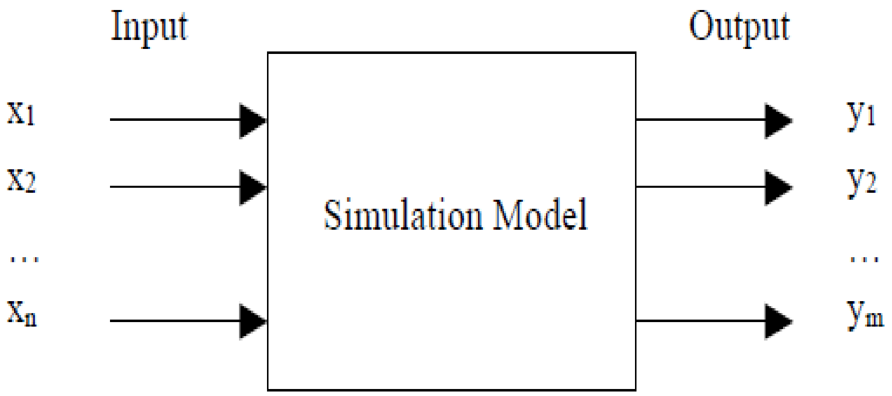


Fig 2.7: A typical simulation model Source: Carson and Maria, (1997)

From figure 2.6, simulation optimization method tends to find the optimal settings for input variables which will ten optimize the output variables. Figure 2.7 shows a typical simulation optimization method.

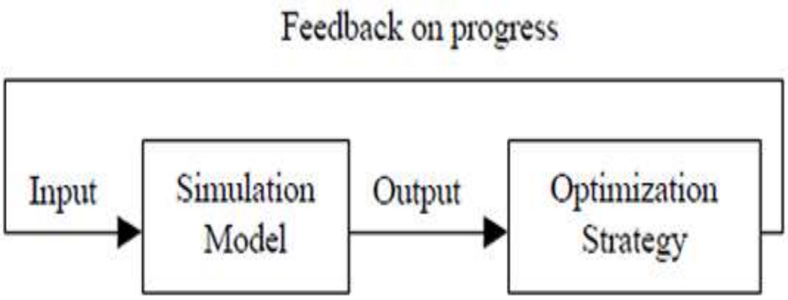


Figure 2.8: A typical simulation optimization method Source: Carson and Maria, (1997)

Several studies have adapted simulation as a solution technique for the optimization of maintenance, Tahvili, Österberg, Silvestrov, and Biteus (2014) explored the use of stochastic simulation, and genetic algorithms for solving complex maintenance planning optimization problems based on discrete event simulation. Alrabghi, Alabdulkarim, and Tiwari 2013) optimized preventive maintenance and spare provision policy under continuous review in a non-identical multi-component manufacturing system through a combined discrete event and continuous simulation model coupled with an optimization engine. The study showed that production dynamics and labour availability have a significant impact on maintenance performance. Gopalakrishnan et al.,(2014) applied discrete event simulation to integrate maintenance policies into a production planning approach in an automotive manufacturing system. The results of the study showed that introducing priority-based planning of maintenance activities has a potential to increase productivity by approximately 5%. Dingzhou, Sun, and Huairui (2013) estimated system cost (availability) in a manufacturing system using discrete event simulation technique and the Optimal Computing Budget Allocation (OCBA) mechanism to try to find the optimal maintenance policies for the system.

2.5 Repairable Systems in Manufacturing

Manufacturing and engineering systems can either be classified as repairable or non-repairable systems. A repairable system as implied by the name is a system that can be repaired back to its operating condition on an event of failure, while non-repairable systems refer to systems in which when a failure occurs, the system is discarded because repairing the system is not economically feasible (Olumuyiwa, 2014).

Recently analysis of repairable systems based on parametric and non-parametric methods are becoming increasingly popular due to their simplicity as well as ability to handle more than just counts of recurrent events (Nelson, 2003, 2005; Trindade and Nathan, 2005). Parametric and non-parametric method of analysis is usually carried out to a repairable system in order to determine whether system failures are becoming more frequent, less frequent or constant using power law process or log linear process for a parametric method and mean cumulative function for a non-parametric method. For other methods see (Faulin, Juan, Alsina, and Ramirez-Marquez, 2010).

Parametric Method:

- Power Law Process

$$u(t)= \lambda\beta t^{\beta-1} \qquad \lambda, \beta > 0 \qquad (2.4)$$

Where

$u(t)$ = failure intensity, i.e. Rate of occurrence of failure

λ = scale parameter (failure function)

β = shape parameter (improvement/degradation)

The parameter ƛ can be used to understand the reliability growth of the system under the following conditions:

- If 0 < ƛ < 1, the failure/repair rate is decreasing. Thus, system is improving over time.
- If ƛ = 1, the failure/repair rate is constant. Thus, system is remaining stable over time.
- If ƛ > 1, the failure/repair rate is increasing. Thus, system is deteriorating over time.

The expected number of failures for the time interval t_1, t_2 is

$$E[N(t_2) - N(t_1)] = \int_{t1}^{t2} u(t)dt. \qquad (2.5)$$

$$[N(t_2) - N(t_1)] = \lambda\,(t_2^{\beta} - t_1^{\beta})\ \lambda, \beta > 0, T_2 \geq T \geq 0 \qquad (2.6)$$

Where

E = Expected failure

$T_{i...N}$ = Time from the start of failure to the end of observation

N = Number of failures

The reliability function for the interval t_1, t_2 is given by

$$[t_2, t_1] = e^{-\lambda}\,(T_2^{\beta} - T_1^{\beta}) \qquad \lambda, \beta > 0, T_2 \geq T \geq 0 \qquad (2.7)$$

- Log linear process

$$u\,(t) = e^{\alpha 0} + \alpha^{1t} \quad -\infty < \alpha_0,\ \alpha_1 < \infty, \qquad t \geq 0 \qquad (2.8)$$

This method gives a good representation of a repairable system with $\alpha_1 > 0$, the expected number of failures for the interval t_1, t_2 is

$$E[N(t_2) - N(t_1)] = \int_{t1}^{t2} u(t)dt. \qquad (2.9)$$

$$[\boldsymbol{N}(\boldsymbol{t}_2)-\boldsymbol{N}(\boldsymbol{t}_1)] = \frac{e\alpha 0}{\alpha 1} \ (\boldsymbol{e}^{\alpha_1 t_2} - \boldsymbol{e}^{\alpha_1 t_1}) \quad -\infty < \boldsymbol{\alpha_0}, \ < \infty, \ \ \boldsymbol{T}_2 \geq \boldsymbol{T} \geq \boldsymbol{0} \qquad (2.10)$$

The reliability function for the interval t_1, t_2 is given by

$$[\boldsymbol{t}_2,\boldsymbol{t}_1]=\boldsymbol{e}^{(e\alpha 1t2 - e\alpha 1t1)} \quad -\infty < \boldsymbol{\alpha_0}, \ < \infty, \ \ \boldsymbol{T}_2 \geq \boldsymbol{T} \geq \boldsymbol{0} \qquad (2.11)$$

Non parametric method:

- Mean cumulative function

$$\boldsymbol{m}(\boldsymbol{t}) = \boldsymbol{E}\ [\ \boldsymbol{N}(\boldsymbol{t})] \qquad \textbf{(2.12)}$$

Where

$$\boldsymbol{N}(\boldsymbol{t}) = \frac{\boldsymbol{1}}{\boldsymbol{n}} \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{n}} \boldsymbol{N}_{\mathrm{i}}\,(\boldsymbol{t}) \qquad (\boldsymbol{2.13})$$

The failure intensity function:

$$m(t) = {}^{dM(t)}\!/_{dt} \qquad (2.14)$$

From the above equation $m(t)\Delta t$ describes the expected number of failures per system within the interval (t, t + Δt).

The effective reliability of a system over a certain time frame during the system's lifetime can be determined based on historical failure data of the system. Historical failure data may exhibit an increasing failure rate (IFR), a constant failure rate (CFR) or a decreasing failure rate (DFR). A system exhibiting an IFR, also called an ageing system, generally comprises components that wear out over the lifetime of the system, which causes the time between consecutive failures to decrease. Systems with IFRs generally require the most attention with respect to planned maintenance, since failures in the system are observed more frequently towards the end of the system's lifetime. Some systems exhibit CFRs, where the failures in the system are observed to be random with inter-failure rates that are exponentially distributed. These systems require less attention when planning maintenance due to the randomness between consecutive failures. In the case where a system exhibits a DFR, the time between consecutive failures is increasing hence fewer failures are observed towards the end of the system's lifetime and so the system may be described as an improving system. For improving systems, it might sometimes be harmful to conduct planned maintenance as the system naturally increases in reliability over time. The well-known bathtub curve, presented in Figure 2.8, may be used to represent the failure rate of a system graphically as a function of the lifetime of the system. Typically, from the start of the lifetime of a system up to a certain time t_1, the system exhibits a DFR. Between times t_1 and $t_2 > t_1$, the system is in its useful stage and exhibits a CFR. The final part of the graph, from t_2 onwards, represents final part of the system's lifetime, which exhibits an IFR.

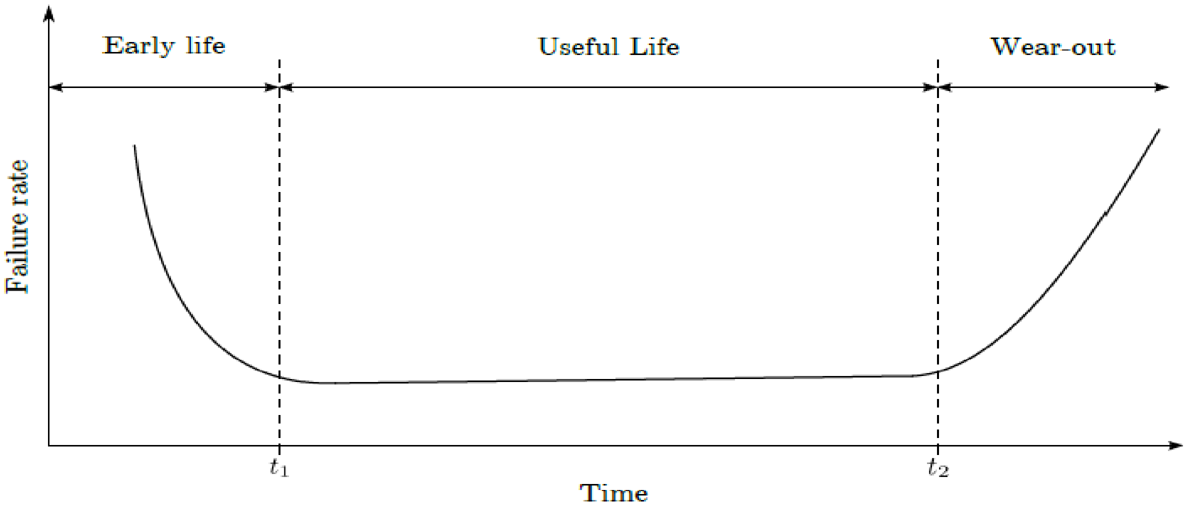


Fig 2.9: The failure rate of a system as a function of time Source: Olumuyiwa,(2014)

2.6 Reliability Lifetime Distribution Models for Repairable Systems

The choice of an appropriate lifetime distribution model for a non-repairable system is usually determined by three main factors, namely:

(i) Whether statistical or physical evidence exist that relates the lifetime distribution model to the failure mechanism on a theoretical basis,

(ii) Whether the lifetime distribution model has previously been used to represent a similar failure mechanism and has proved successful in that context

(iii) Whether the lifetime distribution model is flexible and convenient enough to fit the failure data empirically

Four of the most popular lifetime distribution models for repairable systems are described in this section. In each case, the corresponding PDF, CDF and hazard rate are described. The models reviewed are the exponential model, the Weibull model, the normal model, and the lognormal model.

2.6.1 The Exponential Model

This is a very simple model with only one unknown parameter ƛ that has to be estimated.

The probability density function (PDF) of the exponential lifetime distribution is denoted as follows:

$$f(t) = ƛ\mathrm{e}^{-ƛ\mathrm{t}} \qquad (2.15)$$

The cumulative distribution function (CDF) of the exponential lifetime distribution model is denoted as follows:

$$f(t) = \int_{t}^{\infty} ƛ\mathrm{e}^{-ƛ\mathrm{t}}\, dx = 1 - e^{ƛ\mathrm{t}} \qquad (2.16)$$

From (2.16) the reliability of a system is now calculated as

$$R(t) = 1 - \left(1 - e^{ƛ\mathrm{t}}\right) = e^{ƛ\mathrm{t}} \qquad (2.17)$$

The system reliability can be calculated at any time instant t. With both $f(t)$ and $R(t)$ known, the hazard rate of the system is denoted as:

$$h(t) = \frac{\lambda e^{-\lambda t}}{e^{\lambda t}} = \lambda \qquad (2.18)$$

This results in an exponential failure rate that is reduced to the value λ for all time values t. The exponential distribution is the only lifetime distribution with a constant failure rate. The exponential model is widely used in industry for the modelling of the flat part of the bathtub curve, shown in Figure 2.8, between the time instants t_1 and t_2, due to the model having a constant failure rate. The mean time to failure (MTTF) M for the exponential lifetime distribution may be calculated by

$$M = \frac{1}{\lambda} \qquad (2.19)$$

This model is only applicable when early decreasing failures and late wear-out failures are not taken into account. The exponential model is sometimes also used as proxy for other failure models by approximating an exponential piecewise function for the model under consideration.

2.6.2 The Weibull Model

The Weibull model is another flexible lifetime distribution model that can be used in a wide range of reliability problems. The model has two parameters, namely a scale parameter λ and a shape parameter β. The probability density function (PDF) of the weibull lifetime distribution model is denoted as follows:

$$F(t) = e^{-\lambda t^{\beta}} \qquad (2.20)$$

The cumulative distribution function (CDF) of the weibull lifetime distribution model is denoted as follows:

$$f(t) = \int_{t}^{\infty} e^{-\lambda t^{\beta}} \, dx = 1\text{-}\ e^{-\lambda t^{\beta}} \tag{2.21}$$

From (2.21) the reliability of a system is now calculated as

$$R(t) = 1 - (1 - e^{-\lambda t^{\beta}}) = e^{-\lambda t^{\beta}} \tag{2.22}$$

With both $f(t)$ and $R(t)$ known, the hazard rate of the system is denoted as:

$$h(t) = \frac{1 - e^{-\lambda t\beta}}{e^{-\lambda t\beta}} \tag{2.23}$$

The flexibility of the Weibull model has resulted in the model being applied successfully to a wide range of problems. The Weibull model has also been used as a form of extreme value analysis where the earliest failure time of many competing failures are determined in order to determine system failure.

2.6.3 The Normal Model

The Gaussian distribution, more commonly known as the normal distribution, is probably the most widely-used distribution in statistics. The normal distribution is therefore also adopted as a lifetime distribution model in reliability theory. This model, however, has a left-hand limit extending to negative infinity which is not realistic when modelling failure time data (as time values are normally restricted to non-negative values). For this reason the normal model has been argued by some to be inappropriate for use as a lifetime distribution model (Rothlauf, 2011). The negative left-hand limit may, however, largely be avoided provided that the distribution has a large mean and a relatively small standard deviation. The probability density function (PDF) of the normal lifetime distribution model is denoted as follows:

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2] \tag{2.24}$$

Where μ denotes the mean of the times to failure and σ denotes the standard deviation of the times to failure.

The cumulative distribution function (CDF) of the normal lifetime distribution model is denoted as follows:

$$f(t) = \int_t^\infty \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2]\,\mathrm{dx} \tag{2.25}$$

This may be approximated by

$$f(t) \approx \int_t^0 \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2]\,\mathrm{dx} = \Phi(\mathrm{t})$$

This integral is the standard normal CDF, which may also be written as

$$\Phi(t) = \frac{1}{2}\,[\,1 + \mathrm{erf}\left(\frac{t-\mu}{\sigma}\right)] \tag{2.26}$$

The reliability of a system is now be calculated according to the normal lifetime distribution model as

$$R(t) = 1 - \Phi\left(\frac{t-\mu}{\sigma}\right) = \int_t^\infty \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2]\,\mathrm{dx} \tag{2.27}$$

With both $f(t)$ and $R(t)$ known, the hazard rate of the system is denoted as:

$$h(t) = \frac{\frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2]}{\int_t^\infty \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2]\,\mathrm{dx}} \tag{2.28}$$

The MTTF for the normal lifetime distribution model is simply the mean of the normal distribution, namely M $=\sigma$. The expected number of failures to have occurred by a certain time t may be calculated by taking the integral of (2.28), i.e.

$$H(t) = h(t) = \frac{\frac{1}{\sigma\sqrt{2\pi}}\exp[-\frac{1}{2}(\frac{t-\mu}{\sigma})^2]}{\int_t^\infty \frac{1}{\sigma\sqrt{2\pi}}\exp[-\frac{1}{2}(\frac{t-\mu}{\sigma})^2]\ \mathrm{dx}}dt \tag{2.29}$$

2.6.4 The Lognormal Model

As with the Weibull model, the lognormal model is mainly applied in two-parameter form. The lognormal distribution's two parameters are a shape parameter α, and a median T_{50}, which fulfil the role of a scale parameter. If a system exhibits a time to failure that follows a lognormal distribution, the natural logarithm of such a time to failure has a normal distribution. Therefore, if the natural logarithms of the failure times are taken, the data may be considered normally distributed with mean $\mu = \mathrm{ln}T_{50}$ and with standard deviation σ. After analysis, the failure times may be converted back from logarithmic time to normal time. The probability density function (PDF) of the lognormal lifetime distribution model is denoted as follows:

$$f(t) = \frac{1}{\sigma t\sqrt{2\pi}}\exp[-\frac{1}{2}(Int - \mu)^2 \tag{2.30}$$

The cumulative distribution function (CDF) of the lognormal lifetime distribution model is denoted as follows:

$$f(t) = \int_t^\infty \frac{1}{\sigma x\sqrt{2\pi}}\exp[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2]\ \mathrm{dt} = \Phi\ (\frac{In\,t-\mu}{\sigma}) \tag{2.31}$$

Where Φ is the standard normal cumulative distribution function.

The reliability of a system is now be calculated according to the lognormal lifetime distribution model as

$$R(t) = 1 - \Phi\left(\frac{In\,t - \mu}{\sigma}\right) \tag{2.32}$$

With both $f(t)$ and $R(t)$ known, the hazard rate of the system is denoted as:

$$h(t) = \frac{\frac{1}{\sigma t\sqrt{2\pi}}\exp[-\frac{1}{2}(Int - \mu)^2}{1 - \Phi\left(\frac{In\,t - \mu}{\sigma}\right)} \tag{2.33}$$

Taking the natural logarithm of the failure time data makes for a very convenient model, as the failure time data are now in normal form, which makes it mathematically easier to use. The lognormal model has been employed in the literature to model physical degradation in electronics for failures as a result of as corrosion, diffusion, crack growth etc.

2.7 Failure Data Classification

In order to construct a lifetime distribution model which is capable of making a good prediction of the lifetime of a system, failure data or times-to-failure data are required that are accurate and complete. In practice, however, it is not always possible to obtain complete data or the data may contain some uncertainty, but such data may nevertheless still be useful for the model in some cases. Data such as those mentioned above may be classified into two categories, namely complete data and censored data (Rothlauf, 2011).

2.7.1 Complete Failure Data

A complete set of data contains times to failure for all the systems in a sample. The exact failure time of each of the systems in the sample is therefore known in this case. A graphical representation of the case of complete data is shown in Figure 2.9 for a sample of six systems. Note that each system was observed until a failure occurred.

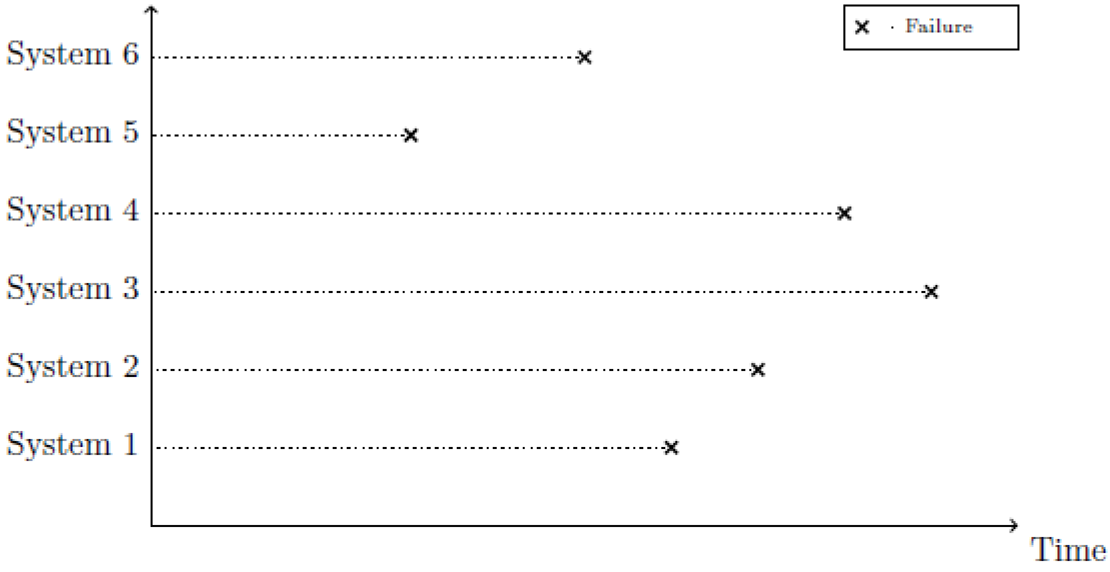


Fig 2.10: A Graphical Representation of a complete Failure Data

Source: Olumuyiwa, (2014)

2.7.2 Right Censored Failure Data

Suspended data, or right-censored data, is the most common case of censoring. In this case, a failure is not observed for every system in the sample. A graphical representation of right censored data is shown in Figure 2.10 for a sample of six systems. In the figure, three of the six systems did not fail and therefore the failure data of these three systems are referred to as right-censored data. The term right-censored is used, because failures are expected to occur to the right of the current lifetimes of these three systems.

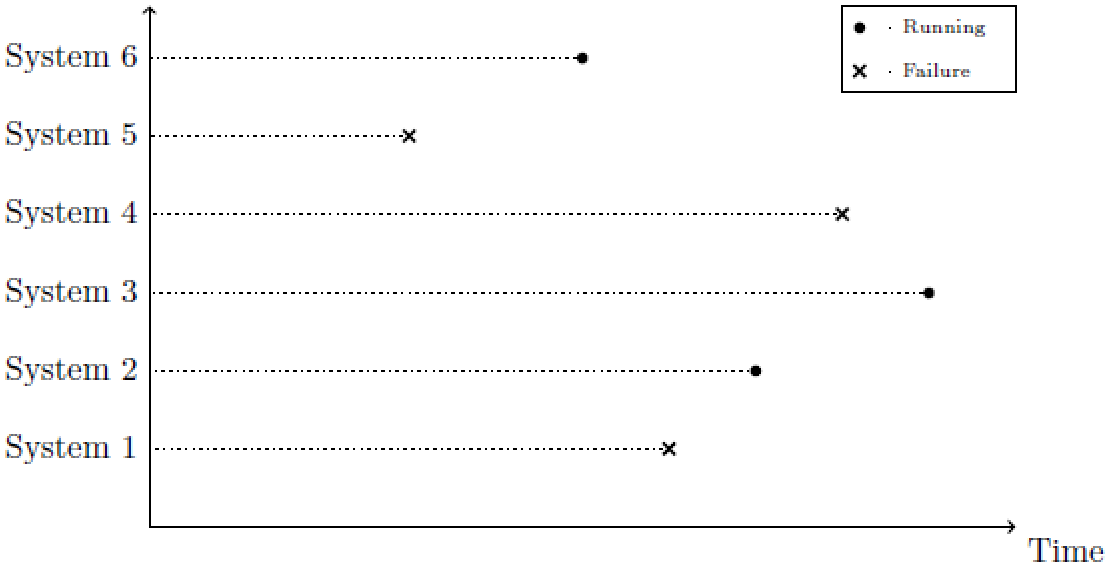


Fig 2.11: A graphical Representation of a Right Censored Failure Data

Source: Olumuyiwa, (2014)

2.7.3 Interval Censored Failure Data

Another type of censored data is interval-censored data. This type of censored data refers to a certain interval during which a system is known to have failed. The exact failure time is uncertain, but it is known in this case that the system failed within a certain time interval during the system's lifetime. This type of data typically arises when the state of a system is not continuously monitored but rather monitored at fixed points in time during the lifetime of the system. In this case, a system will be functioning when performing an inspection of the system, but when performing the next inspection, it may be noticed that the system has already failed. A graphical representation of interval-censored data is shown in Figure 2.11 for a sample of six systems. When it is not possible to continuously monitor a system in order to observe its failures, the interval inspection approach has to be adopted. Interval inspection, however, does not capture as much information as complete data or right-censored data, and is therefore avoided if possible.

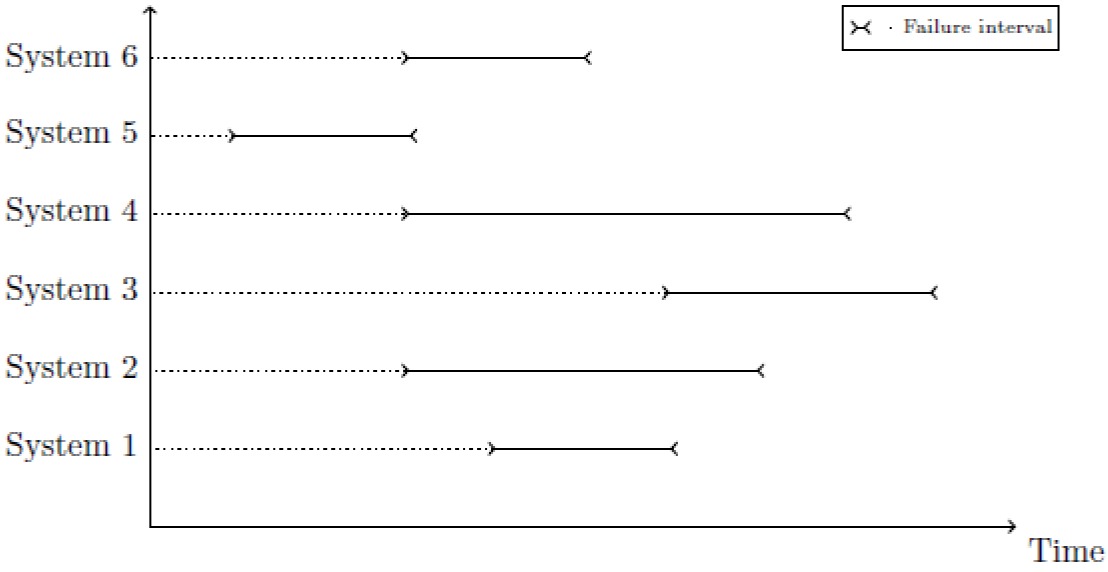


Fig 2.12: A Graphical Representation of an Interval Censored Failure Data

Source: Olumuyiwa, (2014)

2.7.4 Left Censored Failure Data

Left-censored data are similar to interval-censored data in the sense that the exact failure time of the system is not known. A failure in the case of left-censored data is, however, only known to have occurred before a certain time during the system's lifetime. This type of data is typically gathered when the inspection interval is too large, which causes the system to fail before it is inspected for the first time. A graphical representation of the case of left-censored data is shown in Figure 2.12 for a sample of six systems.

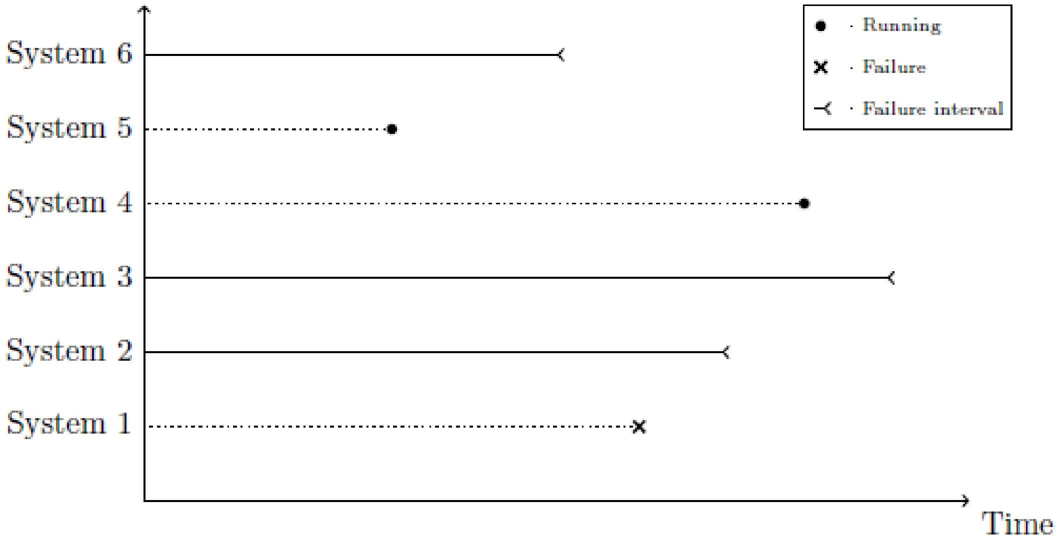


Fig 2.13: A Graphical Representation of a Left Censored Failure Data

Source: Olumuyiwa, (2014)

2.8 Review of Research Works Relevant to the Study

According to Bhamu and Sangwan (2014), the most economic and effective way to carry out literature research is through the use of internet and academic databases. Therefore, Google Scholar and Scopus database was mainly used to start the search for quality academic research papers on the optimization of maintenance applications in manufacturing systems. While reviewing from Google Scholar and Scopus database, more papers were found from cross referencing from publishing groups such as Emerald Insight, Elsevier, Taylor and Francis, Springer, Sage, IEEE and Inderscience publishing groups, and search engines such as Web of Science, Mendeley, Proquest, JSTOR, Index Copernicus and EBSCO, because they contained relevant and required information.

Relevant works gathered were analysed and summarised in the following categories:

- Year of publication
- Author/Authors
- Publication
- Objective of the study

- Solution Method

Marseguerra et al., (2002) in "*Condition-based maintenance optimization by means of genetic algorithms and Monte Carlo simulation"* applied of Genetic Algorithm (GA) to determine the optimal degradation level beyond which preventive maintenance has to be performed in a continuously monitored multi-component system. Monte Carlo (MC) simulation was also used to create a predictive model describing the evolution of the degrading system. The study concluded that the identification of efficient strategies for the maintenance of a plant or of an engineering system is of great importance both from the safety and the financial point of view. And that the advantages of performing condition based preventive maintenance, i.e. maintenance based on the component degradation state, have become more and more evident. A study "*Opportunistic maintenance using genetic algorithms"* by Saranga (2004) applied genetic algorithm to decide whether opportunistic maintenance is cost effective during analysis of manufacturing components that needed opportunistic maintenance. The study concluded that further work needs to be done to see if a customized genetic algorithmic software package can be developed for the purpose of opportunistic maintenance in complex systems. It also identified multi objective genetic algorithm as another area which needs to be explored for a possibility to incorporate optimization of other parameters like downtime, cost of maintenance etc.

Sortrakul, Nachtmann and Cassady (2005) in "*Genetic algorithms for integrated preventive maintenance planning and production scheduling for a single machine"* Developed a heuristics based on genetic algorithms to solve an integrated optimization model for production scheduling and preventive maintenance planning. The results from the study indicated that the proposed genetic algorithms are very efficient for optimizing the integrated problem. Nguyen and Bagajewicz (2008) in **"***Optimization of Preventive Maintenance Scheduling in Processing Plants"* developed of a methodology to optimize both the planning of preventive maintenance and the amount of resources needed to perform maintenance in a process plant using Monte Carlo Simulation and Genetic Algorithm.

M Ali Ilgin and Tunali, (2007) in **“***Joint optimization of spare parts inventory and maintenance policies using genetic algorithms”* develop a simulation optimization approach using genetic algorithms for the optimization of spare parts inventory and maintenance policies. The study carried out a factorial experiment to identify the best values for the GA parameters, including the probabilities of crossover and mutation, the population size, and the number of generations. The results showed that the parameter settings given by the proposed approach achieves a significant cost reduction while increasing the throughput of the manufacturing system.

Lapa et al., (2006) developed a model using genetic algorithm to optimize preventive maintenance policies based on the cost-reliability model. To evaluate the model, the High Pressure Injection System (HPIS) of a typical 4-loop PWR was used as a case study. The results obtained outlined its good performance, allowing specific analysis on the weighting factors of the objective function. The study concluded that applying the proposed cost-reliability model, it is possible to find preventive maintenance policies which provide a high level of reliability with low costs. Samhouri et al.,(2009) in **“***An Intelligent Opportunistic Maintenance (OM) System: A Genetic Algorithm Approach”* presented a methodology on how to decide whether a particular item requires opportunistic maintenance or not using genetic approach. An example of applying opportunistic maintenance strategy in process industry was used to describe the methodology for genetic algorithms. Lee and Ni (2012) in “*Genetic Algorithm for Job Scheduling with Maintenance Consideration in Semiconductor Manufacturing Process*” developed optimization methods which would lead to the best wafer release policy in the chamber tool to maximize the overall yield of the wafers in semiconductor manufacturing system using genetic algorithm. The result showed that job scheduling has to be managed based on the chamber degradation condition and maintenance activities to maximize overall wafer yield.

Bon (2016) in **“***Assembly Line Optimization using Arena Simulation”* improved the productivity assembly line by using ARENA simulation software. Simulation based optimization was implemented to improve the productivity assembly line. Malamura and Murata, (2012) developed a simulation model combining two processes of different origin (the production process and the information flow of maintenance work-orders) in one model. Of which the study aims to extend the capabilities of the conventional reliability analysis techniques by focusing on dynamic aspects of the production system and to present a method for representation of technical, operational, maintenance, organizational and economic aspects by simulation modelling concepts.

The results show that combined strategies customized according to the requirements of the different plant segments devote more attention critical components while avoiding costly over-care where not appropriate. Rezg, Chelbi, and Xie, (2005) in “*Modelling and optimizing a joint inventory control and preventive maintenance strategy for a randomly failing production unit: Analytical and simulation approaches”* developed a mathematical model to evaluate the average cost per time unit in joint optimal inventory control and preventive maintenance strategy for a randomly failing production unit which supplies an assembly. Also Rezg, Xie, and Mati, (2004) in “*Joint optimization of preventive maintenance and inventory control in a production line using simulation”* developed an integrated method for preventive maintenance and inventory control of a production line using simulation based optimization. A methodology combining the simulation and genetic algorithms was proposed jointly to optimize maintenance and inventory control policies. The results show that the joint optimization of maintenance strategy and production control policy leads to a significant reduction of the total cost.

Manuela et al.,(2017) proposed a model for the combined optimization of production/inventory control and preventive maintenance policies with the aim of minimizing the total expected cost per unit time. The model was developing using Simulated Annealing-based algorithm combined with Monte Carlo simulation. The robustness of the developed algorithm was demonstrated by means of repeated runs of different simulated scenarios characterized by diverse sets of cost parameters. Alrabghi et al.,(2013) in *"Simulation Based Optimization of Joint Maintenance and Inventory for Multi-Components Manufacturing Systems"* investigated the impact of production dynamics and labour availability on maintenance performance. Adopting a simulation based optimization, the study shows that production dynamics and labour availability have a significant impact on maintenance performance. Optimization results of Simulated Annealing, Hill Climb and Random solutions are compared. The experiments also show that simulated annealing achieved the best results although the computation time was relatively high.

Roux, Jamali, Kadi, and Châtelet (2008) in developing simulation and optimization platform to analyse maintenance policies performances in manufacturing systems, integrated optimization algorithms and simulation methods to analyse maintenance strategies performances. Tahvili et al.,(2014) in solving complex maintenance planning optimization problems used stochastic simulation, genetic algorithm and multi-criteria fuzzy decision making. Oyarbide-Zubillaga, Goti, and Sanchez (2008) in "*Preventive maintenance optimisation of multi equipment manufacturing systems by combining discrete event simulation and multi-objective evolutionary algorithms"* used discrete event simulation multi-objective evolutionary algorithms to determine the optimal preventive maintenance frequencies for multi-equipment systems under cost and profit criteria. The optimisation results indicate the improvements that can be achieved by means of the discrete event simulation and multi-objective evolutionary algorithms. Doostparast et al.,(2015) in

"Optimisation of PM scheduling for multi-component systems – a simulated annealing approach" adopted simulated annealing to sustain a certain level of availability with the minimal total maintenance-related costs. The computational results demonstrate that the proposed SA approach is quite capable of obtaining high-quality solutions (optimal or near-optimal) for the PM scheduling in reasonable computational times. The results also indicate that the types and the frequencies of the PM actions may vary in different inspection periods.

Ghosh and Roy (2009) in "*Maintenance optimization using probabilistic cost-benefit analysis"* demonstrated an improved technique involving the maximization of reliability-based on benefit-to-cost ratio (BCR). A sensitivity analysis based on the constant failure rate model was also undertaken to study the effect of changing the relative cost parameters on the BCR parameter. Results show that equipments with low failure rates have an appreciable BCR (>5) if the repair and maintenance costs are typically less than 5–10% of the incident cost. Moghaddam and Usher, (2010) developed a model to determine the optimal preventive maintenance and replacement schedules in a repairable and maintainable multi-component system using generational genetic algorithm and simulated annealing. Lapa et al.,(2006) in "*A model for preventive maintenance planning by genetic algorithms based in cost and reliability"* presented a model to optimize preventive maintenance policies based on the cost-reliability model. By applying the proposed cost-reliability model, it was possible to find preventive maintenance policies which provide a high level of reliability with low costs. The study further recommended an investigation of multi-objective genetic algorithm (MOGA) in the search for non-dominated solutions, avoiding the necessity of combining multiple criteria into a unique objective function.

Chung, Lau, Ho, and Ip, (2009) in "*Optimization of system reliability in multi-factory production networks by maintenance approach"* proposed a double tier genetic algorithm approach for multi-factory production networks, aiming to keep the system's reliability in a defined acceptable level, and minimize the make span of the jobs. The results show that when the hazard rate is lower, meaning that the reliability of the system is required to be higher, more frequent maintenance actions are induced. Donoriyanto, Anam, and Pudji (2018) in "*Application of genetic algorithm method on machine maintenance"* applied genetic algorithm to determine the optimal Hell Nailing machine maintenance schedule. The variables used are machine maintenance optimization while the observation variables include data of damage time, data maintenance time, data setup machine data downtime. The result determined optimal machine maintenance schedule in 1 year.

A. Gupta and Lawsirirat (2006) "*Strategically optimum maintenance of monitoring-enabled multi-component systems using continuous-time jump deterioration models"* applied simulation based optimization to analyze strategically optimal maintenance actions for a multi-component system whose deterioration is observed through a monitoring system set in place to support condition-based maintenance. The study found that the framework facilitates analyses at a strategic level the role of degree of response to the deterioration of components for the overall functionality of a multi-component system. Safaei et al.,(2008) developed a multi-objective simulated annealing (MOSA) algorithm to solve a real maintenance workforce scheduling problem.

Alsyouf and Hamdan, (2017) developed a multi-objective optimization model that will be used to compare the performance of maintenance policies based on four performance criteria which are cost, availability, life-time and reliability. The results of the model demonstrated that using the suggested model enables the decision maker the select the most cost effective maintenance policy under different scenarios. Y. Wang and Pham, (2011) in "*A Multi-Objective Optimization of imperfect Preventive Maintenance Policy for Dependent Competing Risk Systems with Hidden Failure"* studied a multi-objective maintenance optimization embedded within the imperfect preventive maintenance (PM) for one single-unit system subject to the dependent competing risks of degradation wear and random shocks. The comparison results show that the optimization solution is consistent between one-objective and multi-objective optimization, and the Pareto frontier for the maintenance optimization problem can provide alternative solutions according to customer preference and resource constraints.

Gopalakrishnan, Skoogh, and Laroque (2013) “*Simulation-based planning of maintenance activities in the automotive industry”* developed an approach to integrate maintenance strategies into a production planning using discrete event simulation. The approach is exemplified in an automotive case study, integrating strategies for reactive maintenance in a simulation model to support decision making on how repair orders should be prioritized to increase production performance. The results show that introducing priority-based planning of maintenance activities has a potential to increase productivity by approximately 5%. Gopalakrishnan et al.,(2014) in “*Simulation-based planning of maintenance activities by a shifting Priority method”* investigating how a shifting priority strategy could be integrated into the scheduling of reactive maintenance using discrete event simulation.

The general summary of relevant research related to this study is presented in table 2.9:

Table 2.9: Summary of Relevant Research

S/N	Year of Publication	Author/Authors	Publication	Objective of Study	Solution Technique
1.	2002	Marseguerra et al., (2002)	Condition-based maintenance optimization by means of genetic algorithms and Monte Carlo simulation	Use of Genetic Algorithm (GA) to determine the optimal degradation level beyond which preventive maintenance has to be performed in a continuously monitored multi-component system.	Genetic Algorithm (GA) and Monte Carlo simulation
2	2004	Saranga (2004)	Opportunistic maintenance using genetic algorithms	Application genetic algorithm to decide whether opportunistic maintenance is cost effective during analysis of manufacturing components that need opportunistic maintenance.	Genetic Algorithm

5.	2005	Sortrakul, Nachtmann, and Cassady (2005)	Genetic algorithms for integrated preventive maintenance planning and production scheduling for a single machine	Development of a heuristics based on genetic algorithms to solve an integrated optimization model for production scheduling and preventive maintenance planning.	Genetic Algorithm
6.	2008	Nguyen and Bagajewicz (2008)	Optimization of Preventive Maintenance Scheduling in Processing Plants	Development of a methodology to optimize both the planning of preventive maintenance and the amount of resources needed to perform maintenance in a process plant	Monte Carlo Simulation Genetic Algorithm
7.	2007	M Ali Ilgin and Tunali, (2007)	Joint optimization of spare parts inventory and maintenance policies using genetic	To develop a simulation optimization approach using genetic algorithms for the optimization	Simulation optimization Genetic Algorithm

			algorithms	of spare parts inventory and maintenance policies	
8.	2006	Lapa et al., (2006)	A model for preventive maintenance planning by genetic algorithms based in cost and reliability	to present a model to optimize preventive maintenance policies based on the cost-reliability model	Genetic Algorithm
9.	2009	Samhouri et al.,(2009)	An Intelligent Opportunistic Maintenance (OM) System: A Genetic Algorithm Approach	To present a methodology on how to decide whether a particular item requires opportunistic maintenance or not	Genetic Algorithm
10.	2012	Lee and Ni (2012)	Genetic Algorithm for Job Scheduling with Maintenance Consideration in Semiconductor Manufacturing Process	To develop optimization methods which would lead to the best wafer release policy in the chamber tool to maximize the overall yield of the wafers in	Genetic Algorithm

				semiconductor manufacturing system.	
11.	2016	Bon (2016)	Assembly Line Optimization using Arena Simulation	To improve the productivity assembly line by using ARENA simulation software	Simulation optimization
12.	2011	Y. Wang and Pham, (2011)	A Multi-Objective Optimization of Imperfect Preventive Maintenance Policy for Dependent Competing Risk Systems With Hidden Failure	To Study a multi-objective maintenance optimization embedded within the imperfect preventive maintenance (PM) for one single-unit system subject to the dependent competing risks of degradation wear and random shocks.	Non-dominated Sorting Genetic Algorithm (NSGAII)
13.	2012	Malamura and Murata, (2012)	Simulation Based Plant Maintenance Planning with	To develop a simulation model combining two processes of different origin	Simulation optimization

			Multiple Maintenance Policy and Evaluation of Production Process Dependability	(the production process and the information flow of maintenance work-orders) in one model opportunities.	
14.	2005	Rezg, Chelbi, and Xie, (2005)	Modelling and optimizing a joint inventory control and preventive maintenance strategy for a randomly failing production unit: Analytical and simulation approaches	To develop a mathematical model to evaluate the average cost per time unit in joint optimal inventory control and preventive maintenance strategy for a randomly failing production unit which supplies an assembly line operating according to a just-in-time configuration.	Simulation optimization
15.	2004	Rezg, Xie, and Mati, (2004)	Joint optimization of preventive maintenance and inventory	To develop an integrated method for preventive maintenance	Simulation optimization

			control in a production line using simulation	and inventory control of a production line	
16.	2012	Raska and Ulyrch (2012)	Simulation Optimization In Manufacturing Systems		Simulation optimization
17.	2015	Amaran et al.,(2015)	Simulation optimization: a review of algorithms and applications		Simulation optimization
18.	2017	Manuela et al.,(2017)	A simulated annealing-based approach for the joint optimization of production/inventory and preventive maintenance policies	To propose a model for the combined optimization of production/inventory control and PM policies with the aim of minimizing the total expected cost per unit time.	Simulated Annealing
19.	2013	Alrabghi et al.,(2013)	Simulation Based Optimization Of Joint Maintenance And Inventory	To investigate the impact of production dynamics and labour availability on	Simulation optimization

			For Multi-Components Manufacturing Systems	maintenance performance.	
20.	2008	Roux, Jamali, Kadi, and Châtelet (2008)	Development of simulation and optimization platform to analyse maintenance policies performances for manufacturing systems	To integrate optimization algorithms and the simulation methods is proposed to analyse maintenance strategies performances.	Simulation optimization
21.	2014	Tahvili et al.,(2014)	Solving complex maintenance planning optimization problems using stochastic simulation and multi-criteria fuzzy decision making		Stochastic simulation multi-criteria fuzzy decision making
22.	2013	Gopalakrishnan, Skoogh, and Laroque (2013)	Simulation-based planning of maintenance Activities in the automotive industry	an approach to integrate maintenance strategies into a production planning approach using	Discrete event simulation

				discrete event simulation.	
23.	2014	Gopalakrishnan et al.,(2014)	Simulation-based planning of maintenance activities by a shifting Priority method	The aim of the paper is to investigate how a shifting priority strategy could be integrated into the scheduling of reactive maintenance using discrete event simulation	Discrete event simulation
24.	2008	Ali, Lee, and Ni, (2008)	Optimized maintenance design for manufacturing performance improvement using simulation	To optimize maintenance design using simulation to analyze the capability of auto part manufacturing production system.	Simulation optimization
25.	2009	Ghosh and Roy (2009)	Maintenance optimization using probabilistic cost-benefit analysis	To demonstrate an improved technique involving the maximization of reliability-based benefit-to-cost ratio	Cost-benefit analysis

				(BCR),	
26.	2006	Lapa et al.,(2006)	A model for preventive maintenance planning by genetic algorithms based in cost and reliability	To present a model to optimize preventive maintenance policies based on the cost-reliability model.	Genetic algorithm
27.	2010	Moghaddam and Usher, (2010)	A new multi-objective optimization model for preventive maintenance and replacement scheduling of multicomponent systems	To develop a model to determine the optimal preventive maintenance and replacement schedules in a repairable and maintainable multi-component system.	generational genetic algorithm, simulated annealing
28	2017	Alsyouf and Hamdan, (2017)	A Multi-Objective Optimization of Maintenance Policies using Weighted Comprehensive Criterion Method (WCCM)	To develop a multi-objective optimization model that will be used to compare the performance of maintenance policies based on four performance criteria which are cost, availability, life-time and	Weighted Comprehensive Criterion Method (WCCM)

				reliability	
29.	2008	Oyarbide-Zubillaga, Goti, and Sanchez (2008)	Preventive maintenance optimisation of multi equipment manufacturing systems by combining discrete event simulation and multi-objective evolutionary algorithms	To determine the optimal preventive maintenance frequencies for multi-equipment systems under cost and profit criteria.	discrete event simulation multi-objective evolutionary algorithms
30.	2009	Chung, Lau, Ho, and Ip, (2009)	Optimization of system reliability in multi-factory production networks by maintenance approach	To propose a double tier genetic algorithm approach for multi-factory production networks, aiming to keep the system's reliability in a defined acceptable level, and minimize the makespan of the jobs.	Genetic algorithms
31.	2015	Doostparast et al.,(2015)	Optimisation of PM	To sustain a certain level of	Simulated annealing

			scheduling for multi-component systems – a simulated annealing approach	availability with the minimal total maintenance-related costs.	
32.	2012	Pleumpirom and Amornsawadwatana (2012)	Multiobjective Optimization of Aircraft Maintenance in Thailand Using Goal Programming: A Decision-Support Model	To develop the multiobjective optimization model in order to evaluate suppliers for aircraft maintenance tasks, using goal programming.	Goal programming
33.	2011	Elmabrouk (2011)	A Linear Programming Technique for the Optimization of the Activities in Maintenance Projects	To develop a framework for crashing total maintenance project time at the least total cost by using Linear Programming (LP) technique.	Linear Programming
34	2010	Roux, Duvivier, Quesnel, and Ramat (2010)	Optimization of preventive maintenance through a combined	The objective is to simultaneously ensure a low frequency of	Simulation optimization

			maintenance-production simulation model	failures by an Efficient periodic preventive maintenance and minimize the unavailability of the system due to preventive maintenance.	
35.	2006	A. Gupta and Lawsirirat (2006)	Strategically optimum maintenance of monitoring-enabled multi-component systems using continuous-time jump deterioration models	To analyze strategically optimal maintenance actions for a multi-component system whose deterioration is observed through a monitoring system set in place to support condition-based maintenance.	Simulation optimization
36.	2016	Alrabghi and Tiwari, (2016)	A novel approach for modelling complex maintenance		Discrete event simulation

			systems using discrete event simulation		
37.	2018	Donoriyanto, Anam, and Pudji (2018)	Application of genetic algorithm method on machine maintenance	To determine the optimal Hell Nailing machine maintenance schedule by using Genetic Algorithm method.	Genetic algorithm
38.	2008	Safaei et al.,(2008)	Multi-objective Simulated Annealing for a Maintenance Workforce Scheduling Problem : A case Study	To develop a multi-objective simulated annealing (MOSA) algorithm to solve a real maintenance workforce scheduling problem (MWSP) with the aim of simultaneously minimizing the workforce cost and the flow time of the work requests.	Multi-objective Simulated Annealing

2.9 Knowledge gap

Following the critical analysis of literature on maintenance and maintenance optimization methods in manufacturing systems, it was found out that despite many technological and management advances that have occurred within maintenance management in manufacturing, there are still some major issues identified in literature that remain unresolved or unaddressed throughout the past years. It was noted that both single objective optimization and multi-objective optimization problems have been intensively studied for several decades. However, relatively little work has been done in the area of multi-objective optimization in maintenance. This has drawn the attention of the researcher. In terms of real economic and business specific maintenance management objectives single objective optimization models are limited in scope. Most of these models take a cost optimization approach, and it doesn't provide any economic justification in relation to other maintenance management constraints such as reliability, availability or output quality in the production line. Hence it doesn't capture all the important aspect of a real life industrial situation. This basically means that maintenance practitioners do not know which of the available optimization approach fits their specific business objectives, and moreover they lack the time and experience to develop an optimization model that satisfies their specific business objectives. However, the need for an efficient periodicity of maintenance for all components of a manufacturing system is far from an easy task to accomplish when considering all the antagonistic criteria of the maintenance and production views of a manufacturing system.

CHAPTER THREE

MATERIALS AND METHODS

3.1 Materials

The basic materials used in this study, comprise the data sets and the software tools for accomplishing statistical analysis, simulation and optimization techniques. Their nature and sources are described as follows:

3.1.1 Data Collection

Data used within this study is categorised under two data source, primary data source and secondary data source.

- **Primary data source:** Necessary data was collected for three years (2015 -2017) historical maintenance records (cost data, records of equipment faults and failures, and factory maintenance compliance sheet), interviews with maintenance staffs and through eight months' direct observation of manufacturing machines and maintenance activities at tummy tummy industries Limited.
- **Secondary data source:** data and information were obtained from the following sources:
  - Academic thesis and dissertations, Journal publications and conference reports
  - Research outputs and project reports.
  - Published textbooks.

The use of this source was cost effective and provided a platform for building the foundation of this study and carrying out optimization analysis.

3.1.2 Software Packages for Data Analysis

The following software packages were used for data analysis in this study:

- Witness 14 Simulation software was used to develop and simulate a discrete event simulation of the maintenance process in this study.
- Lingo was used to test and solve the multi-objective algorithm.
- GaNetXL a decision support system generator for multi-objective optimization of spreadsheet based models by Savić, Bicik, and Morley (2011) was used to solve multi-objective optimization algorithm in the study. GANetXL uses genetic algorithms to solve complex optimization and search problems.
- Matlab was used to optimize the genetic algorithm fitness functions.
- Minitab software was used for statistical analysis and graphical representation.

3.2 Validation Case Study

The validation of this study was carried out in a food and beverage manufacturing company "Tummy Tummy Foods Industries limited" located at Chicason Drive, Umudim, Nnewi, Anambra State, Nigeria. The company was incorporated in December 2008 (with RC No: 793032.) as a manufacturer and marketer of Tummy-Tummy Instant Noodles for human consumption, with production starting in October 2009. The organisational structure is shown in figure 3.1. The production line is connected in series as illustrated in figure 3.2

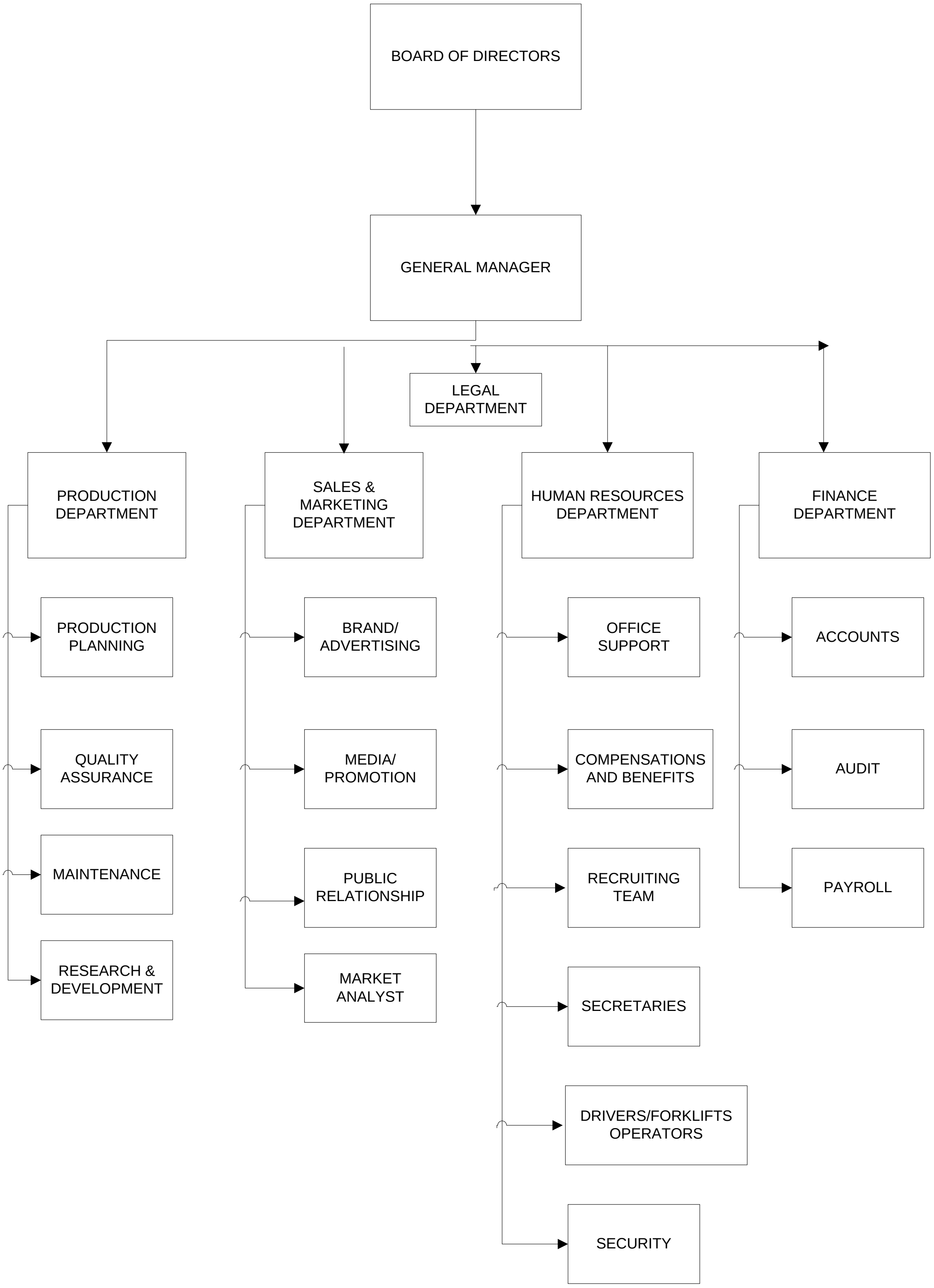


Fig 3.1 Organisational Structure at Tummy Tummy Foods Industries Limited

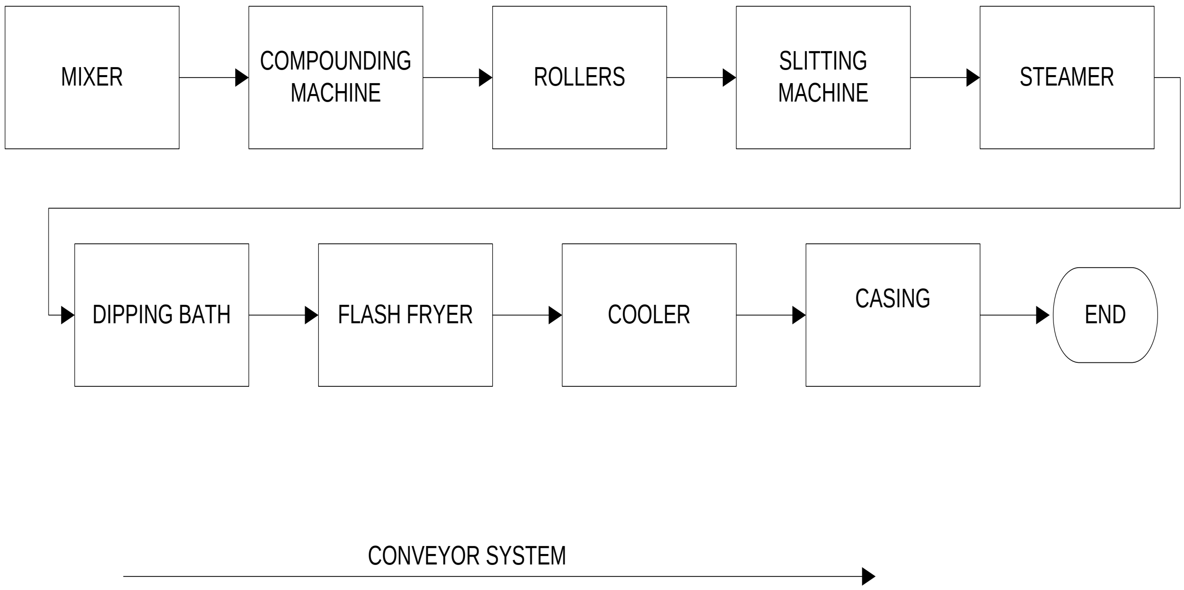


Figure 3.2: Illustration of the Production line

The line consists of the following components:

Mixer: Mixing and kneading occurs at this process, in which the mixing machine stirs up wheat flour and the mixing water. Normally 0.3-0.4kg of the mixing water at a temperature of 20-30˚C is kneaded into the flour for 15-20 minutes. This process gives the dough textiform tissue that generates noodle’s elasticity.

Compounding machine: The dough is put through two rotating rollers, which compounds two noodle belts into one single belt. This process helps distribute ingredients evenly. Occasionally the dough is left for a certain period of time to mature.

Rollers: With the help of pressing rollers, the 10mm thick noodles is flattened repeatedly using four rollers and finally becomes thin at 1mm thickness. This process strengthens the textiform tissue and gives elasticity to the noodles.

Slitter machine: After the rolling process, noodles are put into the slitter, where a rolling blade slits the noodle belt into thin noodles. Most of the instant noodles are wavy. Being pressed up and down gently, the noodles cut out by the slitter become wavy. The wavy-shape gives space in between noodles, which prevent the noodles from sticking together.

Steamer: Pregelatinization process occurs here in the steamer, where the instant noodles are steamed for one to five minutes.

Dipping Bath: The steamed noodles are dipped in seasoning.

Flash Fryer: dehydration process occurs here, most instant noodles are dehydrated either by oil-frying or air-drying.

Fried noodles: Noodles in a metal mold go through frying oil of 140-160˚C for a minute or two. The moisture content of 30-40% in the dough is reduced to 3-6%, and pregelatinization is accelerated. Non-fried noodles: Noodles in a metal mold are placed into air-drier and dehydrated with hot air of approximately 80˚C for more than 30 minutes. “Raw-type instant noodles” are the steamed noodles, which are sterilized with organic acid.

Cooler: After the dehydration process, the noodles, which are at 100˚C, are cooled with air. This cooling process is followed by a series of careful inspection for weight, shape, color, dryness, frying condition, cooling temperature, etc.

Casing: The ready instant noodles are then put into firm bags or containers as required along with the garnish and seasonings and then sealed with aluminium foils.

The company runs a computerized manufacturing process for the manufacturing and processing of noodles for human consumption and adopts corrective maintenance as its preferred maintenance strategy only, which can be described as a reactive, firefighting strategy. The information obtained from the maintenance team of the organisation was that most faults and failures can be fixed manually by the maintenance team in a relatively short period of time. But, there have been incidents and occasions where breakdowns resulted in long unavailability of the manufacturing physical assets. Also observed was the effect of faults and failures on the manufacturing process as depicted in figure 3.3.

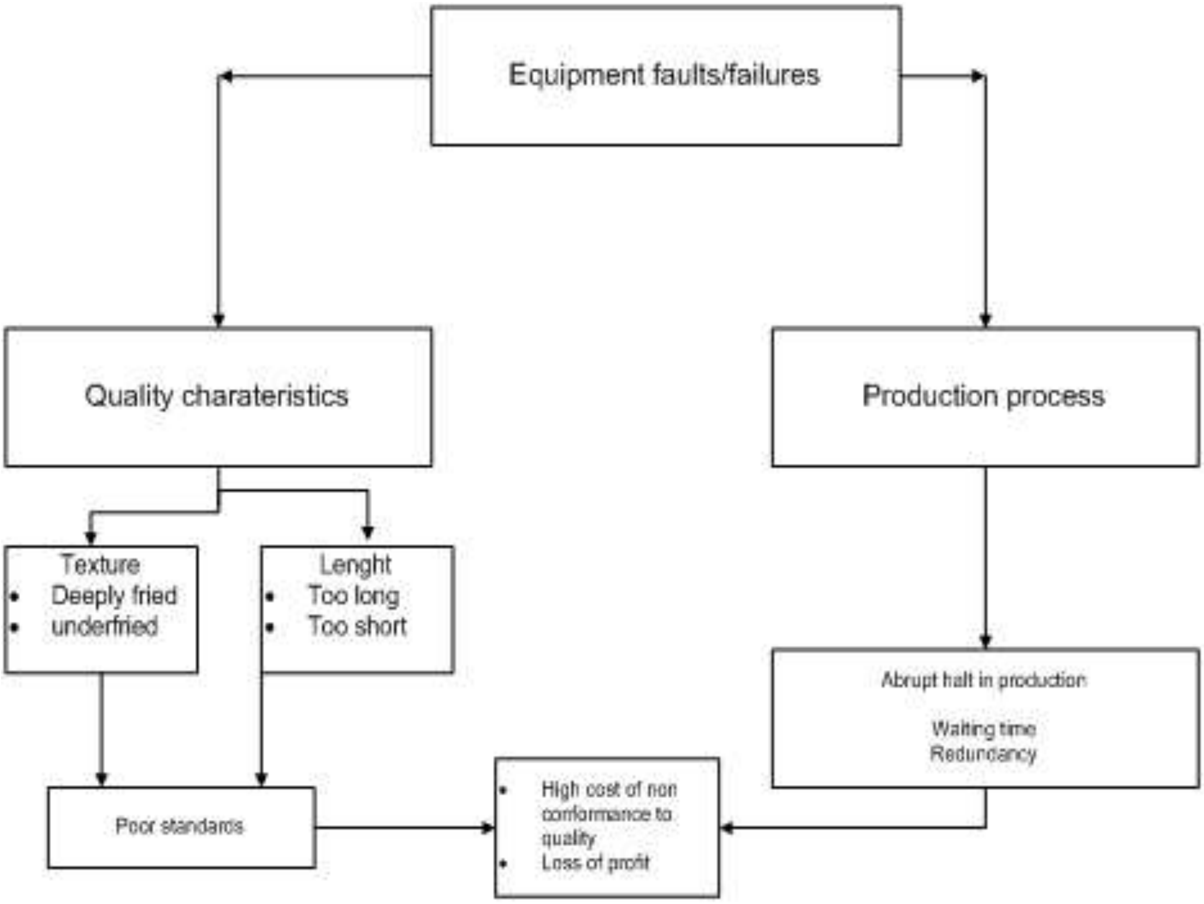


Fig 3.3: Faults/Failures and implications

The implications observed are in contradiction of the maintenance goals and objectives of the organisation which is that in the long run maintenance should ensure equipment availability in order to produce products at the compulsory quantity and quality levels.

The research methodology employed in this study is as follows:

3.2.1 Performance evaluation and downtime analysis of the components

Overall equipment effectiveness (OEE) was used to carry out a maintenance performance evaluation on the manufacturing equipment using three years historical data obtained from the food manufacturing company under study.

Overall equipment effectiveness (OEE) according to (Ahuja and Khamba, 2007; Ahuja and Kumar, 2009) takes into account, the availability rate, quality rate and performance rate of manufacturing equipment and products and is represented as:

$$\text{OEE} = \text{Availability x Performance Rate x Quality Rate} \tag{3.1}$$

Where availability accounts for losses as a result of equipment failure, setup and adjustment and is calculated as the ratio of operating time to loading time and is calculated as follows:

$$\text{Availability} = \frac{\text{Plannedruntime} - \text{Planneddowntime}}{\text{Plannedruntime}} \times 100 \tag{3.2}$$

And performance rate accounting for losses due to idle time and minor stoppages and is calculated as ratio of net operating time to operating time and is calculated as follows:

$$\text{Performance rate} = \frac{\text{Total Actual amount of product}}{\text{Target amount of product}} \times 100 \tag{3.3}$$

Quality rate factors in the defects in process and reduced yield and is defined as ratio of valuable operating time to net operating time and is calculated as follows:

$$\text{Quality rate} = \frac{\text{Processed Quantity} - \text{defective quantity}}{\text{Processed quantity}} \times 100 \tag{3.4}$$

The world class OEE served as a benchmark to evaluate the maintenance performance for the manufacturing organisation and to improve the maintenance policy and affect the continuous improvement in the manufacturing systems. This benchmark guide is shown in table 3.1. In analysis, if the calculated OEE is equal to world class OEE it is interpreted that the manufacturing organisation is in good condition and if the OEE is less then it means that there is a required urgent improvement of maintenance policies and strategies otherwise it will be difficult for the manufacturing organisation to sustain it.

Table 3.1 World class goals for OEE Source: Jain et al., (2013)

OEE Factor	**WORLD CLASS RATE (%)**
Availability	>90.0%
Performance Rate	>95%
Quality Rate	>99%
OEE	85%

Pareto Analysis is used in this case study for downtime analysis. According to Pareto analysis, around 20% of the downtime factors cause 80% of total downtime in manufacturing organisations. To identify these downtimes, a Pareto chart was used.

3.2.2 Repairable systems analysis on the manufacturing components to determine the failure trends

In this study, Parametric and non-parametric method of analysis was carried out on repairable systems under study in order to determine whether system failures are becoming more frequent, less frequent or constant using power law process for parametric method and mean cumulative function for the non-parametric method.

Power Law process (Parametric Method):

$$u(t)= \lambda\beta t^{\beta-1} \qquad \lambda, \beta > 0 \tag{3.5}$$

Where

$u(t)$ = failure intensity, i.e. Rate of occurrence of failure

ƛ = scale parameter (failure function)

β = shape parameter (improvement/degradation)

The parameter ƛ can be used to understand the reliability growth of the system. ƛ <1 implies that there’s reliability growth and ƛ >1 implies that there is reliability degradation.

The expected number of failures for the time interval t_1, t_2 is

$$\mathrm{E}\,[(t_2) - N(t_1)] = \int_{t1}^{t2} u(t)dt. \qquad (3.6)$$

$$\mathrm{E}\,[(t_2) - N(t_1)] = \lambda\,(t_2^{\beta} - t_1^{\beta}) \quad \lambda, \beta > 0, T_2 \geq T \geq 0 \qquad (3.7)$$

Where

E = Expected failure

$T_{i...N}$ = Time from the start of failure to the end of observation

N = Number of failures

The reliability function for the interval t_1, t_2 is given by

$$\mathrm{R}[t_2, t_1] = e^{-\lambda}\,(T_2^{\beta} - T_1^{\beta}) \quad \lambda, \beta > 0, T_2 \geq T \geq 0 \qquad (3.8)$$

Non parametric method:

- Mean cumulative function

$$m(t) = E\,[\,N(t)] \qquad (3.9)$$

Where

$$\boldsymbol{N}(\boldsymbol{t}) = \frac{\mathbf{1}}{\boldsymbol{n}} \sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}} \boldsymbol{N}_{\mathrm{i}}\,(\boldsymbol{t}) \qquad (3.10)$$

The failure intensity function:

$$\boldsymbol{m}(\boldsymbol{t}) = {}^{\boldsymbol{dM}(\boldsymbol{t})}\!/_{\boldsymbol{dt}} \qquad (3.11)$$

From the above equation $m(t)\Delta t$ describes the expected number of failures per system within the interval (t, t + Δt).

3.2.2.1 Parameters of Estimation

To estimate the two parameters in the model, maximum likelihood estimation as illustrated by (Trindade and Nathan, 2005) is applied in this study.

$$\lambda = \frac{\sum_{q=1}^{k} N_q}{\sum_{q=1}^{k} T_q^{\beta} - S_q^{\beta}} \qquad \textbf{(3.12)}$$

$$\beta = \frac{\sum_{q=1}^{k} N_q}{\lambda \sum_{q=0}^{k} \left(T_q^{b}\ \mathrm{In}\ T_q - S_q^{b}\ \mathrm{in}\ S_q\right) - \sum_{q=1}^{k} \sum_{i=1}^{N_q} \mathit{In}\ X_{iq}} \qquad \textbf{(3.13)}$$

Where

K = no of systems,

S and T = start and end times of observation,

N_q = number of failures on the qth system

X_{iq} is the age of the qth system at the ith failure.

3.2.2.1 Tests for equal shapes or scales

Bartlett’s modified likelihood ratio test was applied in the study to test for equal shapes or scales in a pooled failure data using the following equation:

$$\frac{2\log LR}{1+6^{-1}(N-1)^{-1}\left[\sum_{i=1}^{N} m_i^{-1} - \left(\sum_{i=1}^{N} m_i\right)^{-1}\right]} \qquad (3.14)$$

The process for applying these methods is summarised in a flowchart in figure 3.4

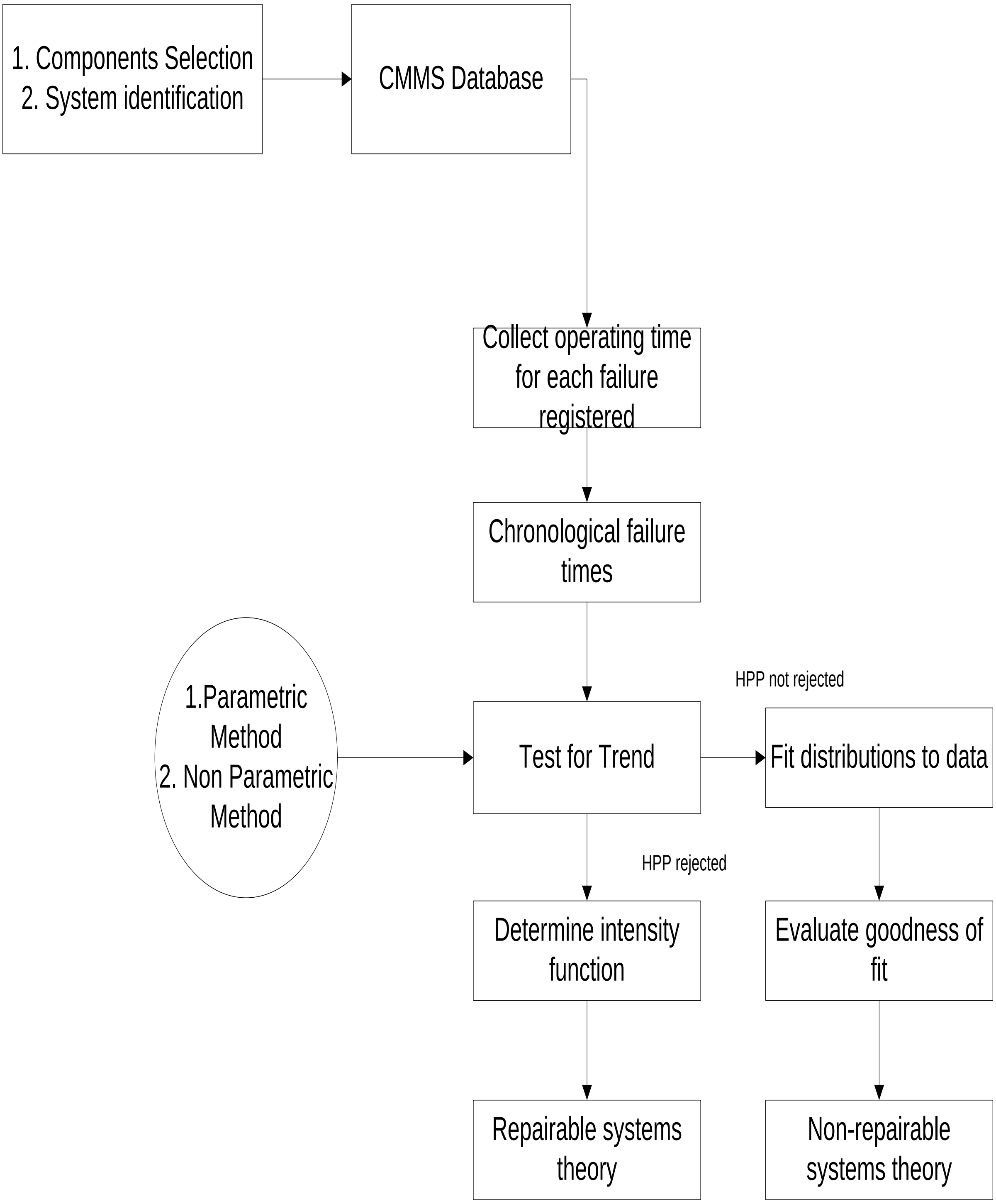


Fig 3.4: Flowchart for repairable systems analysis Source: Louit, Pascual, and Jardine, (2009)

3.2.3 Collection, evaluation and categorization of relevant data from case study for the optimal maintenance strategy.

In order to develop the optimal maintenance strategy, it is necessary to define and identify key essential attributes for the method. The flowchart in figure 3.5 describes the process by which it was achieved.

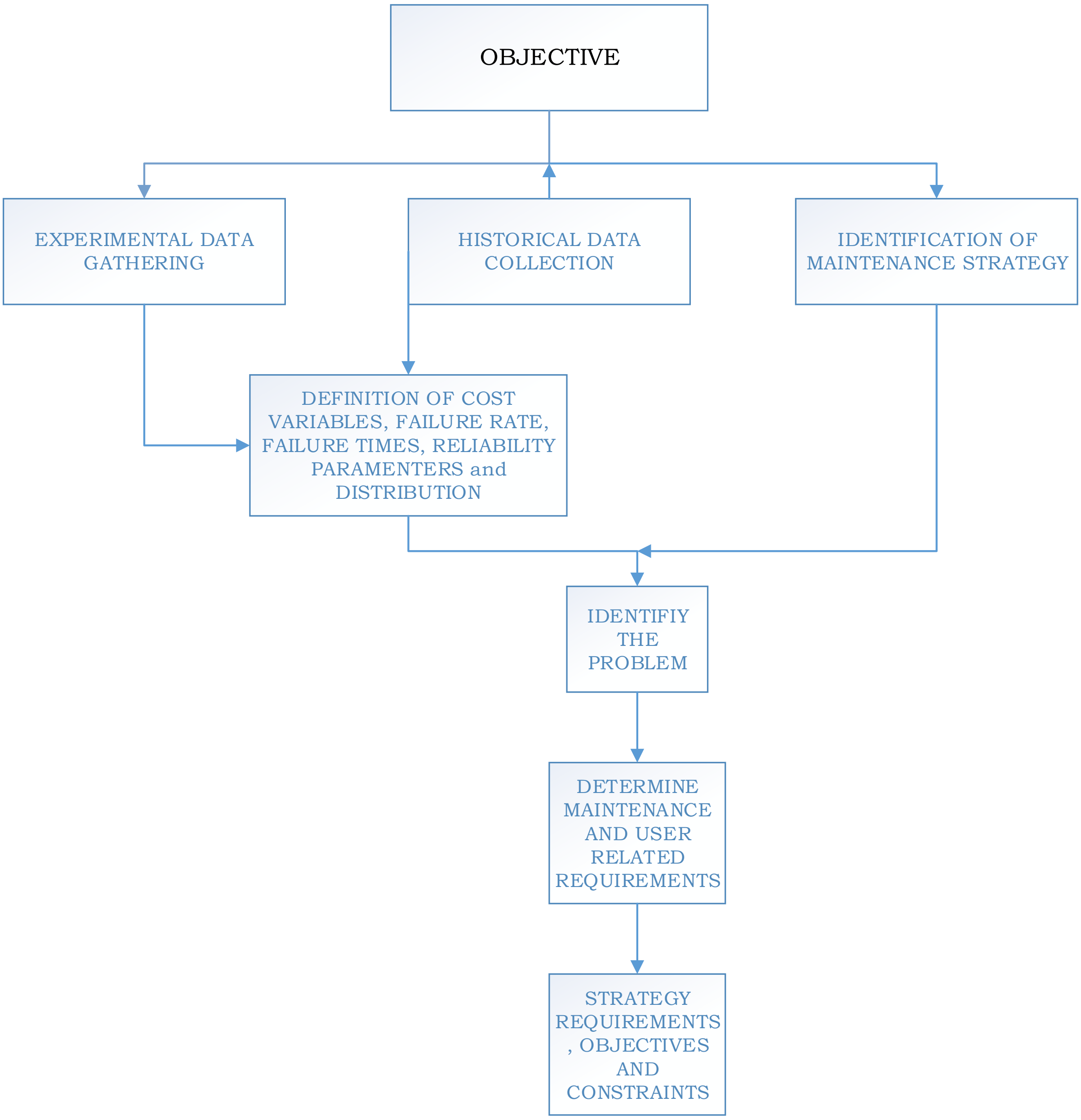


Fig 3.5: Flowchart for collection, evaluation and categorization of relevant data from case study for the optimal maintenance strategy

3.2.4 Development of a multi-objective optimization model for an optimal maintenance strategy based on cost and reliability.

To determine the optimal maintenance strategy, four solution approach were applied: (1) Genetic Algorithm (2) Simulation based optimization method (3) Lingo Solver and (4) GAnetXL

3.2.4.1 Genetic Algorithm

Genetic algorithm as discussed in section 2.4.7, is an efficient method for solving a wide range of analytical and optimization models. It was used because the ability to search from a very large population of potential solutions for a global optimum solution. The following shows the process implemented in this study while applying genetic algorithm:

- The objective function was encoded.
- A fitness function or selection criterion was defined.
- A population of individuals was initialized
- The fitness of all individuals in the population was evaluated.
- A new population was created by performing crossover, mutation fitness proportionate reproduction etc.
- The population was evolved until certain stopping criteria are met.
- The result was decoded to obtain the solution to the problem

The summary of this process is shown in figure 3.6.

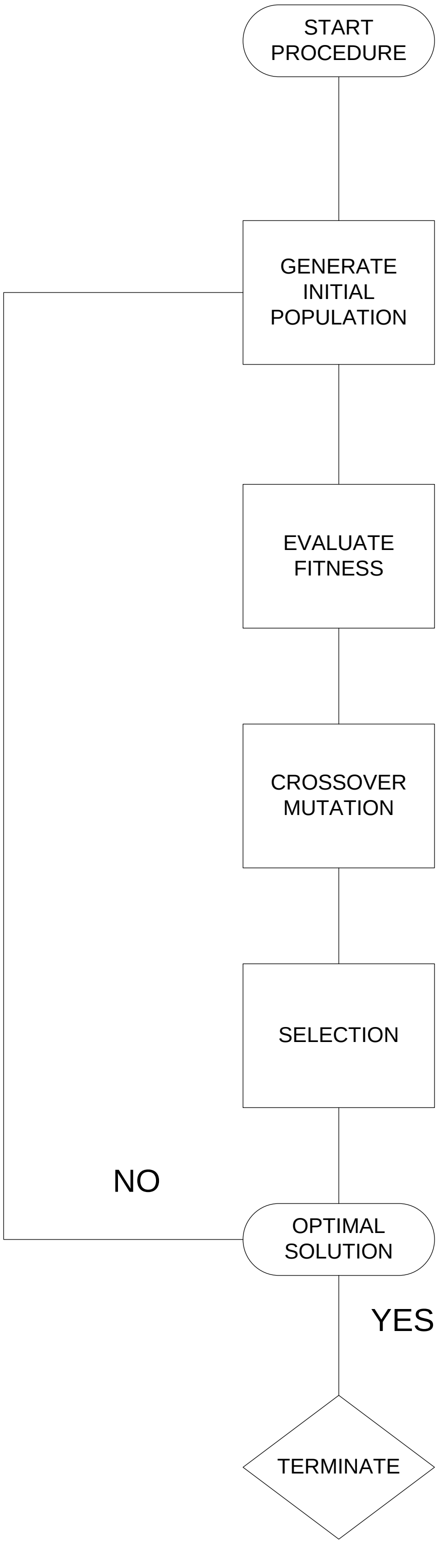


Fig 3.6: Genetic algorithm flowchart Source: Mehmet Ali Ilgin (2006)

3.2.4.2 Simulation based optimization method

Simulation optimization as mentioned in section 2.4.8, is an efficient technique of finding the best input variable values from among all possibilities without explicitly evaluating each possibility. Discrete event simulation (DES) was applied in this study, mainly due to its ability to model stochastic changes in flexible systems. Figure 3.7 illustrated the process implemented in this study while applying simulation optimization.

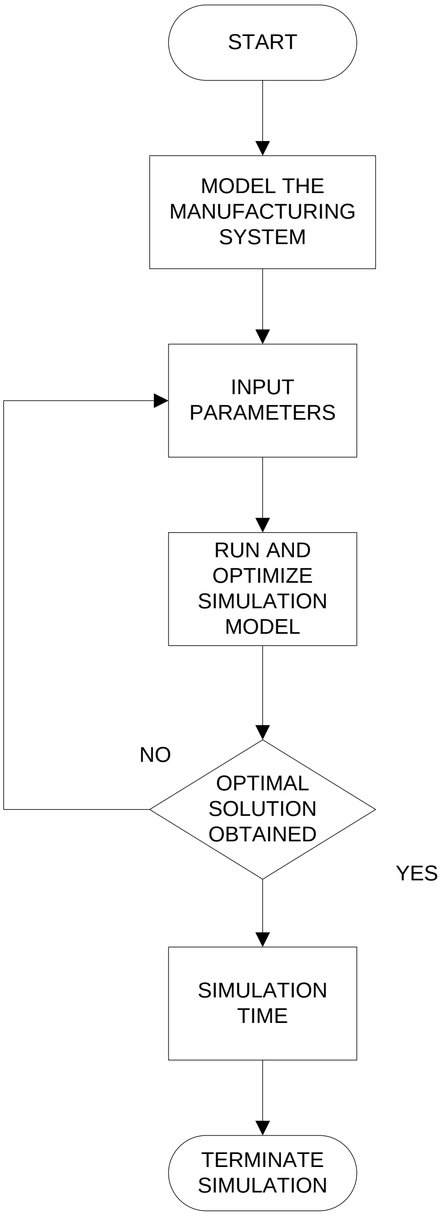


Fig 3.7 Simulation optimization flowchart

3.2.4.3 Lingo/Excel Solver

LINGO is a powerful solver tool designed to solve linear, nonlinear, quadratic and integer optimization models using either the branch and bound algorithm or the GRG algorithm. Excel and LINGO is combined to solve a special case where the multi-objective optimization problem is turned into a single optimization problem. This is achieved in this study by turning the maximisation function of the reliability into a constraint.

3.2.5 Assessment potential contribution and economic implications

Results from optimization solution approach methods were compared and analysed using the industrial case study. Potential contributions were also defined. It will be achieved through the following steps:

- Identification of the key performance indicators
- Determine performance of the validation case study
- Compare performance of optimal solutions with validation case study
- Identify key improvement factors

3.2.6 Development of a generic user interface support system

Implementing one of the solution techniques presented in objective three, a generic user interface support system through the following steps:

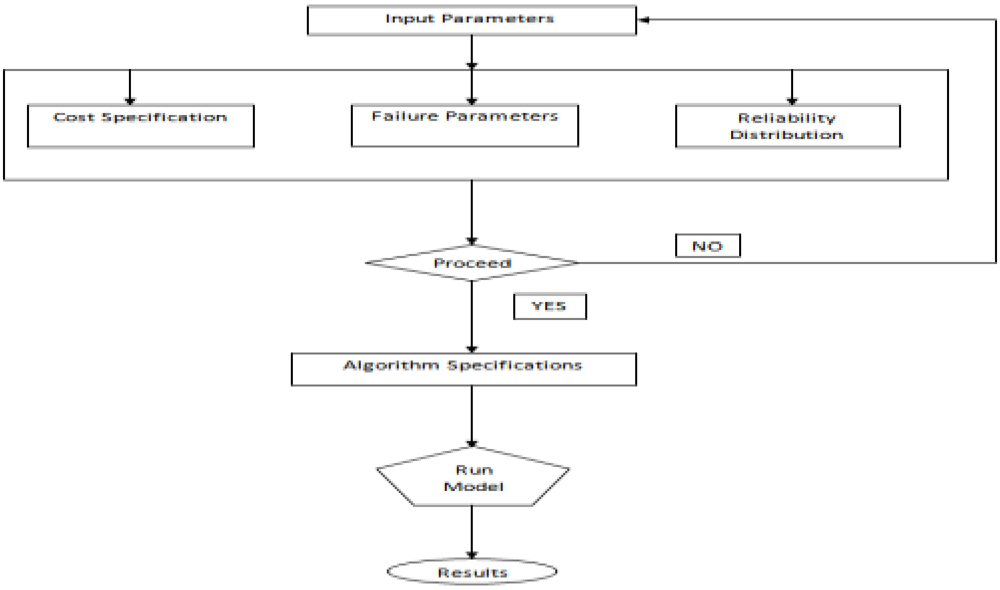


Fig 3.8: User interface support system development flowchart

3.3 Ethical Consideration

The following considerations were upheld during the study:

- The participating manufacturing firm was informed of the purpose and expected benefits of this research study.
- The participating manufacturing firm was offered the choice to indicate whether they would like to receive a report about this research study upon conclusion. A contact detail was provided.

3.4 Test for Normality and Significance of Data Obtained

To determine the distribution of data and if there is any significant difference from the historical and experimental data obtained from the case study, a test of normality was carried out using Shapiro-Wilk Test represented by the following in equation 3.5.

$$W = \frac{(\sum_{i=1}^{n} a_i \, x_{(i)})^2}{\sum_{i=1}^{n}(x_i - x!)^2} \qquad (3.5)$$

Where

$x_{(i)}$ is the *i*th smallest number in the sample

$x!$ Is the sample mean

The coefficient a_I is given by

$$(a_I, \ldots\ldots a_n) = \frac{m^t V^{-1}}{C} \qquad (3.5.1)$$

Where C is a vector norm

$$C = \left(m^{T} V^{-1}\ V^{-1} m \right)^{1/2} \tag{3.5.2}$$

And the vector m,

$$m = \ (m_1 \ldots \ldots \ldots, m_n)^{T} \tag{3.5.3}$$

Wilcoxon Signed Ranks test determines if there are any significant differences between both sets of data and is represented by the equation in 3.6

$$Ws = \ \sum_{i=1}^{n} Z_i\, R_i \tag{3.6}$$

Where

Z_i *is* an indicator variable, which is zero if Xi - m_o is negative, and equals to one if Xi - m_o is positive.

R_i is the rank value

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.1 Presentation of Data

The data used in this study are presented in this section. They included historical data showing records of equipment failure time for thirty-six months as well as observational studies carried out by the researcher for eight months at tummy tummy foods industries limited, Nnewi, Anambra State, Nigeria.

Table 4.1: Conveyor System Historical Time Failures

	Failure Times(hrs)			
Conveyor System (Historical)	153	4772	13100	21895
	188	5122	13677	22500
	233	8260	14512	22780
	530	9205	15300	22966
	968	9810	16431	23044
	1092	10122	17000	23178
	1240	10875	18211	23361
	1778	11022	18657	23450
	2219	11554	19322	23475
	3512	11841	19900	24290
	3791	12200	20690	24850
	3899	12276	20910	24926
	4365	12407	21362	25792
				26100

Table 4.2: Conveyor System Observational Time Failures

	Failure Times(hrs)											
Conveyor System (Observation)	28	432	1072	1690	2160	2680	3192	3785	4303	4782	5289	5679
	52	489	1120	1720	2231	2700	3224	3806	4319	4799	5300	5705
	84	530	1189	1766	2279	2743	3277	3844	4344	4830	5348	5759
	106	599	1226	1790	2300	2770	3300	3890	4390	4879	5390	5790
	120	640	1290	1818	2350	2800	3363	3924	4434	4920	5434	5808
	142	682	1339	1875	2399	2845	3390	3990	4460	4961	5484	5832
	158	710	1381	1900	2431	2880	3430	4010	4499	4990	5500	
	174	732	1400	1924	2460	2910	3486	4063	4524	5024	5576	
	202	770	1416	1980	2499	2960	3514	4100	4577	5058	5599	
	242	830	1499	2000	2522	2991	3560	4154	4600	5088	5640	
	284	899	1534	2040	2561	3020	3598	4189	4641	5100	5690	
	309	932	1580	2074	2590	3080	3620	4215	4697	5148	5720	
	345	981	1601	2100	2619	3103	3690	4260	4723	5190	5788	
	391	1010	1644	2124	2653	3158	3722	4285	4755	5224	5634	

Table 4.3: Mixer System Historical Time Failures

	Failure Times (hrs)									
Mixer System (Historical)	49	348	1080	2089	3654	5640	6387	6830	7440	12200
	76	420	1154	2245	3712	5689	6420	6888	7501	12276
	100	566	1280	2390	3915	5760	6475	6924	8260	12407
	124	642	1360	2421	4365	6000	6521	6970	9205	13100
	148	744	1436	2600	4772	6070	6582	6990	9810	13677
	153	810	1500	2879	5122	6136	6612	7043	10122	14512
	188	855	1675	2900	5345	6180	6656	7085	10875	15300
	200	890	1715	3122	5470	6245	6699	7134	11022	
	233	910	1856	3512	5524	6290	6744	7188	11554	
	282	988	1910	3600	5568	6328	6780	7349	11841	

Table 4.4: Mixer System Observational Time Failures

	Failure Times (hrs)								
Mixer System (Observational)	120	790	1580	2193	2800	3560	4283	4879	5569
	142	830	1601	2231	2845	3598	4303	4924	5599
	158	899	1644	2279	2880	3620	4319	4969	5640
	174	932	1690	2300	2968	3686	4344	4996	5696
	202	981	1720	2350	2998	3729	4396	5020	5724
	242	1010	1766	2399	3030	3785	4439	5055	5788
	284	1072	1790	2431	3080	3816	4466	5083	5630
	309	1120	1818	2460	3103	3844	4499	5120	5679
	345	1189	1875	2499	3158	3895	4529	5148	5705
	391	1226	1900	2522	3192	3924	4570	5196	5750
	432	1290	1924	2561	3224	3989	4600	5224	
	489	1339	1980	2590	3277	4015	4647	5280	
	530	1381	2000	2619	3300	4063	4696	5305	
	599	1400	2040	2653	3363	4106	4728	5348	
	640	1416	2074	2680	3390	4154	4755	5390	
	682	1465	2100	2700	3430	4190	4788	5430	
	710	1499	2124	2743	3486	4215	4799	5484	
	732	1534	2164	2770	3514	4269	4836	5501	

Table 4.5: Roller System Historical Time Failures

	Failure Times (hrs)													
Roller System (Historical)	300	1500	2713	3565	4409	5090	5900	6790	7148	7988	8788	9589	10420	11134
	456	1612	2800	3615	4498	5134	5966	6845	7198	8045	8834	9612	10480	11190
	520	1730	2880	3682	4559	5170	6030	6882	7250	8124	8878	9680	10535	11256
	700	1800	2945	3724	4590	5245	6099	6921	7320	8200	8950	9740	10599	11289
	760	1930	3020	3795	4644	5300	6145	6990	7390	8280	8990	9829	10645	11379
	820	2060	3160	3843	4680	5378	6200	6734	7450	8300	9032	9879	10680	11400
	880	2100	3200	3899	4730	5434	6278	6780	7500	8365	9080	9900	10700	11483
	956	2090	3279	3960	4799	5500	6329	6813	7560	8432	9140	9960	10789	11500
	990	2156	3312	4048	4824	5567	6390	6877	7600	8478	9200	10030	10821	11555
	1050	2260	3190	4122	4860	5620	6457	6905	7660	8544	9280	10140	10889	11631
	1165	2320	3265	4177	4890	5700	6480	6955	7700	8590	9354	10220	10912	11689
	1280	2459	3308	4217	4930	5788	6537	6990	7843	8643	9390	10280	10987	
	1345	2534	3400	4290	4979	5831	6579	7024	7890	8689	9465	10330	11000	
	1420	2677	3488	4366	5060	5890	6722	7088	7934	8740	9522	10390	11067	

Table 4.6: Roller System Observational Time Failures

	Failure Times (hrs)										
Roller System (Observed)	165	640	1180	1699	2238	2710	3300	3884	4466	4969	5535
	200	682	1206	1745	2270	2748	3360	3903	4499	4996	5585
	242	710	1245	1787	2300	2770	3390	3961	4529	5020	5620
	284	732	1298	1800	2345	2808	3430	3990	4570	5055	5677
	309	789	1324	1864	2399	2885	3486	4039	4600	5083	5700
	324	824	1370	1899	2430	2968	3518	4087	4647	5120	5745
	345	878	1398	1934	2467	2998	3560	4127	4696	5187	5790
	372	900	1416	1980	2499	3030	3598	4168	4728	5204	5821
	391	954	1487	2010	2522	3088	3629	4200	4755	5260	
	432	980	1520	2065	2568	3103	3680	4286	4788	5299	
	489	1012	1587	2094	2596	3158	3724	4306	4799	5328	
	530	1068	1605	2130	2619	3199	3776	4344	4836	5386	
	567	1099	1660	2169	2653	3224	3800	4396	4879	5408	
	599	1130	1699	2238	2688	3277	3830	4439	4924	5490	

Table 4.7: Slitter System Historical Time Failures

	Failure Times (hrs)																
Slitter System (Historical)	1628	2696	3408	4280	5400	6479	7590	8390	9452	10500	12030	13239	14480	15599	16573	18048	19200
	1690	2778	3470	4303	5472	6556	7648	8459	9500	10591	12099	13290	14554	15648	16620	18120	19264
	1750	2824	3502	4360	5506	6700	7683	8499	9548	10641	12124	13360	14599	15692	16695	18196	19322
	1791	2880	3568	4390	5592	6740	7720	8544	9580	10720	12200	13483	14676	15729	16740	18255	19410
	1843	2936	3580	4468	5672	6789	7789	8580	9642	10889	12280	13548	14734	15799	16801	18300	
	1880	2990	3634	4500	5700	6834	7814	8694	9728	10949	12356	13629	14896	15876	16882	18386	
	1950	3035	3670	4580	5780	6890	7896	8720	9780	11000	12421	13680	14944	15900	16964	18421	
	2023	3077	3742	4640	5853	6943	7945	8791	9800	11180	12500	13742	14980	15965	17020	18500	
	2099	3124	3790	4681	5899	6980	7999	8869	9860	11250	12580	13800	15036	16000	17090	18575	
	2156	3160	3810	4790	5948	7024	7832	8900	9920	11320	12600	13869	15090	16035	17128	18630	
	2214	3200	3866	4865	5970	7080	7880	8970	9977	11482	12699	13920	15128	16089	17170	18684	
	2287	3255	3900	4910	6049	7145	7930	9000	10024	11500	12758	13990	15189	16154	17220	18741	
	2330	3282	3948	4990	6088	7190	7989	9048	10080	11599	12810	14066	15260	16190	17294	18828	
	2372	3358	3999	5049	6140	7260	8040	9070	10154	11634	12889	14124	15300	16240	17367	18890	
	2400	3417	4051	5124	6199	7324	8079	9148	10199	11686	12923	14191	15399	16289	17410	18934	
	2471	3460	4078	5188	6250	7380	8124	9200	10268	11730	12990	14239	15425	16342	17492	18966	
	2548	3492	4124	5244	6303	7410	8188	9288	10320	11788	13061	14300	15470	16390	17578	19006	
	2599	3530	4180	5290	6380	7486	8278	9324	10389	11830	13110	14376	15500	16444	17612	19079	
	2653	3578	4231	5369	6424	7520	8320	9399	10448	11900	13186	14432	15545	16527	17690	19148	

Table 4.8 Slitter System Observational Time Failures

	Failure Times (hrs)										
Slitter System (Observed)	26	689	1250	1800	2399	2870	3480	4039	4600	5204	5815
	89	728	1298	1869	2430	2968	3519	4090	4647	5265	
	124	770	1318	1899	2467	2998	3567	4127	4696	5299	
	199	820	1368	1934	2499	3030	3608	4168	4728	5328	
	246	878	1390	1987	2522	3080	3629	4207	4755	5386	
	280	900	1436	2010	2560	3109	3685	4286	4780	5412	
	342	949	1480	2064	2596	3158	3724	4310	4799	5490	
	399	980	1520	2094	2619	3199	3776	4352	4836	5535	
	431	1012	1580	2138	2653	3229	3803	4390	4879	5589	
	478	1060	1605	2175	2680	3277	3837	4439	4924	5620	
	501	1099	1660	2229	2710	3307	3884	4466	4969	5677	
	566	1124	1699	2270	2748	3360	3908	4499	4999	5708	
	590	1176	1736	2300	2779	3390	3961	4532	5124	5745	
	634	1206	1787	2340	2800	3438	3990	4570	5187	5790	

Table 4.9 Compounding Machine Historical Time Failures

	Failure Times (hrs)															
Compounding Machine (Historical)	76	1165	2156	3035	3670	4366	5300	6140	7198	8124	8950	9728	10420	11555	12500	13360
	148	1280	2214	3077	3742	4409	5378	6199	7250	8200	8990	9780	10480	11631	12580	13483
	200	1345	2287	3124	3790	4498	5434	6250	7320	8280	9032	9800	10500	11689	12600	13548
	282	1420	2330	3200	3810	4580	5500	6303	7390	8300	9080	9860	10591	11730	12699	13629
	300	1500	2372	3279	3866	4640	5567	6734	7450	8365	9140	9920	10641	11788	12758	13680
	456	1628	2459	3312	3900	4681	5620	6780	7520	8459	9200	9977	10720	11830	12810	13742
	520	1690	2534	3190	3948	4790	5700	6813	7590	8499	9288	10024	10889	11900	12889	13800
	700	1750	2677	3265	3999	4865	5788	6877	7660	8544	9324	10080	10949	12030	12923	13869
	760	1791	2696	3308	4051	4910	5831	6905	7700	8580	9399	10154	11000	12099	12990	13920
	820	1843	2778	3400	4078	4990	5890	6955	7843	8694	9452	10199	11180	12124	13061	13990
	880	1880	2824	3488	4124	5049	5900	6990	7890	8740	9500	10268	11250	12200	13110	14066
	956	1950	2880	3565	4180	5124	5966	7024	7934	8788	9522	10280	11320	12280	13186	
	990	2023	2936	3580	4217	5188	6030	7088	7988	8834	9589	10330	11482	12356	13239	
	1050	2099	2990	3634	4290	5245	6099	7148	8045	8878	9642	10390	11500	12421	13290	

Table 4.10 Compounding Machine Observational Time Failures

Compounding Machine (Observed)	**Failure Times (hrs)**									
	75	689	1210	1787	2340	2968	3480	4127	4755	5490
	120	728	1250	1809	2399	2998	3519	4168	4780	5535
	191	794	1298	1869	2430	3030	3685	4207	4799	5589
	246	818	1318	1899	2467	3080	3724	4286	4836	5620
	280	860	1368	1934	2499	3109	3776	4439	4879	5677
	342	901	1400	1987	2522	3158	3803	4466	4924	5708
	399	949	1467	2009	2560	3199	3837	4499	5187	5745
	436	980	1520	2064	2596	3229	3884	4532	5204	5790
	480	1012	1580	2094	2619	3277	3908	4570	5265	
	501	1067	1605	2108	2653	3307	3961	4600	5299	
	574	1099	1660	2170	2680	3360	3990	4647	5328	
	590	1124	1700	2270	2710	3390	4039	4696	5386	
	634	1176	1736	2300	2748	3438	4090	4728	5412	

Table 4.11 Maintenance Cost Expenditure Summary for 2015

Components	**2015 Cost Expenditures (₦)**							
	Diagnostic Actions	Preventive oil change	Consumable Materials	Inspections	Labour	Administrative	Travel Expenses	Equipment Hire and Transportation
Conveyor System	295,280	118,000	110,230	119,450	50,000	34,670	177,000	240,000
Mixer System	230,000	100211	116,790	111,065	22000	30000	158,000	210,000
Rollers	250,000	112211	117,000	114,680	24000	30000	158,000	200,000
Slitter	255,548	10000	119,500	116,000	28000	30000	158,000	289,000
Compounding Machine	236,880	113,500	151,230	112,355	22000	32000	158,000	265,000

Table 4.12 Maintenance Cost Expenditure Summary for 2016

Components	**2016 Cost Expenditures (₦)**							
	Diagnostic Actions	Preventive oil change	Consumable Materials	Inspections	Labour	Administrative	Travel Expenses	Equipment Hire and Transportation
Conveyor System	273,100	129,000	100,000	122,000	29,565	33,000	160,000	300,000
Mixer System	221,000	23,400	116,000	120,000	-	-	153,000	260,000
Rollers	250,000	-	113,000	114,680	-	-	153,000	260,000
Slitter	200,548	-	122,800	115,000	-	-	153,000	235,000
Compounding Machine	265,880	100,000	117,000	115,300	9,750	13,225	153,000	347,000

Table 4.13 Maintenance Cost Expenditure Summary for 2017

Components	**2017 Cost Expenditures (₦)**							
	Diagnostic Actions	Preventive oil change	Consumable Materials	Inspections	Labour	Administrative	Travel Expenses	Equipment Hire and Transportation
Conveyor System	391,000	212,899	210,689	124,550	90,000	38,000	192,680	380,000
Mixer System	312,000	-	138,000	123,000	-	-	162,000	333,000
Rollers	324,000	-	134,000	124,000	-	-	164,000	212,000
Slitter	244,008	-	138,900	119,000	-	-	160,000	205,000
Compounding Machine	330,100	313,500	140,000	116,000	-	-	160,000	343,000

Table 4.14 Maintenance Cost Expenditure Summary for 2018

Components	**2018 Cost Expenditures (₦)**							
	Diagnostic Actions	Preventive oil change	Consumable Materials	Inspections	Labour	Administrative	Travel Expenses	Equipment Hire and Transportation
Conveyor System	195,090	121,000	120,000	19,450	50,000	34,670	77,000	140,000
Mixer System	100,560	-	20,790	11,065	-	-	62,000	110,000
Rollers	180,000	-	20,000	14,680	-	-	58,000	100,000
Slitter	164,822	-	23,500	16.000	-	-	60,000	189,000
Compounding Machine	162,000	118,330	41,000	12,355	-	-	60,000	165,000

Table 4.15: Summary of Production Variables for 2015

S/No	Category	Months											
		January	February	March	April	May	June	July	August	September	October	November	December
1	Total Time (mins)	44640	40320	44640	43200	44640	43200	44640	44640	43200	44640	43200	44640
2	Downtime (mins)	6900	5380	6757	6792	6880	11498	17222	7005	7065	8220	9611	8140
3	Planned Runtime (mins)	37740	34940	37883	36408	37760	31702	27418	37365	36135	36420	33589	36500
4	Runtime losses (mins)	10153	9539	8865	8884	10007	17373	20097	11061	10732	11436	16022	11716
5	Operating time (mins)	27587	25401	29018	27524	27753	14329	7321	26304	25403	24984	17567	24784
6	Total Units produced	2936	2416	3021	3188	3236	2652	1972	3376	3336	2928	2916	2944
7	Target Unit	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
8	Defected units	183	160	158	256	263	780	381	298	327	326	802	489
9	Availability (A) %	73.10	72.70	76.60	75.60	73.50	45.20	26.70	70.40	70.30	68.60	52.30	67.90
10	Performance rate (P) %	73.40	60.40	75.52	79.70	80.90	66.30	49.30	84.40	83.40	73.20	72.90	73.60
11	Quality rate (Q) %	93.7	93.40	94.80	92.00	91.90	70.60	80.70	91.20	90.20	88.90	72.50	83.40
12	QEE	50.32	40.01	53.05	55.43	54.64	21.15	10.62	54.18	52.88	44.64	27.64	41.67

Table 4.16: Summary of Production Variables for 2016

S/No	Category	Months											
		January	February	March	April	May	June	July	August	September	October	November	December
1	Total Time (mins)	44640	41760	44640	43200	44640	43200	44640	44640	43200	44640	43200	44640
2	Downtime (mins)	8362	6780	6866	6790	7007	8296	6529	7380	7413	6203	6144	6189
3	Planned Runtime (mins)	36278	34980	37774	36410	37633	34904	38111	37260	35787	38437	37056	38451
4	Runtime losses (mins)	11645	8472	8612	8520	8625	11564	8434	10261	6748	8291	8315	7844
5	Operating time (mins)	24633	26508	29162	27890	29008	23340	29677	26999	29039	30146	28741	30607
6	Total Units produced	2944	3176	3204	3152	3356	2932	3172	2988	2940	3167	3048	3184
7	Target Unit	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
8	Defected units	480	137	146	211	300	274	657	296	294	188	230	276
9	Availability (A) %	67.90	75.78	77.20	76.60	70.80	66.87	77.87	72.46	72.76	78.43	77.56	78.60
10	Performance rate (P) %	73.60	79.40	80.10	78.80	83.90	73.30	79.30	74.70	73.47	79.80	76.22	79.60
11	Quality rate (Q) %	83.70	95.70	95.44	93.31	91.05	90.66	90.70	90.10	90.00	94.05	92.47	91.32
12	QEE %	41.82	57.58	59.01	56.32	54.08	44.43	56.00	48.76	48.11	53.99	47.26	52.82

Table 4.17: Summary of Production Variables for 2017

S/No	Category	Months											
		January	**February**	**March**	**April**	**May**	**June**	**July**	**August**	**September**	**October**	**November**	**December**
1	Total Time (mins)	44640	40320	44640	43200	44640	43200	44640	44640	43200	44640	43200	44640
2	Downtime (mins)	6621	6888	7159	7008	6770	6126	6298	8150	8133	6994	6300	6215
3	Planned Runtime (mins)	38019	33432	37481	36192	37870	37074	38342	36490	35067	37646	36900	38425
4	Runtime losses (mins)	9178	11000	10914	9967	9581	8156	9049	11093	10590	10202	8350	8261
5	Operating time (mins)	28841	24432	26567	26225	28289	28918	29293	25397	24477	27444	28550	30164
6	Total Units produced	3372	3271	3200	3249	3346	3412	3385	2994	2997	3589	3360	3421
7	Target Unit	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
8	Defected units	313	380	448	404	354	271	290	442	428	459	286	261
9	Availability (A) %	75.86	73.08	70.88	72.46	74.70	78.00	76.40	69.60	69.80	72.90	77.37	78.50
10	Performance rate (P) %	84.30	81.78	80.00	81.23	83.66	85.30	84.63	74.84	74.92	81.73	84.00	85.52
11	Quality rate (Q) %	90.73	88.39	86.00	87.88	89.42	92.07	91.42	85.25	85.73	87.22	91.49	92.36
12	QEE %	58.02	53.34	48.77	51.72	55.88	61.25	59.11	44.40	44.83	51.97	59.44	62.00

4.2 Preliminary Data Analysis

A preliminary data analysis was carried out on the failure data in other to determine if there are any significant differences between the data sources. First a test of normality was carried out in other to determine the distribution of the data using the Shapiro-Wilk Test. The Shapiro-Wilk Test is more appropriate for small sample sizes (< 50 samples), but can also handle sample sizes as large as 2000. For this reason, this study will use the Shapiro-Wilk test as a numerical means of assessing normality and the formula is denoted as:

4.2.1 Test for normality for Conveyor System Failure Data

Applying equation 3.5, the results are as follows:

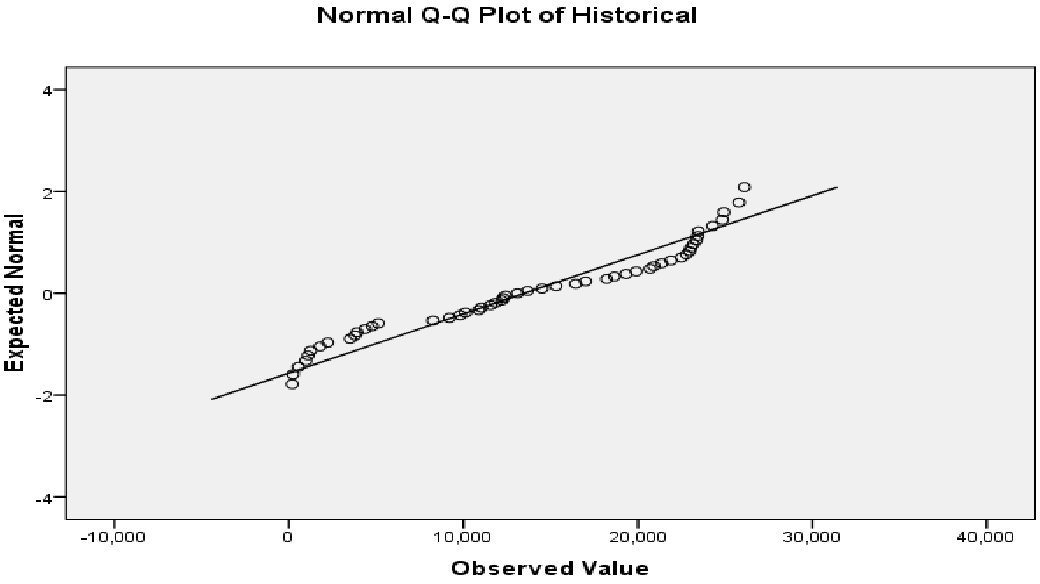
Table 4.18: Results for Normality Test

Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
Historical	53	32.9%	108	67.1%	161	100.0%
Observed	53	32.9%	108	67.1%	161	100.0%

Tests of Normality

	Shapiro-Wilk		
	Statistic	df	Sig.
Historical	.915	53	.001
Observed	.931	53	.004



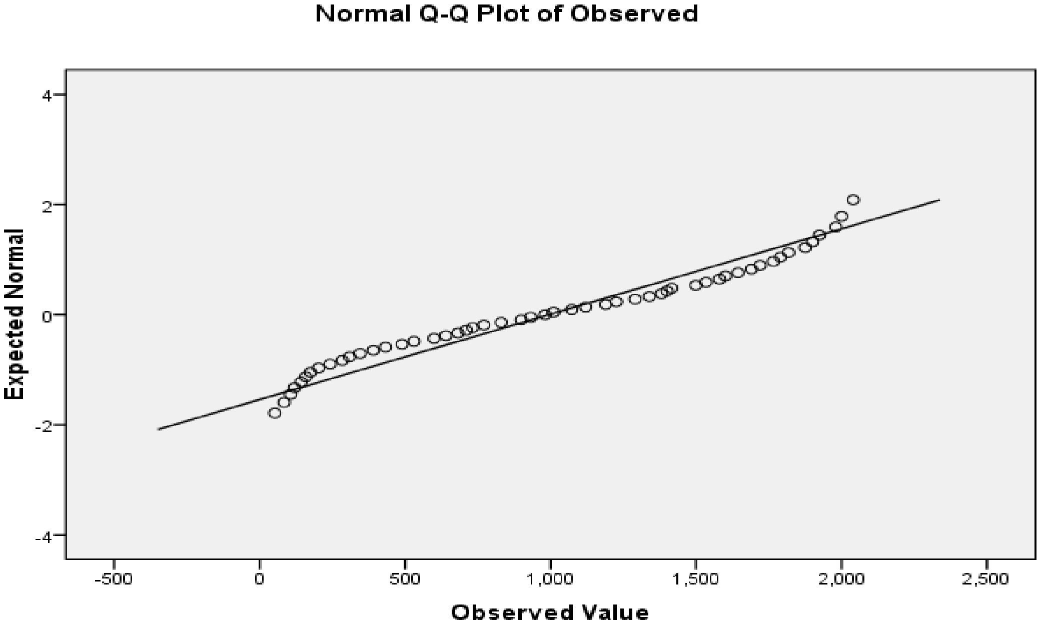


Fig 4.1: Normality Plot for Conveyor System

From table 4.18 it can be seen that the distribution is not a normal distribution since the significance level is below the $\alpha = 0.05$ this is also confirmed in figure 4.1 which show that the data is not a linear fit. Hence a non-parametric test (Wilcoxon Signed Ranks test) was carried out to determine if there are any significant differences between both data. Results are shown in Table 4.19

Table 4.19: Wilcoxon Signed Ranks Test for Significance Conveyor System

Test Statistics[b]

	Observed - Historical
Z	-6.334[a]
Asymp. Sig. (2-tailed)	.000

a. Based on positive ranks.

b. Wilcoxon Signed Ranks Test

The wilcoxon signed test rank show that there is a significant difference in both data collected (Z= -6.334), P = (<0.001)). Hence the observational experimental data will be used for validation purposes while the historical data will be used to establish the baseline situation.

4.2.2 Test for normality for Mixer System Failure Data

Applying equation 3.5, the results are as follows:

Table 4.20: Results for Normality Test

```
EXAMINE VARIABLES=MSH MSO
  /PLOT BOXPLOT STEMLEAF NPPLOT
  /COMPARE GROUP
  /STATISTICS NONE
  /CINTERVAL 95
  /MISSING LISTWISE

  /NOTOTAL.
```

Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
Historical	97	63.0%	57	37.0%	154	100.0%
Observed	97	63.0%	57	37.0%	154	100.0%

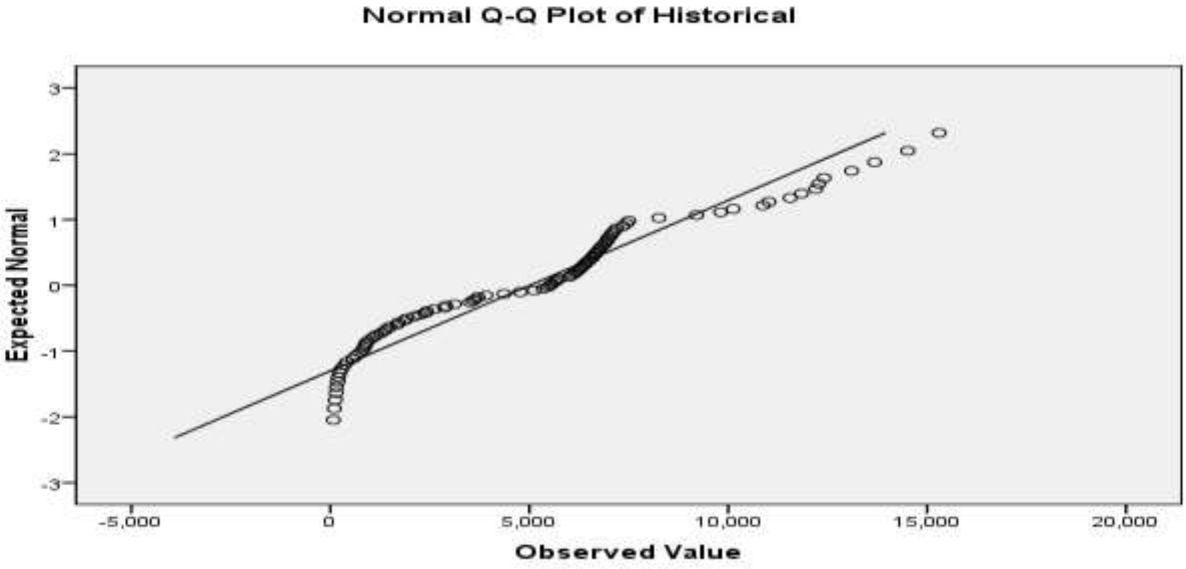
Tests of Normality

	Shapiro-Wilk		
	Statistic	df	Sig.
Historical	.925	97	.000
Observed	.960	97	.004

Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
Historical	97	63.0%	57	37.0%	154	100.0%

From table 4.20 it can be seen that the distribution is not a normal distribution since the significance level is below the α = 0.05 this is also confirmed in figure 4.2 which show that the data is not a linear fit. Hence a non-parametric test (Wilcoxon Signed Ranks test) was carried out to determine if there are any significant differences between both data. Results are shown in Table 4.21



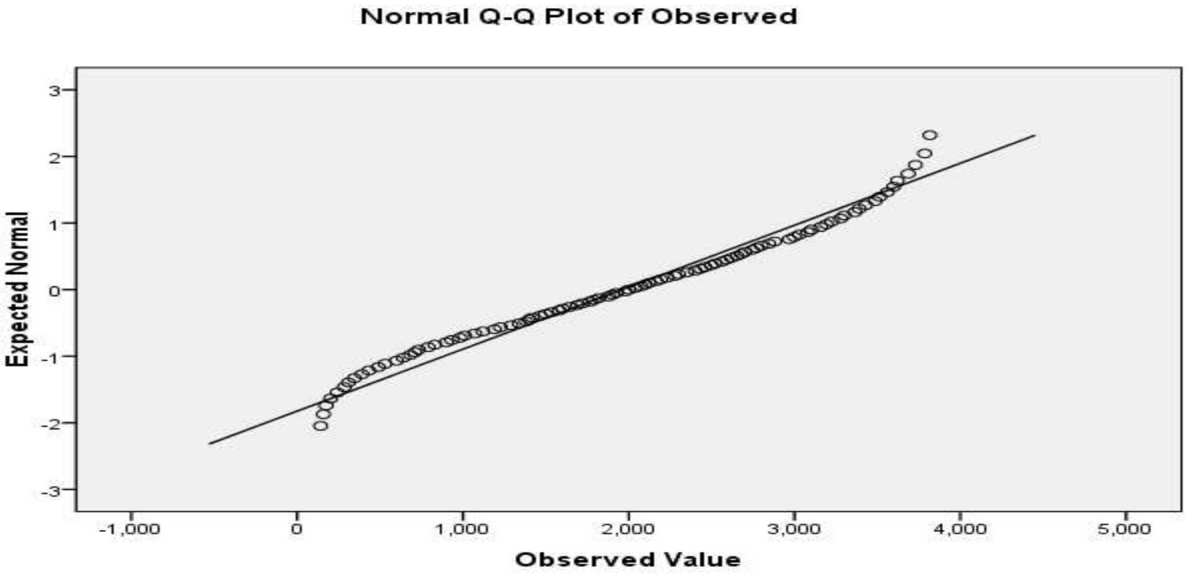


Fig 4.2: Normality Plot for Mixer System

Table 4.21: Wilcoxon Signed Ranks Test for Significance Mixer System

Test Statistics[b]

	Observed - Historical
Z	-8.173[a]
Asymp. Sig. (2-tailed)	.000

a. Based on positive ranks.

b. Wilcoxon Signed Ranks Test

The wilcoxon signed test rank show that there is a significant difference in both data collected (Z= -8.173), P = (<0.001)). Hence the observational experimental data will be used for validation purposes while the historical data will be used to establish the baseline situation

4.2.3 Test for normality for Rollers System Failure Data

Applying equation 3.5, the results are as follows:

Table 4.22: Results for Normality Test

```
EXAMINE VARIABLES=RSH RSO
  /PLOT BOXPLOT STEMLEAF NPPLOT
  /COMPARE GROUP
  /STATISTICS NONE
  /CINTERVAL 95
  /MISSING LISTWISE
```

```
/NOTOTAL.
```

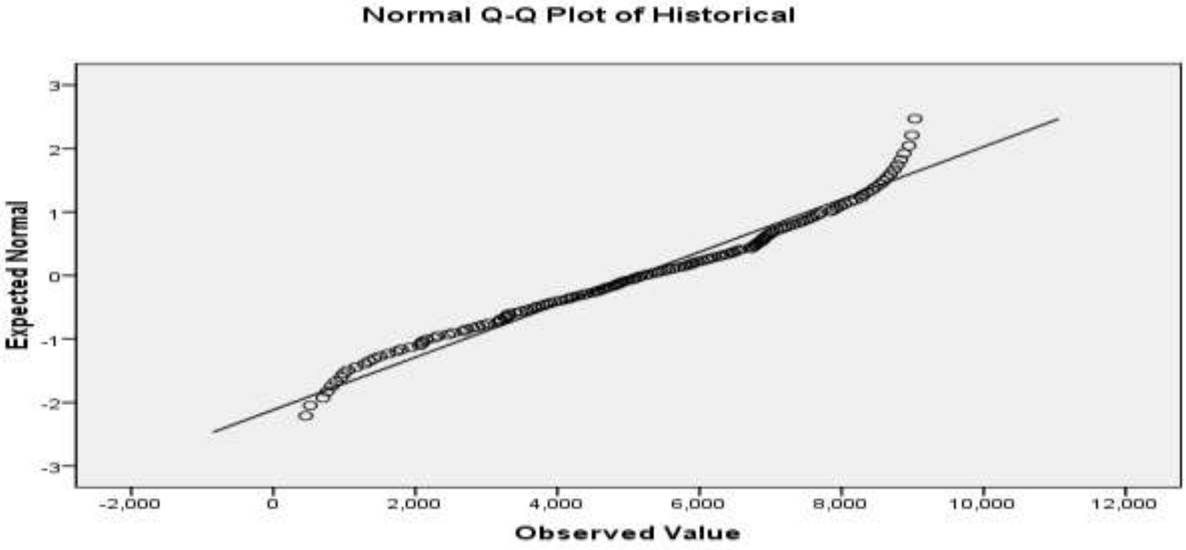
Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
Historical	146	75.6%	47	24.4%	193	100.0%
Observed	146	75.6%	47	24.4%	193	100.0%

Tests of Normality

	Shapiro-Wilk		
	Statistic	df	Sig.
Historical	.961	146	.000
Observed	.952	146	.000

From table 4.22 it can be seen that the distribution is not a normal distribution since the significance level is below the α = 0.05 this is also confirmed in figure 4.3 which show that the data is not a linear fit. Hence a non-parametric test (Wilcoxon Signed Ranks test) was carried out to determine if there are any significant differences between both data. Results are shown in Table 4.23



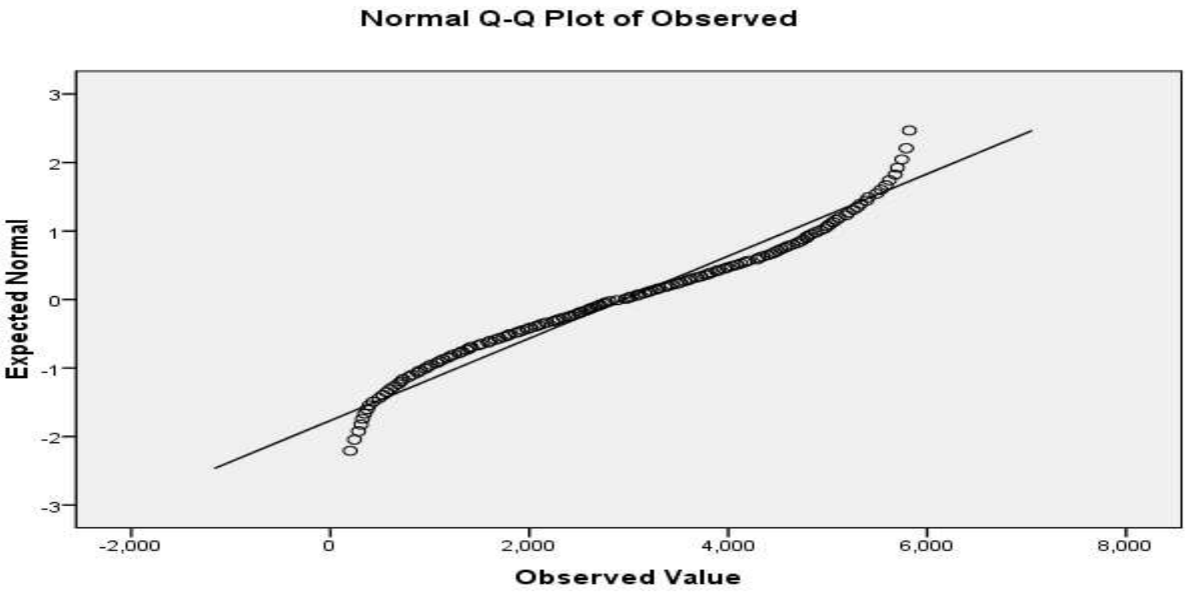


Fig 4.3: Normality Plot for Rollers System

Table 4.23: Wilcoxon Signed Ranks Test for Significance Rollers System

Test Statistics^b

	Observed - Historical
Z	-10.482^a
Asymp. Sig. (2-tailed)	.000

a. Based on positive ranks.

b. Wilcoxon Signed Ranks Test

The wilcoxon signed test rank show that there is a significant difference in both data collected (Z= -10.482), P = (<0.001)). Hence the observational experimental data will be used for validation purposes while the historical data will be used to establish the baseline situation.

4.2.4 Test for normality for Slitter System Failure Data

Applying equation 3.5, the results are as follows:

Table 4.24: Results for Normality Test

```
EXAMINE VARIABLES=SSH SSO
  /PLOT BOXPLOT STEMLEAF NPPLOT
  /COMPARE GROUP
  /STATISTICS NONE
  /CINTERVAL 95
  /MISSING LISTWISE
```

```
/NOTOTAL.
```

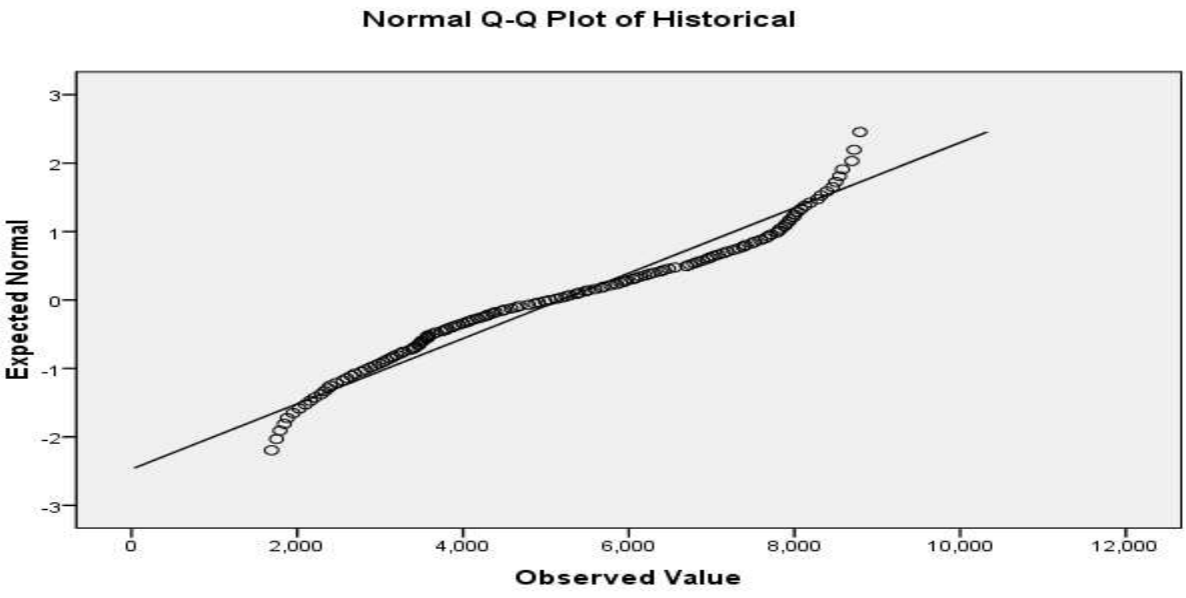
Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
Historical	141	45.8%	167	54.2%	308	100.0%
observed	141	45.8%	167	54.2%	308	100.0%

Tests of Normality

	Shapiro-Wilk		
	Statistic	df	Sig.
Historical	.946	141	.000
observed	.960	141	.000

From table 4.24 it can be seen that the distribution is not a normal distribution since the significance level is below the α = 0.05 this is also confirmed in figure 4.4 which show that the data is not a linear fit. Hence a non-parametric test (Wilcoxon Signed Ranks test) was carried out to determine if there are any significant differences between both data. Results are shown in Table 4.25



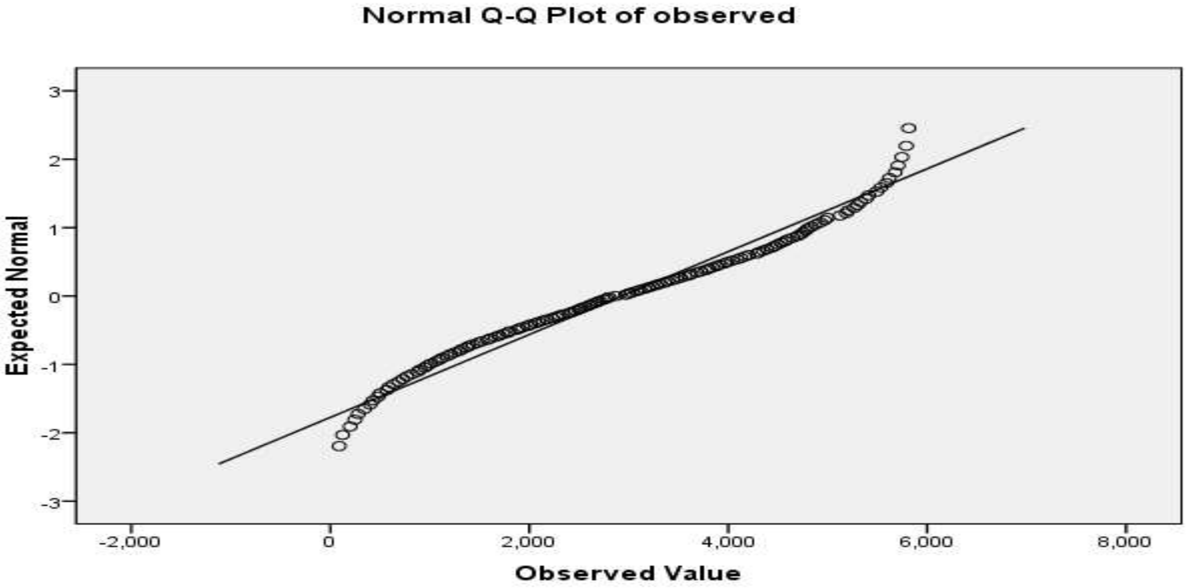


Fig 4.4: Normality Plot for Slitter System

Table 4.25: Wilcoxon Signed Ranks Test for Significance Slitter System

Test Statistics[b]

	observed - Historical
Z	-10.302[a]
Asymp. Sig. (2-tailed)	.000

a. Based on positive ranks.

b. Wilcoxon Signed Ranks Test

The wilcoxon signed test rank show that there is a significant difference in both data collected (Z= -10.302), P = (<0.001)). Hence the observational experimental data will be used for validation purposes while the historical data will be used to establish the baseline situation.

4.2.5 Test for normality for Compounding Machine Failure Data

Applying equation 4.1, the results are as follows:

Table 4.26: Results for Normality Test

```
EXAMINE VARIABLES=CH CO
  /PLOT BOXPLOT STEMLEAF NPPLOT
  /COMPARE GROUP
  /STATISTICS NONE
  /CINTERVAL 95
  /MISSING LISTWISE
```

```
/NOTOTAL.
```

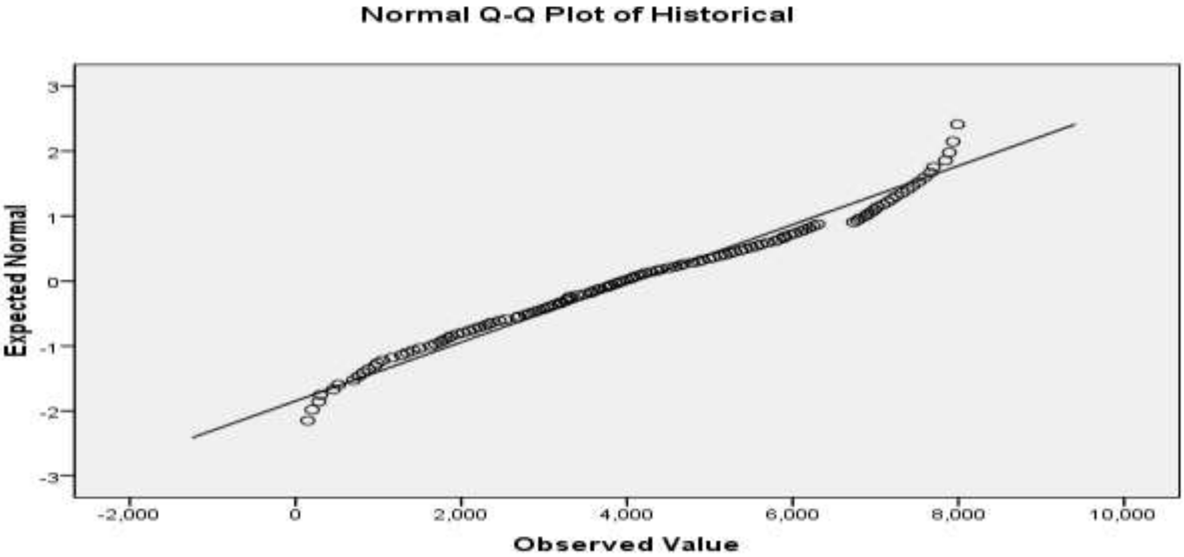
Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
Historical	125	51.2%	119	48.8%	244	100.0%
Observered	125	51.2%	119	48.8%	244	100.0%

Tests of Normality

	Shapiro-Wilk		
	Statistic	df	Sig.
Historical	.966	125	.003
Observered	.955	125	.000

From table 4.26 it can be seen that the distribution is not a normal distribution since the significance level is below the α = 0.05 this is also confirmed in figure 4.5 which show that the data is not a linear fit. Hence a non-parametric test (Wilcoxon Signed Ranks test) was carried out to determine if there are any significant differences between both data. Results are shown in Table 4.27



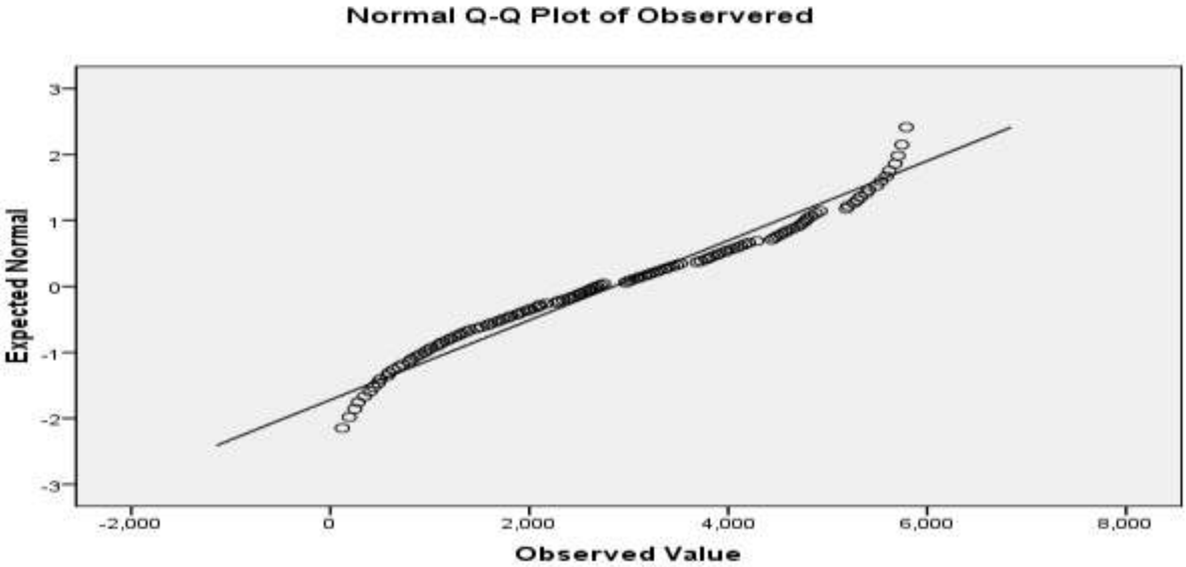


Fig 4.5: Normality Plot for compounding machine

Table 4.27: Wilcoxon Signed Ranks Test for Significance Compounding System

Test Statistics^b

	Observered - Historical
Z	-9.702^a
Asymp. Sig. (2-tailed)	.000

a. Based on positive ranks.

b. Wilcoxon Signed Ranks Test

The wilcoxon signed test rank show that there is a significant difference in both data collected (Z= -9.702), P = (<0.001)). Hence the observational experimental data will be used for validation purposes while the historical data will be used to establish the baseline situation.

4.2.6 Analysis of Maintenance Cost

An analysis of the total cost of maintenance by the company (tummy tummy food industries limited) was carried out using time series analysis. The main goal of this analysis is to create a model that forecasts the future cost of maintenance at a given time parameter with the current maintenance strategy of the company, hence a trend analysis on the historical data, and a corresponding model obtained. A three years forecast of the maintenance cost using the obtained series model was performed with the aim of obtaining the cost of maintenance at the present strategy. A test for stationarity was carried out to validate the use of the model for making forecasts.

Table 4.28: Summary of Maintenance Cost

Total Maintenance Cost	Year Code (t)
₦ 4,211,628	1
₦ 4,531,248	2
₦ 5,935,326	3

Key: Year code (t) represents the year, where 1 is 2015, 2 is 2016 and 3 is 2017

Using table 4.1.11 time series analysis will be performed with the maintenance cost modelled over time and also tested for stationarity.

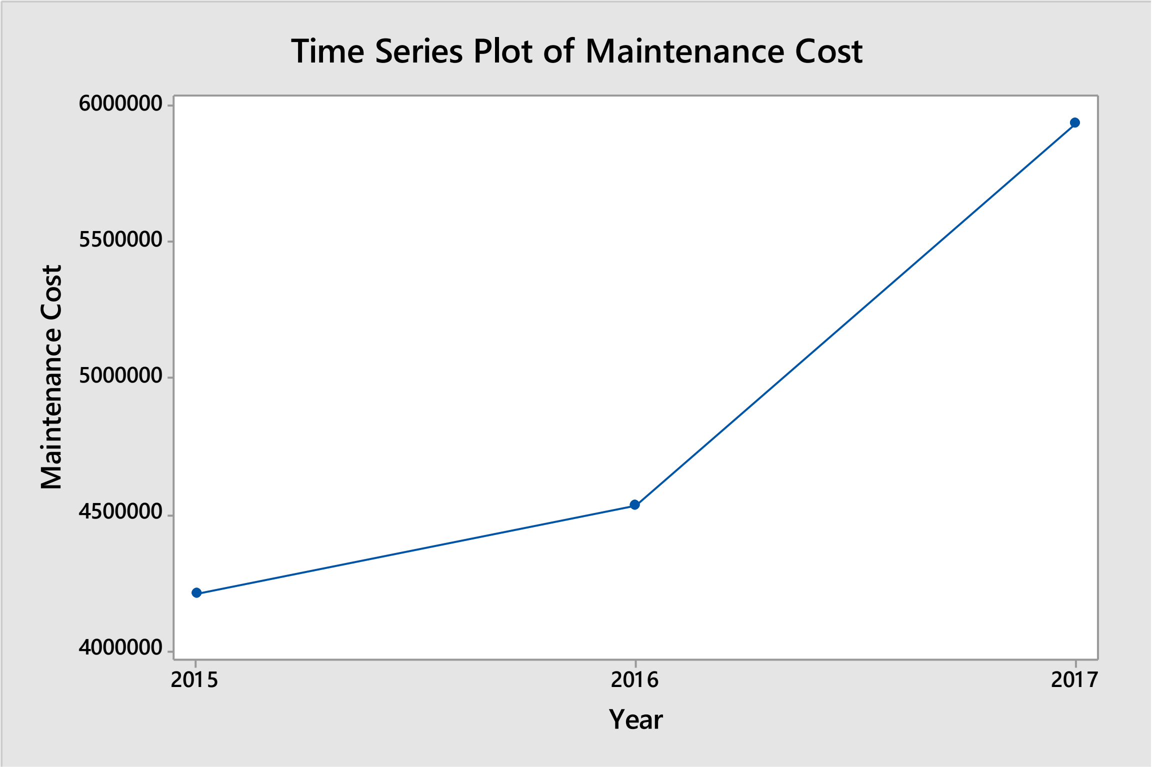


Fig 4.6: Distribution of Total Maintenance Cost (2015 – 2017)

A trend analysis on Total maintenance cost was carried out in order to obtain the fitted model for future forecast and results showed an increasing trend line of total maintenance cost as seen in figure 4.7. The trend fitted model for predicting the total maintenance cost in a given time is expressed as shown in table 4.29.

Table 4.29: Trend Analysis of Total Maintenance Cost

Data Maintenance Cost

Length 3

NMissing 0

Fitted Trend Equation

$$Y_t = 3169036 + 861849 * t \quad (4.1)$$

Where t is the time in year

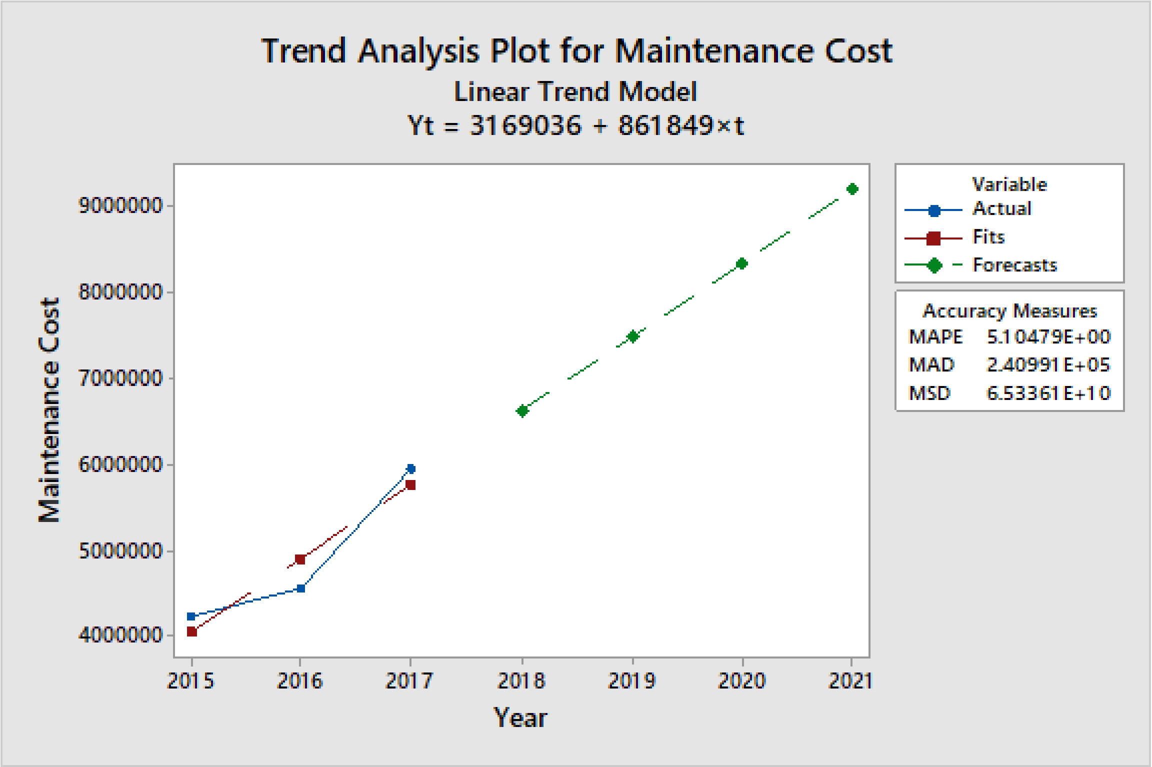


Fig 4.7: Trend Plot of Total Maintenance Cost with Future Forecast

From the analysis it was found that in the year 2021, if the current maintenance strategy is still maintained, the total cost is expected to be about ₦ 9,201,979. Table 4.30 shows the yearly forecast from 2018 to 2021.

Table 4.30: Yearly Forecast

Period	Forecast
2018	₦ 6,616,432
2019	₦ 7,478,281
2020	₦ 8,340,130
2021	₦ 9,201,979

A test of stationarity was carried out on the model using Augmented Dickey-Fuller test in order to determine if it has a unit root and stationary or not. Table 4.31 shows the results from this analysis.

Table 4.31: Test of Stationarity Using Augmented Dickney-Fuller Test

Null Hypothesis: MAINTENANCE_COST has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=1)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		1.726325	0.9106
Test critical values:	1% level	-4.234491	
	5% level	-2.349470	
	10% level	-1.656218	

*MacKinnon (1996) one-sided p-values

The results from table 4.31 found that the series has a unit root and stationary over time since the t- statistics value of 1.726325 was obtained with a p-value of 0.33 which falls on the acceptance region of the hypothesis assuming a 95% confidence level. This implies that the model obtained can be used to make future forecasting behaviour of the process.

4.2.7 Regression Analysis on Estimating Total Maintenance Cost

Using data presented in tables 4.11, 4.12, 4.13, 4.14, a multiple regression analysis was performed with the aim of determine the effects of the different maintenance cost variables on the total maintenance cost of the manufacturing firm. The results are shown in table 4.32, and 4.33.

Table 4.32: Regression Statistics

Regression Statistics	
Multiple R	.811
R Square	.692
Adjusted R Square	.528
Standard Error	55896.22
Observations	5

The result in table 4.32 found an R-square of 69.2% which implies that the maintenance cost variables can only explain about 69.2% of total variation in total maintenance cost. This implies a positive coefficient of determination.

Table 4.33: ANOVA for assessing cost predictors on total maintenance cost

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	1.957E10	4	4.891E9	6.77	0.056
	Residual	.000	0	.		
	Total	1.957E10	4			

a. Predictors: (Constant), x8, x7, x6, x5, x4, x3, x2, x1

b. Dependent Variable: TC

The result displayed in table 4.33 showed an F-value and p-value of 0.056 which falls on the acceptance region of the hypothesis. This result implies that the maintenance cost variables has significant impact on the total maintenance cost.

The resultant multiple regression model equation is as follows:

$$y=197561+1.318x_1+1.5445x_2-0.2989x_3+0.127x_4+0.324x_5+0.0134x_6+0.0987x_7+1.701x_8 \quad (4.1.2)$$

Where y = Total maintenance cost, 197561 is the constant cost incurred when maintenance is carried out. This constant is now termed as downtime cost associated with carrying out maintenance. x_1......x_8 is the cost variables predictors that affecting the total cost of maintenance.

4.2.8 System Reliability

The components failure times was subjected to reliability lifetime distribution models in order to determine which distribution model suits each component. The following reliability life time distribution was tested:

- Weibull Distribution Model
- Lognormal Distribution Model
- Exponential Distribution Model
- Normal Distribution Model

The results are discussed as follows:

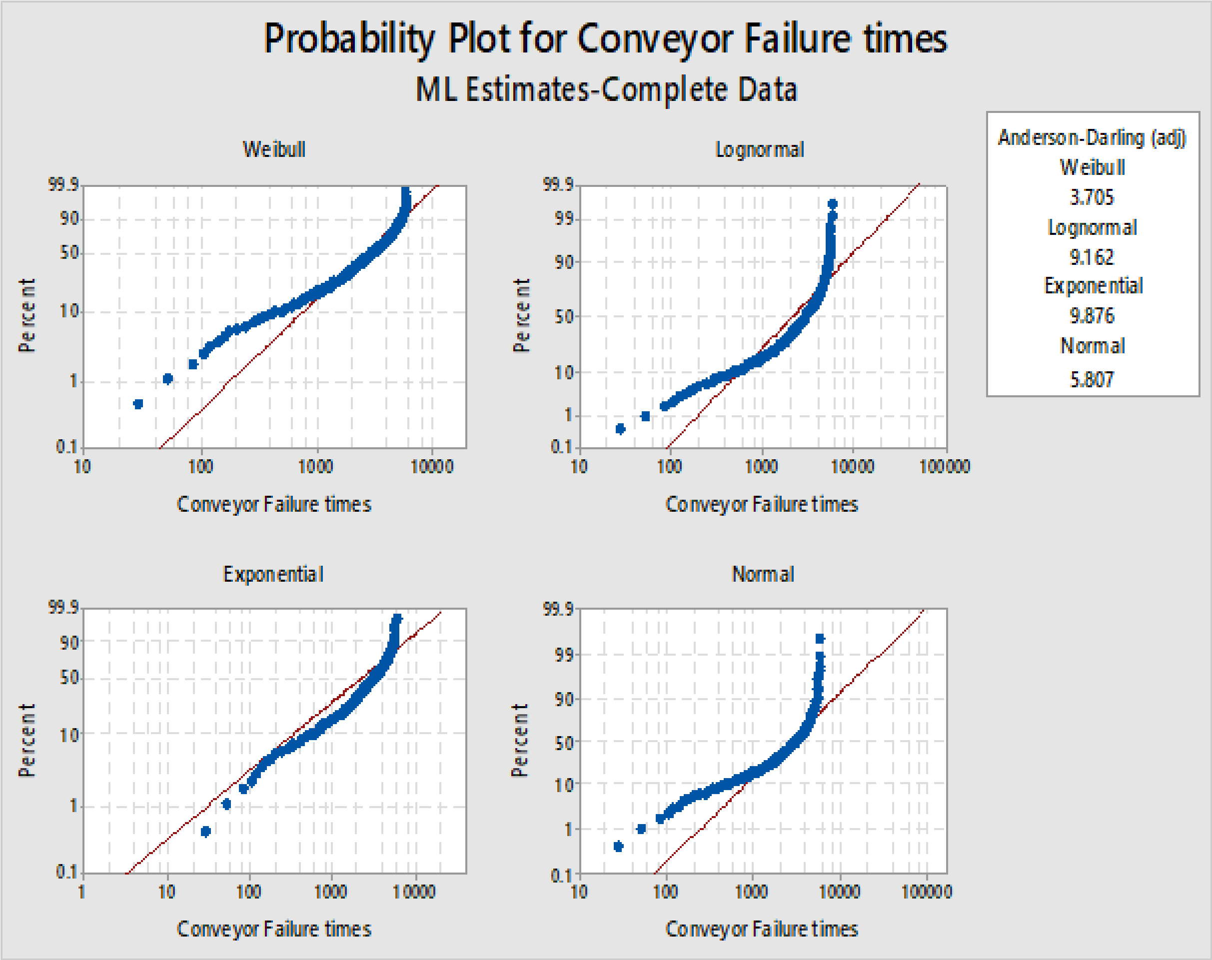


Fig 4.8 Reliability Distribution for Conveyor System

Figure 4.8 show the reliability distribution of the conveyor system, which indicated that the failure data follows a weibull distribution model as indicated by the closeness to fit of the linear line. This is confirmed by Anderson darling goodness of fit value of 3.705, which is the lowest among other distributions in table 4.34.

Table 4.34: Anderson Darling Goodness of Fit

Distribution	Anderson Darling Value
Weibull Distribution Model	3.705
Lognormal Distribution Model	9.162
Exponential Distribution Model	9.876
Normal/Gaussian Distribution Model	5.807

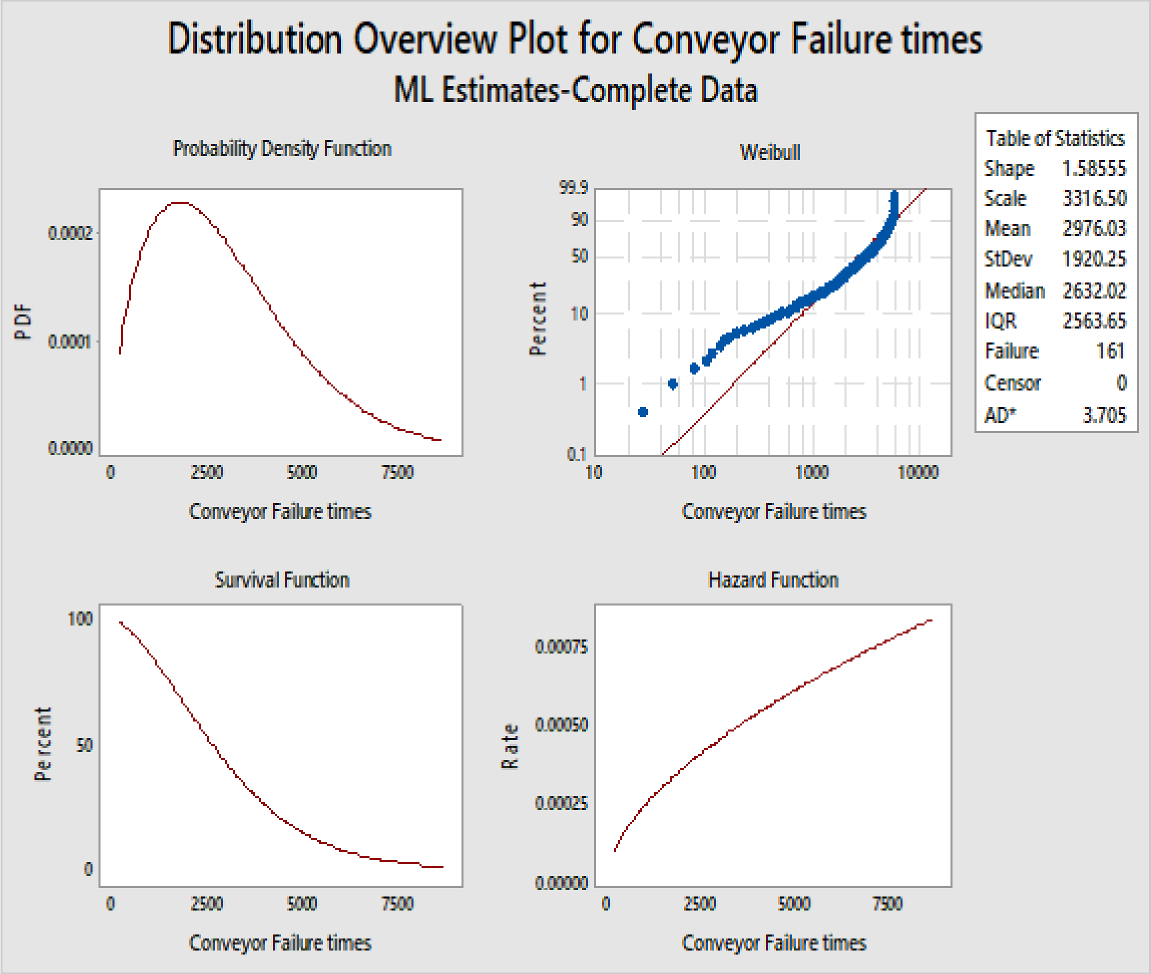


Table of Statistics	
Shape	1.58555
Scale	3316.50
Mean	2976.03
StDev	1920.25
Median	2632.02
IQR	2563.65
Failure	161
Censor	0
AD*	3.705

Fig 4.9: Distribution Overview Plot for Conveyor Failure Times

The distribution is skewed right, as shown in figure 4.9 this indicates an increasing failure rate with the scale parameter of 1.58 which implies that there is reliability degradation thus decreasing the survival function of the component with time.

Figure 4.10 show the reliability distribution of the mixer system, which also indicated that the failure data follows a weibull distribution model as indicated by the closeness to fit of the linear line. This is confirmed by Anderson darling goodness of fit value of 2.736, which is the lowest among other distributions in table 4.35.

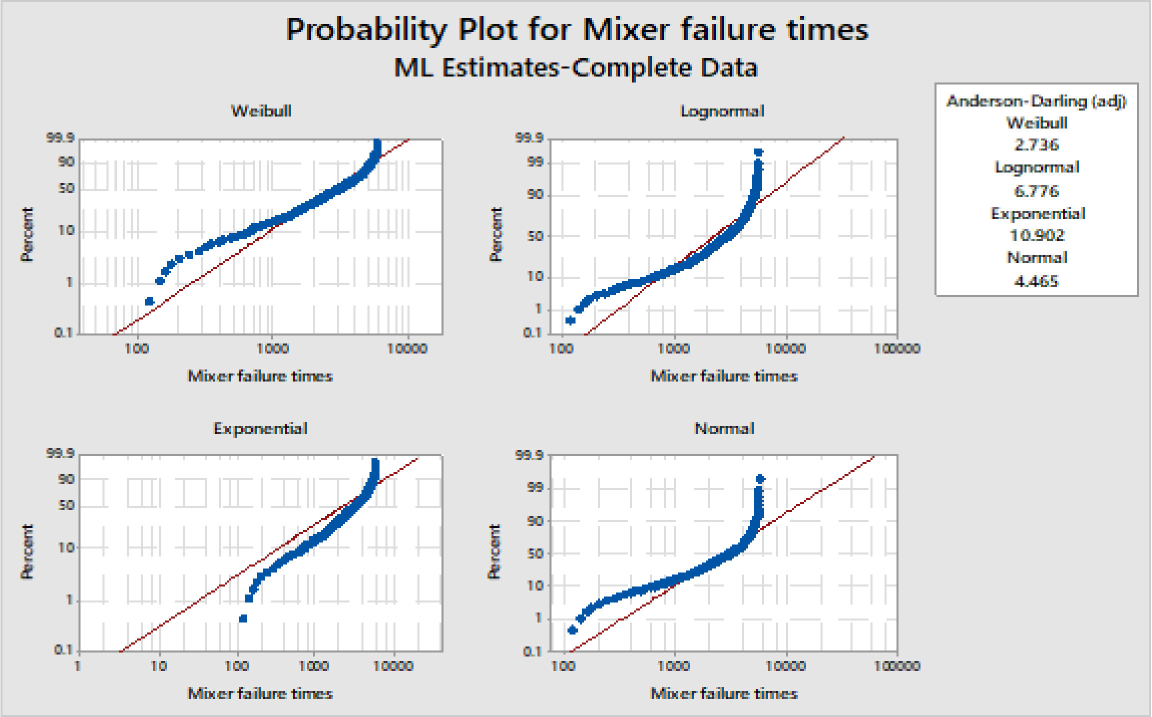


Fig 4.10 Reliability Distribution for Mixer System

Table 4.35: Anderson Darling Goodness of Fit

Distribution	**Anderson Darling Value**
Weibull Distribution Model	2.736
Lognormal Distribution Model	6.776
Exponential Distribution Model	10.902
Normal/Gaussian Distribution Model	4.465

As shown in figure 4.11, the distribution is skewed right, this indicates an increasing failure rate with the scale parameter of 1.76 which implies that there is reliability degradation thus decreasing the survival function of the component with time.

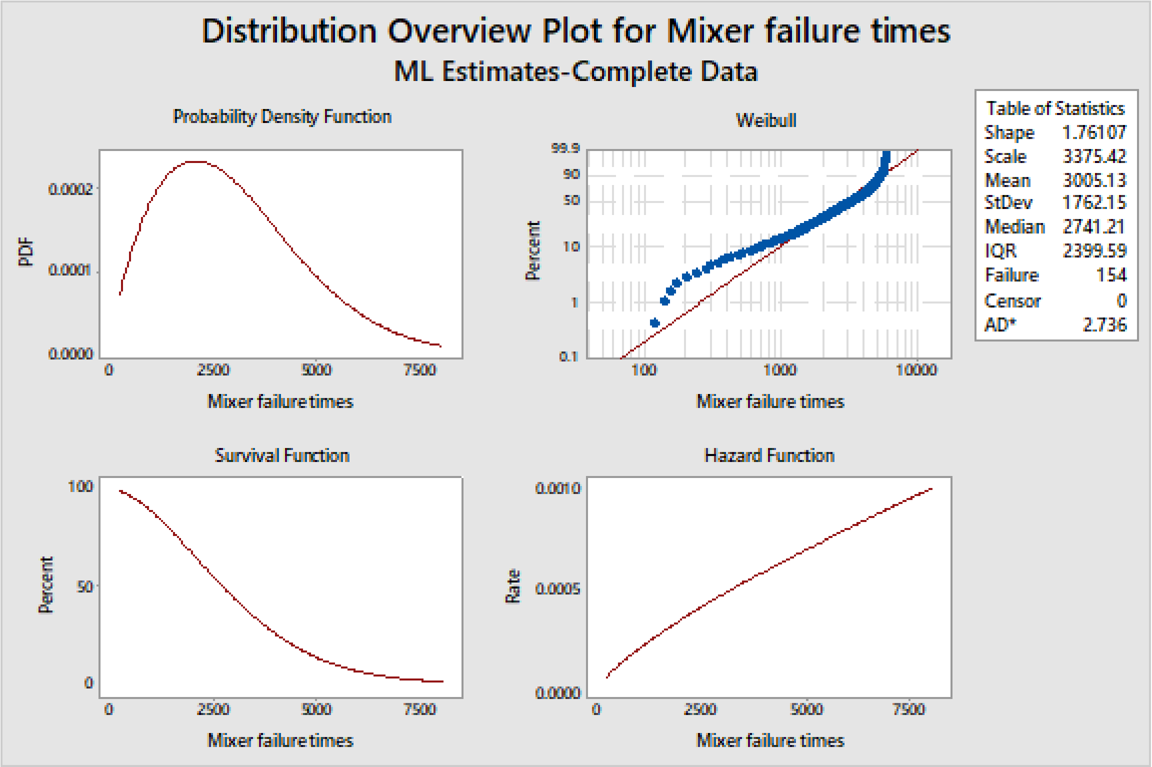


Table of Statistics	
Shape	1.76107
Scale	3375.42
Mean	3005.13
StDev	1762.15
Median	2741.21
IQR	2399.59
Failure	154
Censor	0
AD*	2.736

Fig 4.11: Distribution Overview Plot for Mixer Failure Times

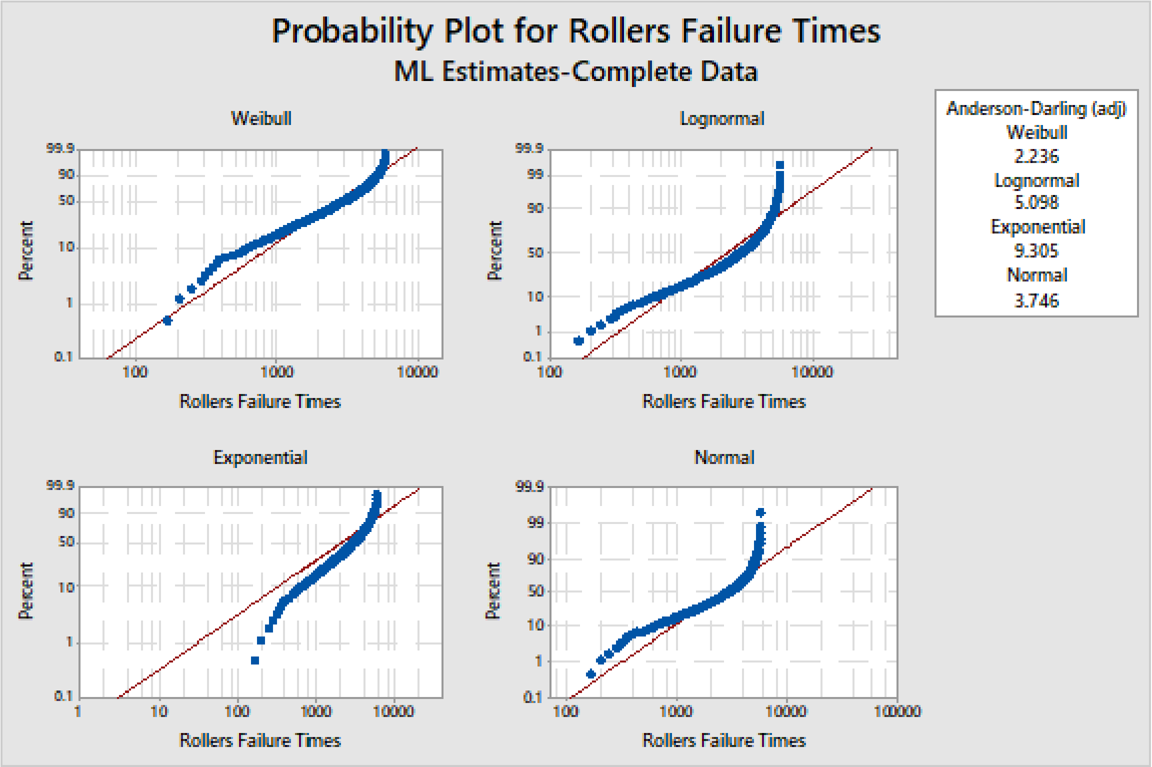


Fig 4.12 Reliability Distribution for Roller System

Figure 4.12 show the reliability distribution of the roller system, which also indicated that the failure data follows a weibull distribution model as indicated by the closeness to fit of the linear line. This is confirmed by Anderson darling goodness of fit value of 2.236, which is the lowest among other distributions in table 4.36.

Table 4.34: Anderson Darling Goodness of Fit

Distribution	Anderson Darling Value
Weibull Distribution Model	2.236
Lognormal Distribution Model	5.098
Exponential Distribution Model	9.305
Normal/Gaussian Distribution Model	3.746

In figure 4.13, the reliability distribution of the roller data, indicates that the distribution is skewed right, this indicates an increasing failure rate with the scale parameter of 1.73 which implies that there is reliability degradation thus decreasing the survival function of the component with time.

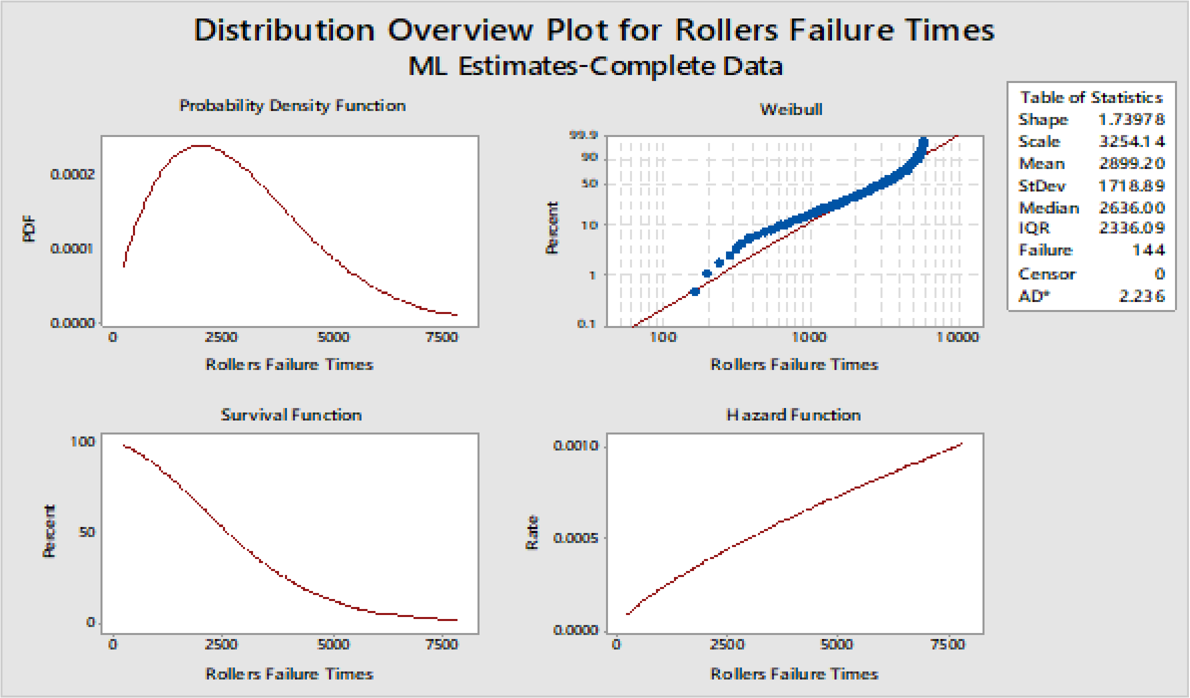


Fig 4.13: Distribution Overview Plot for Roller Failure Times

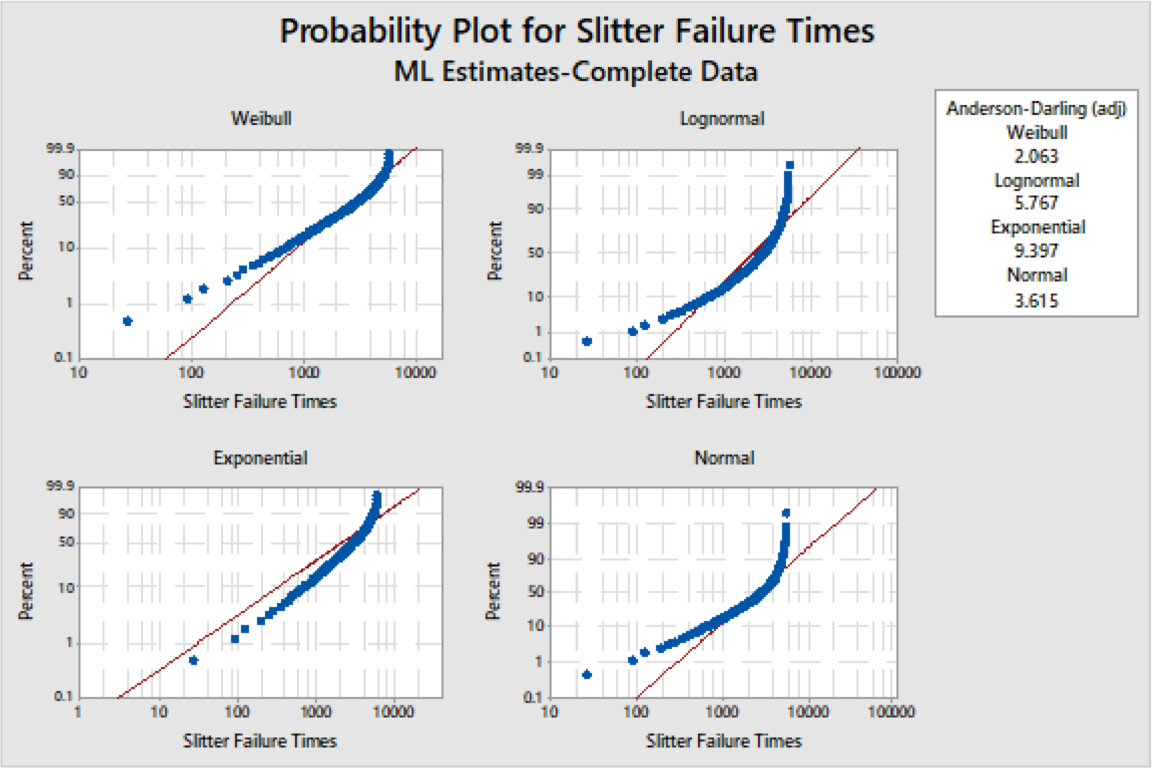


Fig 4.14 Reliability Distribution for Roller System

Figure 4.14 show the reliability distribution of the mixer system, which indicated that the failure data follows a weibull distribution model as indicated by the closeness to fit of the linear line. This is confirmed by Anderson darling goodness of fit value of 2.036, which is the lowest among other distributions in table 4.35.

Table 4.35: Anderson Darling Goodness of Fit

Distribution	**Anderson Darling Value**
Weibull Distribution Model	2.036
Lognormal Distribution Model	5.767
Exponential Distribution Model	9.397
Normal/Gaussian Distribution Model	3.615

In figure 4.15, the reliability distribution of the roller data, indicates that the distribution is skewed right, this indicates an increasing failure rate with the scale parameter of 1.71 which implies that there is reliability degradation thus decreasing the survival function of the component with time.

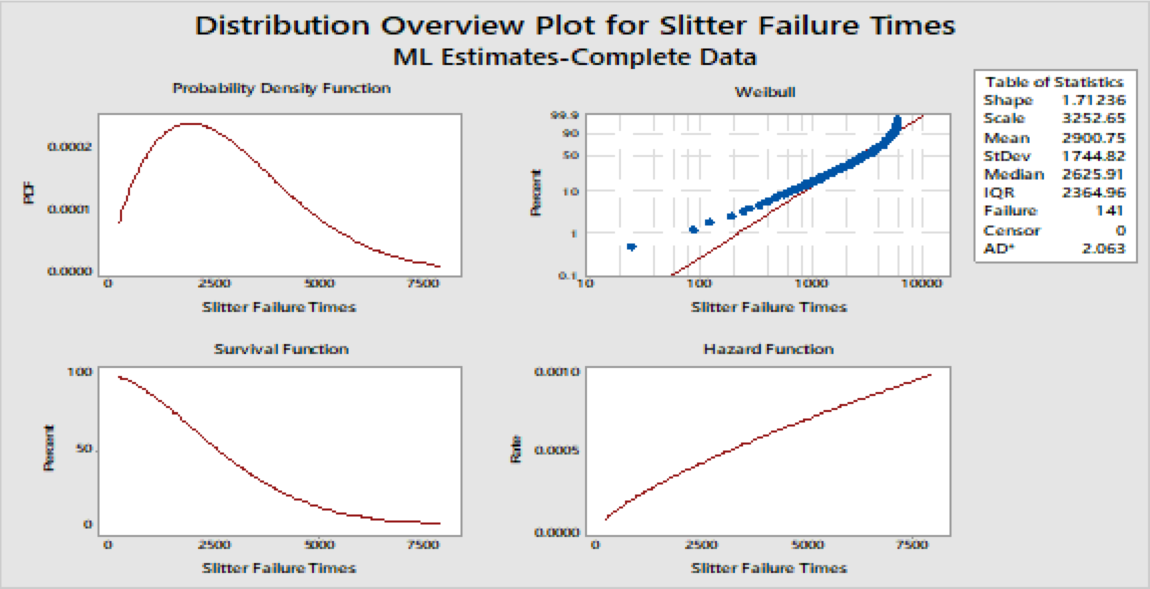


Fig 4.15: Distribution Overview Plot for Slitter Failure Times

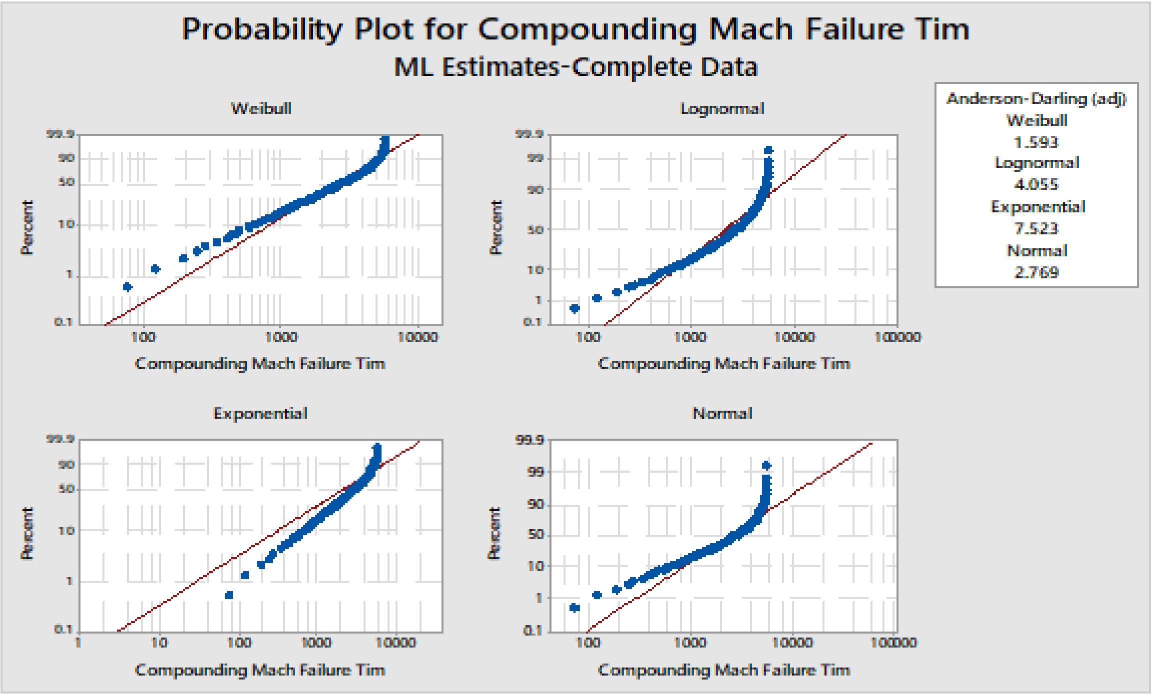


Fig 4.16: Reliability Distribution for Compounding Machine

Figure 4.16 show the reliability distribution of the mixer system, which indicated that the failure data follows a weibull distribution model as indicated by the closeness to fit of the linear line. This is confirmed by Anderson darling goodness of fit value of 1.593, which is the lowest among other distributions in table 4.36.

Table 4.36: Anderson Darling Goodness of Fit

Distribution	**Anderson Darling Value**
Weibull Distribution Model	1.593
Lognormal Distribution Model	4.055
Exponential Distribution Model	7.523
Normal/Gaussian Distribution Model	2.769

The reliability distribution as shown in figure 4.17, is skewed right, this indicates an increasing failure rate with the scale parameter of 1.68 which implies that there is reliability degradation thus decreasing the survival function of the component with time.

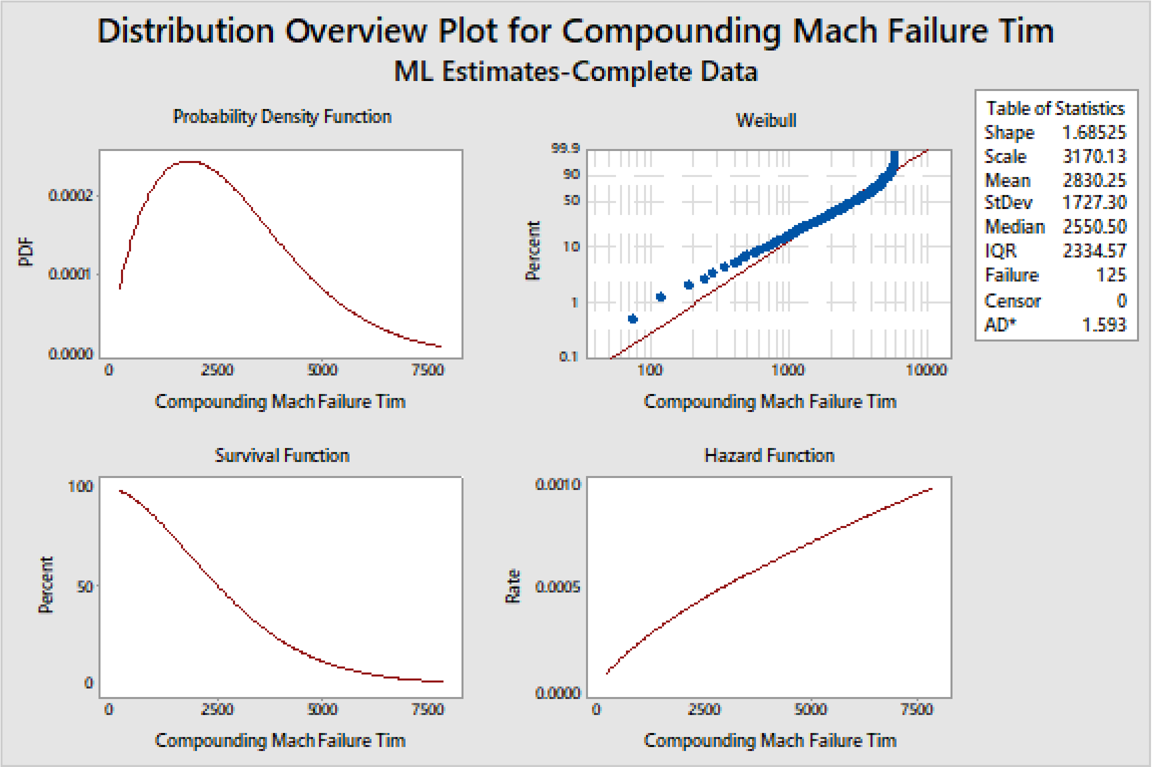


Fig 4.17: Distribution Overview Plot for Slitter Failure Times

The Components failure data used for reliability lifetime distribution model is a complete failure data and a graphical illustration is presented in figure 4.18

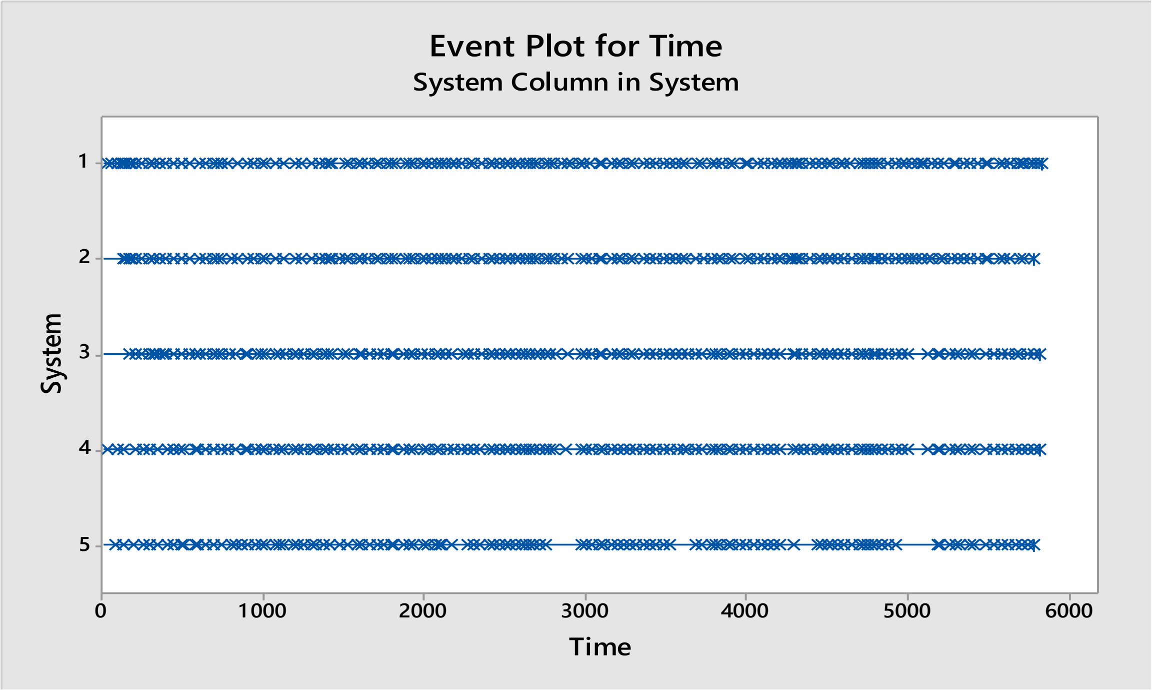


Fig 4.18: Graphical Representation of the Components Failure Data

Keys: 1: Conveyor System, 2: Mixer System, 3: Roller System, 4: Slitter System, 5: Compounding Machine.

4.2.9 Downtime analysis and Performance Evaluation

From the data presented in table 4.15, 4.16 and 4.17, the maintenance variable are summarised in table 4.37

Table 4.37: OEE Variables

MONTHS	**VARIABLES**								
	2015			**2016**			**2017**		
	Avail (%)	Perf (%)	Qual (%)	Avail (%)	Perf (%)	Qual (%)	Avail (%)	Perf (%)	Qual (%)
JANUARY	73.10	73.40	93.70	67.90	73.60	83.70	75.86	84.30	90.73
FEBUARY	72.70	60.40	93.40	75.78	79.40	95.70	73.08	81.78	88.39
MARCH	76.60	73.10	94.80	77.20	80.10	95.44	70.88	80.00	86.00
APRIL	75.60	79.70	92.00	76.60	78.80	93.31	72.46	81.23	87.88
MAY	73.50	80.90	91.90	70.80	83.90	91.05	74.70	83.66	89.42
JUNE	45.20	66.30	70.60	66.87	73.30	90.66	78.00	85.30	92.07
JULY	26.70	49.30	80.70	77.87	79.30	90.70	76.40	84.63	91.42
AUGUST	70.40	84.40	91.20	72.46	74.70	90.10	69.60	74.84	85.25
SEPTEMBER	70.30	83.40	90.20	72.76	73.47	90.00	69.80	74.92	85.73
OCTOBER	68.60	73.20	88.90	78.43	73.20	94.05	72.90	81.73	87.22
NOVEMBER	52.30	72.90	72.50	77.56	65.90	92.47	77.37	84.00	91.49
DECEMBER	67.90	73.60	83.40	78.60	73.60	91.32	78.50	85.52	92.36

N.B: Avail = Availability (%), Perf = Performance (%), Qual = Quality (%).

Table 4.38 OEE Measurements

MONTHS	OVERALL EQUIPMENT EFFECTIVENESS (%)		
	2015	**2016**	**2017**
JANUARY	50.27	41.82	58.02
FEBUARY	40.01	57.58	53.34
MARCH	53.08	59.01	48.77
APRIL	55.43	56.32	51.72
MAY	54.64	54.08	55.88
JUNE	21.15	44.43	61.25
JULY	10.62	56.00	59.11
AUGUST	54.18	48.76	44.40
SEPTEMBER	52.88	48.11	44.83
OCTOBER	44.64	53.99	51.97
NOVEMBER	27.64	47.26	59.44
DECEMBER	41.67	52.82	62.00

The trend analysis showed the overall equipment effectiveness generally hovered between 62% and 41.67% in the year's understudy except for June and July 2015 which went as low as 21.15% and 10.62%, due to low availability of equipment as a result of equipment failure and waiting for spare parts materials to arrive from the manufacturer (see fig 4.19 and 4.20). Overall, the average overall equipment effectiveness is 55.30% which is a low value when compared with OEE world standards as illustrated in figure 4.20. This means that the manufacturing organisation under study is in an average condition and there is a required urgent improvement of maintenance policies and strategies otherwise it will be difficult for the manufacturing organisation to sustain it.

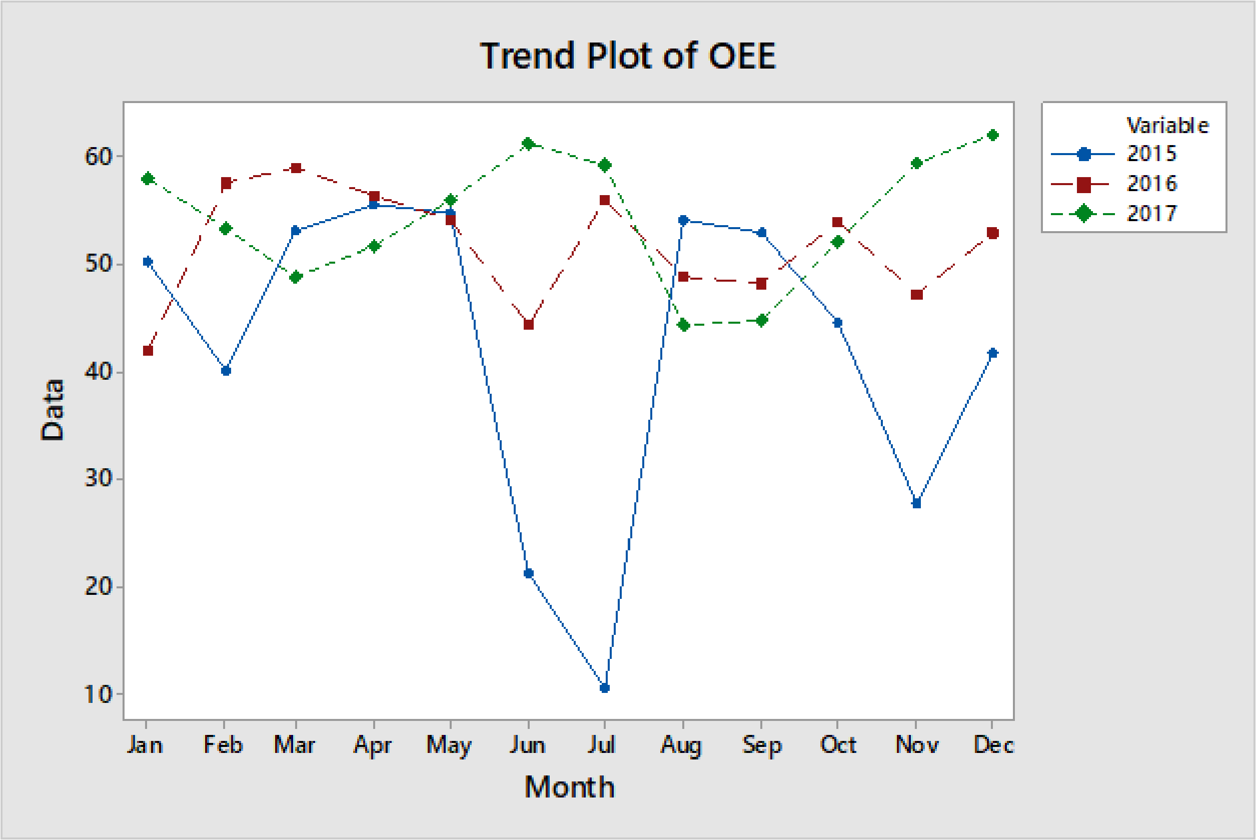


Fig 4.19 OEE Trend Analysis

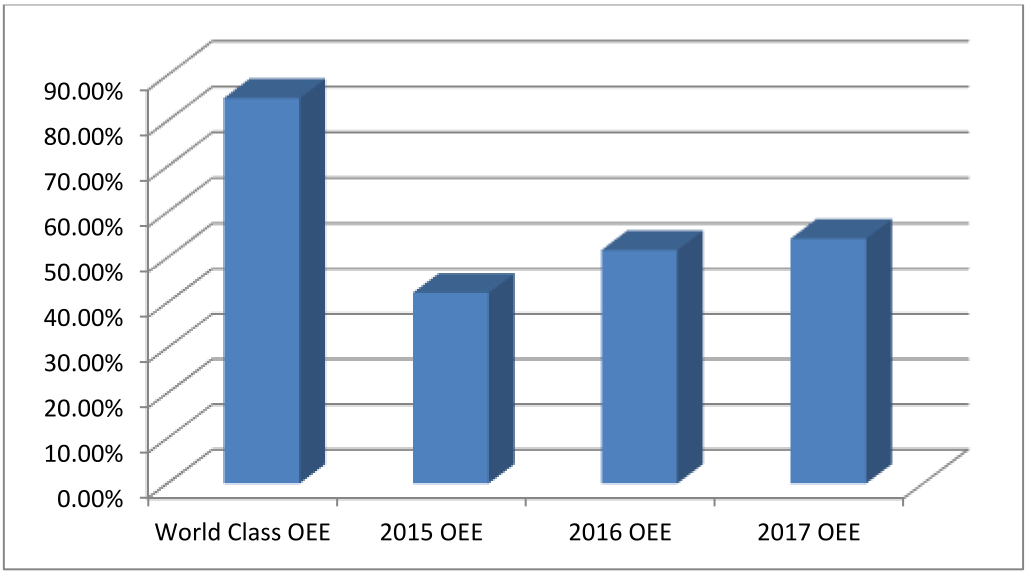


Fig 4.20: Benchmark of Case Study OEE to World Standards

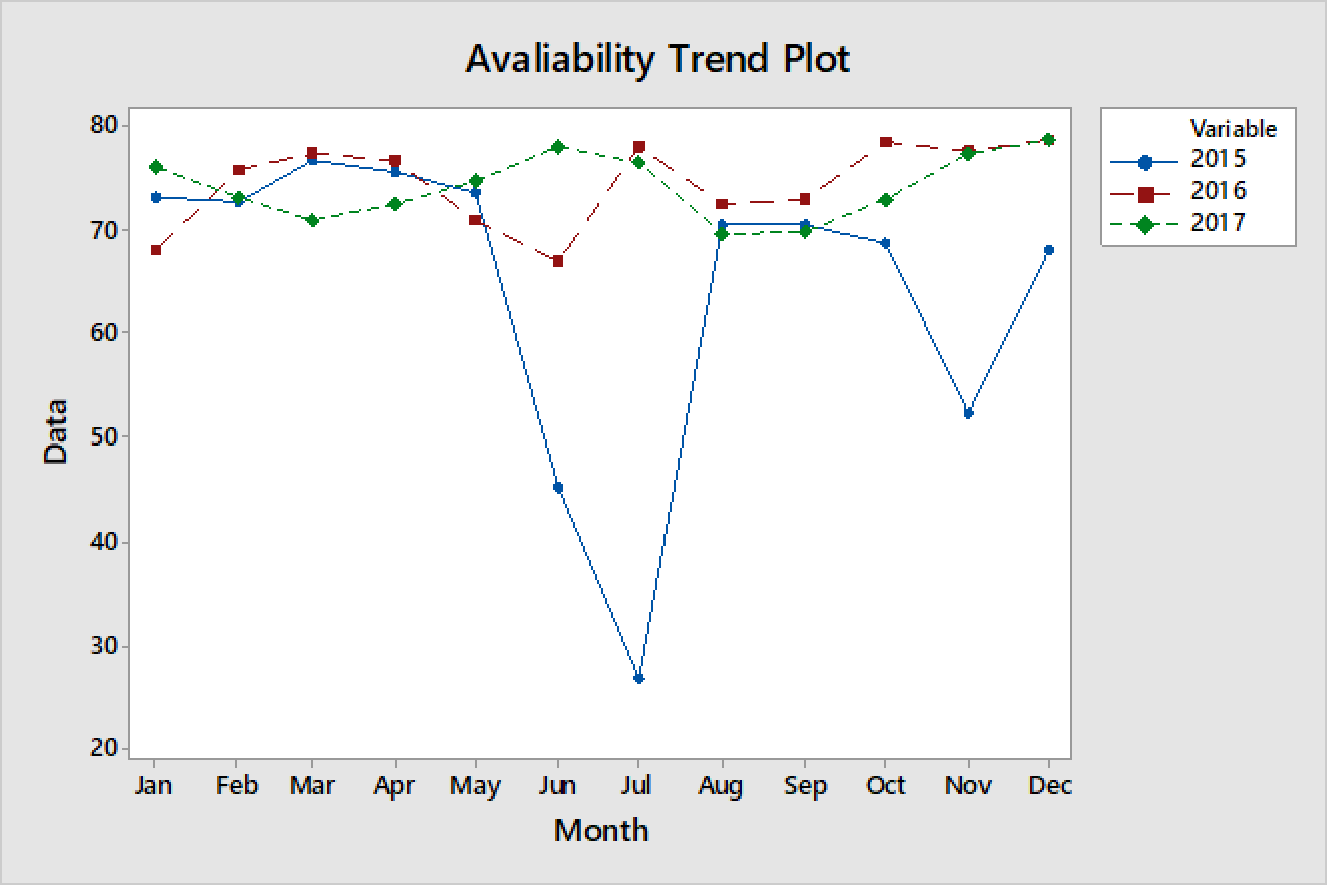


Fig 4.21: Availability Trend Analysis

The average availability for the year under study when compared with the accepted world standards was found to be comparatively lower as illustrated in fig 4.21:

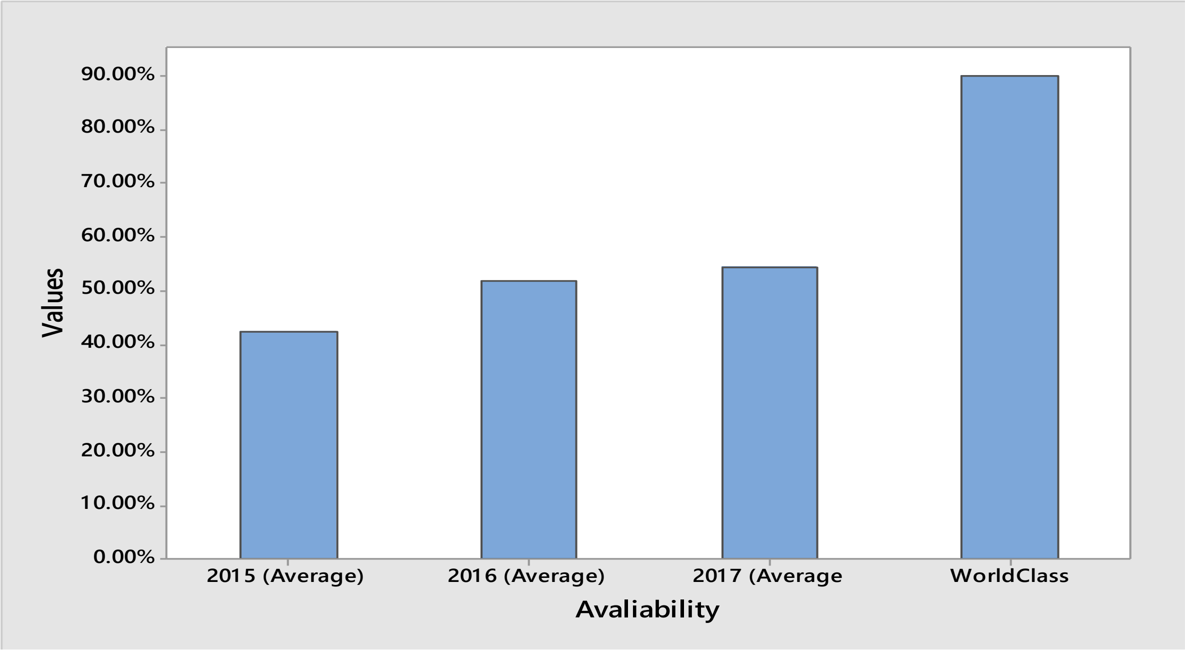


Fig 4.22: Average Availability Benchmark

In order to identify the causes behind these findings in fig 4.22, an analysis of downtime in these years is required using Pareto analysis. Availability is reversely proportional to downtime, and to identify the downtimes that have caused around 80% of total downtime, Pareto chart was drawn. The result is shown in fig 4.23

Table 4.39: Analysis of Downtime Factors

Downtime Factor	**Downtime Minutes**	**Percentage**	**Cumulative Percentage**
Scheduled Maintenance	42723	40.82	40.82
Equipment Failures/Breakdown	25398	24.26	65.08
Waiting for materials to arrive	19856	18.93	84.01
Maintenance job meetings	9723	9.29	93.3
Waiting for maintenance instruction and orders	5153	4.92	98.22
Miscellaneous activities	1797	1.71	100.00

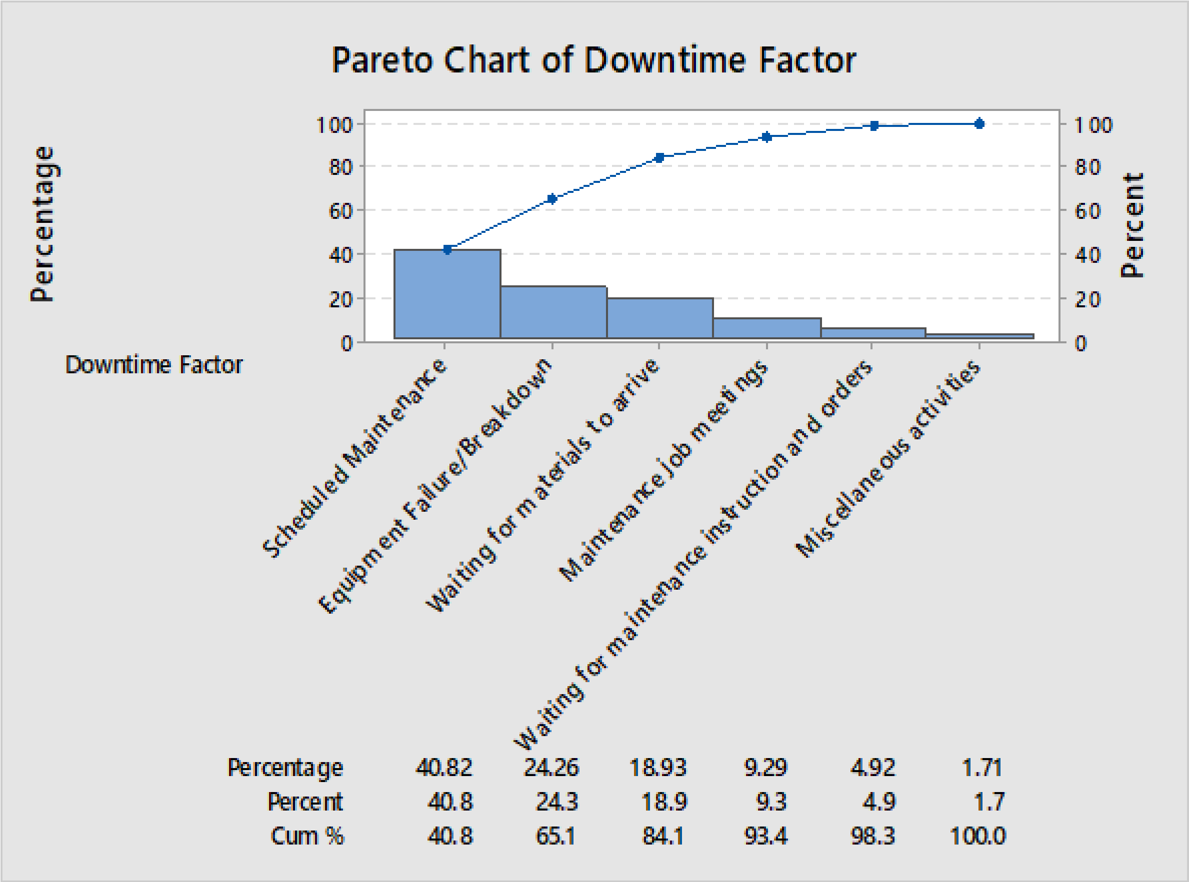


Fig 4.23: Pareto Chart for Downtime Analysis

From table 4.39 and figure 4.23 it has been obtained that scheduled maintenance, equipment failures and breakdown and waiting for materials to arrive have caused 84% of the total downtime. Whereas scheduled maintenance and equipment failures and breakdown was unavoidable, they could be reduced with effective maintenance strategy. The manufacturing organisation under study adopts corrective maintenance as its preferred maintenance strategy only, which can be described as a reactive, firefighting strategy. The information obtained from the maintenance team of the organisation was that most faults and failures can be fixed manually by the maintenance team in a relatively short period of time. But, there have been incidents and occasions where breakdowns resulted in long unavailability of the manufacturing equipment and machines as can be seen in the months of June and July 2015 in table 4.38 and figure 4.21.

Maintenance performance evaluation and downtime analysis is an important area in implementing continuous improvement programs to improve the manufacturing process and consequently overall equipment effectiveness (OEE) is one of the acceptable maintenance performance evaluation methods that are popular in the manufacturing industries to assess the equipment’s effectiveness and performance. It is necessary that in order to improve productivity, the manufacturing organisation under study should look into its manufacturing strategies so that urgent improvement of maintenance policies and strategies can be implemented and adopted thus enhancing productivity levels in the manufacturing organisation.

4.3 Definition and identification of the variables, constraints and objectives for the optimal maintenance strategy.

The goal is to develop a schedule for future maintenance actions for each manufacturing component that is repairable over the period T, where T is the length of the planning scope. The length of planning scope is subsequently divided into J separate intervals of length during which at the end of each interval $\frac{T}{J}$, either preventive maintenance, corrective maintenance action is carried out, or nothing is done. Thus three strategic maintenance schedule are proposed for a multi-component system of N components over T time periods, has N x 3^T possible maintenance schedules. It is assumed that at the end of each interval, the activities (Preventive or corrective actions) carried on each of the components will have a positive effect on the age and the rate of occurrence of failure of the components.

An assumption is made in this study that if preventive maintenance is performed on any component, the effective age of the component is reduced by 30% thus the age reduction factor of preventive maintenance on a component i (α_{pmi}) is assigned a fixed value of 0.7.

To account for the changes in age and rate of failure and if the initial age for each component is equal to zero, let $X_{i,j}$ represent the effective age of component i at the start of period j and $XX_{i,j}$ represent the effective age of component i at the end of period j, then:

$$XX_{i,j} = X_{i,j} + \frac{T}{j} \qquad \text{For } \boldsymbol{i=1,\ldots,N; j=1,\ldots,T} \qquad (4.1)$$

4.3.1 Maintenance Actions

In a scenario where maintenance action is carried out on component i at the end of period j and the preventive maintenance action effectively reduces the age of the component i for the start of the next period then:

$$\boldsymbol{X}_{\boldsymbol{i},j+\mathbf{1}} = \boldsymbol{\alpha}_{\boldsymbol{pmi}} \times \boldsymbol{X}_{\boldsymbol{i,j}} \quad \text{For } \boldsymbol{i} = \mathbf{1}, \dots, \boldsymbol{N}; \boldsymbol{j} = \mathbf{1}, \dots, \boldsymbol{T} \text{ and } (\mathbf{0} \le \boldsymbol{\alpha}_{\boldsymbol{pmi}} \le \mathbf{1}) \qquad (4.2)$$

α_{pmi} indicates the effects of maintenance on the component i thus if $\alpha_{pmi} = 0$ the maintenance action improved the component i to a state of "good-as-new" and if $\alpha_{pmi} = 1$ maintenance action has no effect, and the component i remains in a state of "bad-as-old".

If at the end of period j component i is replaced with another new component, and the component is returned to a state of "good-as-new" then:

$$\boldsymbol{X}_{\boldsymbol{i},j,+\mathbf{1}} = \mathbf{0} \quad \text{For } \boldsymbol{i} = \mathbf{1}, \dots, \boldsymbol{N}; \boldsymbol{j} = \mathbf{1}, \dots, \boldsymbol{T} \qquad (4.3)$$

If no action takes place at the end of period j and the rate of occurrence of failure of component i remains the same as that of the previous period, then:

$$\boldsymbol{X}\mathrm{X}_{\mathrm{i},} = \boldsymbol{X}_{\boldsymbol{i,j}+\boldsymbol{T/J}} \quad \text{for } \boldsymbol{i} = \mathbf{1}, \dots, \boldsymbol{N}; \boldsymbol{j} = \mathbf{1}, \dots, \boldsymbol{T} \qquad (4.4)$$

$$\boldsymbol{X}_{\boldsymbol{i},j+\mathbf{1}} = \boldsymbol{X}\mathrm{X}_{\boldsymbol{i,j}} \quad \text{for } \boldsymbol{i} = \mathbf{1}, \dots, \boldsymbol{N}; \boldsymbol{j} = \mathbf{1}, \dots, \boldsymbol{T} \qquad (4.5)$$

4.3.2 Cost of Maintenance

Taking into account that any maintenance or replacement action that is carried out is associated with cost, the cost of maintenance or replacement of component i at the end of period j include the total sum of failure cost, cost of preventive maintenance, cost of replacement of component and downtime cost.

To calculate for failure cost, the expected number of failures for component i in period j is calculated and multiplied by the cost of failure for component i

$$FC_i = \boldsymbol{F_i} \times [\boldsymbol{N_{i,}}] \quad \text{for } \boldsymbol{i} = \mathbf{1}, \dots\,., \boldsymbol{N}; \boldsymbol{j} = \mathbf{1}, \dots\,., \boldsymbol{T} \qquad (4.6)$$

Where

F_i = cost of failure for component i

$[\boldsymbol{N_{i,j}}]$ = expected number of failures for component i in period j

From equation 4.6

$$[\boldsymbol{N_{i,j}}] = \int_{x_{\mathrm{i,j}}}^{x^{!}{}_{\mathrm{i.j}}} u_{\mathrm{i}}(t)\,dt \quad \text{for } \boldsymbol{i} = \mathbf{1}, \dots\,., \boldsymbol{N}; \boldsymbol{j} = \mathbf{1}, \dots\,., \boldsymbol{T} \qquad (4.7)$$

$$\boldsymbol{E}[\boldsymbol{Ni,j}] = \int_{x_{\mathrm{i,j}}}^{x^{!}{}_{\mathrm{i.j}}} \lambda_i \beta_i t^{\lambda_i - 1}\,dt = \lambda_{\mathrm{i}} \left[(\mathrm{xx}_{\mathrm{i,j}})^{\beta_i} - (\mathrm{x}_{\mathrm{i,j}}) \right] \quad \text{for } \boldsymbol{i} = \mathbf{1}, \dots\,., \boldsymbol{N}; \boldsymbol{j} = \mathbf{1},.., \boldsymbol{T} \qquad (4.8)$$

Therefore FC_i

$$FC_i = \boldsymbol{F_i} \times \lambda_{\mathrm{i}} \left[(\mathrm{xx}_{\mathrm{i,j}})^{\beta_i} - (\mathrm{x}_{\mathrm{i,j}}) \right] \quad \text{for } \boldsymbol{i} = \mathbf{1}, \dots\,., \boldsymbol{N}; \boldsymbol{j} = \mathbf{1},.., \boldsymbol{T} \qquad (4.9)$$

Cost of preventive maintenance PMC_i refers to the cost incurred while component i is maintained. It includes cost of consumables (CCC_i), cost of condition based maintenance ($CCBM_i$), and cost of time based maintenance ($CTBM_i$). Where cost of consumables includes the cost of consumable material and equipment used while carrying out preventive maintenance activities such as cost of lubricating oil (CLO_{ij}), cost of component wires (CCW_{ij}), cost of replacement vital parts (screw nuts, belts etc) ($CRVP_{ij}$), cost lubricating grease (CLG_i). The cost of condition based maintenance includes cost of inspections (CI_{ij}), cost of diagnostic actions (CDA_{ij}), travel cost (CT_{ij}), labour cost (CL_{ij}) and cost of delayed production (CDP_{ij}). While cost of time based maintenance includes the cost of preventive oil change ($CPOC_{ij}$), cost of equipment material change ($CEMC_{ij}$).

Thus $PMC_i = CLO_{ij} + CCW_{ij} + CRVP_{ij} + CLG_{ij} + CI_{ij} + CDA_{ij} + CT_{ij} + CL_{ij} + CDP_{ij} +$

$$CPOC_{ij} + CEMC_{ij} \qquad \text{for } \boldsymbol{i} = \mathbf{1} \ldots \boldsymbol{N};\ \boldsymbol{j} = \mathbf{1},.., \boldsymbol{T} \qquad (4.10)$$

Where $CCC_{ij} = CLO_{ij} + CCW_{ij} + CRVP_{ij} + CLG_{ij}$

$CCBM_{ij} = CI_{ij} + CDA_{ij} + CT_{ij} + CL_{ij} + CDP_{ij}$

$CTBM_{ij} = CPOC_{ij} + CEMC_{ij}$

Thus $PMC_i = CCC_{ij} + CCBM_{ij} + CTBM_{ij} \qquad \text{for } \boldsymbol{i} = \mathbf{1} \ldots \boldsymbol{N};\ \boldsymbol{j} = \mathbf{1},.., \boldsymbol{T} \qquad (4.11)$

Where $PMC_i = PMC_{ij} \qquad \text{for } \boldsymbol{i} = \mathbf{1} \ldots \boldsymbol{N};\ \boldsymbol{j} = \mathbf{1},.., \boldsymbol{T} \qquad (4.12)$

Where $\mathrm{P}MC_{ij} = \begin{cases} 1\ maintained\ at\ j \\ 0\ \ Otherwise \end{cases}$

Cost of corrective maintenance of component *i* is the cost CRC_i incurred when component *i* is replaced at the end of period j with a new component *i*. It includes cost of diagnostic actions, Cost of repair actions, Cost and equipment hire and travel expenses, labour cost and administrative cost.

Thus $CMC_i = CDA_{ij} + CRA_{ij} + CEH_{ij} + TEC_{ij} + LC_{ij} + AC_{ij}$

$$\text{for } \boldsymbol{i} = \mathbf{1} \ldots \boldsymbol{N};\ \boldsymbol{j} = \mathbf{1},.., \boldsymbol{T} \qquad (4.13)$$

$$CMC_i = CMC_{ij} \qquad \text{for } \boldsymbol{i} = \mathbf{1} \ldots \boldsymbol{N};\ \boldsymbol{j} = \mathbf{1},.., \boldsymbol{T} \qquad (4.14)$$

Where $\mathrm{CMC}_{ij} = \begin{cases} 1\ replaced\ at\ j \\ 0\ \ Otherwise \end{cases}$

The cost of downtime of the manufacturing system DC_{ij} is the cost lost when component *i* is maintained or replaced at period *j*

$$DC = DT \ X \ PL \tag{4.15}$$

Where

DT: Average duration for downtime

PL: estimated profit loss per month by the company due to downtime.

For a multi-component manufacturing system, the main issue is to find the optimal maintenance strategy for each component independent of the other components, For example, while the manufacturing system is shut down to carry out an appropriate maintenance action on one component, it may make sense to go ahead and perform preventive maintenance corrective maintenance of some other components, even if they are not at their individual optimum point where maintenance actions would have ordinarily be performed.

Thus from the definitions of each types of cost, the total cost of maintenance is the sum of all the cost defined for component *i* at period *j* and is expressed as follows:

Total Maintenance cost =

$$\sum_{i=1}^{N}\sum_{j=1}^{T} \{ \boldsymbol{F_i} \times \lambda_i [(xx_{i,j})^{\beta_i} - (x_{i,j})^{\beta_i}] + CLO_{ij} + CCW_{ij} + CRVP_{ij} + CLG_{ij} + CI_{ij} + CDA_{ij} + CT_{ij}$$

$$+ CL_{ij} + CDP_{ij} + CPOC_{ij} + CEMC_{ij} + CDA_{ij} + CRA_{ij} + CEH_{ij} + TEC_{ij} + LC_{ij} + AC_{ij} \} + \sum_{j=1}^{T} [DC (1 -$$

$$(PMC_{ij} + CMC_{ij}))]$$

$$\text{for } \boldsymbol{i} = \mathbf{1} \ldots \boldsymbol{N}; \ \boldsymbol{j} = \mathbf{1}, .., \boldsymbol{T} \tag{4.16}$$

This objective function calculates for the total cost of maintenance as a summation of component costs in each period based on any preventive maintenance or corrective maintenance cost, the system downtime cost, and the cost of the expected number of unexpected failures.

4.3.3 System Reliability

Based on the failure time reliability distributions and system configuration, the system reliability is a function of probability of operating without failure over the planning scope. That is the probability of surviving component *i* to the end of period *j* given survival to the start of period *j*. Four reliability distribution models are used in modelling reliability (weibull, lognormal, exponential and normal) in this study, in conjunction with series and parallel system configurations

For weibull distributed failure times, the reliability of the system at the end of period *j* is as

$$R_j = e^{-\lambda t^{\beta}}$$

Where $t = ((xx_{i,j}) - (x_{i,j}))$

Thus R*series* is

$$R_j = \prod_{i=1}^{N} e^{-[\lambda_i [(xx_{i,j})^{\beta_i} - (x_{i,j})^{\beta_i}]]}$$

And $R_{parallel}$ is

$$R_p = 1 - (1 - R_j)^N$$

$$R_p = 1 - (1 - \prod_{i=1}^{N} e^{-[\lambda_i [(xx_{i,j})^{\beta_i} - (x_{i,j})^{\beta_i}]]})^N$$

$$\text{for } \boldsymbol{i} = \mathbf{1} \ldots \boldsymbol{N};\ \boldsymbol{j} = \mathbf{1}, .., \boldsymbol{T} \qquad (4.17)$$

For exponential distributed failure times, the reliability of the system at the end of period *j* is as

$$R_j = e^{-\lambda t}$$

Where $t = ((xx_{i,j}) - (x_{i,j}))$

Thus R*series* is

$$R_j = \prod_{i=1}^{N} e^{-[\lambda_i\,[(xx_{i,j}) \;-\; (x_{i,j})]]}$$

And R_{parallel} is

$$R_p = 1 - (1 - R_j)^N$$

$$R_p = 1 - (1 - \prod_{i=1}^{N} e^{-[\lambda_i\,[(xx_{i,j}) \;-\; (x_{i,j})\,]]}\;)^N$$

for $\boldsymbol{i} = \mathbf{1} \dots \boldsymbol{N}$; $\boldsymbol{j} = \mathbf{1},..,\boldsymbol{T}$ (4.18)

For normal distributed failure times, the reliability of the system at the end of period *j* is as

$$R_j = \int_t^{\infty} f(x)dx$$

$$R_j = \int_t^{\infty} 1/\sigma\sqrt{2\pi}\; e^{-\frac{1}{2}\left(t-{\mu}/{\sigma}\right)^2}\; dx$$

Where $t = ((xx_{i,j}) - (x_{i,j}))$

μ = Mean of normal time to failure

σ = standard deviation of times to failure

Thus R$_{\text{parallel}}$ is

$$R_p = 1 - (1 - R_j)^N$$

$$= 1 - (1 - (\int_t^{\infty} 1/\sigma\sqrt{2\pi}\, e^{-\frac{1}{2}\left(t-\mu/\sigma\right)^2}\, dx))^{N}$$

for $\boldsymbol{i} = \mathbf{1} \ldots \boldsymbol{N}$; $\boldsymbol{j} = \mathbf{1}, .., \boldsymbol{T}$ (4.19)

For lognormal distributed failure times, the reliability of the system at the end of period j is as

$$R_j = \int_t^{\infty} f(x)dx$$

$$R_j = \int_{In(t)}^{\infty} 1/\sigma\sqrt{2\pi}\, e^{-\frac{1}{2}\left(t-\mu/\sigma\right)^2}\, dx$$

Where $t = ((xx_{i,j}) - (x_{i,j}))$

μ = Mean of normal time to failure

σ = standard deviation of times to failure

Thus R$_{\text{parallel}}$ is

$$R_p = 1 - (1 - R_j)^N$$

$$= 1 - (1 - (\int_{In(t)}^{\infty} 1/\sigma\sqrt{2\pi}\, e^{-\frac{1}{2}\left(t-\mu/\sigma\right)^2}\, dx))^{N}$$

for $\boldsymbol{i} = \mathbf{1} \ldots \boldsymbol{N}$; $\boldsymbol{j} = \mathbf{1}, .., \boldsymbol{T}$ (4.20)

The reliability of the system is measured at an instant. Case in point, the reliability of the system would be the reliability at the end of every period.

4.4. Formulation and Development of Optimization Model for an Optimal Maintenance Strategy Based on Cost and Reliability

The parameters, decision variables, cost functions and reliability equations have been defined, the optimization algorithm is presented as a multi-objective mixed integer non linear program optimization problem to minimize total maintenance cost and maximize system reliability:

Minimize Total maintenance cost

$$= \sum_{i=1}^{N}\sum_{j=1}^{T} \{F_i \times \lambda_i [(xx_{i,j})^{\beta_i} - (x_{i,j})^{\beta_i}] + CLO_{ij} + CCW_{ij} + CRVP_{ij} + CLG_{ij} + CI_{ij} + CDA_{ij} + CT_{ij} + CL_{ij} + CDP_{ij} + CPOC_{ij} + CEMC_{ij} + CDA_{ij} + CRA_{ij} + CEH_{ij} + TEC_{ij} + LC_{ij} + AC_{ij}\} + \sum_{j=1}^{T}[DC(1 - (PMC_{ij} + CMC_{ij}))]$$

Maximize Reliability

Weibull distributed:

$$= \prod_{i=1}^{N} e^{-[\lambda_i [(xx_{i,j})^{\beta_i} - (x_{i,j})^{\beta_i}]]}$$

$$= 1 - (1 - \prod_{i=1}^{N} e^{-[\lambda_i [(xx_{i,j})^{\beta_i} - (x_{i,j})^{\beta_i}]]})^{N}$$

Exponential distributed

$$= \prod_{i=1}^{N} e^{-[\lambda_i\,[(xx_{i,j}) \;-\; (x_{i,j})]]}$$

$$= 1 - (1 - \prod_{i=1}^{N} e^{-[\lambda_i\,[(xx_{i,j}) \;-\; (x_{i,j})\,]]}\;)^{N}$$

Normal distributed

$$= \int_{t}^{\infty} 1/\sigma\sqrt{2\pi}\; e^{-\frac{1}{2}\left({}^{t-\mu}/_{\sigma}\right)^{2}\;dx}$$

$$= 1 - \big(1 - (\int_{t}^{\infty} 1/\sigma\sqrt{2\pi}\; e^{-\frac{1}{2}\left({}^{t-\mu}/_{\sigma}\right)^{2}\;dx})\big)\;\mathrm{N}$$

Lognormal distributed

$$= \int_{In(t)}^{\infty} 1/\sigma\sqrt{2\pi}\; e^{-\frac{1}{2}\left({}^{t-\mu}/_{\sigma}\right)^{2}\;dx}$$

$$= 1 - \big(1 - (\int_{In(t)}^{\infty} 1/\sigma\sqrt{2\pi}\; e^{-\frac{1}{2}\left({}^{t-\mu}/_{\sigma}\right)^{2}\;dx})\big)\;\mathrm{N}$$

Subject to

$\boldsymbol{X_{i,j} = 0}$ *For* $\boldsymbol{i = 1, \dots, N; j = 1, \dots, T}$

$XX_{i,j} = X_{i,j} + \frac{T}{j}$ *For* $\boldsymbol{i = 1, \dots, N; j = 1, \dots, T}$

$PMC_{ij} + CMC_{ij} \leq 1$ *For* $\boldsymbol{i = 1, \dots, N; j = 1, \dots, T}$

$PMC_{ij}, CMC_{ij} = 0 \text{ or } 1 \qquad \text{For } \boldsymbol{i = 1, \ldots, N; j = 1, \ldots, T}$

$$X_{ij} = (1 - PMC_{ij-1})(1 - CMC_{ij-1})\, XX_{ij-1} + PMC_{ij-1}\, (\alpha_{pmi} \times XX_{ij-1})$$

$$\text{For } \boldsymbol{i = 1, \ldots, N; j = 1, \ldots, T}$$

(4.21)

The first constraint indicated that the initial age of each component is zero,

The second constraint accounts for the changes in age thus representing the effective age of component i at the end of period j.

The third to fifth constraint specifies that if a component is replaced with another new component then $\boldsymbol{X_{i,j} = 0}$, $CRC_{ij} = 1$, $PMC_{ij} = 0$. If a component is maintained then $CRC_{ij} = 0$, $PMC_{ij} = 1$.

4.4.1 Optimization Algorithm Variables

A representation of the optimization algorithm variables is presented as follows:

N: Number of Components

T: Length of Planning Scope

J: Number of Periodic Intervals

ƛ: Scale Parameter

β: Shape Parameter

µ: Mean

σ: Standard Deviation

$X_{i,j}$: Effective age of component *i* at the start of period *j*

$XX_{i,j}$: Effective age of component *i* at the end of period *j*

α_{pmi}: Age reduction factor of preventive maintenance on component *i*

F_i: Failure cost of component *i*

PMC*i:* Cost of preventive maintenance on component *i*

CMC*i* : Cost of Corrective maintenance on component *i*

DC: Downtime cost

4.4.2 Assumptions

In this section, a number of assumptions are presented and motivated in order to arrive at an optimal strategy formulation for the optimization problem in which the objective is to minimise the total cost of maintenance and to maximise the system-wide reliability. Some of these assumptions are aimed at decreasing the complexity of the problem, thereby making it possible to solve the algorithm efficiently. The optimization complexity is, however, decreased in such a manner so as not to generate maintenance schedules that are unrealistic or unfit for use in practice.

1. Number of Manufacturing Components: A number of components are required to produce an end product in a manufacturing system. Failure of any one of these components typically causes the manufacturing process to be interrupted until the component has been repaired or replaced. Therefore, a failure in one of the components of the manufacturing system typically leads to failure of the entire

manufacturing system. For optimization purposes, all the components of a manufacturing system are considered as a whole in the sense that when the manufacturing system is shut down to carry out an appropriate maintenance action on one component, it may make sense to go ahead and perform preventive maintenance corrective maintenance of some other components, even if they are not at their individual optimum point where maintenance actions would have ordinarily been performed.

2. Frequency of Maintenance: A number of maintenance actions will be carried out on the manufacturing components, including complete overhaul due to corrective maintenance as opposed to just carrying out preventive maintenance. However, the duration of each maintenance activity, which will vary from one component to the other is outside the scope of this study.
3. Reliability after Maintenance: when maintenance is performed on any manufacturing components, the goal is to increase the reliability of the manufacturing component to as good as new or to the state it was operating before maintenance was performed on it. In this study it is assumed that after performing maintenance and the component is back into operation, the component’s reliability will improve to as good a s new or to a state it was operating before.
4. Effect of Maintenance on manufacturing component: in this study, it is assumed that any maintenance action or strategy has a positive effect on the manufacturing component. Thus based on Eygelaar (2018), any preventive maintenance actions carried out reduces the effective age of the manufacturing component by 30% while corrective maintenance results into the component to be as good as new.
5. Resources required for maintenance: in a realistic manufacturing environment, many resources are required to perform effective maintenance on manufacturing

component. These resources include maintenance personnel, finance, spare parts inventory, logistics etc. An optimization algorithm containing all resources is expected to be very complex, hence for the purpose of this study, it is assumed that resources such as maintenance personnel and finance is the required resources to carry out maintenance activities. This is not an unrealistic assumption as the optimization algorithm in this study is expected to produce a schedule for maintenance strategies for the period of thirty six months, meaning that it will be known beforehand that the maintenance of any particular component will occur at a certain period within the scheduling window, thus provisions can be made well in advance of each maintenance active to ensure that the spare parts and maintenance equipment required are indeed available and that all logistics are appropriately taken care of.

6. Independence of component’s failure: it is assumed in this study that failures that occur in a manufacturing system are independent of one another. Hence if a component is taken out of operation due to a failure it is assumed to have little or no effect on the timing of failures of the other components in the manufacturing system.
7. Failure rates of manufacturing components: it is assumed in this study that the failure rates of individual components follow a typical bathtub curve, hence the reliability model incorporated within the optimization algorithm if formulated for components through the different stages.
8. Nature of manufacturing components: within the realm of reliability theory, two main systems prevail, namely repairable systems and non-repairable systems. It is assumed in this study that components in a manufacturing system are repairable system. In a scenario where the manufacturing system has both repairable and non-repairable systems, the optimization algorithm form this study is formulated for repairable systems.

4.5 Validation of the developed Optimization Model with an Industrial Application

The optimization model presented in equation 4.21 is a mixed integer non-linear programming optimization problem. Four different solution techniques are proposed namely Lingo, GAnetXL, Genetic algorithm and simulation based optimization method. These methods as discussed in section 2.4.7 and 2.4.8 are good solution methods for complex optimization problems. Each technique is used to find an optimum solution to the problem using the industrial applied scenario. Data used for this analysis is presented in table 4.39:

The programming language codes written in lingo platform is developed for executing the optimization algorithms in lingo and are as follows:

Table 4.40: Data for Optimization Analysis

T		36 months (3 years)							
DC		₦ 197,561							
N	Component	Shape (β)	Scale (ƛ)	Mean (μ)	Standard Deviation (σ)	α_{pmi}	Failure Cost	Preventive Maintenance Cost	Corrective Maintenance Cost
1	Conveyor System	1.5855	3396.50	2976.03	1920.25	0.7	₦ 884,210	₦387,450	₦ 496,760
2	Mixer system	1.7610	3375.42	3005.13	2741.21	0.7	₦ 366,415	₦93,855	₦ 272,560
3	Roller system	1.7397	3254.14	2899.20	1718.89	0.7	₦ 430,680	₦ 92,680	₦ 338,000
4	Slitter	1.7123	3252..65	2900.75	1744.82	0.7	₦ 513,322	₦ 99,500	₦ 413,822
5	Compounding Machine	1.6852	3170.13	2830.25	1727.30	0.7	₦ 618,685	₦ 231,685	₦ 387,000

4.5.1 Lingo Solver Computational Results

Data from the industrial case was applied to the optimization model developed in section 4.4 and was programmed into lingo 17.0 software and combined with excel solver for an optimal solution. It took approximately 8 hours 42mins on a 2.13 GHz processor to solve the optimization problem for each scenario within the planning scope with 710 variables of which 360 are integer variables and 532 constraints, of which 177 are non-linear. It is important to note here that the solution techniques adopted in the study provided optimal strategies at several pareto fronts, as observed from figure 4.5.1, the solution techniques provided optimal strategies at 50%, lingo and GAnetXL terminated at 90% while genetic algorithm and simulation based optimization terminated just before 60%. At 70% it is expensive to adopt the optimal strategies from lingo and GAnetXL as it is not cost optimal, thus a common ground of 50% was chosen for all solution techniques in this study.

For Lingo, the objective function of the optimal solution is ₦7, 593,578 for which 50% reliability is achieved in the manufacturing system. The achieved reliability of 50% is a product of the reliability of the system at the end of each period and the result is shown in Table 4.41 and the Pareto optimal front is presented in Figure 4.24. The computational programing codes are presented in appendix b. The optimal maintenance strategy for the manufacturing components is as follows:

Table 4.41: Optimal Maintenance Method for Lingo (Cost = ₦7, 593,578, Reliability = 50%)

MONTHLY SCHEDULE																																				
N	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
1	-	-	P	-	-	-	P	C	-	-	-	-	P	C	-	-	-	-	-	P	-	-	C	-	-	P	-	-	-	-	-	P	-	-	-	P
2	-	-	P	-	-	-	-	-	-	-	-	-	P	C	-	-	P	-	-	-	-	-	C	-	-	-	-	-	C	-	-	-	-	-	-	P
3	-	-	P	-	-	-	P	C	-	-	-	-	P	-	-	-	P	-	-	P	-	-	C	-	-	-	-	-	-	-	-	P	-	-	-	P
4	-	-	P	-	-	-	P	C	-	-	-	-	P	-	-	-	-	-	-	-	-	-	C	-	-	-	-	-	C	-	-	P	-	-	-	P
5	-	-	P	-	-	-	-	-	-	-	-	-	P	C	-	-	-	-	-	P	-	-	C	-	-	P	-	-	P	-	-	-	-	-	-	P

KEY: N = Number of Components; 1 = Conveyor System; 2 = Mixer System; 3 = Roller System; 4 = Slitter System; 5= Compounding Machine;

P: Preventive Maintenance; C: Corrective Maintenance

Table 4.41 presents the maintenance schedule generated by lingo 17, of which a combination of three maintenance strategies is presented for each component in the manufacturing system, 1) Preventive Maintenance, 2) Corrective Maintenance and 3) A period whereby nothing is done. The optimal strategy provides an optimal cost solution of ₦7, 593,578 for which 50% reliability is achieved.

Table 4.42 show the reliability optimal solutions with associated cost. Each solution presents an optimal maintenance schedule and 50% was selected for example to show the optimal schedule as presented in table 4.39.

The system reliability for each period for the RR of 50% is shown in Figure 4.24

Table 4.42: Lingo Reliability Optimal Solutions

Reliability	Cost
10%	3,612,769
20%	4,839,280
30%	5,722,900
40%	6,900,431
50%	7,593,578
60%	8,832,608
70%	9,264,333
80%	10,480,991
90%	11,980,000

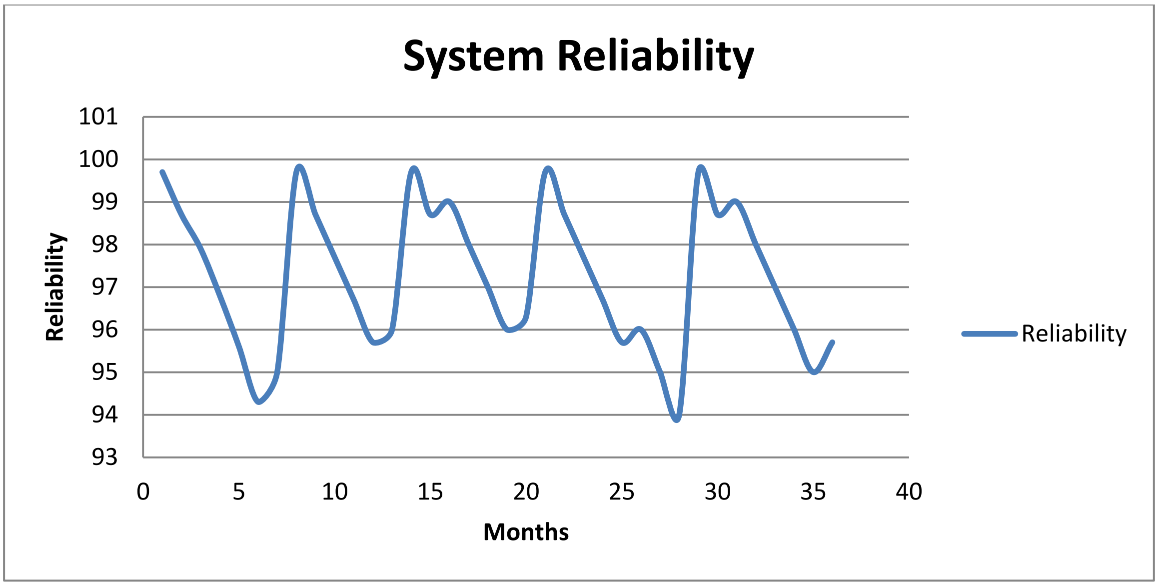


Fig 4.24: System Reliability for Lingo/Excel Solver (Cost = ₦7, 593,578, Reliability = 50%)

The system reliability in figure 4.24 shows that the reliability of the system lies between 94% and 99.7% over the defined planning period of 36 months with average reliability over the planning period being 97.2%. The significant drop at period 6 and 28 is as a result of lack of adequate maintenance action in 4 and 3 consecutive periods.

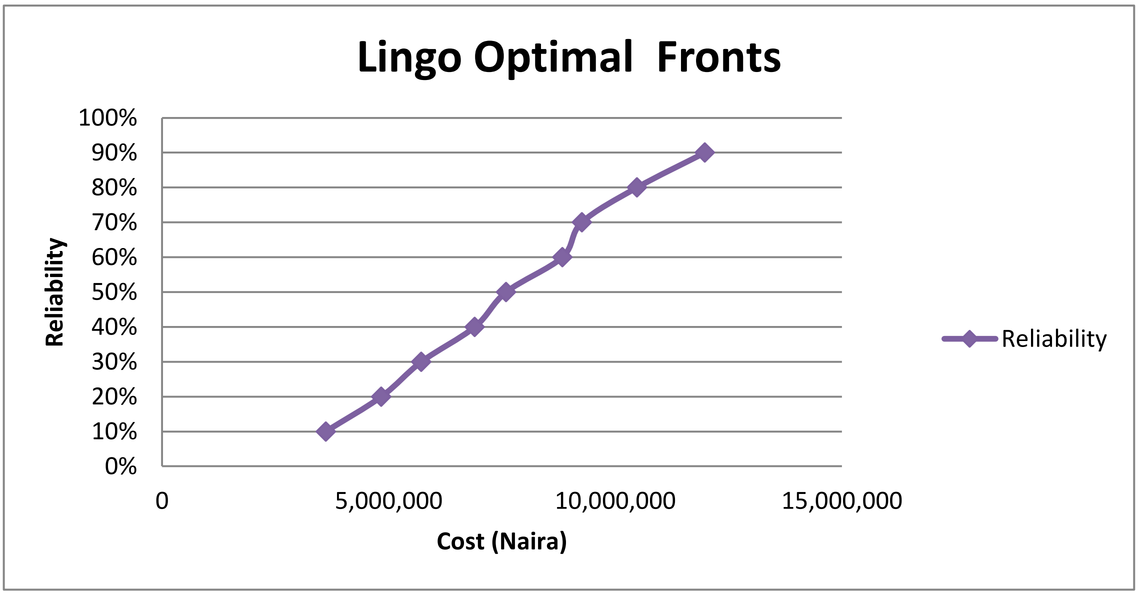


Fig 4.25: Lingo Optimal Fronts for Reliability

From the optimal maintenance strategy one can analyze the effective age of each component. This could be used to track the effective age of the components and then utilize the information to initiate additional monitoring activities. For example, after a component reaches a certain level of effective age, additional monitoring, tests or inspections might be warranted to assist in the detection of imminent failure. Figures 4.26, 4.27, 4.28, 4.29 and 4.30

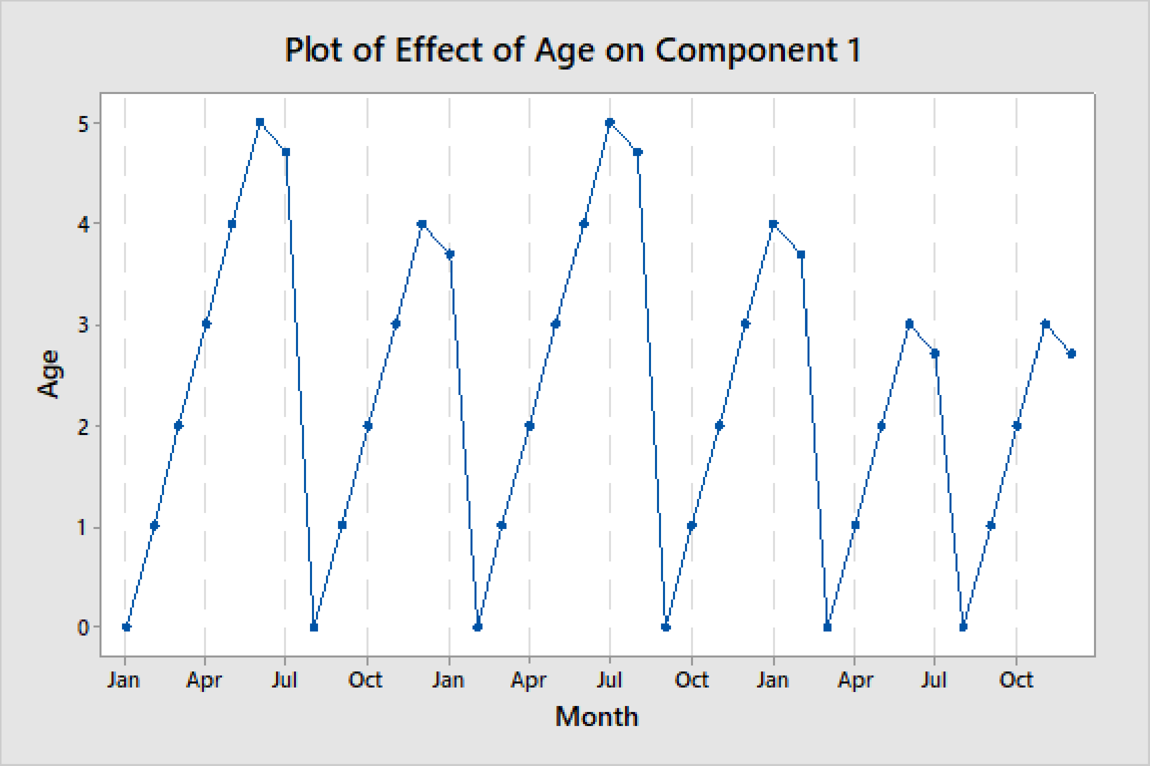


Fig 4.26: Lingo Optimal Maintenance strategy effect on Conveyor System

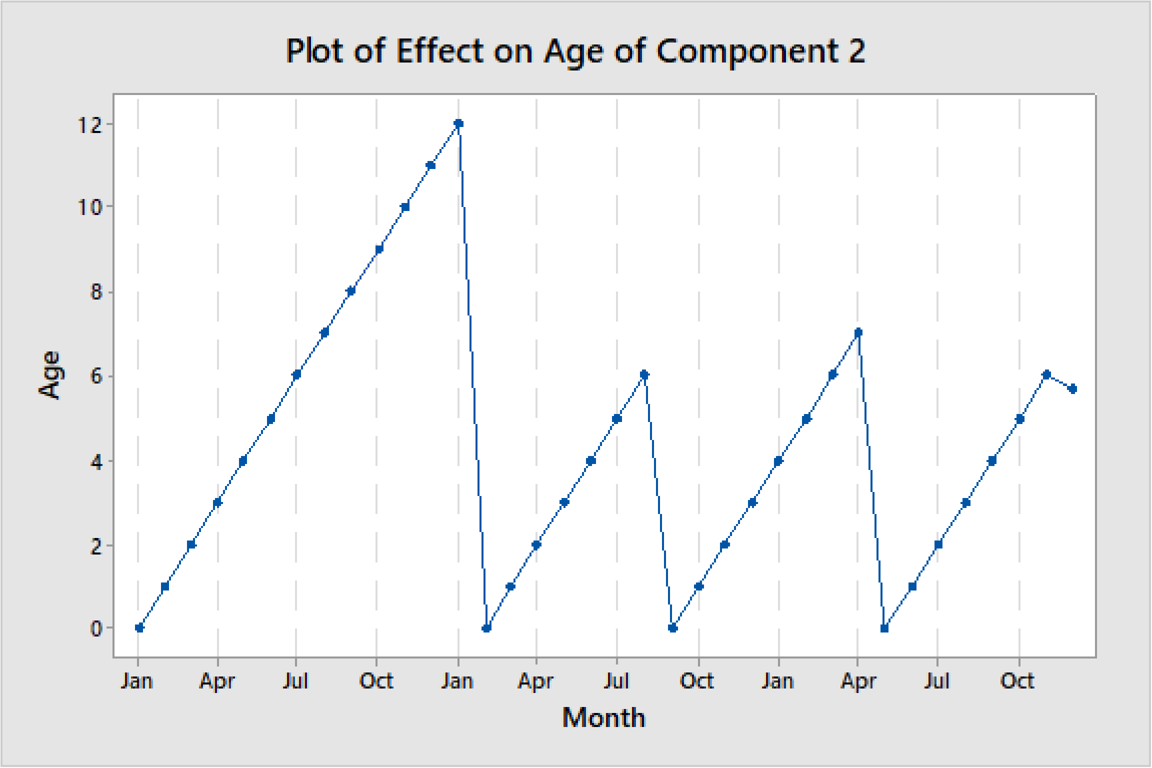


Fig 4.27: Lingo Optimal Maintenance strategy effect on Mixer System

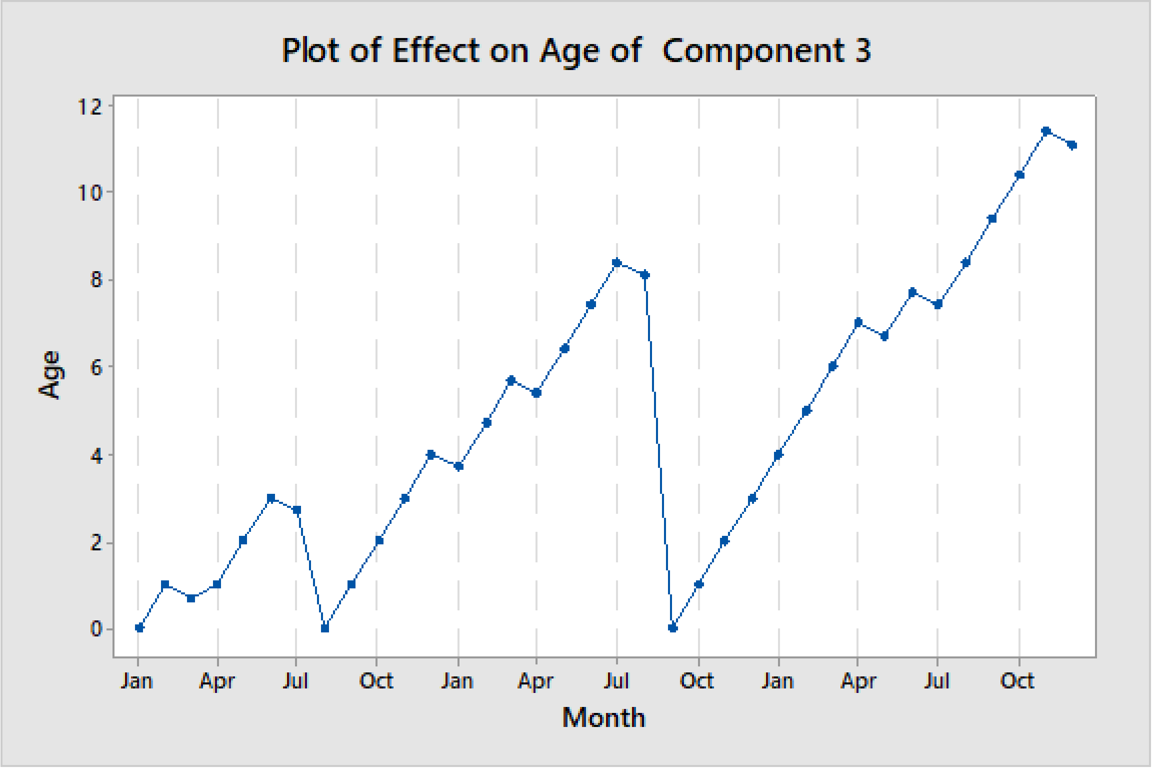


Fig 4.28: Lingo Optimal Maintenance strategy effect on Roller System

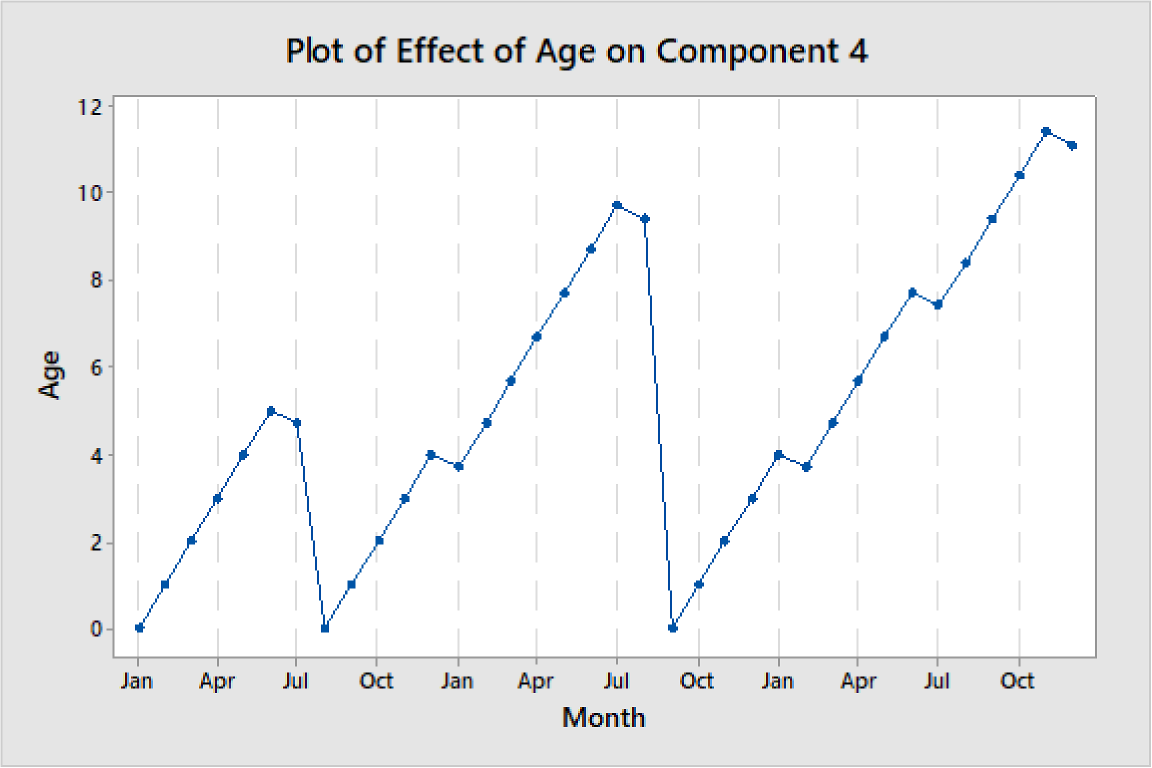


Fig 4.29: Lingo Optimal Maintenance strategy effect on Slitter System

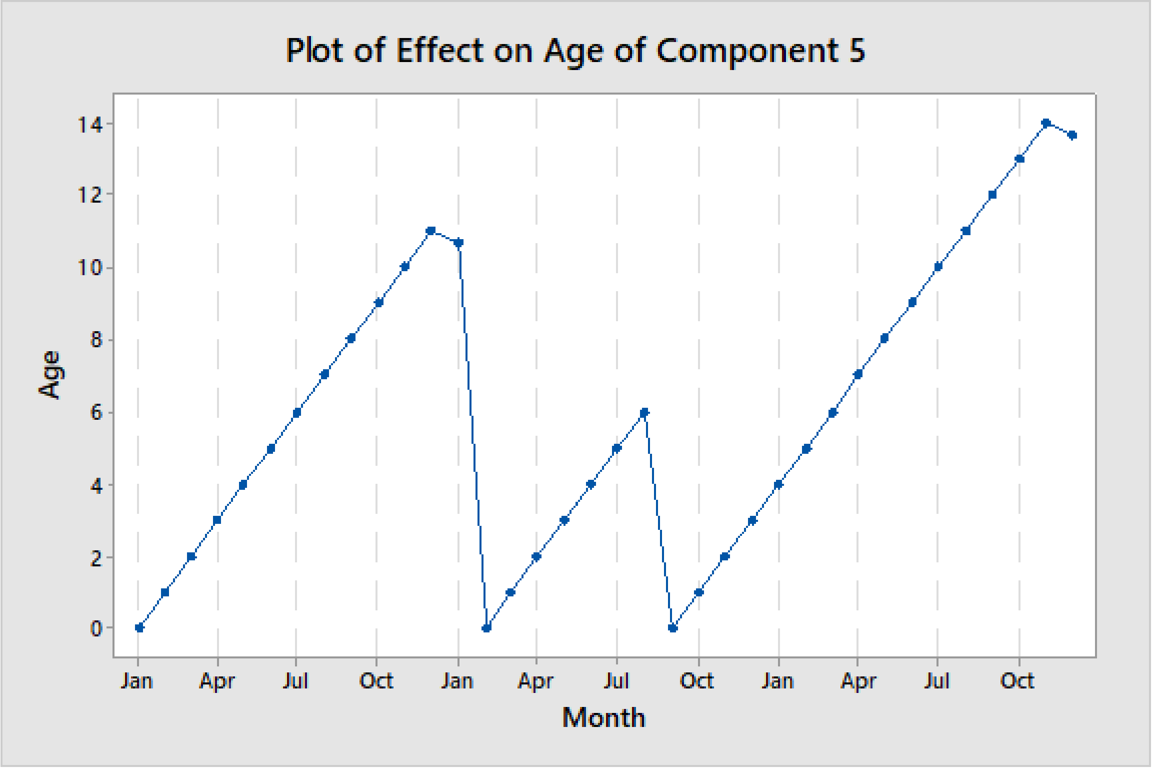


Fig 4.30: Optimal Maintenance strategy effect on Compounding Machine

From figures, one can notice that the minimum age of each component at the beginning of the planning schedule is zero, and that most component undergoes corrective maintenance at a certain point in the planning schedule, thus making the average age of the component to be in the range of 0 months to 14 months as summarised in table 4.43.

Table 4.43: Effective Age of Component with Lingo Solution Output

Components	**Minimum Effective Age (Month)**	**Maximum Effective Age (Month)**
1	0	5
2	0	12
3	0	11.7
4	0	11.7
5	0	14

Another observation is the effect of failure rate on the number of scheduled maintenance, for example when one compares component 1 and 5, it can be observed that component 1 has more scheduled maintenance actions than component 5. This explains the variation in effective ages of the component as component one has higher failure rate than component 5. Thus it is necessary that component 1 receives more attention.

4.5.2 Genetic Algorithm Results

4.5.2.1 GANetXL Computational Results

Non-dominated sorting genetic algorithm was applied using GANetXL software to optimize the problem. Several optimal points called the Pareto optimal points are produced which contains the optimal solutions. The parameters and results of the optimization are as follows:

Table 4.44: Genetic algorithm parameters

Number of Generation: 500
Population Size: 1000
Probability of Selection: 0.2
Probability of Crossover: 0.4
Probability of Mutation: 0.3

The results of the optimal pareto solutions are shown in table 4.43 and fig 4.31:

Table 4.45 show the reliability optimal solutions with associated cost. Each solution presents an optimal maintenance schedule and similarly a point on the pareto front at 50% was selected for example to show the optimal schedule as presented in table 4.46.

Table 4.45: GANetXL Pareto Optimal Solutions

Reliability	Cost
10%	₦3,542,769
20%	₦ 4,139,280
30%	₦ 5,022,900
40%	₦ 6,550,431
50%	₦ 7,349,397
60%	₦ 8,819,608
70%	₦ 9,250,433
80%	₦ 10,463,761
90%	₦ 11,961,210

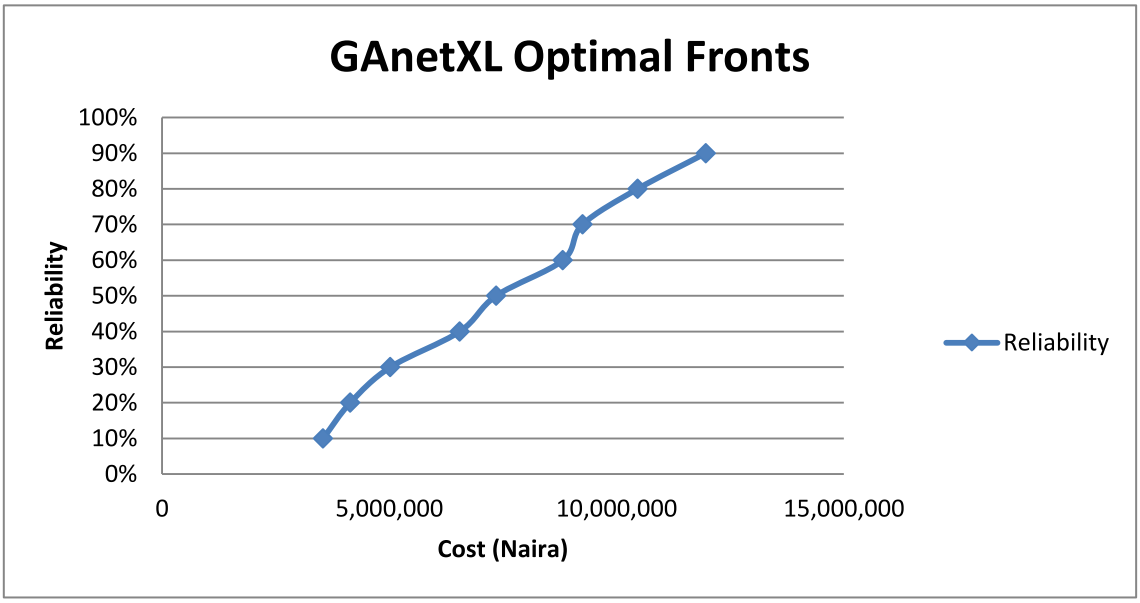


Fig 4.31: GAnetXL Optimal Fronts for Reliability

The optimal maintenance strategy is presented as follows:

Table 4.46: Optimal Maintenance Strategy for GAnetXL (Cost = ₦7, 349,397, Reliability = 50%)

MONTHLY SCHEDULE																																				
N	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
1	-	-	-	**P**	-	-	**P**	-	-	-	-	-	**P**	-	-	-	-	-	-	**P**	-	-	**C**	-	-	-	-	-	**P**	-	-	**P**	-	-	-	**P**
2	-	-	-	-	-	-	**P**	-	-	**C**	-	-	-	**C**	-	-	-	-	-	-	-	-	**C**	-	-	**P**	-	-	-	-	-	**P**	-	-	-	
3	-	-	-	**P**	-	-	-	-	-	**C**	-	-	**P**	-	-	**P**	-	-	-	**P**	-	-	**C**	-	-	-	-	-	**C**	-	-	**P**	-	-	-	**P**
4	-	-	-	**C**	-	-	-	-	-	**C**	-	-	**P**	-	-	**P**	-	-	-	**P**	-	-	**C**	-	-	-	-	-	-	-	-	**P**	-	-	-	**C**
5	-	-	-	-	-	-	**P**	-	-	-	-	-	**P**	**C**	-	-	-	-	-		-	-	**C**	-	-	**P**	-	-	-	-	-	**P**	-	-	-	**P**

KEY: N = Number of Components; 1 = Conveyor System; 2 = Mixer System; 3 = Roller System; 4 = Slitter System; 5= Compounding Machine;

P: Preventive Maintenance; C: Corrective Maintenance

Table 4.46 presents the maintenance schedule generated by GAnetXL, of which a combination of three maintenance strategies is presented for each component in the manufacturing system, 1) Preventive Maintenance, 2) Corrective Maintenance and 3) A period whereby nothing is done. The optimal strategy provides an optimal cost solution of ₦7, 349,397 for which 50% reliability is achieved.

Both lingo and GANetXL gave the same results when applied to the multi-objective optimization algorithm. It can also be seen that GANetXL solutions provided a better quality cost solution in most iterations at 50% reliability. The system reliability is shown in figure 4.32

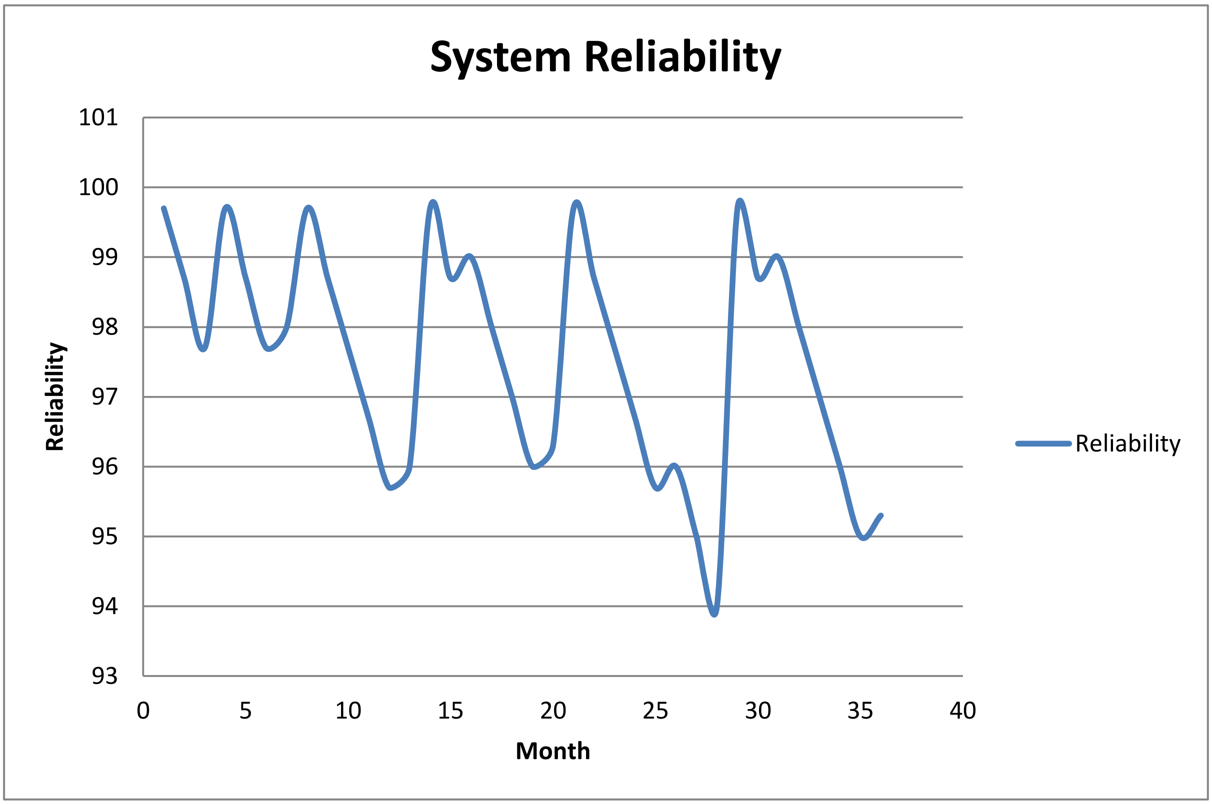


Fig 4.32: System Reliability for GAnetXL (Cost = ₦7, 349,397, Reliability = 50%)

The system reliability shows that the reliability of the system lies between 94% and 99.7%, with average reliability over the planning period being 97.2%. The significant drop at period

28 is as a result of lack of adequate maintenance action for several consecutive periods. The cost savings in the GANetXL solution can be seen as some of the components are allowed to spend longer times in service before being maintained.

4.5.2.2 Genetic Algorithm Fitness Function Results

The programming language codes written in Matlab platform is developed for executing the optimization algorithms through genetic algorithm and are presented in appendix d.

The procedure for implementation is outlined as follows:

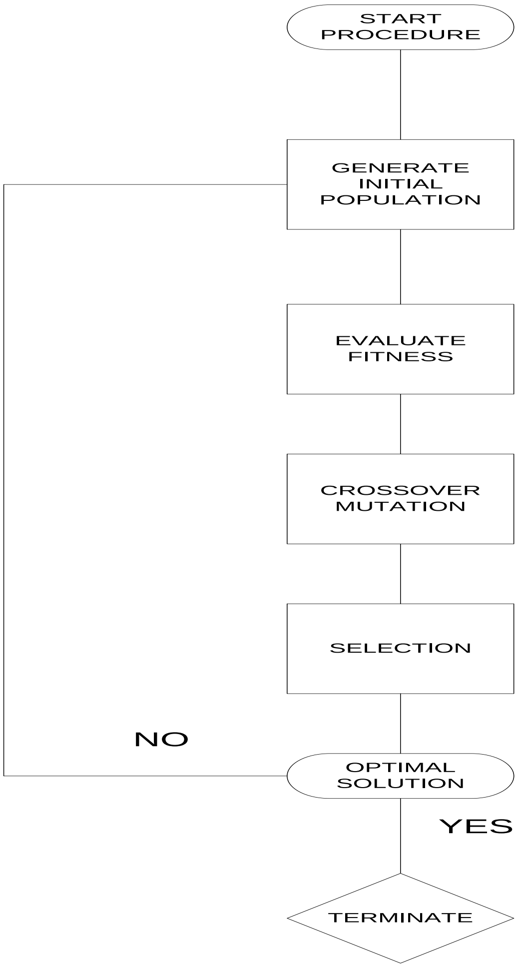


Fig 4.33: Procedure for Implementing Genetic Algorithm Flowchart

The objective of the fitness function is to ensure an appropriate evaluation of good solutions by ranking a solution over other solutions. Considering that the optimization algorithm presented in equation 4.21 has two objectives, the following fitness function proposed by Oke, (2010) was applied in this study,

$$Fitness = W_1 \, (Total\ cost) + W_2 \, (-Reliability) \qquad (4.22)$$

The fitness is based on the weighted summation of the normalized total cost and reliability functions with the condition of W1 + W2 = 1.

MATLAB R2018a programming environment was used to program the fitness function algorithm due to the complexity of the optimal strategy. Different permutation of values between 0 and 1 were used for fitness function with the condition $w_1+w_2 =1$. It took an average of 1 hour 52 minutes on a 2.13 GHz processor to solve each iteration in MATLAB. The pareto fronts for the optimal solution is shown in table 4.43. The farthest case of $W_1 = 0$ and $W_2 = 1$ gave a cost solution of ₦ 11, 821, 480 and a reliability of 57.12%, while $W_1 = 1$ and $W_2 = 0$ gave a cost solution of ₦ 1,849,200 and a reliability of 3.53%.

Table 4.47: Genetic Algorithm Pareto Optimal Solution for Fitness Function

Weights		Fitness Function	
W1	**W2**	**Reliability**	**Cost**
0	1	57.12%	₦ 11, 821, 480
0.1	0.9	54.25%	₦ 10,963,810
0.2	0.8	52.30%	₦ 8, 624, 921
0.3	0.7	50.15%	₦ 7, 757 ,360
0.4	0.6	47.88%	₦ 6, 271, 500
0.5	0.5	43.52%	₦ 5, 105, 822
0.6	0.4	36.29%	₦ 4, 450, 760
0.7	0.3	25.70%	₦ 3, 658, 200
0.8	0.2	15.69%	₦ 2, 800, 261
0.9	0.1	8.22%	₦ 2, 150, 900
1	0	3.55%	₦ 1,849,200

The Pareto optimal front of fitness function obtained is presented in figure 4.34

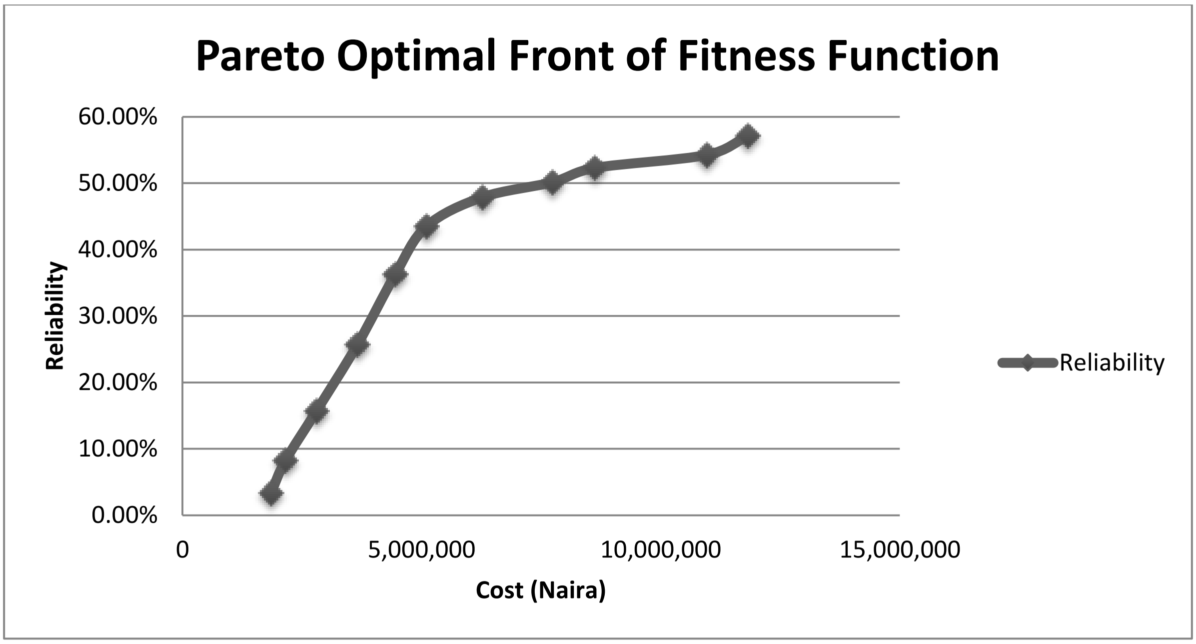


Fig 4.34: **Pareto Optimal Front of Fitness Function**

As shown from the results, several optimal solutions are found and depending on the objective of the maintenance department, any of these points can be taken as an optimal solution. From the Pareto optimal fronts, achieving a higher reliability means the maintenance cost would also increase. An example for the weight of $w_1 = 0.3$ and $w_2 = 0.7$ is shown in table 4.48. The optimal cost solution is ₦ 7, 757, 360 and the reliability is 50.15%. The system reliability is shown in figure 4.35

Table: 4.48: Optimal Maintenance Strategy for Fitness (Cost: ~~N~~ 7, 757, 360, Reliability: 50.15%)

MONTHLY SCHEDULE																																				
N	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
1	-	-	-	P	-	-	P	-	-	C	-	-	P	-	-	-	P	-	-	-	P	-	-	-	-	P	-	-	C	-	P	-	-	-	-	P
2	-	-	-	P	-	-	C	-	-	-	-	-	-	-	C	-	-	-	-	-	C	-	-	-	-	P	-	-	-	-	P	-	-	-	-	P
3	-	-	-	P	-	-	C	-	-	-	-	-	-	-	P	-	P	-	-	-	C	-	-	-	-	-	-	-	-	-	P	-	-	P		P
4	-	-	-	C	-	-	P	-	-	C	-	-	P	-	-	-	-	-	-	-	C	-	-	-	-	P	-	-	-	-	P	-	-	C	-	-
5	-	-	-	P	-	-	-	-	-	-	-	-	P	-	-	-	C	-	-	-		-	-	-	-	P	-	-	C	-	P	-	-	-	-	P

KEY: N = Number of Components; 1 = Conveyor System; 2 = Mixer System; 3 = Roller System; 4 = Slitter System; 5= Compounding Machine;

P: Preventive Maintenance; C: Corrective Maintenance

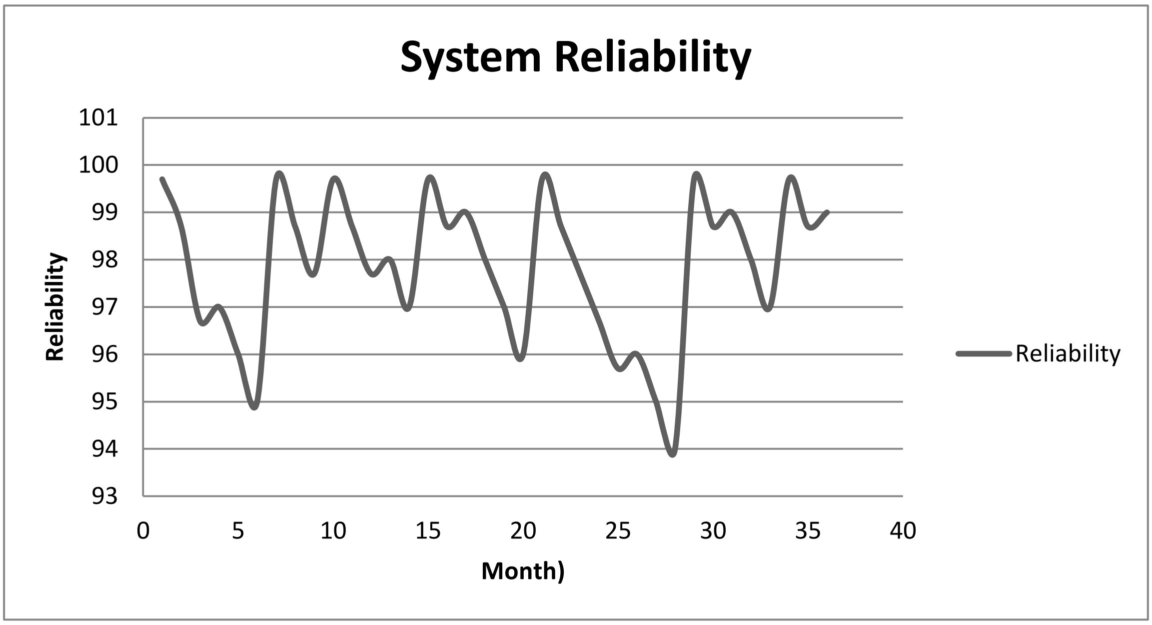


Fig 4.35 System Reliability for Fitness (Cost: ₦ 7, 757, 360, Reliability: 50.15%)

The system reliability shows that the reliability of the system lies between 94% and 99.7%, with average reliability over the planning period being 97.83%. The significant drop at period 27 is as a result of lack of adequate maintenance action for nine consecutive periods

4.5.3 Simulation Based Optimization Results

Discrete event simulation models were developed in Witness 14 simulation software. Witness software has three in-built optimization algorithms namely 1) Hill climb algorithm 2) Random solutions and 3) Simulated Annealing. Hill climb algorithm is a local search heuristic algorithm hence wasn’t applied in this study. Random solutions and simulated annealing are both global search heuristic algorithms, however random solutions doesn’t have the capability to learn from evaluations. Therefore, it is seen as inefficient and unlikely to result in global optimum thus not applicable to this study.

Simulated annealing was then used as the optimization algorithm in witness 14, and it took approximately 5 hours 26mins on a 3.5GHz processor to solve the optimization problem. The programming language codes developed for executing the optimization algorithms through simulated annealing are found in appendix e.

The procedure for implementation is outlined as follows:

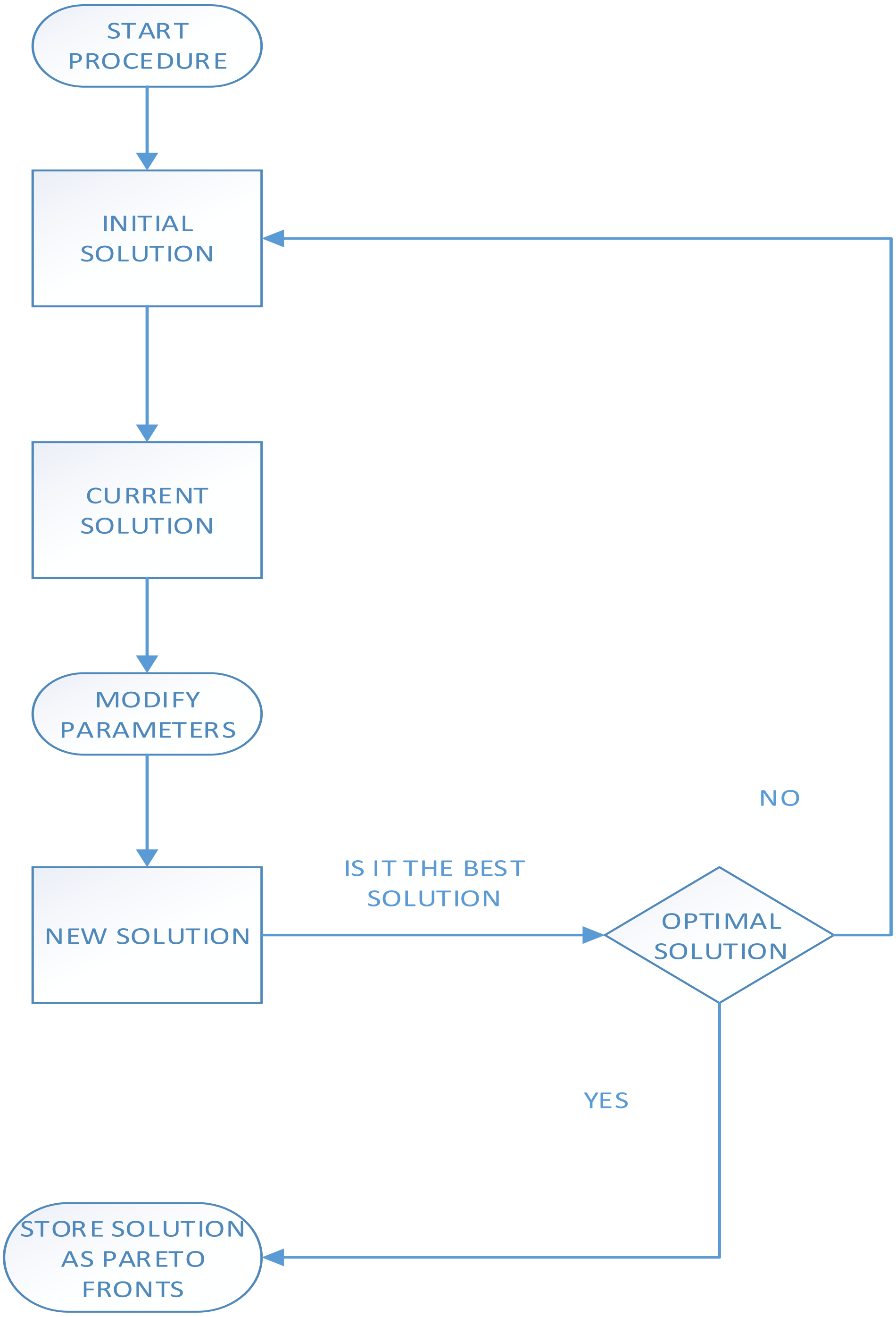


Fig 4.36: Flowchart Procedure for Simulated Annealing Optimization.

Applying the same fitness function described in equation 4.16, the pareto fronts for the optimal solution is presented in table 4.49. The utmost case of $W_1 = 0$ and $W_2 = 1$ gave a cost solution of ₦ 13, 800, 532 and a reliability of 59.29% while $W_1 = 1$ and $W_2 = 0$ gave a cost solution of ₦ 2,358,422 and a reliability of 0.73%.

Table 4.49: Simulation Pareto Optimal Solution for Fitness Function

Weights		**Fitness Function**	
W_1	**W_2**	**Reliability**	**Cost**
0	1	59.29%	₦ 13, 800, 532
0.1	0.9	57.48%	₦ 12,424,999
0.2	0.8	54.32%	₦ 10, 927, 532
0.3	0.7	51.08%	₦ 9, 010 ,493
0.4	0.6	49.04%	₦ 8, 051, 503
0.5	0.5	40.23%	₦ 7, 725, 738
0.6	0.4	35.20%	₦ 6, 369, 840
0.7	0.3	22.06%	₦ 5, 401, 820
0.8	0.2	12.68%	₦ 4, 942, 969
0.9	0.1	6.44%	₦ 3,088,328
1	0	0.73%	₦ 2,358,422

The Pareto optimal front of fitness function obtained is presented in figure 4.37

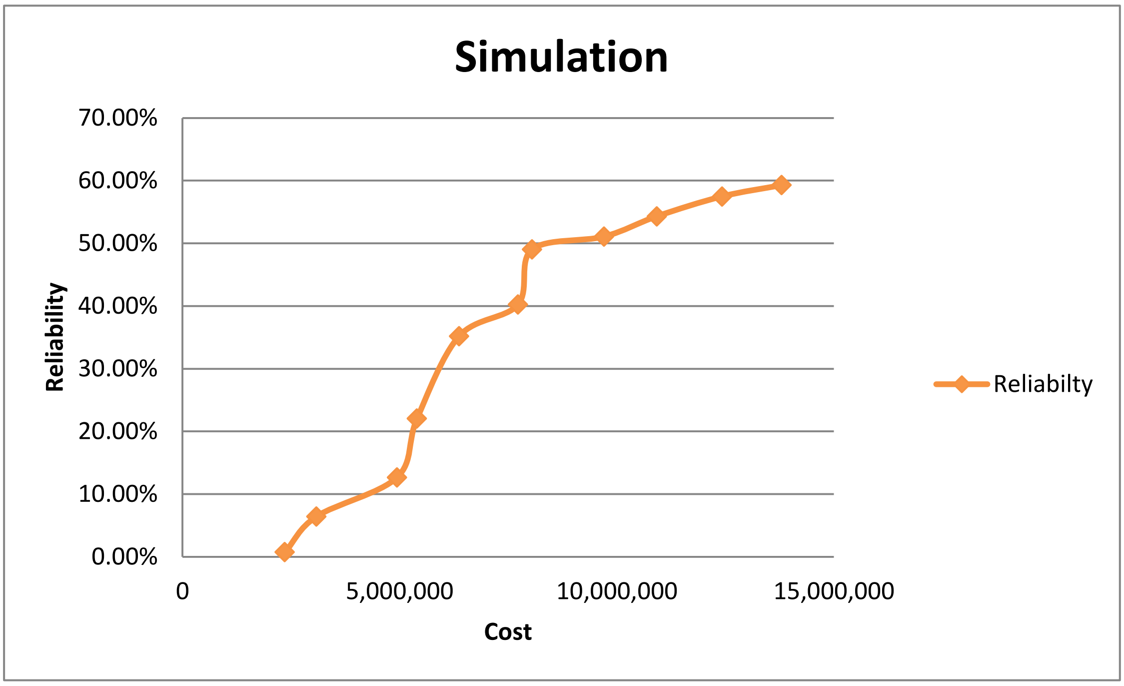


Fig 4.37: **Pareto Optimal Front**

The optimal maintenance strategy for the weight of w_1= 0.3 and w_2 = 0.7 is shown in table 4.50. The optimal cost solution is ₦ 9, 010, 493 and the reliability is 51.08%. Similarly, the system reliability is shown in figure 4.38

Table: 4.50: Optimal Maintenance strategy for Fitness (Cost: ₦ 9, 010, 493, Reliability: 51.08%)

MONTHLY SCHEDULE																																				
N	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
1	-	-	-	-	-	C	-	-	P	-	-	-	C	-	-	P	-	-	C	-	-	P	-	-	C	-	-	-	-	-	C	-	-	-	-	C
2	-	-	-	-	-	C	-	-	P	-	-	-	C	-	-	-	-	-	C	-	-	P	-	-	C	-	-	-	-	-	C	-	-	-	-	C
3	-	-	-	-	-	C	-	-	P	-	-	-	C	-	-	P	-	-	C	-	-	P	-	-	C	-	-	P	-	-	C	-	-	-	-	C
4	-	-	P	-	-	C	-	-	-	-	-	-	C	-	-	P	-	-	C	-	-	-	-	-	C	-	-	-	-	-	C	-	-	-	-	C
5	-	-	P	-	-	C	-	-	-	-	-	-	C	-	-	P	-	-	C	-	-	-	-	-	C	-	-	P	-	-	C	-	-	-	-	C

KEY: N = Number of Components; 1 = Conveyor System; 2 = Mixer System; 3 = Roller System; 4 = Slitter System; 5= Compounding Machine;

P: Preventive Maintenance; C: Corrective Maintenance

The Pareto optimal fronts results produced several optimal choices for a set of required reliability. With system reliability showing that the reliability of the system lies between 95% and 99.7% as shown in figure 4.38 with average reliability over the planning period being 97.8%. The significant drop in reliability at periods 12, and 35 is as a result of no maintenance actions for 3 or more consecutive periods.

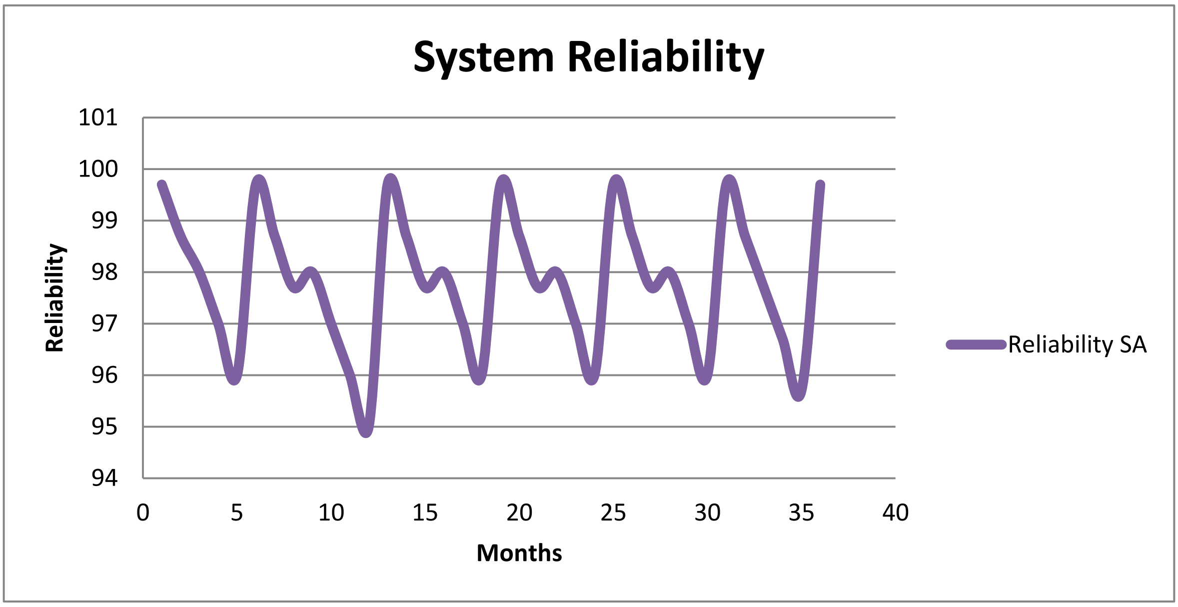


Fig 4.38: System reliability for Simulation based optimization

The effect of this optimal strategy on the age of components are presented in Figures 4.39, 4.40, 4.41, 4.42 and 4.43

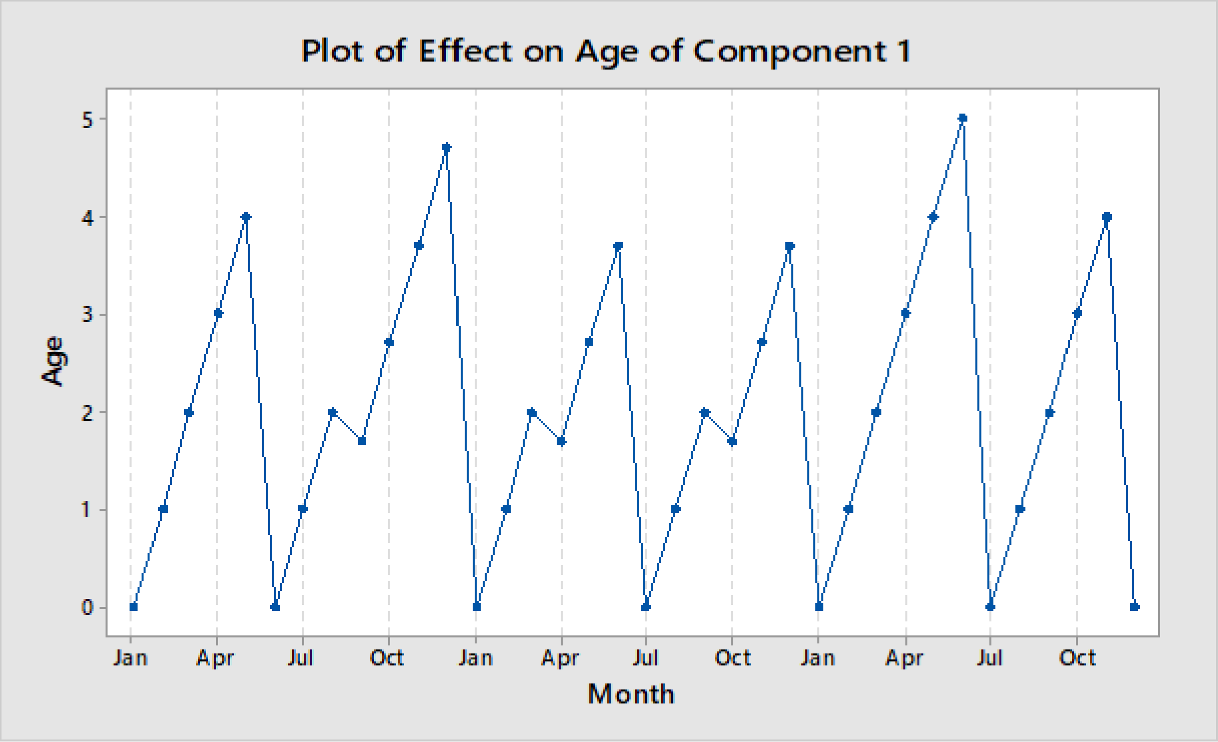


Fig 4.39: Optimal Maintenance strategy effect on Conveyor System

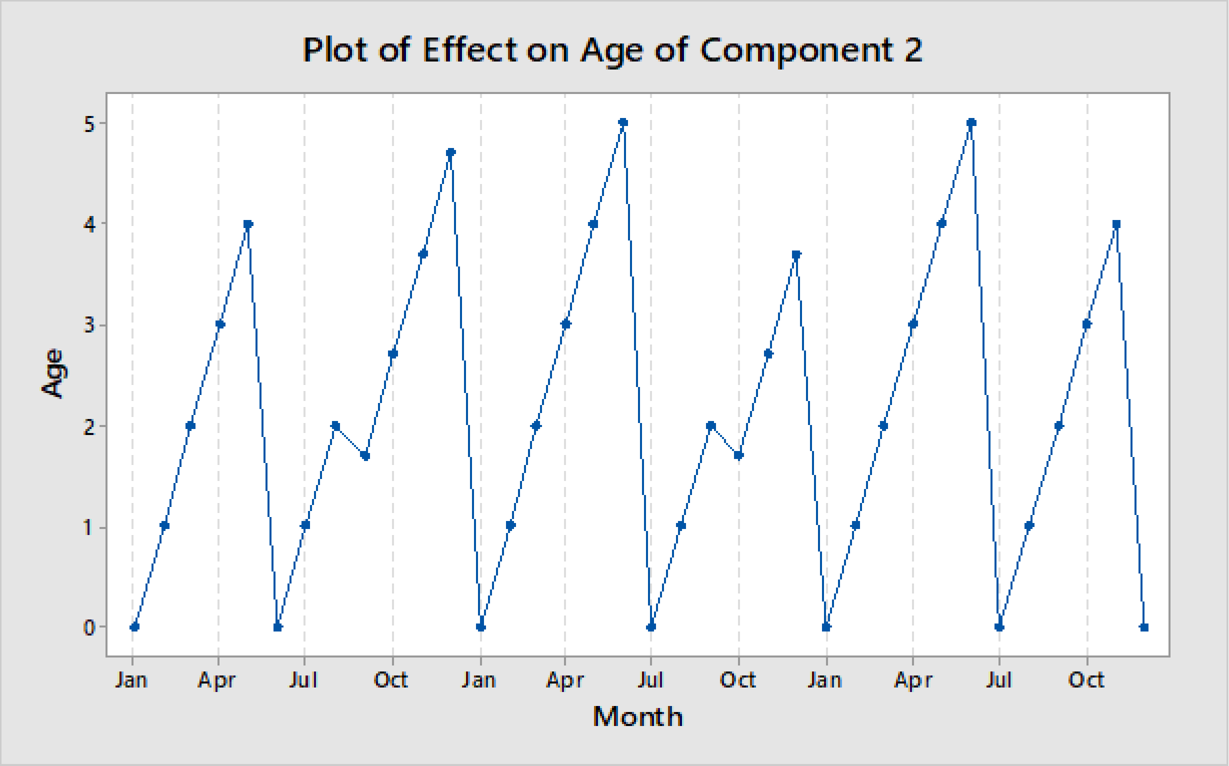


Fig 4.40: Optimal Maintenance strategy effect on Mixer System

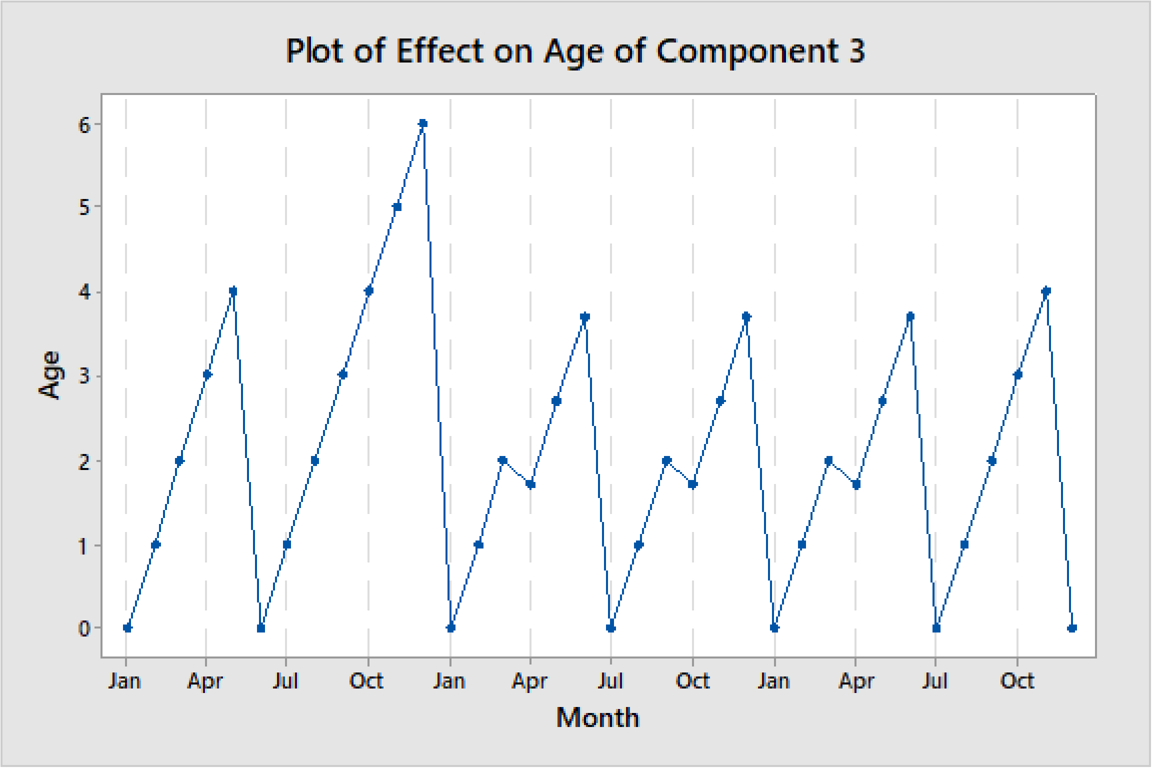


Fig 4.41: Optimal Maintenance strategy effect on Roller System

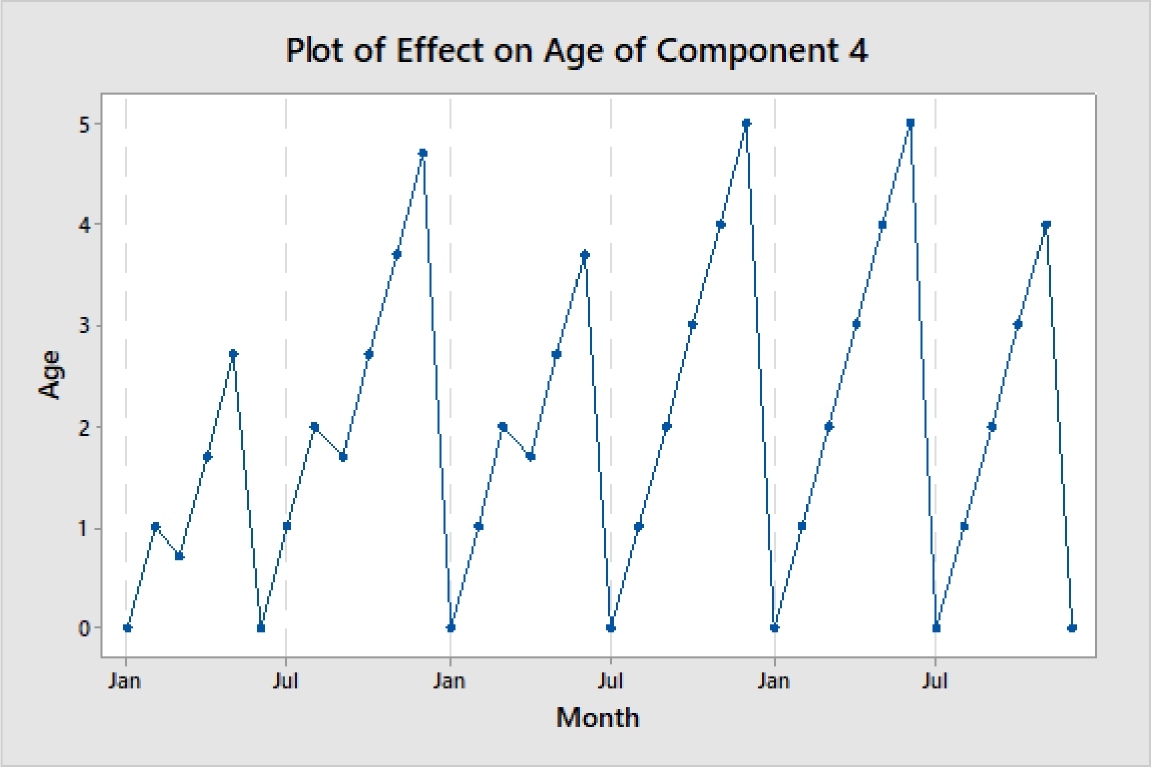


Fig 4.42: Optimal Maintenance strategy effect on Slitter System

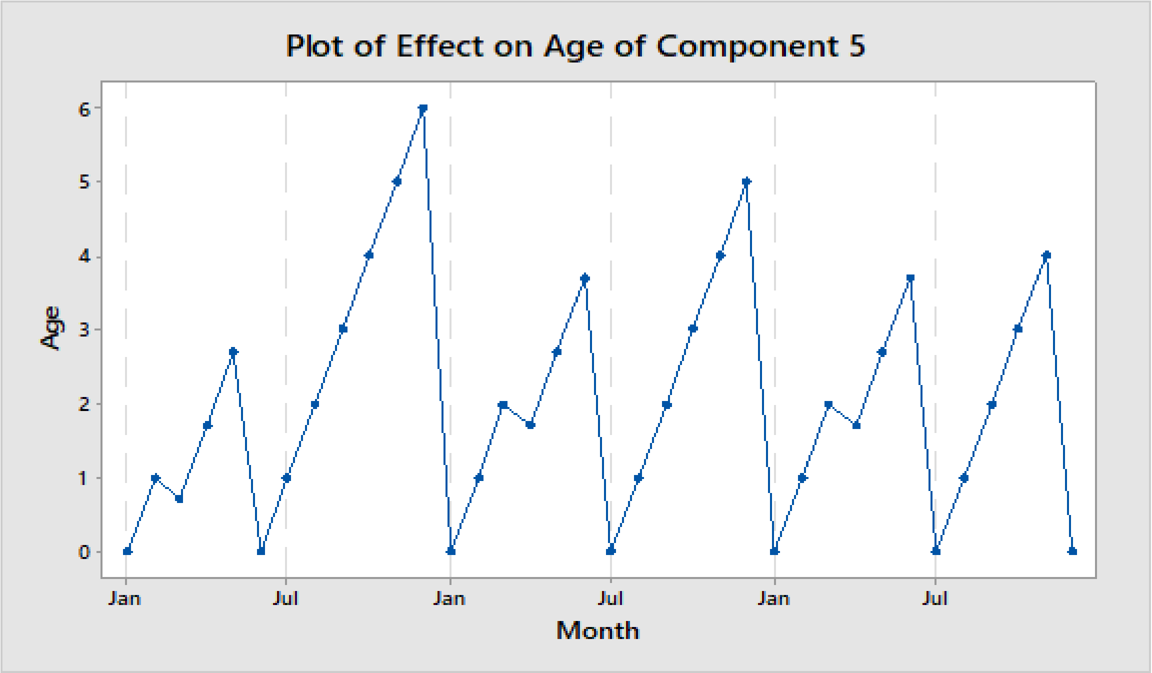


Fig 4.43: Optimal Maintenance strategy effect on Compounding Machine

From figures 4.39, 4.40, 4.41, 4.42 and 4.43, one can notice that the minimum age of each component at the beginning of the planning schedule is zero, and that most more of corrective maintenance is scheduled when compared with preventive maintenance, thus making the average age of the component to be in the range of 0 months to 6 months as summarised in table 4.51

Table 4.51: Effective Age of Component with Simulation Solution Output

Components	Minimum Effective Age (Month)	Maximum Effective Age (Month)
1	0	5
2	0	5
3	0	6
4	0	5
5	0	6

As a result of scheduling more of corrective maintenance, the effective age of the components did not exceed 6 months, thus making this schedule very expensive to run and if the goal is to reduce total maintenance cost, this solution is not the optimal best solution as a maintenance strategy containing more of corrective maintenance actions is very expensive and doesn’t add any optimal value to the reliability of the system.

4.6 Results of assessment of potential contribution and economic implication

The purpose of the optimal maintenance strategy is to find the optimal combination of maintenance actions that meets the objectives of any manufacturing firm. These can either be to maximize reliability or minimize cost. When comparing the solutions in figure 4.44, the Excel/Lingo, GAnetXL/Genetic algorithm solutions are very similar. However, the case of simulation based solution is different because of the deviation in cost solution which is higher due to more Corrective maintenance taking place over the planning period. The difference in computation time is also significant as some technique if selected takes a longer time to solve and will require a longer computational time as the problem becomes bigger.

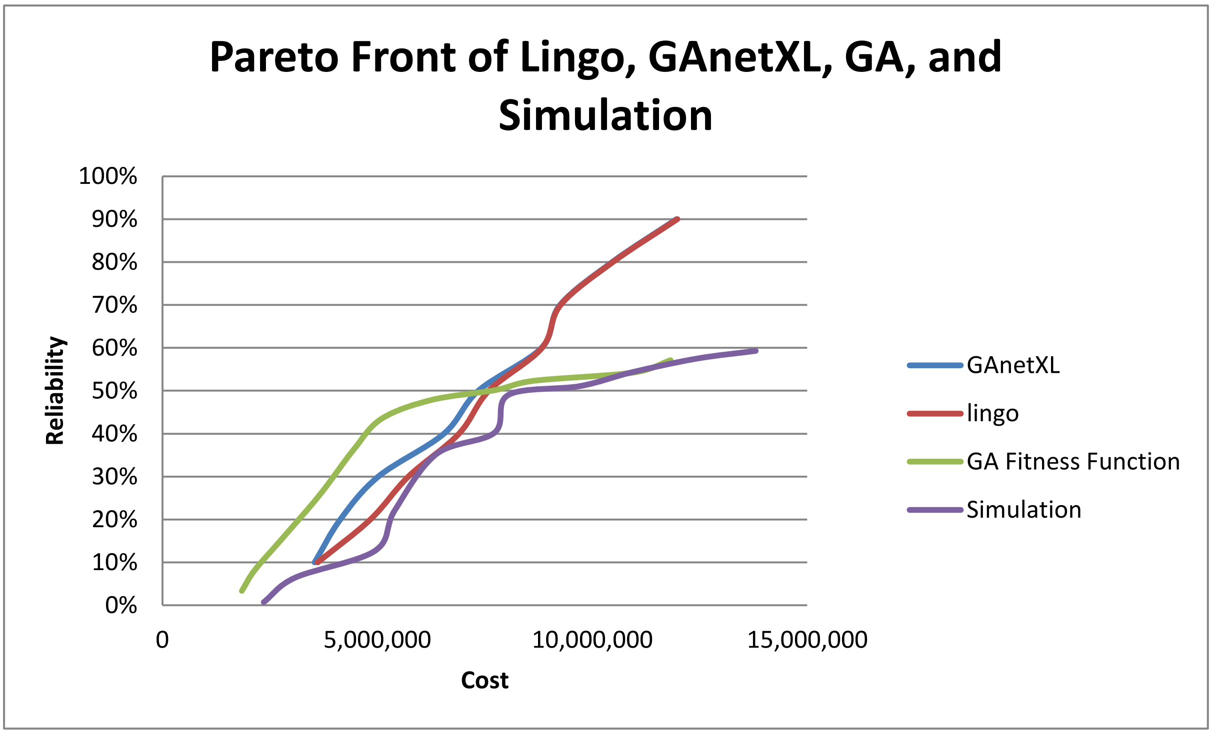
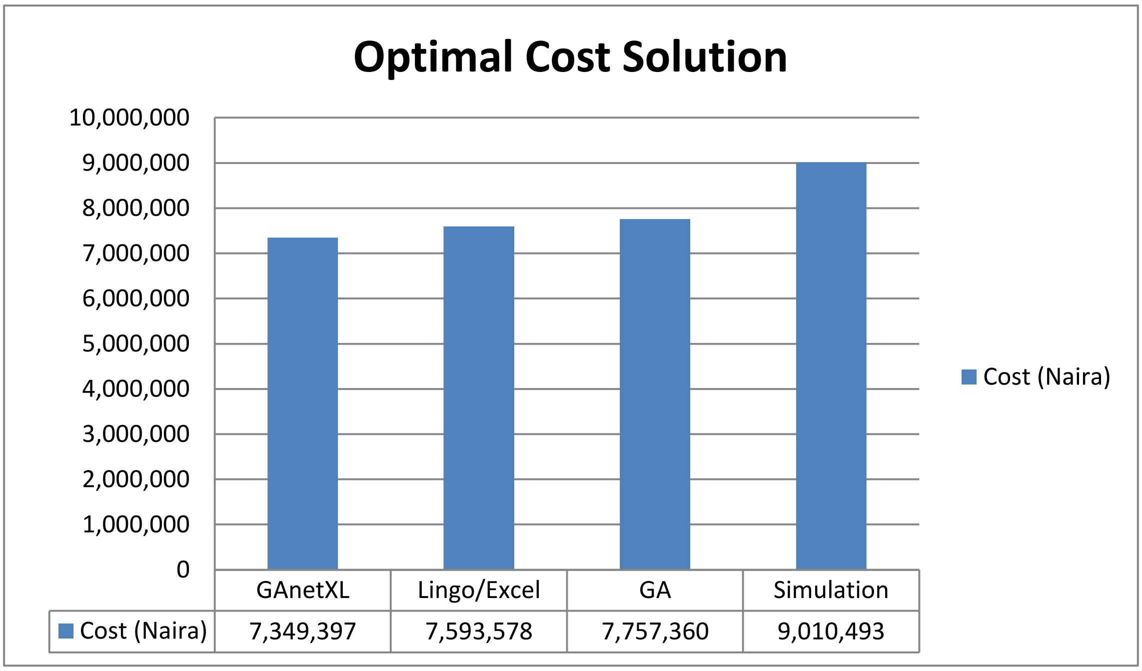


Fig 4.44 Comparison of pareto fronts

At reliability of 50% the optimal solution strategies are compared with their optimal cost and reliability in table 4.52, with optimal cost comparison in figure 4.45

Table: 4.52: Comparison of all solution

Required Reliability	Solution Technique	Optimal Reliability	Total Cost	Average Reliability	Trend
50%	GAnetXL	50%	₦7,349,397	97.5%	-------
	Lingo/Excel	50%	₦7,593,578	97.2%	3.21%
	Genetic Algorithm	50.15%	₦7,757,360	97.83%	5.25%
	Simulation	51.08%	₦9, 010 ,493	97.8%	18.43%



4.45: Comparison of optimal cost solution

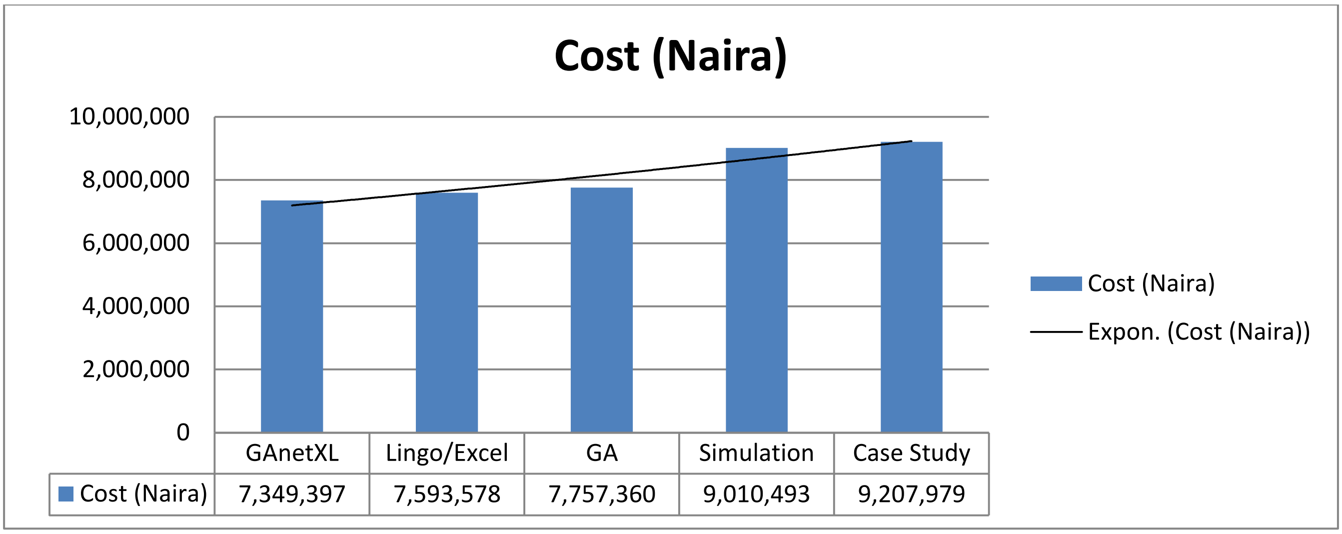
From the above comparison, simulation based solution can be considered to be a poor optimal solution when compared with other solution strategies. It shows that a maintenance strategy containing more of corrective replacement actions is very expensive and doesn’t add any optimal value to the reliability of the system.

4.6.1 Economic Implications

The cost savings and improved reliability is apparent when the optimal solution strategy is compared with the current industrial applied scenario to. The cost savings and improved reliability are illustrated in table 4.53, figure 4.46 and figure 4.47

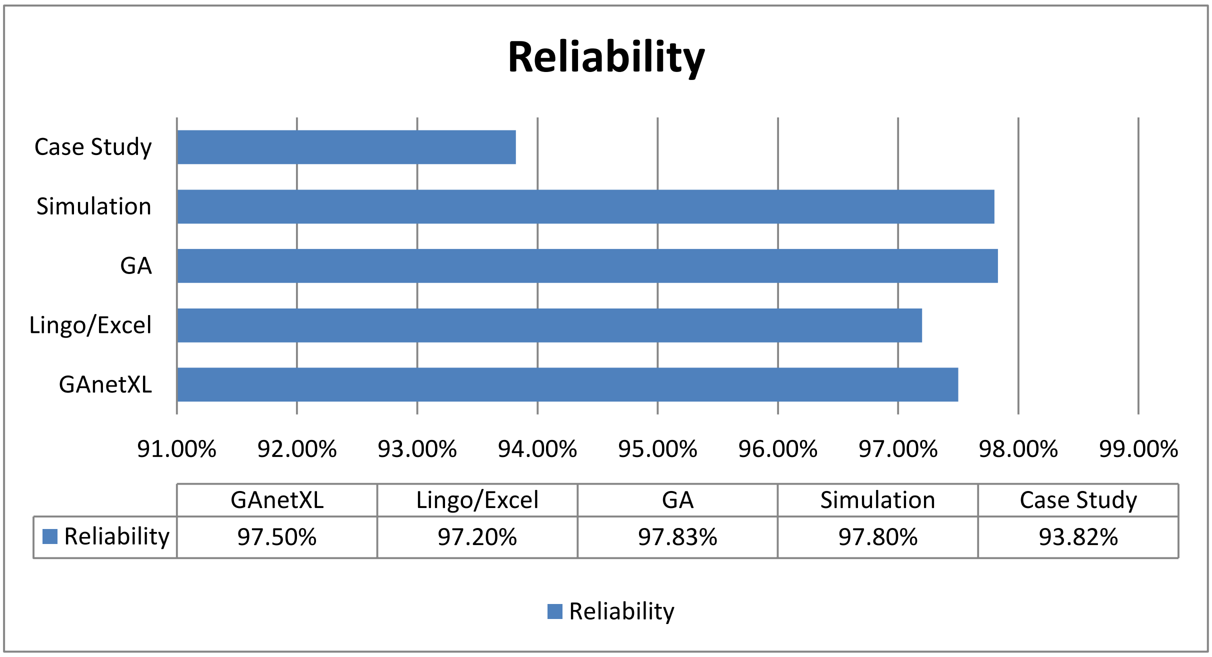
Table 4.53: Economic implication cost savings and reliability improvement

Categories	**Reliability**	**Cost**	**Cost Savings**	**Relaibility Improvement**
Industrial Case Study	93.82%	₦9, 207, 979	----------	--------------
Solution Strategy				
Excel/lingo	97.2%	₦7,593,578	21.26%	3.38%
GAnetXL	97.5%	₦7,349,397	25.28%	3.7%
Genetic Algorithm	97.83%	₦7,757,360	18.69%	4.01%
Simulation	97.8%	₦ 9, 010 ,493	2.19%	3.98%



	GAnetXL	Lingo/Excel	GA	Simulation	Case Study
■ Cost (Naira)	7,349,397	7,593,578	7,757,360	9,010,493	9,207,979

Fig 4.46 Cost Savings



	GAnetXL	Lingo/Excel	GA	Simulation	Case Study
■ Reliability	97.50%	97.20%	97.83%	97.80%	93.82%

Fig 4.47: Reliability Improvement

The developed optimal maintenance strategy provides a better cost savings and improved reliability. In terms of cost GAnetXL provides a better optimal strategy with an improved reliability while in terms of reliability genetic algorithm has better system reliability with an improved cost.

4.7 Development of a Generic User Interface Support System

The models proposed in equation 4.21 of this dissertation are implemented within a newly proposed computerised user interface support tool aimed at facilitating maintenance strategy decisions. The design of this tool is described in detail as follows:

4.7.1 General Considerations in User Interface Support Systems

A user interface support system is a computerised information system that may be employed to support employees of companies with business and organisational decision making activities. Typically, such systems compiles useful information useful information, which is presented to the user, by analysing raw data and documents in order to identify or solve complex problems. The user interface support system developed in this study consists of three main components namely:

- The database: developed to allow input data to be stored in a structure manner.
- A graphical user interface: developed to ensure effective human-computer interaction, thus enabling the user the means of providing the required input and obtaining relevant output.
- A model base: the workhorse of the system, implementing one of the solution techniques developed in the study to provide the relevant output.

An illustration of the interaction between the three components is described in figure 4.48.

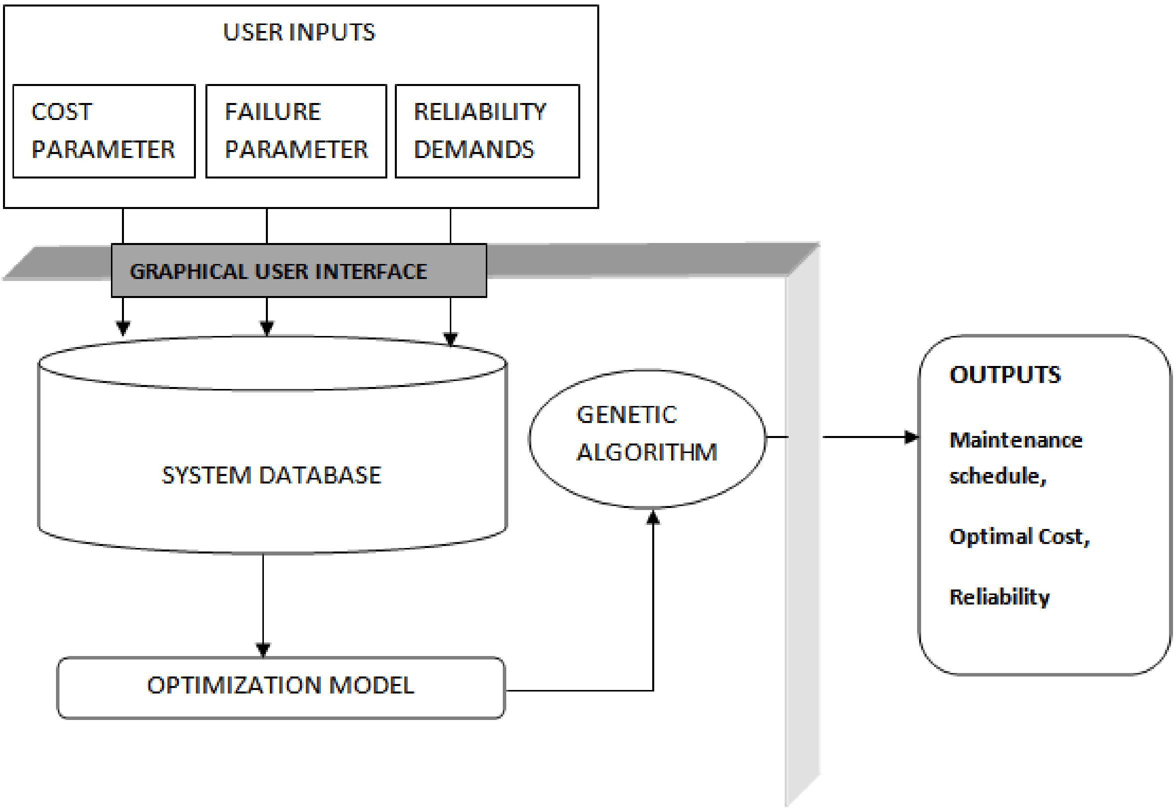


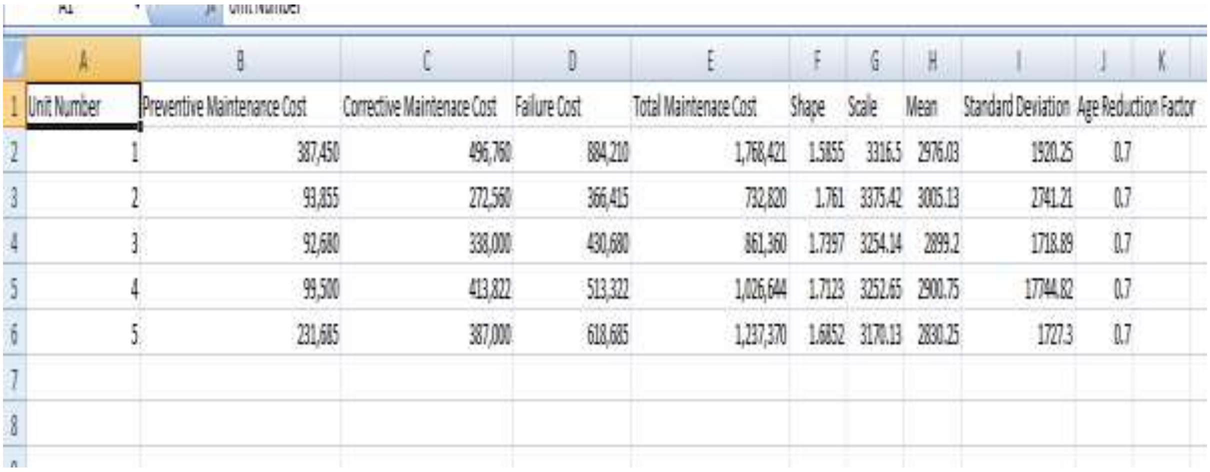
Fig 4.48: A Graphical Overview of the User Support System.

4.7.2 System Development

The software environment within which the user interface support system was developed by this study is a framework supported by RStudio called Shiny. Shiny is an application framework used to construct elegant and powerful applications displaying interactive reports and data visualisations based in R. The framework Shiny was adopted in the development of the user interface support system in this study due to its ability to create elegant GUIs capable of changing dynamically, based on R script files.

4.7.3 Data Preparation

In order to standardise the procedures of the user interface support system, the required input data have to be prepared in a specific format before the user interface support system can be utilised. The system requires one user-specified input file, containing maintenance cost information, information on the age reduction factor and information of reliability parameters. The format required of the file for the system is a comma separated values (CSV) format. An example of the exact required input data required is shown in figure 4.49.



Unit Number	Preventive Maintenance Cost	Corrective Maintenace Cost	Failure Cost	Total Maintenace Cost	Shape	Scale	Mean	Standard Deviation	Age Reduction Factor
1	387,450	496,760	884,210	1,768,421	1.5855	3316.5	2976.03	1920.25	0.7
2	93,855	272,560	366,415	732,820	1.761	3375.42	3005.13	2741.21	0.7
3	92,680	338,000	430,680	861,360	1.7397	3254.14	2899.2	1718.89	0.7
4	99,500	413,822	513,322	1,026,644	1.7123	3252.65	2900.75	17744.82	0.7
5	231,685	387,000	618,685	1,237,370	1.6852	3170.13	2830.25	1727.3	0.7

Fig 4.49: Required Input Format of the Specifications of a Manufacturing System

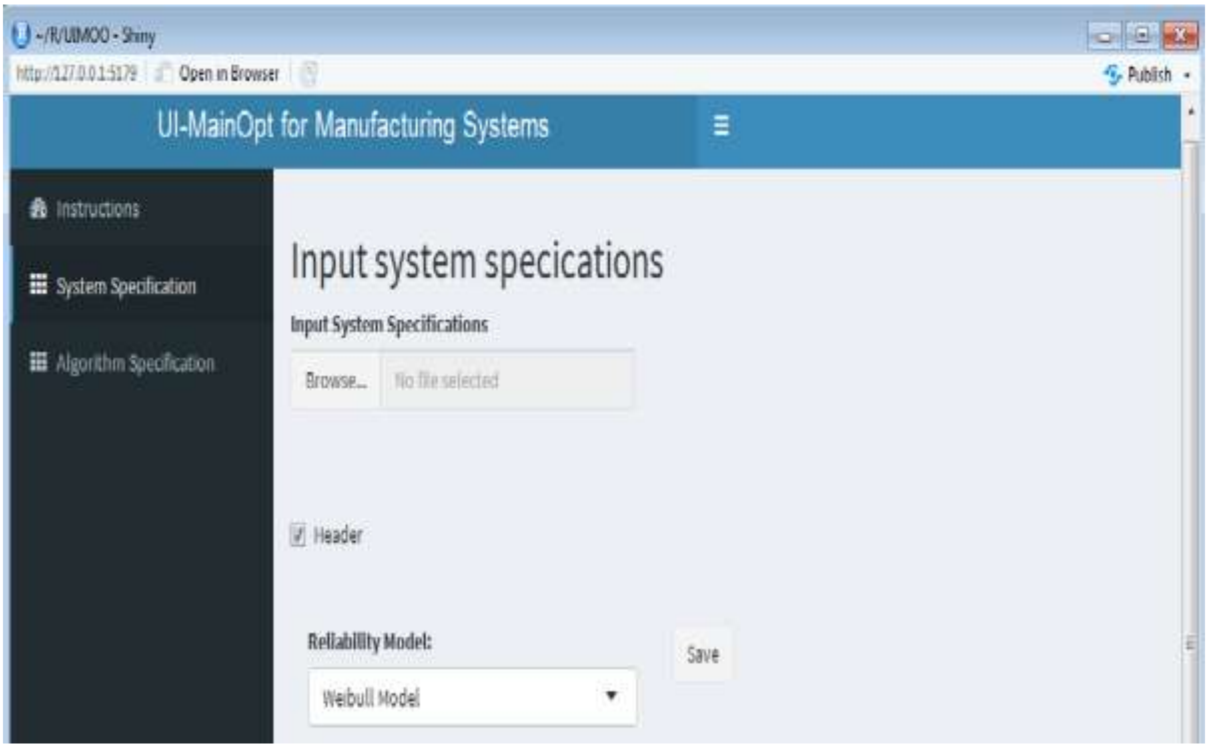
4.7.4 System Walk Through

After having prepared the required input data in the specified format, as described in the previous section, the user interface support system can be utilised to recommend optimal maintenance strategies for the manufacturing system specified. Once the user interface support system is initialised, the user is presented with the “Home screen" shown in Figure 4.50. On this screen, a short introduction to the user interface support system is provided to the user, as well as the steps to be followed in order to utilise the user interface support system to its full potential. After the instructions have been read and understood, the user can navigate to the “System specifications" window on the left-hand side of the screen, which displays the window seen in Figure 4.51. The user may, however, navigate back to the “Instructions" window at any subsequent time if some of the instructions have to be reviewed. In the “System specifications" window, the user can input the input requirement specifications in the format specified above.

Instructions for using the user interface maintenance optimization support system

The purpose of this support system is to aid decision makers and maintenance managers that are tasked with finding an optimal maintenance schedule based on improved cost and equipment reliability in a manufacturing system. The system is capable of providing an optimal maintenance schedule by employing an approximate solution based on the system descriptions provided by the user

Fig 4.50: The “Home screen" presented to the user when the system is initialised



Input system specications

Input System Specifications

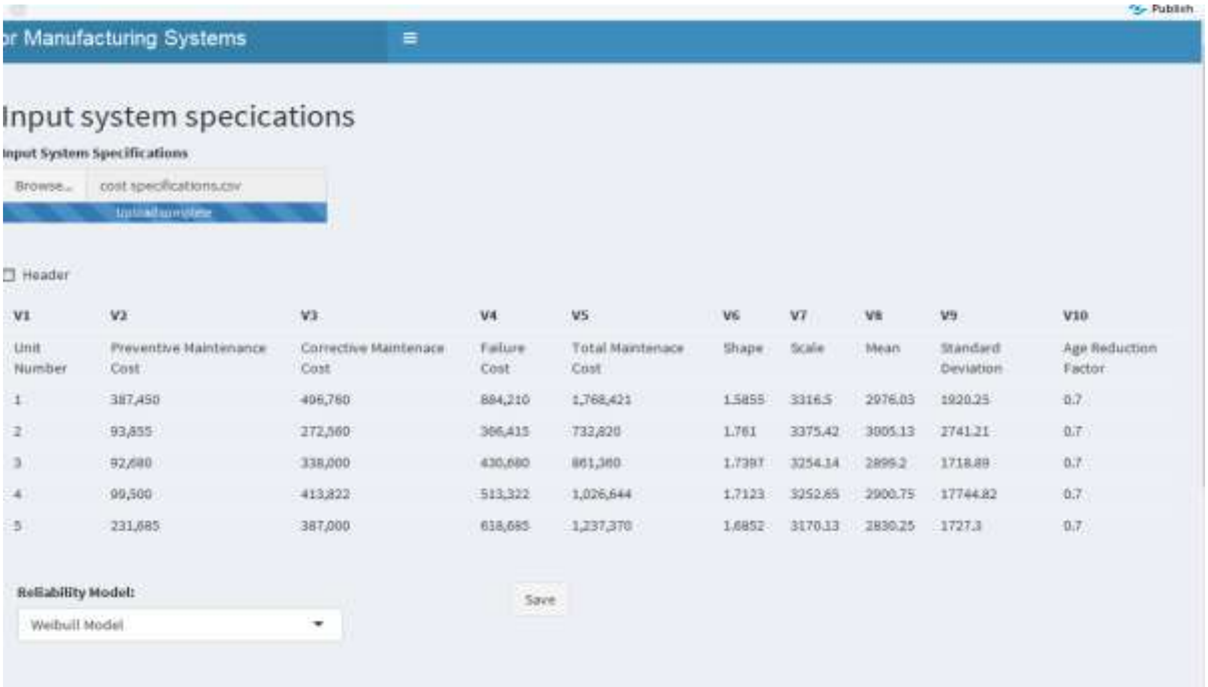
No file selected

☑ Header

Reliability Model: Weibull Model

Fig 4.51: GUI through which the user can upload the input specifications

Once the user uploads the specified input data in the required format, the system generated an overview of the data uploaded, and if the user is satisfied with the data uploaded, he can click on the “save" button which will upload the specifications to the system database. This is illustrated in figures 4.52 and 4.53.



Input system specications

Input System Specifications

Browse... cost specifications.csv

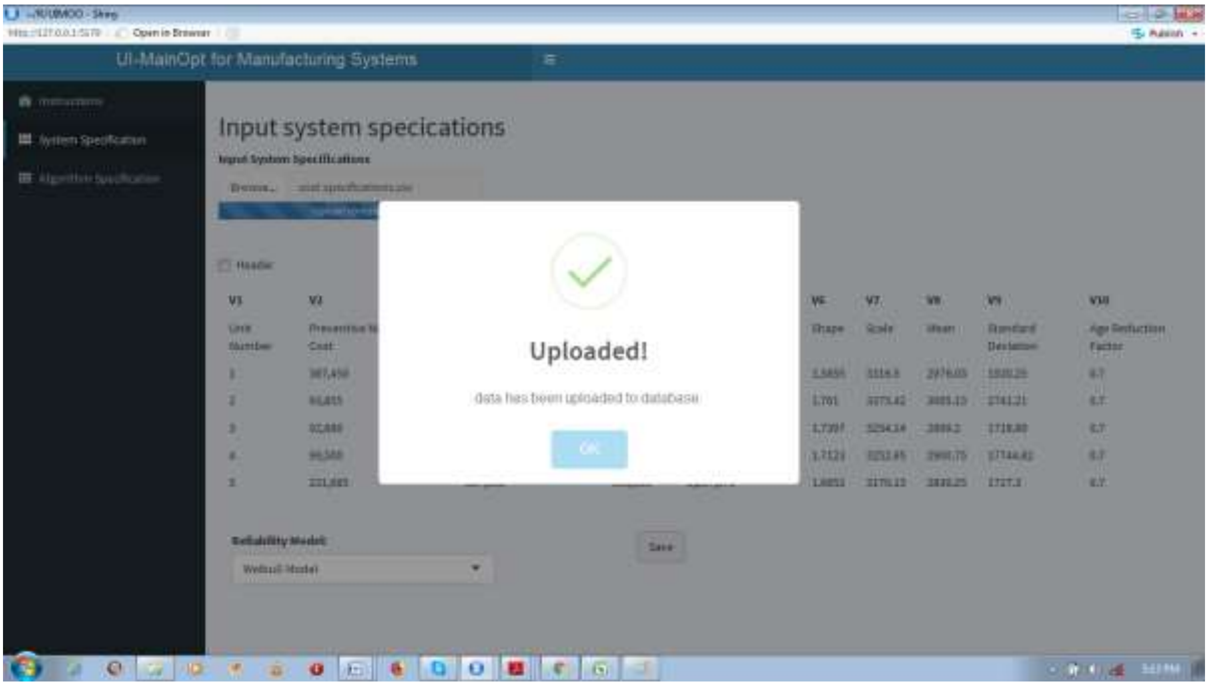
Upload complete

☐ Header

V1	V2	V3	V4	V5	V6	V7	V8	V9	V10
Unit Number	Preventive Maintenance Cost	Corrective Maintenace Cost	Failure Cost	Total Maintenace Cost	Shape	Scale	Mean	Standard Deviation	Age Reduction Factor
1	387,450	496,760	884,210	1,768,421	1.5855	3316.5	2976.03	1920.25	0.7
2	93,855	272,560	366,415	732,820	1.761	3375.42	3005.13	2741.21	0.7
3	92,680	338,000	430,680	861,360	1.7397	3254.14	2899.2	1718.89	0.7
4	99,500	413,822	513,322	1,026,644	1.7123	3252.65	2900.75	17744.82	0.7
5	231,685	387,000	618,685	1,237,370	1.6852	3170.13	2830.25	1727.3	0.7

Reliability Model: Weibull Model

Fig 4.52: An Overview of the Specified System Specification



Input system specications

Input System Specifications

☐ Header

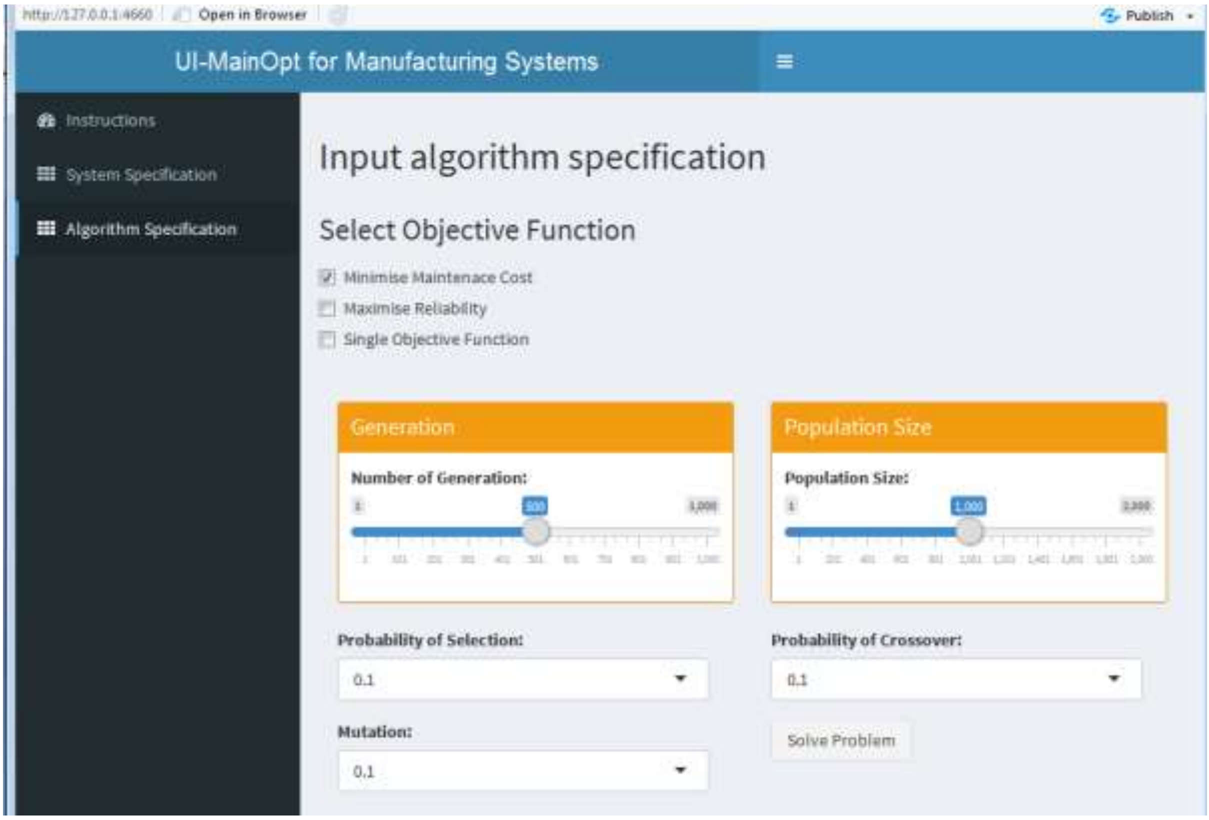
Uploaded!

data has been uploaded to database

Reliability Model: Weibull Model

Fig 4.53: An Overview of the Specified System Specification database upload

After the input specifications have being uploaded successfully the next step is to indicate the algorithm specifications for the model. In the algorithm specification window as shown in figure 4.54, the user is required to specify the objective functions and other requirements of the genetic algorithm. This includes the desired maintenance objective functions (selected by clicking on the radio buttons associated with the objective functions), the number of generations and population size (selected by moving the slider to the associated value), and other required parameters associated to the probability of selection, crossover and mutation (selected by choosing values from the dropdown list).



☑ Minimise Maintenace Cost
☐ Maximise Reliability
☐ Single Objective Function

Number of Generation: 500

Population Size: 1,000

Probability of Selection: 0.1

Probability of Crossover: 0.1

Mutation: 0.1

Fig 4.54: The Algorithm Specification Window

Once the user is satisfied with the selected objective function, and the genetic algorithm parameter values, he/she can click on the solve button. Clicking this button will execute the genetic algorithm and the user interface support system will subsequently be occupied, solving the model. The duration for which the user interface support system may be occupied, depends on various factors such as Central Processing Unit rating, number of generation and population size.

Once the algorithm has found a solution (when the status window disappears), the solutions which includes an optimal schedule, optimal pareto fronts and system reliability will be saved in a comma separated values files through which the user can access the solutions in the user’s personal computer.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

Conclusions drawn with regard to this study are presented in this chapter. It begins with the summary of the findings, specific conclusions from the study, followed by a discussion on the contribution of this research to both theory and practice is presented. This is followed by an evaluation with the objectives. In the end recommendations for further research are made.

5.1 Summary of Findings

The findings from the present research are discussed in details in this chapter.

An optimal maintenance strategy for improved cost and machine reliability in food manufacturing was developed in this study. This strategy is expected to be useful as a decision support tool to obtain optimal cost and machine reliability through an efficient maintenance strategy in food manufacturing and any other manufacturing sectors.

The optimal strategy was obtained through the optimization of a mixed integer nonlinear multi-objective programming model presented in equation (4.21), to minimize total maintenance cost and maximize system reliability using four solution techniques, Lingo, GAnetXL (a decision support system for spreadsheet models), Genetic Algorithm and Simulation based optimization. The programming model was validated using an industrial case study at a food manufacturing company, Tummy Tummy Industries Limited. It was found that the maintenance cost of the industrial case study has an increasing trend over time. The trend equation for predicting maintenance cost at a given time in the industrial case study was presented in equation (4.1).

The trend was found to be stationary and has a unit root using Augmented Dickey-Fuller test which implies that the model obtained can be used to make future forecasting behaviour of the process. Also three years forecast on the maintenance cost of the industrial case study was made, which determined that if the current maintenance strategy is still maintained, the total cost is expected to be about ₦ 9,201,979.

To determine the effects of the different maintenance cost variables on the total maintenance cost of the manufacturing firm, multiple regression analysis was performed. From the result, it was determined that with R-square of 69.2% it implies that the maintenance cost variables can only explain about 69.2% of total variation in total maintenance cost and that that the maintenance cost variables have a significant impact on the total maintenance cost. The model equation was presented in equation (4.1.2).

The components failure times were subjected to reliability lifetime distribution models in order to determine which distribution model suits each component. The results revealed that the failure times follow a Weibull distribution model.

Downtime analysis found that the industrial case study was in an average condition and there is a required urgent improvement of maintenance policies and strategies otherwise it will be difficult for the organization to sustain it. Pareto analysis determined that scheduled maintenance, equipment failures and breakdown and waiting for materials to arrive have caused 84% of the total downtime. And that whereas scheduled maintenance and equipment failures and breakdown was unavoidable, they could be reduced with effective maintenance strategy.

The result of optimizing the mixed integer nonlinear multi-objective programming model using lingo 17 program found that at an optimal pareto front of 50% that the objective function of the optimal solution for maintenance cost is ₦7,593,578 and overall system of 97.2%. The optimal maintenance schedule from this solution was presented in table 4.4.2.

The result of optimizing the mixed integer nonlinear multi-objective programming model using GANetXL found that at an optimal pareto front of 50% that the objective function of the optimal solution for maintenance cost is ₦7,349,397, and overall system reliability of 97.5% The optimal maintenance schedule from this solution was presented in table 4.4.7.

Also the result of optimizing the mixed integer nonlinear multi-objective programming model using Genetic Algorithm found that at an optimal pareto front of 50% that the objective function of the optimal solution for maintenance cost is ₦7,757,360 and overall system of 97.83%. The optimal maintenance schedule from this solution was presented in table 4.4.9.

In addition, the result of optimizing the mixed integer nonlinear multi-objective programming model using Simulation based optimization found that at an optimal pareto front of 50% that the objective function of the optimal solution for maintenance cost is ₦9,010,493 and overall system of 93.82%. The optimal maintenance schedule from this solution was presented in table 4.4.11.

From the developed maintenance strategy, it was observed that it could be used to track the effective age of the components and then utilize the information to initiate additional monitoring activities. Take for example, after a component reaches a certain level of effective age, additional monitoring, tests or inspections might be warranted to assist in the detection of imminent failure

The simulation based optimal solution was considered to be poor at a cost solution of ₦9,010,493 and reliability of 97.8% when compared with Lingo (cost: ₦7,593,578, reliability: 97.2%), GAnetXL (cost: ₦7,349,397, reliability 97.5%) and Genetic Algorithm (cost: ₦7,757,360, 97.83%), as it scheduled more of corrective maintenance actions which is deemed very expensive and did not add any optimal value to the reliability of the system under study. Overall, the developed optimal maintenance strategy provided better cost savings and improved system reliability than the current maintenance cost and machine reliability of the industrial case study (cost: ₦9,207,979, Reliability 93.82%). In terms of cost, GAnetXL provided a better optimal method with improved system reliability while in terms of reliability, Genetic Algorithm had better system reliability with an improved cost and this solution technique was adopted as a model base for the generic user interface support system developed to assist in easy implementation of the optimization model developed in this study for facilitating maintenance strategy decisions.

5.2 Specific Conclusion

- Maintenance of manufacturing components can be improved using reliability parameters and cost of those components.
- For the strategy to be effective, input data needs to be as exact as possible. Therefore, there is a need for manufacturing companies to ensure that failure history and cost of maintenance/ replacement of every component are properly documented to ensure accurate reliability prediction and cost forecasting.
- The impact of various maintenance strategies determines the behaviour of the method. if the impact of the selected strategy on reliability is low, the algorithm allows more replacements to occur to keep reliability high, and therefore this factor must be calculated carefully.

5.3 Achievement of Research Aim

The purpose of this research was to develop an optimal maintenance strategy for improved cost and machine reliability. This aim has been achieved. In chapter 4, an optimization model that was based on improved cost and machine reliability was developed. The output of this model was a maintenance schedule that can be used to achieve a specific reliability at an optimum cost. An attempt was also made to illustrate the effect of the optimal maintenance strategy has on the age to the manufacturing components.

5.3 Contribution to Knowledge

- A multi-objective optimization model was developed, the optimization model was successfully validated with an industrial case study, can contribute to the field of maintenance management in food manufacturing. The model, although considered for the food manufacturing environment, can also be easily modified to suit any other multi-component system.
- The optimal maintenance schedule outputs from the multi-objective optimization model can be used to control cost and reliability of manufacturing components at an efficient level.
- Lingo and matlab codes for the execution of the multi-objective optimization model based on cost and reliability were also presented in this study.
- A Generic User Interface Support System was developed in this study and presented so that it can be employed to support employees of companies with business and organisational decision making activities.

5.4 Recommendations for Further Research

- In this study, cost and reliability were considered as an objective maintenance criterion to improve maintenance performance in food manufacturing. It will be useful to apply other criteria, for example availability, inventory spare parts, maintenance time, risk, OEE, logistics etc to develop an optimal strategy in order to achieve the same purpose of improving maintenance performance.

- In this study, two maintenance strategies was employed, the optimization model can be expanded by applying more maintenance strategies.

- In this study, it can be useful to apply other repairable system models and compare the results achieved with the ones used in this study. It will also be useful to consider adding the non-repairable components of food manufacturing systems to the algorithm, this would necessitate the use other optimization methods.
- Other system configurations such as consecutive k-out-of-n systems, redundancy and stand by system should also be investigated with this model.

5.5 Publication from this Research So Far

I. Igbokwe N.C., and Godwin H.C.,(2017) A Concise Review of Improvement Methodologies in Manufacturing Systems, in the Proceedings of the Nigerian Insititue of Industrial Engineering International Conference, pp. 10-21, University of Ibadan, Ibadan, Nigeria, 2017.

II. Igbokwe N.C., and Godwin H.C.,(2018) Maintenance Performance Evaluation and Downtime analysis of Production Process in a Food Manufacturing Company, in the Proceedings of the Faculty of Engineering International Conference, pp. 74-81, Nnamdi Azikiwe University, Awka, Nigeria, 2018.

III. Igbokwe N.C., and Godwin H.C.,(2018) Evaluating Maintenance and Analysis of Downtime of manufacturing components in a Food Manufacturing Company in Nigeria, Paper Accepted for publication with the Journal of Scientific and Engineering Research

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APPENDIX A

Repairable Systems Analysis

Pooled Components Results

```
System:  Components

Model: Power-Law Process
Estimation Method: Maximum Likelihood


Fixed-point Iterations of the Shape Parameter

Step     Shape    Scale  Log-Likelihood
   0  0.829729  34.4218        -4858.61
   1  0.913841  61.1923        -4854.44
   2  0.913869  61.2030        -4854.44
   3  0.913869  61.2030        -4854.44
   4  0.913869  61.2030        -4854.44


Parameter Estimates

                     Standard     95% Normal CI
Parameter  Estimate     Error     Lower     Upper
Shape      0.913869     0.030  0.856885  0.974642
Scale       61.2030    11.641   42.1571   88.8537


Test for Equal Shape Parameters
Bartlett’s Modified Likelihood Ratio Chi-Square

Test Statistic  50.87
P-Value         0.000
DF                  4


Trend Tests

                   MIL-Hdbk-189         Laplace’s
                TTT-based   Pooled  TTT-based  Pooled
Anderson-Darling
Test Statistic    1873.43  1686.01       -5.46   -0.47
20.67
P-Value             0.020    0.417       0.000   0.636
0.000
DF                   1734     1734
```

Table A1: Data for Pooled Components

Components	Time	MCF1	CLMCF1	CLMCF2	ROCOF1	CLROCOF1	CLROCOF2
1	153	2.310178	1.73207	3.08124	0.013799	0.011009	0.017295
1	188	2.788728	2.115638	3.67596	0.013556	0.010941	0.016796
1	233	3.392946	2.605713	4.418016	0.013308	0.01087	0.016292
1	530	7.190432	5.784661	8.93783	0.012398	0.010592	0.014513
1	968	12.46874	10.37195	14.98942	0.011771	0.010375	0.013356
1	1092	13.9207	11.65635	16.62493	0.01165	0.01033	0.013139
1	1240	15.63529	13.18281	18.54401	0.011523	0.01028	0.012916
1	1778	21.7338	18.68179	25.2844	0.011171	0.010131	0.012317
1	2219	26.61173	23.14139	30.6025	0.01096	0.01003	0.011975
1	3512	40.48518	36.0262	45.49604	0.010535	0.009787	0.01134
1	3791	43.4146	38.77484	48.60956	0.010466	0.00974	0.011245
1	3899	44.54352	39.83611	49.8072	0.01044	0.009723	0.011211
1	4365	49.3847	44.39862	54.93074	0.010339	0.009649	0.011079
1	4772	53.57646	48.36216	59.35295	0.01026	0.009587	0.010981
1	5122	57.15649	51.75538	63.12125	0.010198	0.009535	0.010907
1	8260	88.45662	81.60343	95.88535	0.009787	0.009132	0.010488
1	9205	97.66122	90.40341	105.5017	0.009696	0.00903	0.010411
1	9810	103.5109	95.99386	111.6167	0.009643	0.008968	0.010369
1	10122	106.5154	98.86387	114.7591	0.009617	0.008937	0.010349
1	10875	113.7342	105.7546	122.3159	0.009558	0.008865	0.010304
1	11022	115.1384	107.094	123.787	0.009546	0.008851	0.010296
1	11554	120.2067	111.9255	129.1008	0.009508	0.008804	0.010268
1	11841	122.9326	114.5217	131.9612	0.009488	0.008779	0.010254
1	12200	126.3343	117.7594	135.5335	0.009463	0.008748	0.010237
1	12276	127.0533	118.4435	136.289	0.009458	0.008742	0.010234
1	12407	128.2918	119.6213	137.5907	0.00945	0.008731	0.010228
1	13100	134.825	125.8286	144.4645	0.009406	0.008674	0.010198
1	13677	140.2418	130.9667	150.1738	0.009371	0.00863	0.010175
1	14512	148.0462	138.3549	158.4163	0.009323	0.008568	0.010145
1	15300	155.3758	145.2771	166.1765	0.009281	0.008512	0.010118
1	16431	165.8396	155.1303	177.2882	0.009224	0.008437	0.010084
1	17000	171.0802	160.052	182.8683	0.009197	0.008401	0.010068
1	18211	182.1842	170.4508	194.7252	0.009142	0.008328	0.010036
1	18657	186.2574	174.2554	199.0861	0.009123	0.008302	0.010026
1	19322	192.3153	179.9037	205.5832	0.009096	0.008265	0.01001
1	19900	197.566	184.7899	211.2255	0.009073	0.008234	0.009997
1	20690	204.7215	191.4344	218.9307	0.009042	0.008193	0.009981
1	20910	206.7099	193.278	221.0752	0.009034	0.008181	0.009976
1	21362	210.7896	197.0567	225.4796	0.009018	0.008159	0.009967
1	21895	215.5909	201.497	230.6705	0.008998	0.008132	0.009957
1	22500	221.0285	206.5175	236.5593	0.008977	0.008104	0.009945
1	22780	223.5409	208.834	239.2835	0.008968	0.00809	0.00994
1	22966	225.2083	210.3704	241.0927	0.008962	0.008082	0.009937
1	23044	225.9072	211.0142	241.8514	0.008959	0.008078	0.009936
1	23178	227.1074	212.1193	243.1545	0.008954	0.008072	0.009933
1	23361	228.7455	213.627	244.934	0.008948	0.008064	0.00993
1	23450	229.5418	214.3596	245.7993	0.008945	0.00806	0.009929

1	23475	229.7654	214.5653	246.0423	0.008945	0.008059	0.009928
1	24290	237.0445	221.2533	253.9628	0.008918	0.008022	0.009914
1	24850	242.0339	225.8287	259.402	0.008901	0.007998	0.009905
1	24926	242.7103	226.4485	260.1399	0.008899	0.007995	0.009904
1	25792	250.4051	233.4897	268.5458	0.008872	0.007959	0.009891
1	26100	253.1364	235.9852	271.5341	0.008863	0.007947	0.009886
2	49	0.816097	0.573069	1.162187	0.015221	0.011383	0.020352
2	76	1.218824	0.87782	1.692296	0.014656	0.011239	0.019112
2	100	1.566252	1.146003	2.14061	0.014313	0.011149	0.018377
2	124	1.9065	1.412298	2.573639	0.014051	0.011078	0.017821
2	148	2.241087	1.677075	2.99478	0.013838	0.01102	0.017377
2	153	2.310178	1.73207	3.08124	0.013799	0.011009	0.017295
2	188	2.788728	2.115638	3.67596	0.013556	0.010941	0.016796
2	200	2.950963	2.246637	3.876096	0.013484	0.010921	0.016649
2	233	3.392946	2.605713	4.418016	0.013308	0.01087	0.016292
2	282	4.039526	3.136121	5.20317	0.013091	0.010806	0.015858
2	348	4.895468	3.846198	6.230986	0.012856	0.010736	0.015395
2	420	5.813396	4.616122	7.321206	0.012649	0.010672	0.014993
2	566	7.635497	6.165347	9.45621	0.012328	0.010569	0.014381
2	642	8.56728	6.96654	10.53583	0.012195	0.010525	0.014131
2	744	9.80314	8.03708	11.95727	0.012041	0.010472	0.013846
2	810	10.59493	8.727185	12.86239	0.011954	0.010441	0.013685
2	855	11.13158	9.196639	13.47362	0.011898	0.010421	0.013584
2	890	11.54728	9.561199	13.94592	0.011857	0.010407	0.01351
2	910	11.78419	9.769303	14.21465	0.011834	0.010398	0.013469
2	988	12.70396	10.57947	15.25508	0.011751	0.010368	0.013318
2	1080	13.78084	11.53228	16.46782	0.011661	0.010334	0.013159
2	1154	14.64127	12.29664	17.43295	0.011595	0.010308	0.013042
2	1280	16.09558	13.59424	19.05716	0.011492	0.010268	0.012861
2	1360	17.01249	14.41576	20.07697	0.011432	0.010243	0.012758
2	1436	17.87925	15.19459	21.03824	0.011378	0.010221	0.012666
2	1500	18.60609	15.8493	21.84239	0.011336	0.010203	0.012594
2	1675	20.58026	17.63441	24.01822	0.011228	0.010157	0.012413
2	1715	21.02894	18.04144	24.51115	0.011206	0.010147	0.012375
2	1856	22.6035	19.47342	26.23671	0.01113	0.010112	0.012249
2	1910	23.20376	20.02072	26.89287	0.011102	0.0101	0.012204
2	2089	25.18329	21.83072	29.05073	0.011017	0.010059	0.012066
2	2245	26.89654	23.40314	30.9114	0.010949	0.010025	0.011958
2	2390	28.47979	24.86074	32.62568	0.01089	0.009995	0.011865
2	2421	28.81719	25.17188	32.9904	0.010878	0.009988	0.011846
2	2600	30.75828	26.96533	35.08474	0.010811	0.009953	0.011744
2	2879	33.76116	29.75036	38.31268	0.010717	0.009899	0.011601
2	2900	33.98614	29.9595	38.55398	0.01071	0.009895	0.011591
2	3122	36.35613	32.16635	41.09163	0.010642	0.009855	0.011492
2	3512	40.48518	36.0262	45.49604	0.010535	0.009787	0.01134
2	3600	41.41125	36.89426	46.48125	0.010512	0.009772	0.011309
2	3654	41.97855	37.42643	47.08434	0.010499	0.009763	0.01129
2	3712	42.58707	37.99759	47.73089	0.010485	0.009753	0.011271
2	3915	44.71054	39.99321	49.98428	0.010437	0.00972	0.011206
2	4365	49.3847	44.39862	54.93074	0.010339	0.009649	0.011079

2	4772	53.57646	48.36216	59.35295	0.01026	0.009587	0.010981
2	5122	57.15649	51.75538	63.12125	0.010198	0.009535	0.010907
2	5345	59.42642	53.91021	65.50705	0.010161	0.009503	0.010864
2	5470	60.69521	55.1157	66.83956	0.01014	0.009485	0.010841
2	5524	61.24256	55.63594	67.41417	0.010132	0.009478	0.010831
2	5568	61.6882	56.05961	67.88192	0.010125	0.009471	0.010823
2	5640	62.41678	56.75245	68.64647	0.010114	0.009461	0.010811
2	5689	62.91216	57.22364	69.16618	0.010106	0.009455	0.010802
2	5760	63.62931	57.90593	69.91839	0.010095	0.009445	0.010791
2	6000	66.0479	60.20833	72.45384	0.01006	0.009412	0.010752
2	6070	66.75174	60.87873	73.19131	0.01005	0.009403	0.010742
2	6136	67.41471	61.51035	73.88584	0.01004	0.009394	0.010732
2	6180	67.85636	61.93118	74.34842	0.010034	0.009388	0.010725
2	6245	68.50829	62.55249	75.03116	0.010025	0.009379	0.010716
2	6290	68.95929	62.98237	75.5034	0.010019	0.009373	0.010709
2	6328	69.33991	63.34522	75.90191	0.010014	0.009368	0.010704
2	6387	69.93049	63.9083	76.52016	0.010006	0.009361	0.010696
2	6420	70.26061	64.22309	76.86571	0.010001	0.009356	0.010691
2	6475	70.81049	64.74749	77.44123	0.009994	0.009349	0.010684
2	6521	71.27007	65.18584	77.92219	0.009988	0.009343	0.010677
2	6582	71.87909	65.7668	78.55946	0.00998	0.009335	0.010669
2	6612	72.17843	66.05237	78.87266	0.009976	0.009331	0.010665
2	6656	72.61725	66.47106	79.33175	0.00997	0.009326	0.01066
2	6699	73.04586	66.88004	79.78012	0.009965	0.00932	0.010654
2	6744	73.49415	67.30784	80.24904	0.009959	0.009314	0.010648
2	6780	73.85259	67.64994	80.62395	0.009955	0.00931	0.010644
2	6830	74.35016	68.12486	81.14434	0.009948	0.009303	0.010638
2	6888	74.92695	68.67545	81.74752	0.009941	0.009296	0.010631
2	6924	75.28474	69.01702	82.12166	0.009937	0.009292	0.010626
2	6970	75.74169	69.45329	82.59945	0.009931	0.009286	0.010621
2	6990	75.94028	69.64291	82.80709	0.009928	0.009283	0.010618
2	7043	76.46632	70.1452	83.35705	0.009922	0.009277	0.010612
2	7085	76.88293	70.54305	83.79259	0.009917	0.009272	0.010607
2	7134	77.36871	71.00698	84.30041	0.009911	0.009265	0.010601
2	7188	77.90373	71.51798	84.85966	0.009905	0.009259	0.010595
2	7349	79.49683	73.0398	86.5247	0.009886	0.009239	0.010577
2	7440	80.39595	73.89884	87.46429	0.009875	0.009228	0.010568
2	7501	80.99813	74.47422	88.09352	0.009868	0.009221	0.010561
2	8260	88.45662	81.60343	95.88535	0.009787	0.009132	0.010488
2	9205	97.66122	90.40341	105.5017	0.009696	0.00903	0.010411
2	9810	103.5109	95.99386	111.6167	0.009643	0.008968	0.010369
2	10122	106.5154	98.86387	114.7591	0.009617	0.008937	0.010349
2	10875	113.7342	105.7546	122.3159	0.009558	0.008865	0.010304
2	11022	115.1384	107.094	123.787	0.009546	0.008851	0.010296
2	11554	120.2067	111.9255	129.1008	0.009508	0.008804	0.010268
2	11841	122.9326	114.5217	131.9612	0.009488	0.008779	0.010254
2	12200	126.3343	117.7594	135.5335	0.009463	0.008748	0.010237
2	12276	127.0533	118.4435	136.289	0.009458	0.008742	0.010234
2	12407	128.2918	119.6213	137.5907	0.00945	0.008731	0.010228
2	13100	134.825	125.8286	144.4645	0.009406	0.008674	0.010198

2	13677	140.2418	130.9667	150.1738	0.009371	0.00863	0.010175
2	14512	148.0462	138.3549	158.4163	0.009323	0.008568	0.010145
2	15300	155.3758	145.2771	166.1765	0.009281	0.008512	0.010118
3	76	1.218824	0.87782	1.692296	0.014656	0.011239	0.019112
3	148	2.241087	1.677075	2.99478	0.013838	0.01102	0.017377
3	200	2.950963	2.246637	3.876096	0.013484	0.010921	0.016649
3	282	4.039526	3.136121	5.20317	0.013091	0.010806	0.015858
3	300	4.274527	3.330237	5.486571	0.013021	0.010786	0.01572
3	456	6.267138	4.999501	7.856187	0.01256	0.010644	0.014821
3	520	7.066347	5.678771	8.79297	0.012419	0.010598	0.014552
3	700	9.27194	7.575904	11.34767	0.012105	0.010494	0.013963
3	760	9.995625	8.204556	12.17769	0.012019	0.010464	0.013806
3	820	10.7144	8.831581	12.99862	0.011941	0.010437	0.013662
3	880	11.42866	9.457089	13.81125	0.011869	0.010411	0.01353
3	956	12.32741	10.24737	14.82966	0.011784	0.01038	0.013378
3	990	12.72746	10.60022	15.2816	0.011749	0.010367	0.013315
3	1050	13.43059	11.2219	16.07399	0.011689	0.010345	0.013209
3	1165	14.76876	12.41011	17.57568	0.011585	0.010305	0.013025
3	1280	16.09558	13.59424	19.05716	0.011492	0.010268	0.012861
3	1345	16.84093	14.26186	19.88639	0.011443	0.010248	0.012777
3	1420	17.69711	15.03075	20.83646	0.011389	0.010226	0.012685
3	1500	18.60609	15.8493	21.84239	0.011336	0.010203	0.012594
3	1628	20.05188	17.15569	23.437	0.011256	0.010169	0.012459
3	1690	20.74862	17.78709	24.20325	0.01122	0.010153	0.012399
3	1750	21.4208	18.3973	24.9412	0.011186	0.010138	0.012343
3	1791	21.87897	18.81382	25.4435	0.011164	0.010128	0.012306
3	1843	22.45878	19.34157	26.07837	0.011136	0.010115	0.01226
3	1880	22.87047	19.71673	26.52865	0.011117	0.010107	0.012229
3	1950	23.64745	20.42574	27.37732	0.011082	0.01009	0.012172
3	2023	24.45518	21.16408	28.25806	0.011047	0.010074	0.012115
3	2099	25.29344	21.93165	29.17054	0.011012	0.010057	0.012059
3	2156	25.92042	22.50661	29.85203	0.010987	0.010044	0.012018
3	2214	26.55693	23.09103	30.54304	0.010962	0.010032	0.011978
3	2287	27.35602	23.82573	31.4094	0.010931	0.010016	0.01193
3	2330	27.82569	24.25805	31.91802	0.010914	0.010007	0.011903
3	2372	28.28371	24.68	32.41363	0.010897	0.009998	0.011876
3	2459	29.23027	25.55306	33.43664	0.010863	0.009981	0.011824
3	2534	30.04395	26.30468	34.31477	0.010835	0.009966	0.01178
3	2677	31.58968	27.73519	35.97986	0.010784	0.009938	0.011702
3	2696	31.79452	27.92501	36.20022	0.010777	0.009934	0.011693
3	2778	32.67713	28.74357	37.14899	0.01075	0.009918	0.011651
3	2824	33.17126	29.2023	37.67965	0.010734	0.00991	0.011628
3	2880	33.77188	29.76032	38.32418	0.010716	0.009899	0.011601
3	2936	34.3715	30.31787	38.96712	0.010699	0.009889	0.011575
3	2990	34.94877	30.85505	39.58562	0.010682	0.009879	0.01155
3	3035	35.42914	31.30237	40.09996	0.010668	0.009871	0.01153
3	3077	35.87693	31.71961	40.57914	0.010655	0.009863	0.011511
3	3124	36.37741	32.1862	41.11438	0.010642	0.009855	0.011491
3	3200	37.18533	32.94003	41.97776	0.01062	0.009841	0.011459
3	3279	38.02339	33.72274	42.87249	0.010597	0.009827	0.011428

3	3312	38.37294	34.04944	43.24544	0.010588	0.009821	0.011415
3	3190	37.07912	32.84089	41.8643	0.010622	0.009843	0.011464
3	3265	37.875	33.5841	42.71413	0.010601	0.00983	0.011433
3	3308	38.33059	34.00985	43.20026	0.010589	0.009822	0.011416
3	3400	39.30364	34.91989	44.23772	0.010564	0.009806	0.011381
3	3488	40.23227	35.78928	45.22683	0.010541	0.009791	0.011348
3	3565	41.04316	36.54914	46.08976	0.010521	0.009778	0.011321
3	3580	41.20095	36.69707	46.2576	0.010517	0.009776	0.011316
3	3634	41.76852	37.22938	46.8611	0.010504	0.009766	0.011297
3	3670	42.1465	37.58403	47.26282	0.010495	0.00976	0.011285
3	3742	42.9015	38.29284	48.06483	0.010477	0.009748	0.011261
3	3790	43.40414	38.76501	48.59845	0.010466	0.009741	0.011245
3	3810	43.61341	38.96165	48.82055	0.010461	0.009737	0.011239
3	3866	44.19886	39.51199	49.44168	0.010448	0.009728	0.011221
3	3900	44.55396	39.84593	49.81827	0.01044	0.009723	0.011211
3	3948	45.05482	40.31713	50.34925	0.010429	0.009715	0.011196
3	3999	45.58641	40.81746	50.91256	0.010418	0.009707	0.011181
3	4051	46.12783	41.32726	51.48603	0.010406	0.009698	0.011166
3	4078	46.40871	41.59183	51.78344	0.0104	0.009694	0.011158
3	4124	46.88688	42.04238	52.28961	0.01039	0.009687	0.011145
3	4180	47.46838	42.59052	52.90491	0.010378	0.009678	0.011129
3	4217	47.85222	42.95247	53.31091	0.01037	0.009672	0.011119
3	4290	48.60868	43.66611	54.1107	0.010355	0.00966	0.011099
3	4366	49.39504	44.40839	54.94166	0.010339	0.009649	0.011079
3	4409	49.83944	44.82805	55.41105	0.01033	0.009642	0.011068
3	4498	50.75805	45.69597	56.38089	0.010313	0.009628	0.011046
3	4580	51.60303	46.4948	57.27248	0.010297	0.009616	0.011026
3	4640	52.22047	47.07881	57.92369	0.010285	0.009607	0.011012
3	4681	52.642	47.47764	58.36812	0.010277	0.0096	0.011002
3	4790	53.76111	48.53701	59.54749	0.010257	0.009584	0.010977
3	4865	54.52986	49.26514	60.3572	0.010243	0.009573	0.010961
3	4910	54.99062	49.70171	60.84235	0.010235	0.009566	0.010951
3	4990	55.80886	50.47727	61.70359	0.010221	0.009554	0.010934
3	5049	56.41158	51.04879	62.33775	0.01021	0.009546	0.010922
3	5124	57.17688	51.77473	63.14269	0.010198	0.009535	0.010906
3	5188	57.82918	52.3937	63.82854	0.010187	0.009525	0.010894
3	5245	58.40954	52.9446	64.43858	0.010177	0.009517	0.010883
3	5300	58.96903	53.47582	65.02651	0.010168	0.009509	0.010872
3	5378	59.76162	54.22863	65.85916	0.010155	0.009498	0.010857
3	5434	60.33006	54.76869	66.45614	0.010146	0.00949	0.010847
3	5500	60.99935	55.40476	67.15886	0.010136	0.009481	0.010835
3	5567	61.67808	56.04999	67.87129	0.010125	0.009472	0.010823
3	5620	62.21448	56.56005	68.4342	0.010117	0.009464	0.010814
3	5700	63.02332	57.32938	69.28278	0.010104	0.009453	0.010801
3	5788	63.91192	58.17486	70.21476	0.010091	0.009441	0.010786
3	5831	64.3457	58.58769	70.6696	0.010085	0.009435	0.010779
3	5890	64.94043	59.15382	71.29311	0.010076	0.009427	0.01077
3	5900	65.04119	59.24974	71.39873	0.010074	0.009426	0.010768
3	5966	65.70578	59.88253	72.09531	0.010065	0.009417	0.010758
3	6030	66.34963	60.49571	72.77001	0.010056	0.009408	0.010748

3	6099	67.04312	61.15631	73.49658	0.010046	0.009399	0.010737
3	6140	67.45487	61.54861	73.92791	0.01004	0.009393	0.010731
3	6199	68.04698	62.11284	74.54807	0.010032	0.009385	0.010722
3	6250	68.55842	62.60026	75.08365	0.010025	0.009379	0.010715
3	6303	69.08952	63.10652	75.63976	0.010017	0.009372	0.010707
3	6734	73.39455	67.21279	80.14486	0.00996	0.009316	0.01065
3	6780	73.85259	67.64994	80.62395	0.009955	0.00931	0.010644
3	6813	74.18102	67.96341	80.96745	0.00995	0.009306	0.01064
3	6877	74.81759	68.57105	81.63316	0.009942	0.009298	0.010632
3	6905	75.09592	68.83676	81.92422	0.009939	0.009294	0.010628
3	6955	75.59271	69.31105	82.44368	0.009933	0.009288	0.010622
3	6990	75.94028	69.64291	82.80709	0.009928	0.009283	0.010618
3	7024	76.27778	69.96517	83.15994	0.009924	0.009279	0.010614
3	7088	76.91268	70.57146	83.82369	0.009916	0.009271	0.010607
3	7148	77.50745	71.13949	84.44544	0.009909	0.009264	0.0106
3	7198	78.00277	71.61257	84.96318	0.009903	0.009258	0.010594
3	7250	78.51758	72.10432	85.50127	0.009897	0.009251	0.010588
3	7320	79.2101	72.76587	86.22504	0.009889	0.009243	0.010581
3	7390	79.90205	73.42694	86.94816	0.009881	0.009234	0.010573
3	7450	80.4947	73.99319	87.56747	0.009874	0.009227	0.010567
3	7520	81.1856	74.65336	88.28942	0.009866	0.009219	0.010559
3	7590	81.87595	75.31306	89.01074	0.009858	0.00921	0.010552
3	7660	82.56575	75.97229	89.73145	0.00985	0.009202	0.010545
3	7700	82.95968	76.34877	90.14302	0.009846	0.009197	0.010541
3	7843	84.36654	77.69345	91.61279	0.00983	0.00918	0.010527
3	7890	84.82845	78.13497	92.09534	0.009825	0.009175	0.010522
3	7934	85.26067	78.54812	92.54685	0.009821	0.00917	0.010518
3	7988	85.79083	79.05492	93.10067	0.009815	0.009164	0.010513
3	8045	86.35011	79.58957	93.6849	0.009809	0.009157	0.010507
3	8124	87.12468	80.33005	94.49403	0.009801	0.009148	0.0105
3	8200	87.86923	81.04186	95.27178	0.009793	0.009139	0.010493
3	8280	88.65233	81.79054	96.08979	0.009785	0.00913	0.010486
3	8300	88.848	81.97761	96.29419	0.009783	0.009128	0.010484
3	8365	89.48366	82.58534	96.95818	0.009776	0.009121	0.010478
3	8459	90.40216	83.46351	97.91765	0.009767	0.00911	0.01047
3	8499	90.79274	83.83694	98.32566	0.009763	0.009106	0.010467
3	8544	91.23196	84.25687	98.78448	0.009758	0.009101	0.010463
3	8580	91.58319	84.59268	99.15138	0.009755	0.009097	0.01046
3	8694	92.69459	85.65527	100.3124	0.009744	0.009084	0.010451
3	8740	93.1427	86.08369	100.7805	0.009739	0.009079	0.010447
3	8788	93.61006	86.53053	101.2688	0.009735	0.009074	0.010443
3	8834	94.05775	86.95855	101.7365	0.00973	0.009069	0.01044
3	8878	94.48579	87.36777	102.1837	0.009726	0.009064	0.010436
3	8950	95.18582	88.03702	102.9151	0.009719	0.009057	0.01043
3	8990	95.57452	88.40862	103.3212	0.009716	0.009052	0.010427
3	9032	95.98249	88.79863	103.7475	0.009712	0.009048	0.010424
3	9080	96.44854	89.24416	104.2345	0.009707	0.009043	0.01042
3	9140	97.0308	89.80078	104.8429	0.009702	0.009036	0.010416
3	9200	97.61274	90.35706	105.4511	0.009696	0.00903	0.010412
3	9288	98.46566	91.17235	106.3424	0.009688	0.009021	0.010405

3	9324	98.81438	91.50566	106.7069	0.009685	0.009017	0.010402
3	9399	99.54051	92.1997	107.4658	0.009678	0.009009	0.010397
3	9452	100.0533	92.68983	108.0018	0.009674	0.009004	0.010393
3	9500	100.5176	93.13351	108.4871	0.009669	0.008999	0.01039
3	9522	100.7303	93.33679	108.7094	0.009668	0.008997	0.010388
3	9589	101.3778	93.9556	109.3863	0.009662	0.00899	0.010384
3	9642	101.8898	94.44482	109.9216	0.009657	0.008984	0.01038
3	9728	102.7199	95.2381	110.7896	0.00965	0.008976	0.010374
3	9780	103.2216	95.71744	111.3141	0.009645	0.008971	0.010371
3	9800	103.4145	95.90173	111.5158	0.009644	0.008969	0.01037
3	9860	103.993	96.45439	112.1207	0.009639	0.008963	0.010366
3	9920	104.5711	97.00673	112.7254	0.009633	0.008957	0.010362
3	9977	105.1201	97.53114	113.2996	0.009629	0.008951	0.010358
3	10024	105.5726	97.96334	113.7728	0.009625	0.008946	0.010355
3	10080	106.1114	98.47803	114.3365	0.00962	0.008941	0.010351
3	10154	106.8231	99.15773	115.081	0.009614	0.008933	0.010347
3	10199	107.2557	99.57083	115.5336	0.009611	0.008929	0.010344
3	10268	107.9186	100.2039	116.2273	0.009605	0.008922	0.01034
3	10280	108.0338	100.3139	116.3479	0.009604	0.008921	0.010339
3	10330	108.5139	100.7723	116.8503	0.0096	0.008916	0.010336
3	10390	109.0898	101.3221	117.4529	0.009595	0.008911	0.010332
3	10420	109.3776	101.5969	117.7542	0.009593	0.008908	0.01033
3	10480	109.953	102.1463	118.3565	0.009588	0.008902	0.010327
3	10500	110.1448	102.3293	118.5572	0.009586	0.0089	0.010326
3	10591	111.0168	103.1617	119.4701	0.009579	0.008892	0.01032
3	10641	111.4957	103.6187	119.9715	0.009575	0.008887	0.010317
3	10720	112.2519	104.3403	120.7634	0.009569	0.008879	0.010313
3	10889	113.868	105.8823	122.4561	0.009556	0.008864	0.010303
3	10949	114.4413	106.4291	123.0567	0.009552	0.008858	0.0103
3	11000	114.9284	106.8937	123.567	0.009548	0.008853	0.010297
3	11180	116.6458	108.5315	125.3668	0.009535	0.008837	0.010288
3	11250	117.3131	109.1676	126.0663	0.00953	0.008831	0.010284
3	11320	117.98	109.8034	126.7654	0.009525	0.008824	0.01028
3	11482	119.522	111.273	128.3825	0.009513	0.00881	0.010272
3	11500	119.6932	111.4362	128.5621	0.009512	0.008808	0.010271
3	11555	120.2163	111.9345	129.1107	0.009508	0.008804	0.010268
3	11631	120.9386	112.6227	129.8686	0.009502	0.008797	0.010264
3	11689	121.4897	113.1476	130.4468	0.009498	0.008792	0.010262
3	11730	121.879	113.5184	130.8554	0.009495	0.008788	0.01026
3	11788	122.4296	114.0428	131.4333	0.009491	0.008783	0.010257
3	11830	122.8282	114.4224	131.8516	0.009488	0.00878	0.010255
3	11900	123.4923	115.0546	132.5487	0.009484	0.008774	0.010251
3	12030	124.7246	116.2276	133.8426	0.009475	0.008762	0.010245
3	12099	125.3782	116.8497	134.5291	0.00947	0.008757	0.010242
3	12124	125.6149	117.0749	134.7778	0.009468	0.008754	0.010241
3	12200	126.3343	117.7594	135.5335	0.009463	0.008748	0.010237
3	12280	127.0912	118.4794	136.3288	0.009458	0.008741	0.010234
3	12356	127.8098	119.163	137.084	0.009453	0.008735	0.01023
3	12421	128.4241	119.7471	137.7297	0.009449	0.00873	0.010227
3	12500	129.1703	120.4567	138.5143	0.009444	0.008723	0.010224

3	12580	129.9256	121.1747	139.3085	0.009438	0.008716	0.01022
3	12600	130.1144	121.3541	139.507	0.009437	0.008715	0.010219
3	12699	131.0483	122.2417	140.4894	0.009431	0.008707	0.010215
3	12758	131.6046	122.7703	141.0747	0.009427	0.008702	0.010212
3	12810	132.0947	123.2359	141.5904	0.009424	0.008698	0.01021
3	12889	132.839	123.9429	142.3737	0.009419	0.008691	0.010207
3	12923	133.1592	124.247	142.7107	0.009417	0.008689	0.010205
3	12990	133.79	124.846	143.3747	0.009412	0.008683	0.010203
3	13061	134.4581	125.4803	144.0782	0.009408	0.008678	0.0102
3	13110	134.919	125.9179	144.5636	0.009405	0.008674	0.010198
3	13186	135.6336	126.5961	145.3162	0.0094	0.008668	0.010195
3	13239	136.1317	127.0689	145.841	0.009397	0.008664	0.010193
3	13290	136.6109	127.5235	146.3458	0.009394	0.00866	0.01019
3	13360	137.2683	128.1472	147.0386	0.00939	0.008654	0.010188
3	13483	138.4228	129.2422	148.2555	0.009382	0.008645	0.010183
3	13548	139.0325	129.8203	148.8984	0.009378	0.00864	0.01018
3	13629	139.7919	130.5403	149.6993	0.009374	0.008633	0.010177
3	13680	140.2699	130.9933	150.2034	0.00937	0.008629	0.010175
3	13742	140.8508	131.5438	150.8162	0.009367	0.008625	0.010173
3	13800	141.394	132.0585	151.3893	0.009363	0.00862	0.010171
3	13869	142.0399	132.6705	152.071	0.009359	0.008615	0.010168
3	13920	142.5172	133.1225	152.5748	0.009356	0.008611	0.010166
3	13990	143.172	133.7427	153.2661	0.009352	0.008606	0.010164
3	14066	143.8826	134.4155	154.0164	0.009348	0.0086	0.010161
4	300	4.274527	3.330237	5.486571	0.013021	0.010786	0.01572
4	456	6.267138	4.999501	7.856187	0.01256	0.010644	0.014821
4	520	7.066347	5.678771	8.79297	0.012419	0.010598	0.014552
4	700	9.27194	7.575904	11.34767	0.012105	0.010494	0.013963
4	760	9.995625	8.204556	12.17769	0.012019	0.010464	0.013806
4	820	10.7144	8.831581	12.99862	0.011941	0.010437	0.013662
4	880	11.42866	9.457089	13.81125	0.011869	0.010411	0.01353
4	956	12.32741	10.24737	14.82966	0.011784	0.01038	0.013378
4	990	12.72746	10.60022	15.2816	0.011749	0.010367	0.013315
4	1050	13.43059	11.2219	16.07399	0.011689	0.010345	0.013209
4	1165	14.76876	12.41011	17.57568	0.011585	0.010305	0.013025
4	1280	16.09558	13.59424	19.05716	0.011492	0.010268	0.012861
4	1345	16.84093	14.26186	19.88639	0.011443	0.010248	0.012777
4	1420	17.69711	15.03075	20.83646	0.011389	0.010226	0.012685
4	1500	18.60609	15.8493	21.84239	0.011336	0.010203	0.012594
4	1612	19.87171	16.9926	23.23862	0.011266	0.010173	0.012475
4	1730	21.19696	18.19398	24.6956	0.011197	0.010143	0.012361
4	1800	21.97943	18.9052	25.55356	0.011159	0.010126	0.012298
4	1930	23.42571	20.22327	27.13526	0.011092	0.010095	0.012188
4	2060	24.86361	21.5379	28.70285	0.01103	0.010065	0.012088
4	2100	25.30445	21.94174	29.18251	0.011012	0.010056	0.012058
4	2090	25.19431	21.84081	29.06271	0.011016	0.010059	0.012065
4	2156	25.92042	22.50661	29.85203	0.010987	0.010044	0.012018
4	2260	27.06073	23.5541	31.08939	0.010942	0.010022	0.011948
4	2320	27.71653	24.15754	31.79985	0.010918	0.010009	0.011909
4	2459	29.23027	25.55306	33.43664	0.010863	0.009981	0.011824

4	2534	30.04395	26.30468	34.31477	0.010835	0.009966	0.01178
4	2677	31.58968	27.73519	35.97986	0.010784	0.009938	0.011702
4	2713	31.97769	28.09479	36.39722	0.010772	0.009931	0.011684
4	2800	32.91354	28.963	37.40292	0.010742	0.009914	0.01164
4	2880	33.77188	29.76032	38.32418	0.010716	0.009899	0.011601
4	2945	34.46777	30.40743	39.0703	0.010696	0.009887	0.01157
4	3020	35.26908	31.1533	39.92862	0.010673	0.009873	0.011536
4	3160	36.76032	32.54338	41.52368	0.010631	0.009848	0.011476
4	3200	37.18533	32.94003	41.97776	0.01062	0.009841	0.011459
4	3279	38.02339	33.72274	42.87249	0.010597	0.009827	0.011428
4	3312	38.37294	34.04944	43.24544	0.010588	0.009821	0.011415
4	3190	37.07912	32.84089	41.8643	0.010622	0.009843	0.011464
4	3265	37.875	33.5841	42.71413	0.010601	0.00983	0.011433
4	3308	38.33059	34.00985	43.20026	0.010589	0.009822	0.011416
4	3400	39.30364	34.91989	44.23772	0.010564	0.009806	0.011381
4	3488	40.23227	35.78928	45.22683	0.010541	0.009791	0.011348
4	3565	41.04316	36.54914	46.08976	0.010521	0.009778	0.011321
4	3615	41.5689	37.04213	46.64888	0.010509	0.00977	0.011303
4	3682	42.27242	37.70222	47.39662	0.010492	0.009758	0.011281
4	3724	42.71287	38.1157	47.8645	0.010482	0.009751	0.011267
4	3795	43.45646	38.81417	48.65399	0.010465	0.00974	0.011244
4	3843	43.9585	39.28601	49.18671	0.010453	0.009732	0.011228
4	3899	44.54352	39.83611	49.8072	0.01044	0.009723	0.011211
4	3960	45.17996	40.43488	50.48187	0.010426	0.009713	0.011192
4	4048	46.09661	41.29786	51.45297	0.010407	0.009699	0.011166
4	4122	46.8661	42.02279	52.26762	0.01039	0.009687	0.011145
4	4177	47.43725	42.56116	52.87197	0.010379	0.009678	0.01113
4	4217	47.85222	42.95247	53.31091	0.01037	0.009672	0.011119
4	4290	48.60868	43.66611	54.1107	0.010355	0.00966	0.011099
4	4366	49.39504	44.40839	54.94166	0.010339	0.009649	0.011079
4	4409	49.83944	44.82805	55.41105	0.01033	0.009642	0.011068
4	4498	50.75805	45.69597	56.38089	0.010313	0.009628	0.011046
4	4559	51.38675	46.2903	57.04432	0.010301	0.009619	0.011031
4	4590	51.70598	46.59216	57.38108	0.010295	0.009614	0.011023
4	4644	52.26161	47.11773	57.96706	0.010284	0.009606	0.011011
4	4680	52.63173	47.46791	58.35728	0.010277	0.0096	0.011002
4	4730	53.14536	47.95404	58.89868	0.010268	0.009593	0.010991
4	4799	53.85342	48.62442	59.64473	0.010255	0.009583	0.010975
4	4824	54.10974	48.86717	59.91474	0.010251	0.009579	0.01097
4	4860	54.47865	49.21662	60.30327	0.010244	0.009574	0.010962
4	4890	54.78589	49.50771	60.62679	0.010239	0.009569	0.010955
4	4930	55.19529	49.89567	61.05781	0.010231	0.009563	0.010947
4	4979	55.69642	50.37068	61.58526	0.010223	0.009556	0.010936
4	5060	56.52389	51.1553	62.45589	0.010209	0.009544	0.010919
4	5090	56.83007	51.44571	62.77795	0.010203	0.00954	0.010913
4	5134	57.27885	51.87147	63.24992	0.010196	0.009533	0.010904
4	5170	57.64579	52.21966	63.63574	0.01019	0.009528	0.010897
4	5245	58.40954	52.9446	64.43858	0.010177	0.009517	0.010883
4	5300	58.96903	53.47582	65.02651	0.010168	0.009509	0.010872
4	5378	59.76162	54.22863	65.85916	0.010155	0.009498	0.010857

4	5434	60.33006	54.76869	66.45614	0.010146	0.00949	0.010847
4	5500	60.99935	55.40476	67.15886	0.010136	0.009481	0.010835
4	5567	61.67808	56.04999	67.87129	0.010125	0.009472	0.010823
4	5620	62.21448	56.56005	68.4342	0.010117	0.009464	0.010814
4	5700	63.02332	57.32938	69.28278	0.010104	0.009453	0.010801
4	5788	63.91192	58.17486	70.21476	0.010091	0.009441	0.010786
4	5831	64.3457	58.58769	70.6696	0.010085	0.009435	0.010779
4	5890	64.94043	59.15382	71.29311	0.010076	0.009427	0.01077
4	5900	65.04119	59.24974	71.39873	0.010074	0.009426	0.010768
4	5966	65.70578	59.88253	72.09531	0.010065	0.009417	0.010758
4	6030	66.34963	60.49571	72.77001	0.010056	0.009408	0.010748
4	6099	67.04312	61.15631	73.49658	0.010046	0.009399	0.010737
4	6145	67.50507	61.59644	73.98049	0.010039	0.009393	0.01073
4	6200	68.05701	62.1224	74.55857	0.010031	0.009385	0.010722
4	6278	68.83905	62.86776	75.37751	0.010021	0.009375	0.010711
4	6329	69.34993	63.35477	75.91239	0.010014	0.009368	0.010704
4	6390	69.96051	63.93692	76.55159	0.010005	0.00936	0.010695
4	6457	70.63057	64.5759	77.25293	0.009996	0.009351	0.010686
4	6480	70.86045	64.79514	77.49352	0.009993	0.009348	0.010683
4	6537	71.42986	65.33826	78.0894	0.009986	0.009341	0.010675
4	6579	71.84915	65.73823	78.52813	0.00998	0.009336	0.01067
4	6722	73.27502	67.09872	80.01983	0.009962	0.009317	0.010651
4	6790	73.95213	67.74494	80.72806	0.009953	0.009309	0.010643
4	6845	74.49937	68.26728	81.30038	0.009946	0.009302	0.010636
4	6882	74.8673	68.6185	81.68514	0.009942	0.009297	0.010631
4	6921	75.25493	68.98856	82.09049	0.009937	0.009292	0.010627
4	6990	75.94028	69.64291	82.80709	0.009928	0.009283	0.010618
4	6734	73.39455	67.21279	80.14486	0.00996	0.009316	0.01065
4	6780	73.85259	67.64994	80.62395	0.009955	0.00931	0.010644
4	6813	74.18102	67.96341	80.96745	0.00995	0.009306	0.01064
4	6877	74.81759	68.57105	81.63316	0.009942	0.009298	0.010632
4	6905	75.09592	68.83676	81.92422	0.009939	0.009294	0.010628
4	6955	75.59271	69.31105	82.44368	0.009933	0.009288	0.010622
4	6990	75.94028	69.64291	82.80709	0.009928	0.009283	0.010618
4	7024	76.27778	69.96517	83.15994	0.009924	0.009279	0.010614
4	7088	76.91268	70.57146	83.82369	0.009916	0.009271	0.010607
4	7148	77.50745	71.13949	84.44544	0.009909	0.009264	0.0106
4	7198	78.00277	71.61257	84.96318	0.009903	0.009258	0.010594
4	7250	78.51758	72.10432	85.50127	0.009897	0.009251	0.010588
4	7320	79.2101	72.76587	86.22504	0.009889	0.009243	0.010581
4	7390	79.90205	73.42694	86.94816	0.009881	0.009234	0.010573
4	7450	80.4947	73.99319	87.56747	0.009874	0.009227	0.010567
4	7500	80.98826	74.46479	88.08321	0.009868	0.009221	0.010561
4	7560	81.58016	75.03039	88.70168	0.009862	0.009214	0.010555
4	7600	81.97453	75.40727	89.11374	0.009857	0.009209	0.010551
4	7660	82.56575	75.97229	89.73145	0.00985	0.009202	0.010545
4	7700	82.95968	76.34877	90.14302	0.009846	0.009197	0.010541
4	7843	84.36654	77.69345	91.61279	0.00983	0.00918	0.010527
4	7890	84.82845	78.13497	92.09534	0.009825	0.009175	0.010522
4	7934	85.26067	78.54812	92.54685	0.009821	0.00917	0.010518

4	7988	85.79083	79.05492	93.10067	0.009815	0.009164	0.010513
4	8045	86.35011	79.58957	93.6849	0.009809	0.009157	0.010507
4	8124	87.12468	80.33005	94.49403	0.009801	0.009148	0.0105
4	8200	87.86923	81.04186	95.27178	0.009793	0.009139	0.010493
4	8280	88.65233	81.79054	96.08979	0.009785	0.00913	0.010486
4	8300	88.848	81.97761	96.29419	0.009783	0.009128	0.010484
4	8365	89.48366	82.58534	96.95818	0.009776	0.009121	0.010478
4	8432	90.13842	83.21135	97.64215	0.009769	0.009113	0.010473
4	8478	90.58771	83.64091	98.11147	0.009765	0.009108	0.010469
4	8544	91.23196	84.25687	98.78448	0.009758	0.009101	0.010463
4	8590	91.68074	84.68594	99.25328	0.009754	0.009096	0.010459
4	8643	92.19754	85.18005	99.79316	0.009749	0.00909	0.010455
4	8689	92.64587	85.60869	100.2615	0.009744	0.009085	0.010451
4	8740	93.1427	86.08369	100.7805	0.009739	0.009079	0.010447
4	8788	93.61006	86.53053	101.2688	0.009735	0.009074	0.010443
4	8834	94.05775	86.95855	101.7365	0.00973	0.009069	0.01044
4	8878	94.48579	87.36777	102.1837	0.009726	0.009064	0.010436
4	8950	95.18582	88.03702	102.9151	0.009719	0.009057	0.01043
4	8990	95.57452	88.40862	103.3212	0.009716	0.009052	0.010427
4	9032	95.98249	88.79863	103.7475	0.009712	0.009048	0.010424
4	9080	96.44854	89.24416	104.2345	0.009707	0.009043	0.01042
4	9140	97.0308	89.80078	104.8429	0.009702	0.009036	0.010416
4	9200	97.61274	90.35706	105.4511	0.009696	0.00903	0.010412
4	9280	98.38815	91.09826	106.2614	0.009689	0.009022	0.010406
4	9354	99.10489	91.78334	107.0105	0.009682	0.009014	0.0104
4	9390	99.4534	92.11644	107.3747	0.009679	0.00901	0.010398
4	9465	100.1791	92.81002	108.1333	0.009673	0.009003	0.010392
4	9522	100.7303	93.33679	108.7094	0.009668	0.008997	0.010388
4	9589	101.3778	93.9556	109.3863	0.009662	0.00899	0.010384
4	9612	101.6	94.16794	109.6186	0.00966	0.008988	0.010382
4	9680	102.2567	94.79542	110.3052	0.009654	0.008981	0.010378
4	9740	102.8357	95.34874	110.9106	0.009649	0.008975	0.010374
4	9829	103.6941	96.16889	111.8082	0.009641	0.008966	0.010368
4	9879	104.1761	96.62933	112.3122	0.009637	0.008961	0.010364
4	9900	104.3784	96.82265	112.5239	0.009635	0.008959	0.010363
4	9960	104.9564	97.37477	113.1284	0.00963	0.008953	0.010359
4	10030	105.6303	98.0185	113.8332	0.009624	0.008946	0.010355
4	10140	106.6885	99.02918	114.9402	0.009615	0.008935	0.010348
4	10220	107.4575	99.76354	115.7447	0.009609	0.008927	0.010343
4	10280	108.0338	100.3139	116.3479	0.009604	0.008921	0.010339
4	10330	108.5139	100.7723	116.8503	0.0096	0.008916	0.010336
4	10390	109.0898	101.3221	117.4529	0.009595	0.008911	0.010332
4	10420	109.3776	101.5969	117.7542	0.009593	0.008908	0.01033
4	10480	109.953	102.1463	118.3565	0.009588	0.008902	0.010327
4	10535	110.4803	102.6495	118.9084	0.009584	0.008897	0.010324
4	10599	111.0935	103.2348	119.5504	0.009579	0.008891	0.01032
4	10645	111.534	103.6552	120.0116	0.009575	0.008886	0.010317
4	10680	111.8691	103.975	120.3625	0.009572	0.008883	0.010315
4	10700	112.0605	104.1577	120.563	0.009571	0.008881	0.010314
4	10789	112.912	104.9702	121.4547	0.009564	0.008873	0.010309

4	10821	113.218	105.2622	121.7752	0.009562	0.00887	0.010307
4	10889	113.868	105.8823	122.4561	0.009556	0.008864	0.010303
4	10912	114.0878	106.0919	122.6863	0.009555	0.008862	0.010302
4	10987	114.8042	106.7753	123.4369	0.009549	0.008855	0.010298
4	11000	114.9284	106.8937	123.567	0.009548	0.008853	0.010297
4	11067	115.5679	107.5036	124.2371	0.009543	0.008847	0.010294
4	11134	116.2071	108.1132	124.907	0.009538	0.008841	0.01029
4	11190	116.7412	108.6224	125.4667	0.009534	0.008836	0.010287
4	11256	117.3702	109.2222	126.1262	0.009529	0.00883	0.010284
4	11289	117.6847	109.5219	126.4558	0.009527	0.008827	0.010282
4	11379	118.5418	110.3389	127.3545	0.00952	0.008819	0.010277
4	11400	118.7417	110.5294	127.5642	0.009519	0.008817	0.010276
4	11483	119.5315	111.2821	128.3925	0.009513	0.00881	0.010272
4	11500	119.6932	111.4362	128.5621	0.009512	0.008808	0.010271
4	11555	120.2163	111.9345	129.1107	0.009508	0.008804	0.010268
4	11631	120.9386	112.6227	129.8686	0.009502	0.008797	0.010264
4	11689	121.4897	113.1476	130.4468	0.009498	0.008792	0.010262
5	1628	20.05188	17.15569	23.437	0.011256	0.010169	0.012459
5	1690	20.74862	17.78709	24.20325	0.01122	0.010153	0.012399
5	1750	21.4208	18.3973	24.9412	0.011186	0.010138	0.012343
5	1791	21.87897	18.81382	25.4435	0.011164	0.010128	0.012306
5	1843	22.45878	19.34157	26.07837	0.011136	0.010115	0.01226
5	1880	22.87047	19.71673	26.52865	0.011117	0.010107	0.012229
5	1950	23.64745	20.42574	27.37732	0.011082	0.01009	0.012172
5	2023	24.45518	21.16408	28.25806	0.011047	0.010074	0.012115
5	2099	25.29344	21.93165	29.17054	0.011012	0.010057	0.012059
5	2156	25.92042	22.50661	29.85203	0.010987	0.010044	0.012018
5	2214	26.55693	23.09103	30.54304	0.010962	0.010032	0.011978
5	2287	27.35602	23.82573	31.4094	0.010931	0.010016	0.01193
5	2330	27.82569	24.25805	31.91802	0.010914	0.010007	0.011903
5	2372	28.28371	24.68	32.41363	0.010897	0.009998	0.011876
5	2400	28.58867	24.96113	32.7434	0.010886	0.009993	0.011859
5	2471	29.3606	25.67339	33.57737	0.010859	0.009978	0.011817
5	2548	30.19561	26.44488	34.47831	0.01083	0.009963	0.011773
5	2599	30.74746	26.95532	35.07309	0.010812	0.009953	0.011744
5	2653	31.33077	27.49533	35.70122	0.010792	0.009942	0.011715
5	2696	31.79452	27.92501	36.20022	0.010777	0.009934	0.011693
5	2778	32.67713	28.74357	37.14899	0.01075	0.009918	0.011651
5	2824	33.17126	29.2023	37.67965	0.010734	0.00991	0.011628
5	2880	33.77188	29.76032	38.32418	0.010716	0.009899	0.011601
5	2936	34.3715	30.31787	38.96712	0.010699	0.009889	0.011575
5	2990	34.94877	30.85505	39.58562	0.010682	0.009879	0.01155
5	3035	35.42914	31.30237	40.09996	0.010668	0.009871	0.01153
5	3077	35.87693	31.71961	40.57914	0.010655	0.009863	0.011511
5	3124	36.37741	32.1862	41.11438	0.010642	0.009855	0.011491
5	3160	36.76032	32.54338	41.52368	0.010631	0.009848	0.011476
5	3200	37.18533	32.94003	41.97776	0.01062	0.009841	0.011459
5	3255	37.76897	33.48505	42.60096	0.010604	0.009831	0.011437
5	3282	38.05518	33.75245	42.90641	0.010596	0.009827	0.011426
5	3358	38.85971	34.50458	43.76454	0.010576	0.009813	0.011397

5	3417	39.4832	35.08792	44.42904	0.01056	0.009803	0.011374
5	3460	39.93702	35.51277	44.91245	0.010548	0.009796	0.011359
5	3492	40.27443	35.82877	45.27171	0.01054	0.00979	0.011347
5	3530	40.67476	36.20384	45.69781	0.01053	0.009784	0.011333
5	3578	41.17991	36.67735	46.23523	0.010518	0.009776	0.011316
5	3408	39.38815	34.99897	44.32777	0.010562	0.009805	0.011378
5	3470	40.04249	35.61153	45.02476	0.010546	0.009794	0.011355
5	3502	40.37982	35.92749	45.3839	0.010537	0.009789	0.011343
5	3568	41.07472	36.57872	46.12334	0.01052	0.009778	0.01132
5	3580	41.20095	36.69707	46.2576	0.010517	0.009776	0.011316
5	3634	41.76852	37.22938	46.8611	0.010504	0.009766	0.011297
5	3670	42.1465	37.58403	47.26282	0.010495	0.00976	0.011285
5	3742	42.9015	38.29284	48.06483	0.010477	0.009748	0.011261
5	3790	43.40414	38.76501	48.59845	0.010466	0.009741	0.011245
5	3810	43.61341	38.96165	48.82055	0.010461	0.009737	0.011239
5	3866	44.19886	39.51199	49.44168	0.010448	0.009728	0.011221
5	3900	44.55396	39.84593	49.81827	0.01044	0.009723	0.011211
5	3948	45.05482	40.31713	50.34925	0.010429	0.009715	0.011196
5	3999	45.58641	40.81746	50.91256	0.010418	0.009707	0.011181
5	4051	46.12783	41.32726	51.48603	0.010406	0.009698	0.011166
5	4078	46.40871	41.59183	51.78344	0.0104	0.009694	0.011158
5	4124	46.88688	42.04238	52.28961	0.01039	0.009687	0.011145
5	4180	47.46838	42.59052	52.90491	0.010378	0.009678	0.011129
5	4231	47.99738	43.08938	53.46442	0.010367	0.00967	0.011115
5	4280	48.50512	43.56839	54.00124	0.010357	0.009662	0.011102
5	4303	48.74327	43.79312	54.25296	0.010352	0.009658	0.011096
5	4360	49.33301	44.34981	54.87612	0.01034	0.009649	0.011081
5	4390	49.64312	44.64265	55.20371	0.010334	0.009645	0.011073
5	4468	50.44858	45.40352	56.05423	0.010319	0.009633	0.011053
5	4500	50.77867	45.71546	56.40266	0.010312	0.009628	0.011045
5	4580	51.60303	46.4948	57.27248	0.010297	0.009616	0.011026
5	4640	52.22047	47.07881	57.92369	0.010285	0.009607	0.011012
5	4681	52.642	47.47764	58.36812	0.010277	0.0096	0.011002
5	4790	53.76111	48.53701	59.54749	0.010257	0.009584	0.010977
5	4865	54.52986	49.26514	60.3572	0.010243	0.009573	0.010961
5	4910	54.99062	49.70171	60.84235	0.010235	0.009566	0.010951
5	4990	55.80886	50.47727	61.70359	0.010221	0.009554	0.010934
5	5049	56.41158	51.04879	62.33775	0.01021	0.009546	0.010922
5	5124	57.17688	51.77473	63.14269	0.010198	0.009535	0.010906
5	5188	57.82918	52.3937	63.82854	0.010187	0.009525	0.010894
5	5244	58.39936	52.93494	64.42788	0.010177	0.009517	0.010883
5	5290	58.86734	53.37926	64.91966	0.01017	0.009511	0.010874
5	5369	59.67022	54.1418	65.76315	0.010157	0.0095	0.010859
5	5400	59.985	54.44084	66.09377	0.010152	0.009495	0.010853
5	5472	60.71549	55.13497	66.86085	0.01014	0.009485	0.01084
5	5506	61.06016	55.46256	67.2227	0.010135	0.00948	0.010834
5	5592	61.93115	56.29062	68.13688	0.010121	0.009468	0.010819
5	5672	62.74034	57.06019	68.98593	0.010109	0.009457	0.010805
5	5700	63.02332	57.32938	69.28278	0.010104	0.009453	0.010801
5	5780	63.83119	58.09803	70.1301	0.010092	0.009442	0.010787

5	5853	64.56753	58.79883	70.90218	0.010081	0.009432	0.010775
5	5899	65.03111	59.24015	71.38817	0.010075	0.009426	0.010768
5	5948	65.52459	59.70999	71.90541	0.010067	0.009419	0.01076
5	5970	65.74604	59.92086	72.1375	0.010064	0.009416	0.010757
5	6049	66.54066	60.67766	72.97017	0.010053	0.009405	0.010745
5	6088	66.93261	61.05103	73.38081	0.010047	0.0094	0.010739
5	6140	67.45487	61.54861	73.92791	0.01004	0.009393	0.010731
5	6199	68.04698	62.11284	74.54807	0.010032	0.009385	0.010722
5	6250	68.55842	62.60026	75.08365	0.010025	0.009379	0.010715
5	6303	69.08952	63.10652	75.63976	0.010017	0.009372	0.010707
5	6380	69.86045	63.84151	76.44684	0.010007	0.009361	0.010697
5	6424	70.30062	64.26124	76.90759	0.010001	0.009356	0.010691
5	6479	70.85046	64.78561	77.48307	0.009994	0.009349	0.010683
5	6556	71.61957	65.51922	78.28791	0.009983	0.009339	0.010673
5	6700	73.05582	66.88955	79.79054	0.009965	0.00932	0.010654
5	6740	73.45431	67.26982	80.20737	0.00996	0.009315	0.010649
5	6789	73.94218	67.73544	80.71765	0.009953	0.009309	0.010643
5	6834	74.38995	68.16284	81.18595	0.009948	0.009303	0.010637
5	6890	74.94683	68.69443	81.76831	0.009941	0.009296	0.01063
5	6943	75.47351	69.19725	82.31904	0.009934	0.009289	0.010624
5	6980	75.84099	69.54811	82.70327	0.00993	0.009285	0.010619
5	7024	76.27778	69.96517	83.15994	0.009924	0.009279	0.010614
5	7080	76.83334	70.4957	83.74076	0.009917	0.009272	0.010608
5	7145	77.47773	71.1111	84.41436	0.00991	0.009264	0.0106
5	7190	77.92354	71.5369	84.88036	0.009904	0.009259	0.010595
5	7260	78.61655	72.19886	85.6047	0.009896	0.00925	0.010587
5	7324	79.24966	72.80366	86.26638	0.009889	0.009242	0.01058
5	7380	79.80323	73.33253	86.8449	0.009882	0.009235	0.010574
5	7410	80.09964	73.61573	87.15465	0.009879	0.009232	0.010571
5	7486	80.85009	74.33277	87.93884	0.00987	0.009223	0.010563
5	7520	81.1856	74.65336	88.28942	0.009866	0.009219	0.010559
5	7590	81.87595	75.31306	89.01074	0.009858	0.00921	0.010552
5	7648	82.44754	75.85931	89.60795	0.009852	0.009203	0.010546
5	7683	82.79229	76.18879	89.96813	0.009848	0.009199	0.010542
5	7720	83.15658	76.53696	90.34873	0.009844	0.009195	0.010539
5	7789	83.83554	77.1859	91.05806	0.009836	0.009187	0.010532
5	7814	84.08142	77.42091	91.31492	0.009834	0.009184	0.010529
5	7896	84.88741	78.19132	92.15692	0.009825	0.009174	0.010521
5	7945	85.36869	78.65138	92.65969	0.009819	0.009168	0.010517
5	7999	85.89879	79.15812	93.21345	0.009814	0.009162	0.010512
5	7832	84.2584	77.59008	91.49982	0.009832	0.009182	0.010528
5	7880	84.7302	78.04105	91.99269	0.009826	0.009176	0.010523
5	7930	85.22138	78.51057	92.50581	0.009821	0.00917	0.010518
5	7989	85.80064	79.0643	93.11092	0.009815	0.009163	0.010513
5	8040	86.30106	79.54268	93.63367	0.009809	0.009158	0.010508
5	8079	86.68355	79.90833	94.03322	0.009805	0.009153	0.010504
5	8124	87.12468	80.33005	94.49403	0.009801	0.009148	0.0105
5	8188	87.75171	80.92951	95.14902	0.009794	0.009141	0.010494
5	8278	88.63276	81.77183	96.06935	0.009785	0.00913	0.010486
5	8320	89.04363	82.16465	96.49854	0.009781	0.009126	0.010482

5	8390	89.72803	82.81898	97.21345	0.009773	0.009118	0.010476
5	8459	90.40216	83.46351	97.91765	0.009767	0.00911	0.01047
5	8499	90.79274	83.83694	98.32566	0.009763	0.009106	0.010467
5	8544	91.23196	84.25687	98.78448	0.009758	0.009101	0.010463
5	8580	91.58319	84.59268	99.15138	0.009755	0.009097	0.01046
5	8694	92.69459	85.65527	100.3124	0.009744	0.009084	0.010451
5	8720	92.94789	85.89745	100.577	0.009741	0.009081	0.010449
5	8791	93.63927	86.55845	101.2993	0.009734	0.009074	0.010443
5	8869	94.39825	87.28408	102.0923	0.009727	0.009065	0.010437
5	8900	94.69974	87.57231	102.4073	0.009724	0.009062	0.010434
5	8970	95.38019	88.22284	103.1182	0.009717	0.009054	0.010429
5	9000	95.67167	88.50149	103.4228	0.009715	0.009051	0.010427
5	9048	96.13786	88.94717	103.9099	0.00971	0.009046	0.010423
5	9070	96.35146	89.15136	104.1331	0.009708	0.009044	0.010421
5	9148	97.10841	89.87497	104.924	0.009701	0.009036	0.010415
5	9200	97.61274	90.35706	105.4511	0.009696	0.00903	0.010412
5	9288	98.46566	91.17235	106.3424	0.009688	0.009021	0.010405
5	9324	98.81438	91.50566	106.7069	0.009685	0.009017	0.010402
5	9399	99.54051	92.1997	107.4658	0.009678	0.009009	0.010397
5	9452	100.0533	92.68983	108.0018	0.009674	0.009004	0.010393
5	9500	100.5176	93.13351	108.4871	0.009669	0.008999	0.01039
5	9548	100.9816	93.57698	108.9722	0.009665	0.008994	0.010387
5	9580	101.2908	93.8725	109.2954	0.009662	0.008991	0.010384
5	9642	101.8898	94.44482	109.9216	0.009657	0.008984	0.01038
5	9728	102.7199	95.2381	110.7896	0.00965	0.008976	0.010374
5	9780	103.2216	95.71744	111.3141	0.009645	0.008971	0.010371
5	9800	103.4145	95.90173	111.5158	0.009644	0.008969	0.01037
5	9860	103.993	96.45439	112.1207	0.009639	0.008963	0.010366
5	9920	104.5711	97.00673	112.7254	0.009633	0.008957	0.010362
5	9977	105.1201	97.53114	113.2996	0.009629	0.008951	0.010358
5	10024	105.5726	97.96334	113.7728	0.009625	0.008946	0.010355
5	10080	106.1114	98.47803	114.3365	0.00962	0.008941	0.010351
5	10154	106.8231	99.15773	115.081	0.009614	0.008933	0.010347
5	10199	107.2557	99.57083	115.5336	0.009611	0.008929	0.010344
5	10268	107.9186	100.2039	116.2273	0.009605	0.008922	0.01034
5	10320	108.4179	100.6807	116.7498	0.009601	0.008917	0.010337
5	10389	109.0802	101.313	117.4429	0.009595	0.008911	0.010332
5	10448	109.6462	101.8533	118.0353	0.009591	0.008905	0.010329
5	10500	110.1448	102.3293	118.5572	0.009586	0.0089	0.010326
5	10591	111.0168	103.1617	119.4701	0.009579	0.008892	0.01032
5	10641	111.4957	103.6187	119.9715	0.009575	0.008887	0.010317
5	10720	112.2519	104.3403	120.7634	0.009569	0.008879	0.010313
5	10889	113.868	105.8823	122.4561	0.009556	0.008864	0.010303
5	10949	114.4413	106.4291	123.0567	0.009552	0.008858	0.0103
5	11000	114.9284	106.8937	123.567	0.009548	0.008853	0.010297
5	11180	116.6458	108.5315	125.3668	0.009535	0.008837	0.010288
5	11250	117.3131	109.1676	126.0663	0.00953	0.008831	0.010284
5	11320	117.98	109.8034	126.7654	0.009525	0.008824	0.01028
5	11482	119.522	111.273	128.3825	0.009513	0.00881	0.010272
5	11500	119.6932	111.4362	128.5621	0.009512	0.008808	0.010271

5	11599	120.6345	112.333	129.5495	0.009505	0.0088	0.010266
5	11634	120.9671	112.6499	129.8985	0.009502	0.008797	0.010264
5	11686	121.4612	113.1204	130.4169	0.009499	0.008792	0.010262
5	11730	121.879	113.5184	130.8554	0.009495	0.008788	0.01026
5	11788	122.4296	114.0428	131.4333	0.009491	0.008783	0.010257
5	11830	122.8282	114.4224	131.8516	0.009488	0.00878	0.010255
5	11900	123.4923	115.0546	132.5487	0.009484	0.008774	0.010251
5	12030	124.7246	116.2276	133.8426	0.009475	0.008762	0.010245
5	12099	125.3782	116.8497	134.5291	0.00947	0.008757	0.010242
5	12124	125.6149	117.0749	134.7778	0.009468	0.008754	0.010241
5	12200	126.3343	117.7594	135.5335	0.009463	0.008748	0.010237
5	12280	127.0912	118.4794	136.3288	0.009458	0.008741	0.010234
5	12356	127.8098	119.163	137.084	0.009453	0.008735	0.01023
5	12421	128.4241	119.7471	137.7297	0.009449	0.00873	0.010227
5	12500	129.1703	120.4567	138.5143	0.009444	0.008723	0.010224
5	12580	129.9256	121.1747	139.3085	0.009438	0.008716	0.01022
5	12600	130.1144	121.3541	139.507	0.009437	0.008715	0.010219
5	12699	131.0483	122.2417	140.4894	0.009431	0.008707	0.010215
5	12758	131.6046	122.7703	141.0747	0.009427	0.008702	0.010212
5	12810	132.0947	123.2359	141.5904	0.009424	0.008698	0.01021
5	12889	132.839	123.9429	142.3737	0.009419	0.008691	0.010207
5	12923	133.1592	124.247	142.7107	0.009417	0.008689	0.010205
5	12990	133.79	124.846	143.3747	0.009412	0.008683	0.010203
5	13061	134.4581	125.4803	144.0782	0.009408	0.008678	0.0102
5	13110	134.919	125.9179	144.5636	0.009405	0.008674	0.010198
5	13186	135.6336	126.5961	145.3162	0.0094	0.008668	0.010195
5	13239	136.1317	127.0689	145.841	0.009397	0.008664	0.010193
5	13290	136.6109	127.5235	146.3458	0.009394	0.00866	0.01019
5	13360	137.2683	128.1472	147.0386	0.00939	0.008654	0.010188
5	13483	138.4228	129.2422	148.2555	0.009382	0.008645	0.010183
5	13548	139.0325	129.8203	148.8984	0.009378	0.00864	0.01018
5	13629	139.7919	130.5403	149.6993	0.009374	0.008633	0.010177
5	13680	140.2699	130.9933	150.2034	0.00937	0.008629	0.010175
5	13742	140.8508	131.5438	150.8162	0.009367	0.008625	0.010173
5	13800	141.394	132.0585	151.3893	0.009363	0.00862	0.010171
5	13869	142.0399	132.6705	152.071	0.009359	0.008615	0.010168
5	13920	142.5172	133.1225	152.5748	0.009356	0.008611	0.010166
5	13990	143.172	133.7427	153.2661	0.009352	0.008606	0.010164
5	14066	143.8826	134.4155	154.0164	0.009348	0.0086	0.010161
5	14124	144.4247	134.9287	154.589	0.009345	0.008596	0.010159
5	14191	145.0506	135.5212	155.2502	0.009341	0.008591	0.010156
5	14239	145.4989	135.9454	155.7238	0.009338	0.008588	0.010154
5	14300	146.0685	136.4843	156.3256	0.009335	0.008583	0.010152
5	14376	146.7778	137.1553	157.0753	0.009331	0.008578	0.01015
5	14432	147.3002	137.6494	157.6276	0.009327	0.008574	0.010148
5	14480	147.7478	138.0728	158.1008	0.009325	0.00857	0.010146
5	14554	148.4377	138.725	158.8304	0.009321	0.008565	0.010143
5	14599	148.8571	139.1215	159.2739	0.009318	0.008561	0.010142
5	14676	149.5744	139.7995	160.0328	0.009314	0.008556	0.010139
5	14734	150.1145	140.3099	160.6043	0.009311	0.008552	0.010137

5	14896	151.6222	141.7341	162.2	0.009302	0.00854	0.010132
5	14944	152.0686	142.1557	162.6727	0.009299	0.008537	0.01013
5	14980	152.4033	142.4718	163.0272	0.009298	0.008534	0.010129
5	15036	152.9239	142.9633	163.5785	0.009295	0.00853	0.010127
5	15090	153.4257	143.437	164.1101	0.009292	0.008527	0.010125
5	15128	153.7788	143.7702	164.4841	0.00929	0.008524	0.010124
5	15189	154.3454	144.3049	165.0844	0.009286	0.00852	0.010122
5	15260	155.0046	144.9269	165.783	0.009283	0.008515	0.01012
5	15300	155.3758	145.2771	166.1765	0.009281	0.008512	0.010118
5	15399	156.2944	146.1434	167.1504	0.009275	0.008505	0.010115
5	15425	156.5355	146.3708	167.4061	0.009274	0.008504	0.010114
5	15470	156.9528	146.7643	167.8486	0.009272	0.008501	0.010113
5	15500	157.2309	147.0265	168.1436	0.00927	0.008498	0.010112
5	15545	157.648	147.4196	168.5861	0.009268	0.008495	0.010111
5	15599	158.1484	147.8913	169.1169	0.009265	0.008492	0.010109
5	15648	158.6023	148.319	169.5986	0.009263	0.008488	0.010107
5	15692	159.0098	148.703	170.0311	0.00926	0.008486	0.010106
5	15729	159.3524	149.0258	170.3947	0.009259	0.008483	0.010105
5	15799	160.0004	149.6361	171.0826	0.009255	0.008478	0.010103
5	15876	160.7129	150.3071	171.8391	0.009251	0.008473	0.0101
5	15900	160.9349	150.5161	172.0749	0.00925	0.008472	0.0101
5	15965	161.5361	151.0821	172.7134	0.009247	0.008467	0.010098
5	16000	161.8597	151.3867	173.0572	0.009245	0.008465	0.010097
5	16035	162.1832	151.6912	173.4009	0.009243	0.008463	0.010096
5	16089	162.6823	152.1608	173.9312	0.00924	0.008459	0.010094
5	16154	163.2828	152.7259	174.5694	0.009237	0.008455	0.010092
5	16190	163.6153	153.0387	174.9228	0.009236	0.008453	0.010091
5	16240	164.077	153.473	175.4137	0.009233	0.008449	0.01009
5	16289	164.5294	153.8984	175.8947	0.009231	0.008446	0.010088
5	16342	165.0185	154.3584	176.4149	0.009228	0.008443	0.010087
5	16390	165.4614	154.7748	176.8859	0.009226	0.00844	0.010085
5	16444	165.9595	155.243	177.4158	0.009223	0.008436	0.010084
5	16527	166.7249	155.9623	178.2301	0.009219	0.008431	0.010081
5	16573	167.1489	156.3608	178.6814	0.009217	0.008428	0.01008
5	16620	167.5821	156.7677	179.1424	0.009215	0.008425	0.010079
5	16695	168.273	157.4167	179.878	0.009211	0.00842	0.010076
5	16740	168.6875	157.806	180.3193	0.009209	0.008417	0.010075
5	16801	169.2491	158.3334	180.9175	0.009206	0.008413	0.010073
5	16882	169.9947	159.0332	181.7116	0.009202	0.008408	0.010071
5	16964	170.7491	159.7413	182.5155	0.009198	0.008403	0.010069
5	17020	171.2642	160.2246	183.0644	0.009196	0.0084	0.010067
5	17090	171.9077	160.8283	183.7504	0.009193	0.008395	0.010066
5	17128	172.257	161.156	184.1228	0.009191	0.008393	0.010065
5	17170	172.643	161.518	184.5343	0.009189	0.00839	0.010063
5	17220	173.1024	161.9487	185.0242	0.009187	0.008387	0.010062
5	17294	173.7821	162.586	185.7492	0.009183	0.008383	0.01006
5	17367	174.4523	163.2142	186.4643	0.00918	0.008378	0.010058
5	17410	174.847	163.5841	186.8854	0.009178	0.008376	0.010057
5	17492	175.5994	164.2891	187.6885	0.009174	0.008371	0.010055
5	17578	176.3883	165.028	188.5306	0.00917	0.008366	0.010053

5	17612	176.7	165.3199	188.8635	0.009169	0.008363	0.010052
5	17690	177.4151	165.9895	189.6271	0.009165	0.008359	0.01005
5	18048	180.6934	169.057	193.1307	0.009149	0.008338	0.010041
5	18120	181.352	169.6729	193.8351	0.009146	0.008333	0.010039
5	18196	182.047	170.3226	194.5785	0.009143	0.008329	0.010037
5	18255	182.5864	170.8268	195.1556	0.009141	0.008325	0.010035
5	18300	182.9977	171.2111	195.5957	0.009139	0.008323	0.010034
5	18386	183.7834	171.9452	196.4367	0.009135	0.008318	0.010032
5	18421	184.1031	172.2439	196.7789	0.009133	0.008316	0.010031
5	18500	184.8245	172.9176	197.5514	0.00913	0.008311	0.010029
5	18575	185.5092	173.5569	198.2846	0.009127	0.008307	0.010028
5	18630	186.0111	174.0254	198.8222	0.009125	0.008304	0.010026
5	18684	186.5038	174.4853	199.35	0.009122	0.008301	0.010025
5	18741	187.0237	174.9705	199.9072	0.00912	0.008298	0.010024
5	18828	187.8169	175.7106	200.7574	0.009116	0.008293	0.010022
5	18890	188.382	176.2377	201.3632	0.009114	0.008289	0.01002
5	18934	188.783	176.6117	201.7932	0.009112	0.008287	0.010019
5	18966	189.0746	176.8835	202.1058	0.00911	0.008285	0.010018
5	19006	189.4389	177.2233	202.4966	0.009109	0.008283	0.010017
5	19079	190.1038	177.8431	203.2098	0.009106	0.008279	0.010016
5	19148	190.732	178.4286	203.8838	0.009103	0.008275	0.010014
5	19200	191.2053	178.8696	204.3917	0.009101	0.008272	0.010013
5	19264	191.7877	179.4122	205.0167	0.009098	0.008268	0.010011
5	19322	192.3153	179.9037	205.5832	0.009096	0.008265	0.01001
5	19410	193.1156	180.649	206.4425	0.009092	0.00826	0.010008

NB

- MCF: Mean Cumulative Function
- CLMCF: Confidence Limit for Mean Cumulative Function
- ROCOF: Failure Rate
- CLROCOF: Confidence Limit for Failure Rate

```
System:  Conveyor System

Model: Power-Law Process
Estimation Method: Maximum Likelihood


Parameter Estimates

                     Standard    95% Normal CI
Parameter  Estimate     Error     Lower    Upper
```

```
Shape      0.857656     0.119  0.653541  1.12552
Scale       260.516   171.686   71.5934  947.972


Trend Tests

                MIL-Hdbk-189  Laplace’s  Anderson-Darling
Test Statistic        121.26       0.19              1.25
P-Value                0.237      0.848             0.250
DF                       104
```

Nonparametric Growth Curve: Time

```
System:  Conveyor System

Nonparametric Estimates


Table of Mean Cumulative Function

             Mean             95% Normal
       Cumulative  Standard       CI
 Time    Function     Error  Lower  Upper  System
  153           1         *      *      *       1
  188           2         *      *      *       1
  233           3         *      *      *       1
  530           4         *      *      *       1
  968           5         *      *      *       1
 1092           6         *      *      *       1
 1240           7         *      *      *       1
 1778           8         *      *      *       1
 2219           9         *      *      *       1
 3512          10         *      *      *       1
 3791          11         *      *      *       1
 3899          12         *      *      *       1
 4365          13         *      *      *       1
 4772          14         *      *      *       1
 5122          15         *      *      *       1
 8260          16         *      *      *       1
 9205          17         *      *      *       1
 9810          18         *      *      *       1
10122          19         *      *      *       1
10875          20         *      *      *       1
11022          21         *      *      *       1
11554          22         *      *      *       1
11841          23         *      *      *       1
12200          24         *      *      *       1
12276          25         *      *      *       1
12407          26         *      *      *       1
```

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13100          27          *       *       *       1
13677          28          *       *       *       1
14512          29          *       *       *       1
15300          30          *       *       *       1
16431          31          *       *       *       1
17000          32          *       *       *       1
18211          33          *       *       *       1
18657          34          *       *       *       1
19322          35          *       *       *       1
19900          36          *       *       *       1
20690          37          *       *       *       1
20910          38          *       *       *       1
21362          39          *       *       *       1
21895          40          *       *       *       1
22500          41          *       *       *       1
22780          42          *       *       *       1
22966          43          *       *       *       1
23044          44          *       *       *       1
23178          45          *       *       *       1
23361          46          *       *       *       1
23450          47          *       *       *       1
23475          48          *       *       *       1
24290          49          *       *       *       1
24850          50          *       *       *       1
24926          51          *       *       *       1
25792          52          *       *       *       1
26100          53          *       *       *       1
```

Table A2: Data for Conveyor System

MCF1	CLMCF1	CLMCF2	ROCOF1	CLROCOF1	CLROCOF2
0.6335	0.1855	2.1641	0.0035512	0.0013526	0.0093237
0.7559	0.2319	2.4642	0.0034486	0.0013753	0.0086474
0.9087	0.2926	2.8216	0.0033449	0.0013991	0.0079969
1.8388	0.7124	4.7460	0.0029756	0.0014892	0.0059454
3.0825	1.3649	6.9614	0.0027311	0.0015511	0.0048088
3.4182	1.5541	7.5182	0.0026846	0.0015625	0.0046127
3.8119	1.7818	8.1547	0.0026365	0.0015740	0.0044163
5.1924	2.6235	10.2766	0.0025047	0.0016024	0.0039150
6.2791	3.3254	11.8562	0.0024269	0.0016156	0.0036455
9.3092	5.4205	15.9877	0.0022734	0.0016263	0.0031779
9.9400	5.8772	16.8113	0.0022488	0.0016250	0.0031119
10.1823	6.0542	17.1252	0.0022398	0.0016243	0.0030886
11.2176	6.8198	18.4513	0.0022041	0.0016197	0.0029994
12.1089	7.4900	19.5763	0.0021763	0.0016141	0.0029344
12.8667	8.0670	20.5223	0.0021545	0.0016083	0.0028862
19.3850	13.2323	28.3987	0.0020128	0.0015337	0.0026415
21.2723	14.7738	30.6292	0.0019820	0.0015080	0.0026050
22.4659	15.7551	32.0351	0.0019641	0.0014915	0.0025866
23.0773	16.2593	32.7544	0.0019554	0.0014830	0.0025783

24.5422	17.4704	34.4765	0.0019355	0.0014627	0.0025612
24.8264	17.7058	34.8107	0.0019318	0.0014588	0.0025583
25.8507	18.5549	36.0151	0.0019189	0.0014447	0.0025488
26.4004	19.0111	36.6619	0.0019122	0.0014372	0.0025442
27.0855	19.5797	37.4685	0.0019041	0.0014280	0.0025390
27.2301	19.6998	37.6389	0.0019024	0.0014260	0.0025380
27.4791	19.9066	37.9323	0.0018995	0.0014227	0.0025363
28.7904	20.9953	39.4797	0.0018849	0.0014053	0.0025282
29.8746	21.8950	40.7624	0.0018734	0.0013912	0.0025227
31.4323	23.1858	42.6117	0.0018576	0.0013714	0.0025163
32.8905	24.3914	44.3512	0.0018437	0.0013534	0.0025116
34.9651	26.0998	46.8417	0.0018251	0.0013287	0.0025069
36.0010	26.9493	48.0930	0.0018163	0.0013168	0.0025052
38.1897	28.7349	50.7554	0.0017986	0.0012925	0.0025028
38.9904	29.3848	51.7362	0.0017924	0.0012839	0.0025023
40.1794	30.3460	53.1993	0.0017835	0.0012714	0.0025018
41.2081	31.1739	54.4721	0.0017760	0.0012608	0.0025017
42.6072	32.2941	56.2138	0.0017662	0.0012468	0.0025018
42.9955	32.6037	56.6994	0.0017635	0.0012430	0.0025019
43.7914	33.2368	57.6977	0.0017582	0.0012354	0.0025022
44.7268	33.9778	58.8763	0.0017520	0.0012265	0.0025027
45.7847	34.8119	60.2162	0.0017452	0.0012167	0.0025033
46.2730	35.1954	60.8371	0.0017422	0.0012122	0.0025037
46.5968	35.4493	61.2498	0.0017401	0.0012093	0.0025040
46.7325	35.5555	61.4230	0.0017393	0.0012081	0.0025041
46.9655	35.7378	61.7205	0.0017379	0.0012060	0.0025043
47.2833	35.9861	62.1271	0.0017359	0.0012032	0.0025045
47.4378	36.1067	62.3249	0.0017350	0.0012018	0.0025047
47.4812	36.1405	62.3804	0.0017347	0.0012014	0.0025047
48.8915	37.2365	64.1945	0.0017263	0.0011892	0.0025061
49.8567	37.9820	65.4437	0.0017207	0.0011810	0.0025071
49.9874	38.0827	65.6135	0.0017200	0.0011799	0.0025072
51.4733	39.2225	67.5504	0.0017116	0.0011677	0.0025090
52.0000	39.6244	68.2407	0.0017087	0.0011634	0.0025096

```
System:  Mixer

Model: Power-Law Process
Estimation Method: Maximum Likelihood


Parameter Estimates

                     Standard      95% Normal CI
Parameter  Estimate     Error      Lower     Upper
Shape      0.593573     0.061   0.485958  0.725019
Scale       7.00057     5.624    1.44960   33.8078


Trend Tests
```

```
                MIL-Hdbk-189  Laplace’s  Anderson-Darling
Test Statistic        323.46      -6.08             20.28
P-Value                0.000      0.000             0.000
DF                       192
```

Mean Cumulative Function for Time

Nonparametric Growth Curve: Time

```
System:  Mixer

Nonparametric Estimates


Table of Mean Cumulative Function

             Mean              95% Normal
       Cumulative  Standard        CI
 Time    Function     Error  Lower  Upper  System
   49           1         *      *      *       1
   76           2         *      *      *       1
  100           3         *      *      *       1
  124           4         *      *      *       1
  148           5         *      *      *       1
  153           6         *      *      *       1
  188           7         *      *      *       1
  200           8         *      *      *       1
  233           9         *      *      *       1
  282          10         *      *      *       1
  348          11         *      *      *       1
  420          12         *      *      *       1
  566          13         *      *      *       1
  642          14         *      *      *       1
  744          15         *      *      *       1
  810          16         *      *      *       1
  855          17         *      *      *       1
  890          18         *      *      *       1
  910          19         *      *      *       1
  988          20         *      *      *       1
 1080          21         *      *      *       1
 1154          22         *      *      *       1
 1280          23         *      *      *       1
 1360          24         *      *      *       1
 1436          25         *      *      *       1
 1500          26         *      *      *       1
```

```
1675         27          *       *       *        1
1715         28          *       *       *        1
1856         29          *       *       *        1
1910         30          *       *       *        1
2089         31          *       *       *        1
2245         32          *       *       *        1
2390         33          *       *       *        1
2421         34          *       *       *        1
2600         35          *       *       *        1
2879         36          *       *       *        1
2900         37          *       *       *        1
3122         38          *       *       *        1
3512         39          *       *       *        1
3600         40          *       *       *        1
3654         41          *       *       *        1
3712         42          *       *       *        1
3915         43          *       *       *        1
4365         44          *       *       *        1
4772         45          *       *       *        1
5122         46          *       *       *        1
5345         47          *       *       *        1
5470         48          *       *       *        1
5524         49          *       *       *        1
5568         50          *       *       *        1
5640         51          *       *       *        1
5689         52          *       *       *        1
5760         53          *       *       *        1
6000         54          *       *       *        1
6070         55          *       *       *        1
6136         56          *       *       *        1
6180         57          *       *       *        1
6245         58          *       *       *        1
6290         59          *       *       *        1
6328         60          *       *       *        1
6387         61          *       *       *        1
6420         62          *       *       *        1
6475         63          *       *       *        1
6521         64          *       *       *        1
6582         65          *       *       *        1
6612         66          *       *       *        1
6656         67          *       *       *        1
6699         68          *       *       *        1
6744         69          *       *       *        1
6780         70          *       *       *        1
6830         71          *       *       *        1
6888         72          *       *       *        1
6924         73          *       *       *        1
6970         74          *       *       *        1
6990         75          *       *       *        1
7043         76          *       *       *        1
7085         77          *       *       *        1
```

```
 7134         78          *      *      *       1
 7188         79          *      *      *       1
 7349         80          *      *      *       1
 7440         81          *      *      *       1
 7501         82          *      *      *       1
 8260         83          *      *      *       1
 9205         84          *      *      *       1
 9810         85          *      *      *       1
10122         86          *      *      *       1
10875         87          *      *      *       1
11022         88          *      *      *       1
11554         89          *      *      *       1
11841         90          *      *      *       1
12200         91          *      *      *       1
12276         92          *      *      *       1
12407         93          *      *      *       1
13100         94          *      *      *       1
13677         95          *      *      *       1
14512         96          *      *      *       1
15300         97          *      *      *       1
```

Table A3 Data for Mixer System

MCF1	**CLMCF1**	**CLMCF2**	**ROCOF1**	**CLROCOF1**	**CLROCOF2**
3.1740	1.5593	6.461	0.0384489	0.0228170	0.0647904
4.1186	2.1268	7.976	0.0321671	0.0200220	0.0516794
4.8473	2.5819	9.100	0.0287721	0.0184419	0.0448889
5.5074	3.0052	10.093	0.0263635	0.0172852	0.0402096
6.1173	3.4047	10.991	0.0245342	0.0163842	0.0367383
6.2392	3.4855	11.168	0.0242052	0.0162199	0.0361217
7.0507	4.0301	12.335	0.0222611	0.0152336	0.0325305
7.3144	4.2096	12.709	0.0217083	0.0149480	0.0315258
8.0085	4.6874	13.683	0.0204018	0.0142632	0.0291823
8.9692	5.3609	15.006	0.0188790	0.0134458	0.0265076
10.1617	6.2144	16.616	0.0173324	0.0125916	0.0238582
11.3617	7.0908	18.205	0.0160571	0.0118661	0.0217282
13.5628	8.7382	21.051	0.0142235	0.0107834	0.0187610
14.6160	9.5425	22.387	0.0135135	0.0103494	0.0176449
15.9529	10.5768	24.062	0.0127274	0.0098576	0.0164327
16.7784	11.2222	25.085	0.0122953	0.0095817	0.0157774
17.3256	11.6528	25.760	0.0120281	0.0094089	0.0153762
17.7431	11.9827	26.273	0.0118335	0.0092821	0.0150863
17.9787	12.1694	26.561	0.0117271	0.0092123	0.0149284
18.8781	12.8854	27.658	0.0113416	0.0089571	0.0143610
19.9026	13.7070	28.899	0.0109386	0.0086858	0.0137755
20.7011	14.3516	29.860	0.0106479	0.0084873	0.0133584
22.0144	15.4192	31.431	0.0102087	0.0081824	0.0127369
22.8210	16.0793	32.390	0.0099603	0.0080071	0.0123899
23.5696	16.6946	33.276	0.0097426	0.0078517	0.0120888
24.1876	17.2045	34.005	0.0095714	0.0077283	0.0118540

25.8250	18.5633	35.927	0.0091516	0.0074211	0.0112858
26.1893	18.8671	36.353	0.0090643	0.0073563	0.0111689
27.4468	19.9196	37.818	0.0087778	0.0071416	0.0107890
27.9180	20.3154	38.366	0.0086761	0.0070645	0.0106554
29.4427	21.6012	40.131	0.0083659	0.0068267	0.0102521
30.7286	22.6912	41.613	0.0081246	0.0066388	0.0099429
31.8917	23.6809	42.949	0.0079205	0.0064778	0.0096846
32.1366	23.8898	43.230	0.0078791	0.0064449	0.0096325
33.5264	25.0779	44.821	0.0076540	0.0062646	0.0093515
35.6175	26.8735	47.207	0.0073434	0.0060120	0.0089696
35.7715	27.0061	47.382	0.0073217	0.0059942	0.0089433
37.3725	28.3869	49.203	0.0071055	0.0058154	0.0086818
40.0772	30.7286	52.270	0.0067736	0.0055367	0.0082867
40.6702	31.2434	52.941	0.0067058	0.0054791	0.0082070
41.0312	31.5570	53.350	0.0066653	0.0054447	0.0081596
41.4166	31.8918	53.786	0.0066228	0.0054084	0.0081098
42.7464	33.0485	55.290	0.0064810	0.0052869	0.0079449
45.5982	35.5337	58.513	0.0062007	0.0050440	0.0076226
48.0760	37.6966	61.313	0.0059800	0.0048505	0.0073725
50.1389	39.4985	63.646	0.0058104	0.0047006	0.0071823
51.4234	40.6207	65.099	0.0057107	0.0046119	0.0070712
52.1338	41.2413	65.903	0.0056573	0.0045643	0.0070119
52.4387	41.5076	66.249	0.0056347	0.0045442	0.0069870
52.6863	41.7239	66.529	0.0056166	0.0045280	0.0069669
53.0896	42.0761	66.986	0.0055873	0.0045018	0.0069346
53.3629	42.3148	67.296	0.0055677	0.0044843	0.0069129
53.7572	42.6591	67.743	0.0055397	0.0044592	0.0068821
55.0757	43.8101	69.238	0.0054486	0.0043774	0.0067819
55.4562	44.1422	69.670	0.0054229	0.0043544	0.0067538
55.8133	44.4538	70.076	0.0053992	0.0043330	0.0067277
56.0505	44.6607	70.345	0.0053835	0.0043189	0.0067106
56.3997	44.9653	70.742	0.0053607	0.0042983	0.0066856
56.6406	45.1754	71.016	0.0053450	0.0042842	0.0066686
56.8435	45.3523	71.246	0.0053320	0.0042724	0.0066543
57.1575	45.6260	71.603	0.0053119	0.0042543	0.0066324
57.3326	45.7786	71.803	0.0053008	0.0042443	0.0066203
57.6236	46.0323	72.134	0.0052824	0.0042277	0.0066003
57.8663	46.2438	72.410	0.0052673	0.0042140	0.0065838
58.1869	46.5232	72.775	0.0052474	0.0041960	0.0065621
58.3442	46.6602	72.954	0.0052377	0.0041873	0.0065516
58.5744	46.8606	73.216	0.0052236	0.0041745	0.0065363
58.7987	47.0560	73.472	0.0052099	0.0041622	0.0065214
59.0328	47.2598	73.739	0.0051958	0.0041494	0.0065061
59.2197	47.4225	73.952	0.0051845	0.0041392	0.0064939
59.4785	47.6478	74.247	0.0051691	0.0041252	0.0064771
59.7778	47.9082	74.588	0.0051513	0.0041091	0.0064579
59.9630	48.0694	74.800	0.0051404	0.0040992	0.0064461
60.1992	48.2748	75.069	0.0051266	0.0040867	0.0064312
60.3017	48.3639	75.186	0.0051207	0.0040813	0.0064247
60.5726	48.5995	75.495	0.0051050	0.0040671	0.0064077

60.7868	48.7857	75.740	0.0050926	0.0040559	0.0063944
61.0360	49.0023	76.025	0.0050784	0.0040430	0.0063790
61.3098	49.2402	76.338	0.0050629	0.0040288	0.0063623
62.1212	49.9450	77.266	0.0050175	0.0039876	0.0063133
62.5767	50.3403	77.787	0.0049924	0.0039649	0.0062864
62.8807	50.6041	78.136	0.0049759	0.0039498	0.0062686
66.5833	53.8092	82.390	0.0047847	0.0037756	0.0060636
71.0050	57.6167	87.504	0.0045787	0.0035874	0.0058439
73.7392	59.9585	90.687	0.0044617	0.0034804	0.0057198
75.1224	61.1392	92.304	0.0044053	0.0034287	0.0056601
78.3911	63.9184	96.141	0.0042787	0.0033128	0.0055262
79.0184	64.4498	96.880	0.0042554	0.0032915	0.0055016
81.2605	66.3446	99.530	0.0041747	0.0032176	0.0054163
82.4527	67.3488	100.944	0.0041332	0.0031797	0.0053727
83.9275	68.5879	102.698	0.0040834	0.0031341	0.0053201
84.2374	68.8479	103.067	0.0040731	0.0031247	0.0053093
84.7699	69.2941	103.702	0.0040555	0.0031087	0.0052908
87.5493	71.6158	107.028	0.0039669	0.0030277	0.0051975
89.8181	73.5015	109.757	0.0038980	0.0029649	0.0051249
93.0337	76.1592	113.647	0.0038053	0.0028803	0.0050273
96.0000	78.5952	117.259	0.0037244	0.0028067	0.0049421

Parametric Growth Curve: Time

```
System:  Rollers

Model: Power-Law Process
Estimation Method: Maximum Likelihood


Parameter Estimates

                     Standard    95% Normal CI
Parameter  Estimate     Error     Lower    Upper
Shape       1.26149     0.091  1.09510  1.45316
Scale       181.037    55.428  99.3472  329.899


Trend Tests

                MIL-Hdbk-189  Laplace’s  Anderson-Darling
Test Statistic        304.40       2.17              3.11
P-Value                0.002      0.030             0.024
DF                       384
```

Nonparametric Growth Curve: Time

```
System:  Rollers

Nonparametric Estimates


Table of Mean Cumulative Function

                 Mean             95% Normal
       Cumulative  Standard       CI
 Time     Function      Error  Lower  Upper  System
  300            1          *      *      *       1
  456            2          *      *      *       1
  520            3          *      *      *       1
  700            4          *      *      *       1
  760            5          *      *      *       1
  820            6          *      *      *       1
  880            7          *      *      *       1
  956            8          *      *      *       1
  990            9          *      *      *       1
 1050           10          *      *      *       1
 1165           11          *      *      *       1
 1280           12          *      *      *       1
 1345           13          *      *      *       1
 1420           14          *      *      *       1
 1500           15          *      *      *       1
 1612           16          *      *      *       1
 1730           17          *      *      *       1
 1800           18          *      *      *       1
 1930           19          *      *      *       1
 2060           20          *      *      *       1
 2090           21          *      *      *       1
 2100           22          *      *      *       1
 2156           23          *      *      *       1
 2260           24          *      *      *       1
 2320           25          *      *      *       1
 2459           26          *      *      *       1
 2534           27          *      *      *       1
 2677           28          *      *      *       1
 2713           29          *      *      *       1
 2800           30          *      *      *       1
 2880           31          *      *      *       1
 2945           32          *      *      *       1
 3020           33          *      *      *       1
 3160           34          *      *      *       1
 3190           35          *      *      *       1
 3200           36          *      *      *       1
 3265           37          *      *      *       1
 3279           38          *      *      *       1
 3308           39          *      *      *       1
 3312           40          *      *      *       1
```

3400	41	*	*	*	1
3488	42	*	*	*	1
3565	43	*	*	*	1
3615	44	*	*	*	1
3682	45	*	*	*	1
3724	46	*	*	*	1
3795	47	*	*	*	1
3843	48	*	*	*	1
3899	49	*	*	*	1
3960	50	*	*	*	1
4048	51	*	*	*	1
4122	52	*	*	*	1
4177	53	*	*	*	1
4217	54	*	*	*	1
4290	55	*	*	*	1
4366	56	*	*	*	1
4409	57	*	*	*	1
4498	58	*	*	*	1
4559	59	*	*	*	1
4590	60	*	*	*	1
4644	61	*	*	*	1
4680	62	*	*	*	1
4730	63	*	*	*	1
4799	64	*	*	*	1
4824	65	*	*	*	1
4860	66	*	*	*	1
4890	67	*	*	*	1
4930	68	*	*	*	1
4979	69	*	*	*	1
5060	70	*	*	*	1
5090	71	*	*	*	1
5134	72	*	*	*	1
5170	73	*	*	*	1
5245	74	*	*	*	1
5300	75	*	*	*	1
5378	76	*	*	*	1
5434	77	*	*	*	1
5500	78	*	*	*	1
5567	79	*	*	*	1
5620	80	*	*	*	1
5700	81	*	*	*	1
5788	82	*	*	*	1
5831	83	*	*	*	1
5890	84	*	*	*	1
5900	85	*	*	*	1
5966	86	*	*	*	1
6030	87	*	*	*	1
6099	88	*	*	*	1
6145	89	*	*	*	1
6200	90	*	*	*	1
6278	91	*	*	*	1

6329	92	*	*	*	1
6390	93	*	*	*	1
6457	94	*	*	*	1
6480	95	*	*	*	1
6537	96	*	*	*	1
6579	97	*	*	*	1
6722	98	*	*	*	1
6734	99	*	*	*	1
6780	100	*	*	*	1
6790	101	*	*	*	1
6813	102	*	*	*	1
6845	103	*	*	*	1
6877	104	*	*	*	1
6882	105	*	*	*	1
6905	106	*	*	*	1
6921	107	*	*	*	1
6955	108	*	*	*	1
6990	109	*	*	*	1
6990	110	*	*	*	1
7024	111	*	*	*	1
7088	112	*	*	*	1
7148	113	*	*	*	1
7198	114	*	*	*	1
7250	115	*	*	*	1
7320	116	*	*	*	1
7390	117	*	*	*	1
7450	118	*	*	*	1
7500	119	*	*	*	1
7560	120	*	*	*	1
7600	121	*	*	*	1
7660	122	*	*	*	1
7700	123	*	*	*	1
7843	124	*	*	*	1
7890	125	*	*	*	1
7934	126	*	*	*	1
7988	127	*	*	*	1
8045	128	*	*	*	1
8124	129	*	*	*	1
8200	130	*	*	*	1
8280	131	*	*	*	1
8300	132	*	*	*	1
8365	133	*	*	*	1
8432	134	*	*	*	1
8478	135	*	*	*	1
8544	136	*	*	*	1
8590	137	*	*	*	1
8643	138	*	*	*	1
8689	139	*	*	*	1
8740	140	*	*	*	1
8788	141	*	*	*	1
8834	142	*	*	*	1

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 8878          143          *      *      *       1
 8950          144          *      *      *       1
 8990          145          *      *      *       1
 9032          146          *      *      *       1
 9080          147          *      *      *       1
 9140          148          *      *      *       1
 9200          149          *      *      *       1
 9280          150          *      *      *       1
 9354          151          *      *      *       1
 9390          152          *      *      *       1
 9465          153          *      *      *       1
 9522          154          *      *      *       1
 9589          155          *      *      *       1
 9612          156          *      *      *       1
 9680          157          *      *      *       1
 9740          158          *      *      *       1
 9829          159          *      *      *       1
 9879          160          *      *      *       1
 9900          161          *      *      *       1
 9960          162          *      *      *       1
10030          163          *      *      *       1
10140          164          *      *      *       1
10220          165          *      *      *       1
10280          166          *      *      *       1
10330          167          *      *      *       1
10390          168          *      *      *       1
10420          169          *      *      *       1
10480          170          *      *      *       1
10535          171          *      *      *       1
10599          172          *      *      *       1
10645          173          *      *      *       1
10680          174          *      *      *       1
10700          175          *      *      *       1
10789          176          *      *      *       1
10821          177          *      *      *       1
10889          178          *      *      *       1
10912          179          *      *      *       1
10987          180          *      *      *       1
11000          181          *      *      *       1
11067          182          *      *      *       1
11134          183          *      *      *       1
11190          184          *      *      *       1
11256          185          *      *      *       1
11289          186          *      *      *       1
11379          187          *      *      *       1
11400          188          *      *      *       1
11483          189          *      *      *       1
11500          190          *      *      *       1
11555          191          *      *      *       1
11631          192          *      *      *       1
```

Table A4 Data for Rollers

MCF1	CLMCF1	CLMCF2	ROCOF1	CLROCOF1	CLROCOF2
1.891	0.969	3.691	0.0079519	0.0046746	0.0135269
3.207	1.767	5.819	0.0088720	0.0056025	0.0140494
3.785	2.134	6.714	0.0091820	0.0059287	0.0142205
5.507	3.268	9.280	0.0099241	0.0067351	0.0146231
6.109	3.677	10.151	0.0101399	0.0069759	0.0147388
6.723	4.099	11.027	0.0103433	0.0072055	0.0148476
7.350	4.536	11.910	0.0105361	0.0074252	0.0149504
8.159	5.107	13.037	0.0107668	0.0076906	0.0150735
8.527	5.369	13.544	0.0108657	0.0078052	0.0151262
9.184	5.841	14.442	0.0110341	0.0080015	0.0152161
10.471	6.777	16.179	0.0113381	0.0083591	0.0153787
11.791	7.753	17.932	0.0116207	0.0086950	0.0155308
12.552	8.322	18.931	0.0117722	0.0088763	0.0156129
13.441	8.993	20.089	0.0119404	0.0090785	0.0157045
14.403	9.725	21.331	0.0121128	0.0092865	0.0157991
15.773	10.778	23.082	0.0123430	0.0095657	0.0159267
17.243	11.921	24.940	0.0125731	0.0098458	0.0160560
18.127	12.615	26.049	0.0127042	0.0100057	0.0161306
19.794	13.933	28.121	0.0129380	0.0102912	0.0162655
21.491	15.289	30.209	0.0131604	0.0105631	0.0163964
22.019	15.713	30.855	0.0132268	0.0106441	0.0164360
21.886	15.607	30.693	0.0132103	0.0106240	0.0164262
22.762	16.313	31.761	0.0133181	0.0107556	0.0164910
24.156	17.443	33.451	0.0134832	0.0109570	0.0165918
24.967	18.106	34.430	0.0135759	0.0110698	0.0166492
26.869	19.666	36.709	0.0137840	0.0113225	0.0167807
27.907	20.524	37.946	0.0138927	0.0114539	0.0168508
29.908	22.187	40.316	0.0140936	0.0116956	0.0169832
30.416	22.611	40.915	0.0141429	0.0117547	0.0170163
31.652	23.646	42.368	0.0142601	0.0118946	0.0170960
32.797	24.609	43.708	0.0143655	0.0120198	0.0171691
33.733	25.400	44.801	0.0144496	0.0121192	0.0172283
34.821	26.320	46.066	0.0145450	0.0122312	0.0172964
36.869	28.063	48.439	0.0147183	0.0124333	0.0174233
37.459	28.567	49.119	0.0147668	0.0124894	0.0174595
38.629	29.568	50.467	0.0148613	0.0125982	0.0175310
39.120	29.990	51.031	0.0149003	0.0126428	0.0175609
37.311	28.440	48.949	0.0147547	0.0124755	0.0174504
38.421	29.390	50.227	0.0148447	0.0125791	0.0175183
39.061	29.938	50.962	0.0148956	0.0126374	0.0175572
40.436	31.121	52.539	0.0150028	0.0127594	0.0176405
41.761	32.264	54.053	0.0151034	0.0128729	0.0177203
42.927	33.273	55.383	0.0151898	0.0129696	0.0177901
43.688	33.932	56.248	0.0152453	0.0130312	0.0178356
44.712	34.822	57.411	0.0153186	0.0131121	0.0178965
45.356	35.383	58.141	0.0153641	0.0131620	0.0179348
46.450	36.336	59.379	0.0154402	0.0132448	0.0179995

47.192	36.984	60.217	0.0154910	0.0132997	0.0180434
48.061	37.744	61.198	0.0155498	0.0133628	0.0180946
49.012	38.577	62.268	0.0156130	0.0134302	0.0181506
50.390	39.787	63.817	0.0157030	0.0135252	0.0182314
51.554	40.812	65.124	0.0157776	0.0136031	0.0182997
52.424	41.578	66.098	0.0158323	0.0136597	0.0183505
53.058	42.138	66.808	0.0158718	0.0137004	0.0183875
54.219	43.164	68.106	0.0159432	0.0137732	0.0184552
55.433	44.239	69.461	0.0160166	0.0138472	0.0185259
56.123	44.850	70.229	0.0160577	0.0138883	0.0185659
57.556	46.122	71.824	0.0161418	0.0139717	0.0186491
58.542	46.999	72.921	0.0161988	0.0140275	0.0187062
59.045	47.446	73.479	0.0162275	0.0140554	0.0187353
59.923	48.228	74.453	0.0162772	0.0141034	0.0187861
60.509	48.751	75.103	0.0163101	0.0141350	0.0188199
61.326	49.480	76.008	0.0163555	0.0141783	0.0188671
62.456	50.490	77.259	0.0164176	0.0142369	0.0189322
62.867	50.857	77.714	0.0164399	0.0142578	0.0189559
63.460	51.387	78.369	0.0164719	0.0142877	0.0189900
63.954	51.829	78.915	0.0164984	0.0143123	0.0190184
64.615	52.421	79.645	0.0165336	0.0143448	0.0190564
65.426	53.148	80.540	0.0165764	0.0143840	0.0191029
66.771	54.355	82.024	0.0166465	0.0144476	0.0191801
67.271	54.804	82.575	0.0166723	0.0144708	0.0192087
68.006	55.464	83.384	0.0167098	0.0145043	0.0192507
68.608	56.005	84.047	0.0167404	0.0145315	0.0192851
69.866	57.136	85.431	0.0168035	0.0145870	0.0193569
70.791	57.970	86.448	0.0168494	0.0146270	0.0194096
72.108	59.156	87.895	0.0169139	0.0146825	0.0194844
73.056	60.012	88.937	0.0169598	0.0147216	0.0195383
74.177	61.023	90.167	0.0170134	0.0147669	0.0196018
75.319	62.055	91.419	0.0170674	0.0148119	0.0196664
76.225	62.873	92.412	0.0171097	0.0148469	0.0197175
77.596	64.113	93.914	0.0171731	0.0148987	0.0197947
79.110	65.484	95.573	0.0172420	0.0149542	0.0198798
79.853	66.156	96.385	0.0172754	0.0149809	0.0199214
80.873	67.081	97.502	0.0173210	0.0150169	0.0199785
81.046	67.238	97.691	0.0173286	0.0150230	0.0199882
82.192	68.276	98.944	0.0173791	0.0150625	0.0200521
83.306	69.286	100.162	0.0174277	0.0151001	0.0201141
84.510	70.379	101.479	0.0174796	0.0151398	0.0201810
85.315	71.109	102.358	0.0175140	0.0151659	0.0202256
86.279	71.985	103.412	0.0175548	0.0151967	0.0202789
87.651	73.230	104.911	0.0176123	0.0152395	0.0203546
88.550	74.047	105.893	0.0176496	0.0152670	0.0204040
89.628	75.027	107.070	0.0176939	0.0152995	0.0204632
90.815	76.106	108.367	0.0177423	0.0153345	0.0205282
91.223	76.477	108.813	0.0177588	0.0153463	0.0205505
92.237	77.399	109.919	0.0177995	0.0153754	0.0206057
92.985	78.079	110.736	0.0178293	0.0153966	0.0206464

95.542	80.405	113.529	0.0179299	0.0154670	0.0207849
96.762	81.515	114.862	0.0179771	0.0154995	0.0208507
97.752	82.416	115.943	0.0180151	0.0155254	0.0209039
98.419	83.023	116.671	0.0180405	0.0155427	0.0209397
99.123	83.663	117.441	0.0180672	0.0155607	0.0209774
100.372	84.799	118.804	0.0181141	0.0155921	0.0210440
95.757	80.600	113.764	0.0179382	0.0154728	0.0207965
96.583	81.352	114.666	0.0179702	0.0154948	0.0208411
97.176	81.892	115.314	0.0179930	0.0155104	0.0208730
98.329	82.941	116.573	0.0180371	0.0155404	0.0209349
98.834	83.400	117.125	0.0180562	0.0155533	0.0209619
99.738	84.222	118.112	0.0180903	0.0155762	0.0210102
100.372	84.799	118.804	0.0181141	0.0155921	0.0210440
100.988	85.360	119.478	0.0181371	0.0156074	0.0210768
102.150	86.417	120.747	0.0181802	0.0156358	0.0211386
103.242	87.411	121.941	0.0182203	0.0156620	0.0211964
104.154	88.240	122.938	0.0182535	0.0156836	0.0212445
105.104	89.104	123.976	0.0182879	0.0157058	0.0212945
106.386	90.270	125.378	0.0183339	0.0157352	0.0213618
107.671	91.439	126.784	0.0183796	0.0157641	0.0214290
108.775	92.443	127.992	0.0184185	0.0157885	0.0214865
109.696	93.281	129.001	0.0184507	0.0158086	0.0215344
110.805	94.288	130.214	0.0184892	0.0158325	0.0215918
111.545	94.961	131.025	0.0185148	0.0158482	0.0216300
112.657	95.971	132.243	0.0185529	0.0158714	0.0216873
113.399	96.646	133.057	0.0185781	0.0158868	0.0217255
116.062	99.064	135.977	0.0186678	0.0159405	0.0218615
116.941	99.862	136.941	0.0186969	0.0159579	0.0219062
117.764	100.609	137.844	0.0187242	0.0159739	0.0219479
118.776	101.527	138.955	0.0187574	0.0159934	0.0219991
119.846	102.497	140.131	0.0187923	0.0160137	0.0220530
121.332	103.845	141.765	0.0188404	0.0160415	0.0221276
122.766	105.144	143.341	0.0188863	0.0160678	0.0221992
124.279	106.514	145.006	0.0189343	0.0160950	0.0222745
124.658	106.857	145.424	0.0189463	0.0161018	0.0222933
125.890	107.973	146.781	0.0189849	0.0161235	0.0223543
127.164	109.125	148.185	0.0190246	0.0161456	0.0224170
128.040	109.917	149.151	0.0190517	0.0161606	0.0224600
129.298	111.054	150.539	0.0190903	0.0161818	0.0225216
130.177	111.848	151.509	0.0191172	0.0161965	0.0225645
131.191	112.764	152.630	0.0191479	0.0162133	0.0226138
132.072	113.559	153.604	0.0191745	0.0162277	0.0226566
133.051	114.442	154.686	0.0192039	0.0162435	0.0227039
133.974	115.274	155.706	0.0192314	0.0162582	0.0227484
134.859	116.072	156.686	0.0192577	0.0162722	0.0227909
135.707	116.836	157.625	0.0192827	0.0162855	0.0228316
137.097	118.088	159.165	0.0193235	0.0163070	0.0228980
137.870	118.784	160.022	0.0193461	0.0163188	0.0229348
138.683	119.516	160.924	0.0193696	0.0163312	0.0229734
139.613	120.352	161.957	0.0193965	0.0163451	0.0230175

140.778	121.399	163.251	0.0194299	0.0163624	0.0230725
141.945	122.447	164.547	0.0194632	0.0163795	0.0231274
143.504	123.846	166.281	0.0195073	0.0164021	0.0232005
144.949	125.142	167.890	0.0195479	0.0164227	0.0232678
145.653	125.773	168.675	0.0195675	0.0164326	0.0233006
147.122	127.089	170.313	0.0196083	0.0164530	0.0233686
148.241	128.090	171.561	0.0196391	0.0164684	0.0234202
149.558	129.268	173.032	0.0196751	0.0164863	0.0234808
150.010	129.672	173.538	0.0196875	0.0164924	0.0235015
151.350	130.869	175.036	0.0197238	0.0165103	0.0235628
152.535	131.927	176.362	0.0197557	0.0165259	0.0236167
154.295	133.497	178.333	0.0198027	0.0165488	0.0236965
155.286	134.380	179.444	0.0198290	0.0165615	0.0237412
155.702	134.751	179.912	0.0198400	0.0165668	0.0237600
156.894	135.811	181.249	0.0198714	0.0165819	0.0238135
158.286	137.049	182.813	0.0199078	0.0165993	0.0238758
160.479	138.998	185.280	0.0199647	0.0166262	0.0239735
162.078	140.416	187.081	0.0200057	0.0166456	0.0240442
163.279	141.481	188.435	0.0200364	0.0166599	0.0240972
164.281	142.369	189.566	0.0200618	0.0166718	0.0241412
165.486	143.435	190.926	0.0200922	0.0166859	0.0241940
166.089	143.969	191.608	0.0201074	0.0166929	0.0242203
167.296	145.036	192.973	0.0201376	0.0167068	0.0242729
168.405	146.015	194.227	0.0201652	0.0167195	0.0243210
169.696	147.155	195.690	0.0201971	0.0167341	0.0243769
170.626	147.975	196.744	0.0202200	0.0167445	0.0244169
171.334	148.599	197.547	0.0202374	0.0167524	0.0244474
171.739	148.956	198.006	0.0202473	0.0167569	0.0244648
173.543	150.544	200.055	0.0202912	0.0167767	0.0245420
174.192	151.115	200.793	0.0203069	0.0167837	0.0245697
175.574	152.330	202.365	0.0203402	0.0167986	0.0246284
176.042	152.741	202.898	0.0203514	0.0168036	0.0246483
177.570	154.082	204.638	0.0203879	0.0168199	0.0247129
177.835	154.315	204.940	0.0203942	0.0168227	0.0247241
179.203	155.513	206.500	0.0204266	0.0168370	0.0247816
180.572	156.713	208.064	0.0204589	0.0168512	0.0248390
181.719	157.716	209.374	0.0204858	0.0168629	0.0248869
183.072	158.899	210.922	0.0205173	0.0168767	0.0249432
183.749	159.491	211.697	0.0205330	0.0168835	0.0249713
185.599	161.105	213.816	0.0205757	0.0169020	0.0250478
186.031	161.482	214.312	0.0205856	0.0169063	0.0250656
187.741	162.973	216.275	0.0206247	0.0169231	0.0251359
188.092	163.278	216.677	0.0206327	0.0169265	0.0251502
189.228	164.266	217.982	0.0206584	0.0169376	0.0251967
190.799	165.632	219.790	0.0206939	0.0169527	0.0252607
192.000	166.675	221.173	0.0207208	0.0169641	0.0253094

Parametric Growth Curve: Time

```
System:  Slitter Machine

Model: Power-Law Process
Estimation Method: Maximum Likelihood

Parameter Estimates
                     Standard    95% Normal CI
Parameter  Estimate     Error    Lower    Upper
Shape       1.19341     0.068  1.06711  1.33466
Scale       159.943    44.468  92.7498  275.815
Trend Tests

                MIL-Hdbk-189  Laplace’s  Anderson-Darling
Test Statistic        514.49       0.86              3.14
P-Value                0.003      0.389             0.023
DF                       614
```

Mean Cumulative Function for Time

Nonparametric Growth Curve: Time

```
System:  Slitter Machine

Nonparametric Estimates


Table of Mean Cumulative Function

             Mean             95% Normal
       Cumulative  Standard       CI
 Time    Function     Error  Lower  Upper  System
 1628           1         *      *      *       1
 1690           2         *      *      *       1
 1750           3         *      *      *       1
 1791           4         *      *      *       1
 1843           5         *      *      *       1
 1880           6         *      *      *       1
 1950           7         *      *      *       1
 2023           8         *      *      *       1
 2099           9         *      *      *       1
 2156          10         *      *      *       1
 2214          11         *      *      *       1
 2287          12         *      *      *       1
 2330          13         *      *      *       1
 2372          14         *      *      *       1
 2400          15         *      *      *       1
```

```
2471          16          *       *       *        1
2548          17          *       *       *        1
2599          18          *       *       *        1
2653          19          *       *       *        1
2696          20          *       *       *        1
2778          21          *       *       *        1
2824          22          *       *       *        1
2880          23          *       *       *        1
2936          24          *       *       *        1
2990          25          *       *       *        1
3035          26          *       *       *        1
3077          27          *       *       *        1
3124          28          *       *       *        1
3160          29          *       *       *        1
3200          30          *       *       *        1
3255          31          *       *       *        1
3282          32          *       *       *        1
3358          33          *       *       *        1
3408          34          *       *       *        1
3417          35          *       *       *        1
3460          36          *       *       *        1
3470          37          *       *       *        1
3492          38          *       *       *        1
3502          39          *       *       *        1
3530          40          *       *       *        1
3568          41          *       *       *        1
3578          42          *       *       *        1
3580          43          *       *       *        1
3634          44          *       *       *        1
3670          45          *       *       *        1
3742          46          *       *       *        1
3790          47          *       *       *        1
3810          48          *       *       *        1
3866          49          *       *       *        1
3900          50          *       *       *        1
3948          51          *       *       *        1
3999          52          *       *       *        1
4051          53          *       *       *        1
4078          54          *       *       *        1
4124          55          *       *       *        1
4180          56          *       *       *        1
4231          57          *       *       *        1
4280          58          *       *       *        1
4303          59          *       *       *        1
4360          60          *       *       *        1
4390          61          *       *       *        1
4468          62          *       *       *        1
4500          63          *       *       *        1
4580          64          *       *       *        1
4640          65          *       *       *        1
4681          66          *       *       *        1
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4790          67         *        *        *         1
4865          68         *        *        *         1
4910          69         *        *        *         1
4990          70         *        *        *         1
5049          71         *        *        *         1
5124          72         *        *        *         1
5188          73         *        *        *         1
5244          74         *        *        *         1
5290          75         *        *        *         1
5369          76         *        *        *         1
5400          77         *        *        *         1
5472          78         *        *        *         1
5506          79         *        *        *         1
5592          80         *        *        *         1
5672          81         *        *        *         1
5700          82         *        *        *         1
5780          83         *        *        *         1
5853          84         *        *        *         1
5899          85         *        *        *         1
5948          86         *        *        *         1
5970          87         *        *        *         1
6049          88         *        *        *         1
6088          89         *        *        *         1
6140          90         *        *        *         1
6199          91         *        *        *         1
6250          92         *        *        *         1
6303          93         *        *        *         1
6380          94         *        *        *         1
6424          95         *        *        *         1
6479          96         *        *        *         1
6556          97         *        *        *         1
6700          98         *        *        *         1
6740          99         *        *        *         1
6789         100         *        *        *         1
6834         101         *        *        *         1
6890         102         *        *        *         1
6943         103         *        *        *         1
6980         104         *        *        *         1
7024         105         *        *        *         1
7080         106         *        *        *         1
7145         107         *        *        *         1
7190         108         *        *        *         1
7260         109         *        *        *         1
7324         110         *        *        *         1
7380         111         *        *        *         1
7410         112         *        *        *         1
7486         113         *        *        *         1
7520         114         *        *        *         1
7590         115         *        *        *         1
7648         116         *        *        *         1
7683         117         *        *        *         1
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 7720         118          *      *      *       1
 7789         119          *      *      *       1
 7814         120          *      *      *       1
 7832         121          *      *      *       1
 7880         122          *      *      *       1
 7896         123          *      *      *       1
 7930         124          *      *      *       1
 7945         125          *      *      *       1
 7989         126          *      *      *       1
 7999         127          *      *      *       1
 8040         128          *      *      *       1
 8079         129          *      *      *       1
 8124         130          *      *      *       1
 8188         131          *      *      *       1
 8278         132          *      *      *       1
 8320         133          *      *      *       1
 8390         134          *      *      *       1
 8459         135          *      *      *       1
 8499         136          *      *      *       1
 8544         137          *      *      *       1
 8580         138          *      *      *       1
 8694         139          *      *      *       1
 8720         140          *      *      *       1
 8791         141          *      *      *       1
 8869         142          *      *      *       1
 8900         143          *      *      *       1
 8970         144          *      *      *       1
 9000         145          *      *      *       1
 9048         146          *      *      *       1
 9070         147          *      *      *       1
 9148         148          *      *      *       1
 9200         149          *      *      *       1
 9288         150          *      *      *       1
 9324         151          *      *      *       1
 9399         152          *      *      *       1
 9452         153          *      *      *       1
 9500         154          *      *      *       1
 9548         155          *      *      *       1
 9580         156          *      *      *       1
 9642         157          *      *      *       1
 9728         158          *      *      *       1
 9780         159          *      *      *       1
 9800         160          *      *      *       1
 9860         161          *      *      *       1
 9920         162          *      *      *       1
 9977         163          *      *      *       1
10024         164          *      *      *       1
10080         165          *      *      *       1
10154         166          *      *      *       1
10199         167          *      *      *       1
10268         168          *      *      *       1
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10320       169       *     *     *     1
10389       170       *     *     *     1
10448       171       *     *     *     1
10500       172       *     *     *     1
10591       173       *     *     *     1
10641       174       *     *     *     1
10720       175       *     *     *     1
10889       176       *     *     *     1
10949       177       *     *     *     1
11000       178       *     *     *     1
11180       179       *     *     *     1
11250       180       *     *     *     1
11320       181       *     *     *     1
11482       182       *     *     *     1
11500       183       *     *     *     1
11599       184       *     *     *     1
11634       185       *     *     *     1
11686       186       *     *     *     1
11730       187       *     *     *     1
11788       188       *     *     *     1
11830       189       *     *     *     1
11900       190       *     *     *     1
12030       191       *     *     *     1
12099       192       *     *     *     1
12124       193       *     *     *     1
12200       194       *     *     *     1
12280       195       *     *     *     1
12356       196       *     *     *     1
12421       197       *     *     *     1
12500       198       *     *     *     1
12580       199       *     *     *     1
12600       200       *     *     *     1
12699       201       *     *     *     1
12758       202       *     *     *     1
12810       203       *     *     *     1
12889       204       *     *     *     1
12923       205       *     *     *     1
12990       206       *     *     *     1
13061       207       *     *     *     1
13110       208       *     *     *     1
13186       209       *     *     *     1
13239       210       *     *     *     1
13290       211       *     *     *     1
13360       212       *     *     *     1
13483       213       *     *     *     1
13548       214       *     *     *     1
13629       215       *     *     *     1
13680       216       *     *     *     1
13742       217       *     *     *     1
13800       218       *     *     *     1
13869       219       *     *     *     1
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13920          220          *       *       *       1
13990          221          *       *       *       1
14066          222          *       *       *       1
14124          223          *       *       *       1
14191          224          *       *       *       1
14239          225          *       *       *       1
14300          226          *       *       *       1
14376          227          *       *       *       1
14432          228          *       *       *       1
14480          229          *       *       *       1
14554          230          *       *       *       1
14599          231          *       *       *       1
14676          232          *       *       *       1
14734          233          *       *       *       1
14896          234          *       *       *       1
14944          235          *       *       *       1
14980          236          *       *       *       1
15036          237          *       *       *       1
15090          238          *       *       *       1
15128          239          *       *       *       1
15189          240          *       *       *       1
15260          241          *       *       *       1
15300          242          *       *       *       1
15399          243          *       *       *       1
15425          244          *       *       *       1
15470          245          *       *       *       1
15500          246          *       *       *       1
15545          247          *       *       *       1
15599          248          *       *       *       1
15648          249          *       *       *       1
15692          250          *       *       *       1
15729          251          *       *       *       1
15799          252          *       *       *       1
15876          253          *       *       *       1
15900          254          *       *       *       1
15965          255          *       *       *       1
16000          256          *       *       *       1
16035          257          *       *       *       1
16089          258          *       *       *       1
16154          259          *       *       *       1
16190          260          *       *       *       1
16240          261          *       *       *       1
16289          262          *       *       *       1
16342          263          *       *       *       1
16390          264          *       *       *       1
16444          265          *       *       *       1
16527          266          *       *       *       1
16573          267          *       *       *       1
16620          268          *       *       *       1
16695          269          *       *       *       1
16740          270          *       *       *       1
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16801          271          *      *      *        1
16882          272          *      *      *        1
16964          273          *      *      *        1
17020          274          *      *      *        1
17090          275          *      *      *        1
17128          276          *      *      *        1
17170          277          *      *      *        1
17220          278          *      *      *        1
17294          279          *      *      *        1
17367          280          *      *      *        1
17410          281          *      *      *        1
17492          282          *      *      *        1
17578          283          *      *      *        1
17612          284          *      *      *        1
17690          285          *      *      *        1
18048          286          *      *      *        1
18120          287          *      *      *        1
18196          288          *      *      *        1
18255          289          *      *      *        1
18300          290          *      *      *        1
18386          291          *      *      *        1
18421          292          *      *      *        1
18500          293          *      *      *        1
18575          294          *      *      *        1
18630          295          *      *      *        1
18684          296          *      *      *        1
18741          297          *      *      *        1
18828          298          *      *      *        1
18890          299          *      *      *        1
18934          300          *      *      *        1
18966          301          *      *      *        1
19006          302          *      *      *        1
19079          303          *      *      *        1
19148          304          *      *      *        1
19200          305          *      *      *        1
19264          306          *      *      *        1
19322          307          *      *      *        1
```

Table A5 Data for Slitter

MCF1	**CLMCF1**	**CLMCF2**	**ROCOF1**	**CLROCOF1**	**CLROCOF2**
15.94359	11.24358	22.60828	0.011687	0.009139	0.014946
16.67085	11.81211	23.52817	0.011772	0.009247	0.014988
17.37959	12.36861	24.42071	0.011852	0.009348	0.015027
17.86662	12.75237	25.0319	0.011905	0.009415	0.015054
18.48741	13.24308	25.80853	0.011971	0.009499	0.015087
18.93121	13.59491	26.36211	0.012017	0.009558	0.01511
19.77542	14.26648	27.41162	0.012103	0.009667	0.015152
20.66208	14.97494	28.50907	0.012189	0.009777	0.015196
21.59178	15.72108	29.65477	0.012276	0.009889	0.01524

22.29335	16.28628	30.51608	0.01234	0.009971	0.015272
23.01092	16.86621	31.39428	0.012404	0.010052	0.015305
23.91924	17.60288	32.50207	0.012482	0.010152	0.015345
24.45692	18.04027	33.15588	0.012527	0.01021	0.015368
24.98396	18.46993	33.79537	0.01257	0.010266	0.015391
25.33632	18.75769	34.22219	0.012599	0.010303	0.015406
26.23336	19.49203	35.30618	0.01267	0.010395	0.015443
27.21185	20.29586	36.48452	0.012745	0.010492	0.015483
27.86311	20.83245	37.26652	0.012794	0.010555	0.015509
28.55538	21.40417	38.09583	0.012845	0.01062	0.015536
29.10858	21.86201	38.75717	0.012885	0.010672	0.015557
30.16825	22.74135	40.02063	0.01296	0.010768	0.015598
30.76536	23.23817	40.73072	0.013001	0.010822	0.01562
31.49483	23.84638	41.59643	0.013051	0.010885	0.015647
32.22704	24.45823	42.4635	0.013099	0.010948	0.015674
32.93566	25.05163	43.30088	0.013146	0.011007	0.015699
33.52808	25.54866	43.99964	0.013184	0.011056	0.01572
34.08253	26.0146	44.65259	0.013219	0.011101	0.01574
34.70473	26.53832	45.38413	0.013258	0.011151	0.015762
35.18254	26.9411	45.94509	0.013287	0.011189	0.015778
35.71467	27.39028	46.56899	0.013319	0.011231	0.015797
36.44845	28.01071	47.42792	0.013363	0.011287	0.015822
36.80956	28.31646	47.85002	0.013385	0.011314	0.015834
37.82906	29.1812	49.03973	0.013444	0.01139	0.015868
38.62361	29.85663	49.9649	0.01349	0.011448	0.015895
39.20437	30.35113	50.64004	0.013522	0.01149	0.015914
39.63746	30.72035	51.14293	0.013546	0.011521	0.015928
40.15276	31.16012	51.74063	0.013575	0.011557	0.015945
40.8052	31.71768	52.49642	0.01361	0.011602	0.015966
38.50224	29.75337	49.82369	0.013483	0.01144	0.015891
39.33963	30.4664	50.79715	0.01353	0.0115	0.015918
39.77297	30.83594	51.30017	0.013554	0.01153	0.015932
40.66914	31.60134	52.33889	0.013603	0.011593	0.015961
40.83243	31.74096	52.52793	0.013612	0.011604	0.015967
41.56853	32.37103	53.37928	0.013651	0.011654	0.01599
42.06044	32.79265	53.94746	0.013677	0.011687	0.016006
43.04705	33.63963	55.08529	0.013729	0.011753	0.016037
43.70685	34.20703	55.84491	0.013763	0.011795	0.016058
43.98224	34.44408	56.16168	0.013777	0.011813	0.016066
44.75482	35.10981	57.0494	0.013816	0.011862	0.01609
45.22495	35.51543	57.58894	0.013839	0.011892	0.016105
45.89	36.08986	58.35136	0.013872	0.011933	0.016125
46.59834	36.70249	59.16234	0.013906	0.011976	0.016147
47.32237	37.32955	59.9902	0.013941	0.01202	0.016169
47.69902	37.65608	60.42043	0.013959	0.012042	0.01618
48.34183	38.21389	61.15402	0.013989	0.01208	0.0162
49.12625	38.89546	62.04809	0.014026	0.012126	0.016223
49.84241	39.51854	62.8633	0.014059	0.012167	0.016245
50.53206	40.11929	63.64741	0.01409	0.012206	0.016265
50.8563	40.40198	64.01575	0.014105	0.012224	0.016275

51.66129	41.10448	64.92938	0.014141	0.012269	0.016298
52.08579	41.47532	65.4107	0.014159	0.012292	0.016311
53.19211	42.44298	66.66357	0.014208	0.012351	0.016343
53.64707	42.84142	67.17817	0.014227	0.012375	0.016356
54.7872	43.84115	68.46621	0.014276	0.012435	0.016389
55.64484	44.59433	69.43367	0.014312	0.012479	0.016414
56.23213	45.11065	70.09547	0.014336	0.012509	0.016431
57.79828	46.48971	71.85764	0.0144	0.012586	0.016476
58.87992	47.44395	73.07244	0.014444	0.012639	0.016506
59.53046	48.01855	73.80223	0.014469	0.01267	0.016525
60.68982	49.04383	75.10127	0.014515	0.012724	0.016557
61.54716	49.80304	76.06067	0.014548	0.012763	0.016581
62.63979	50.77184	77.28189	0.014589	0.012813	0.016612
63.57462	51.60178	78.32545	0.014624	0.012854	0.016638
64.39443	52.3304	79.23966	0.014655	0.01289	0.016661
65.06912	52.93058	79.99138	0.014679	0.012919	0.01668
66.23046	53.96481	81.28397	0.014722	0.012968	0.016712
66.68708	54.37184	81.79174	0.014738	0.012988	0.016724
67.74958	55.31976	82.97226	0.014776	0.013031	0.016754
68.25226	55.76863	83.53031	0.014793	0.013052	0.016767
69.52641	56.9075	84.94349	0.014838	0.013103	0.016802
70.71508	57.97138	86.26019	0.014879	0.01315	0.016835
71.13188	58.34474	86.72152	0.014893	0.013166	0.016846
72.32493	59.41433	88.04096	0.014933	0.013212	0.016879
73.41637	60.39396	89.24672	0.014969	0.013253	0.016908
74.10548	61.01303	90.00738	0.014992	0.013278	0.016927
74.84068	61.67395	90.81838	0.015016	0.013305	0.016947
75.17116	61.97119	91.18274	0.015027	0.013317	0.016956
76.35979	63.04104	92.4924	0.015065	0.01336	0.016988
76.94769	63.57063	93.13966	0.015084	0.013381	0.017004
77.73269	64.27821	94.00341	0.015109	0.013408	0.017025
78.62493	65.08306	94.98447	0.015137	0.013439	0.017049
79.39751	65.78047	95.83337	0.015161	0.013465	0.01707
80.20168	66.5069	96.71642	0.015185	0.013492	0.017091
81.37233	67.56526	98.0009	0.015221	0.013531	0.017123
82.0425	68.17161	98.73572	0.015241	0.013553	0.017141
82.88147	68.93114	99.65508	0.015266	0.01358	0.017163
84.05834	69.99743	100.9438	0.015301	0.013617	0.017194
86.26639	72.00062	103.3587	0.015366	0.013685	0.017253
86.88138	72.55914	104.0307	0.015384	0.013704	0.01727
87.6357	73.24453	104.8545	0.015405	0.013726	0.01729
88.32938	73.87515	105.6117	0.015425	0.013747	0.017308
89.19385	74.66147	106.5549	0.015449	0.013772	0.017331
90.01327	75.40723	107.4484	0.015472	0.013795	0.017353
90.58603	75.92876	108.0727	0.015488	0.013812	0.017368
91.26791	76.54991	108.8157	0.015507	0.013831	0.017386
92.13697	77.34196	109.7622	0.015531	0.013855	0.017409
93.14735	78.26337	110.8619	0.015558	0.013883	0.017435
93.8479	78.90257	111.6241	0.015577	0.013902	0.017454
94.93931	79.89896	112.8109	0.015606	0.013931	0.017483

95.93897	80.81215	113.8973	0.015633	0.013958	0.017509
96.81505	81.61289	114.8489	0.015656	0.01398	0.017532
97.28491	82.04252	115.3591	0.015668	0.013993	0.017544
98.47686	83.13291	116.6529	0.015699	0.014023	0.017576
99.01087	83.62164	117.2322	0.015713	0.014036	0.01759
100.1118	84.62965	118.4261	0.015741	0.014064	0.017618
101.0254	85.46667	119.4165	0.015764	0.014086	0.017642
101.5774	85.97256	120.0147	0.015778	0.014099	0.017657
102.1615	86.50799	120.6474	0.015793	0.014114	0.017672
103.2521	87.50825	121.8285	0.01582	0.014139	0.0177
103.6477	87.87121	122.2568	0.01583	0.014149	0.017711
104.9471	89.06381	123.6629	0.015862	0.014179	0.017745
105.7248	89.77795	124.5042	0.015881	0.014197	0.017765
106.5829	90.56624	125.4321	0.015902	0.014216	0.017787
103.9327	88.13273	122.5653	0.015837	0.014155	0.017718
104.6933	88.83086	123.3884	0.015856	0.014173	0.017738
105.4866	89.55922	124.2466	0.015875	0.014191	0.017759
106.4239	90.42016	125.2602	0.015898	0.014212	0.017783
107.2352	91.16566	126.1373	0.015917	0.014231	0.017804
107.8563	91.73654	126.8085	0.015932	0.014244	0.01782
108.5736	92.3961	127.5836	0.015949	0.01426	0.017839
109.5951	93.33571	128.687	0.015974	0.014282	0.017865
111.0343	94.6601	130.2409	0.016007	0.014313	0.017902
111.7069	95.27937	130.9668	0.016023	0.014327	0.01792
112.8295	96.31319	132.178	0.016049	0.014351	0.017949
113.9377	97.3343	133.3734	0.016075	0.014373	0.017977
114.581	97.92718	134.067	0.016089	0.014386	0.017994
115.3054	98.59497	134.848	0.016106	0.014401	0.018012
115.8854	99.12982	135.4732	0.016119	0.014412	0.018027
117.7253	100.8271	137.4556	0.01616	0.014448	0.018074
118.1456	101.2149	137.9083	0.016169	0.014456	0.018085
119.2945	102.2754	139.1457	0.016195	0.014478	0.018114
120.5588	103.4428	140.5068	0.016222	0.014502	0.018147
121.0618	103.9075	141.0483	0.016233	0.014511	0.01816
122.199	104.9581	142.2721	0.016258	0.014532	0.018188
122.6869	105.4089	142.797	0.016268	0.014541	0.018201
123.4682	106.131	143.6375	0.016285	0.014555	0.018221
123.8266	106.4623	144.023	0.016293	0.014562	0.01823
125.0985	107.6382	145.3909	0.01632	0.014584	0.018262
125.9476	108.4235	146.304	0.016338	0.014599	0.018284
127.3866	109.7548	147.851	0.016368	0.014624	0.01832
127.9761	110.3002	148.4846	0.01638	0.014634	0.018335
129.2055	111.438	149.8058	0.016405	0.014654	0.018366
130.0755	112.2434	150.7406	0.016423	0.014669	0.018388
130.8642	112.9736	151.588	0.016439	0.014682	0.018407
131.6537	113.7047	152.436	0.016455	0.014695	0.018427
132.1804	114.1925	153.0018	0.016466	0.014703	0.01844
133.202	115.1388	154.099	0.016487	0.01472	0.018466
134.621	116.4535	155.6228	0.016515	0.014742	0.018501
135.4803	117.2497	156.5454	0.016532	0.014755	0.018523

135.811	117.5562	156.9004	0.016539	0.01476	0.018531
136.8039	118.4765	157.9664	0.016558	0.014776	0.018556
137.7979	119.398	159.0334	0.016578	0.014791	0.01858
138.7434	120.2745	160.0482	0.016596	0.014805	0.018604
139.5238	120.9981	160.8858	0.016611	0.014816	0.018623
140.4545	121.8612	161.8846	0.016629	0.01483	0.018646
141.6859	123.0033	163.2061	0.016652	0.014848	0.018676
142.4356	123.6987	164.0105	0.016667	0.014859	0.018695
143.5863	124.7663	165.2453	0.016688	0.014875	0.018723
144.4546	125.5718	166.1768	0.016705	0.014887	0.018744
145.6079	126.642	167.4142	0.016726	0.014903	0.018773
146.5953	127.5582	168.4736	0.016745	0.014917	0.018797
147.4665	128.3667	169.4081	0.016761	0.014928	0.018818
148.993	129.7834	171.0458	0.016789	0.014949	0.018855
149.8328	130.5629	171.9467	0.016804	0.01496	0.018875
151.1613	131.7961	173.3718	0.016828	0.014977	0.018908
154.0095	134.4402	176.4274	0.016879	0.015014	0.018976
155.0228	135.381	177.5144	0.016897	0.015026	0.019001
155.885	136.1814	178.4394	0.016912	0.015037	0.019021
158.934	139.0122	181.7107	0.016965	0.015074	0.019094
160.1223	140.1155	182.9857	0.016986	0.015088	0.019122
161.312	141.2201	184.2624	0.017006	0.015102	0.019151
164.0708	143.7814	187.2233	0.017053	0.015134	0.019216
164.3778	144.0664	187.5528	0.017058	0.015137	0.019223
166.068	145.6355	189.3671	0.017087	0.015156	0.019263
166.6662	146.1908	190.0093	0.017097	0.015163	0.019277
167.5556	147.0164	190.9642	0.017111	0.015173	0.019298
168.3088	147.7156	191.7729	0.017124	0.015181	0.019315
169.3024	148.6379	192.8399	0.01714	0.015192	0.019339
170.0225	149.3062	193.6132	0.017152	0.015199	0.019355
171.2239	150.4212	194.9035	0.017171	0.015212	0.019383
173.4585	152.4949	197.304	0.017208	0.015235	0.019435
174.6465	153.5972	198.5804	0.017227	0.015248	0.019462
175.0772	153.9968	199.0433	0.017234	0.015252	0.019472
176.3878	155.2126	200.4518	0.017254	0.015265	0.019502
177.769	156.4938	201.9365	0.017276	0.015279	0.019534
179.0828	157.7123	203.3489	0.017297	0.015292	0.019564
180.2076	158.7555	204.5585	0.017314	0.015303	0.01959
181.5763	160.0245	206.0306	0.017336	0.015317	0.019621
182.964	161.311	207.5235	0.017357	0.01533	0.019652
183.3112	161.6329	207.897	0.017362	0.015333	0.01966
185.0314	163.2272	209.7482	0.017389	0.015349	0.019699
186.0578	164.1783	210.853	0.017404	0.015359	0.019722
186.9631	165.0172	211.8277	0.017418	0.015367	0.019742
188.34	166.2927	213.3103	0.017439	0.01538	0.019773
188.933	166.842	213.9491	0.017448	0.015385	0.019786
190.1026	167.9252	215.2089	0.017465	0.015396	0.019813
191.3433	169.074	216.5457	0.017483	0.015407	0.01984
192.2003	169.8674	217.4693	0.017496	0.015414	0.019859
193.5307	171.0989	218.9035	0.017516	0.015426	0.019889

194.4594	171.9584	219.9048	0.017529	0.015434	0.019909
195.3537	172.7859	220.8692	0.017542	0.015442	0.019929
196.5823	173.9225	222.1945	0.01756	0.015452	0.019956
198.7441	175.9218	224.5272	0.017591	0.01547	0.020003
199.8881	176.9795	225.7621	0.017608	0.01548	0.020028
201.3151	178.2985	227.303	0.017628	0.015491	0.020059
202.2145	179.1296	228.2744	0.017641	0.015499	0.020079
203.3087	180.1405	229.4566	0.017656	0.015508	0.020102
204.3332	181.0869	230.5636	0.017671	0.015516	0.020125
205.553	182.2134	231.8822	0.017688	0.015525	0.020151
206.4554	183.0465	232.8579	0.0177	0.015533	0.02017
207.695	184.1908	234.1986	0.017717	0.015542	0.020197
209.0423	185.434	235.6561	0.017736	0.015553	0.020226
210.0713	186.3834	236.7699	0.01775	0.01556	0.020248
211.2611	187.4808	238.0579	0.017766	0.01557	0.020273
212.1142	188.2674	238.9816	0.017778	0.015576	0.020291
213.1991	189.2675	240.1567	0.017793	0.015584	0.020314
214.552	190.5144	241.6225	0.017811	0.015594	0.020343
215.5498	191.4337	242.704	0.017824	0.015601	0.020364
216.4057	192.2221	243.6318	0.017836	0.015608	0.020382
217.7261	193.4381	245.0638	0.017853	0.015617	0.020409
218.5298	194.178	245.9355	0.017864	0.015623	0.020426
219.906	195.4446	247.4289	0.017882	0.015633	0.020455
220.9436	196.3993	248.5551	0.017896	0.01564	0.020477
223.8458	199.0682	251.7073	0.017934	0.015661	0.020537
224.7068	199.8597	252.643	0.017945	0.015667	0.020554
225.353	200.4535	253.3454	0.017953	0.015671	0.020568
226.3587	201.3776	254.4388	0.017966	0.015678	0.020588
227.3293	202.269	255.4943	0.017979	0.015684	0.020608
228.0126	202.8966	256.2377	0.017987	0.015689	0.020622
229.1103	203.9043	257.4321	0.018001	0.015696	0.020645
230.3889	205.0778	258.824	0.018018	0.015705	0.020671
231.1098	205.7393	259.6089	0.018027	0.01571	0.020686
232.8956	207.3771	261.5541	0.018049	0.015721	0.020722
233.3649	207.8075	262.0656	0.018055	0.015724	0.020731
234.1776	208.5525	262.9514	0.018065	0.01573	0.020748
234.7197	209.0493	263.5423	0.018072	0.015733	0.020759
235.5332	209.7948	264.4293	0.018082	0.015738	0.020775
236.5099	210.6896	265.4946	0.018094	0.015745	0.020795
237.3968	211.5019	266.4622	0.018105	0.01575	0.020813
238.1937	212.2315	267.3318	0.018115	0.015755	0.020828
238.8641	212.8452	268.0636	0.018123	0.015759	0.020842
240.1333	214.0067	269.4495	0.018139	0.015767	0.020867
241.5306	215.285	270.9759	0.018156	0.015776	0.020895
241.9664	215.6835	271.4521	0.018161	0.015779	0.020904
243.1474	216.7633	272.7429	0.018176	0.015786	0.020927
243.7837	217.3449	273.4386	0.018183	0.01579	0.02094
244.4202	217.9266	274.1347	0.018191	0.015794	0.020952
245.4029	218.8244	275.2096	0.018203	0.0158	0.020972
246.5865	219.9054	276.5048	0.018217	0.015807	0.020995

247.2425	220.5044	277.2228	0.018225	0.015811	0.021008
248.154	221.3364	278.2208	0.018236	0.015816	0.021026
249.0478	222.1521	279.1998	0.018246	0.015821	0.021043
250.0152	223.0346	280.2596	0.018258	0.015827	0.021062
250.8918	223.8341	281.2203	0.018268	0.015832	0.021079
251.8786	224.7338	282.3021	0.01828	0.015838	0.021098
253.3966	226.1173	283.9669	0.018298	0.015847	0.021128
254.2385	226.8843	284.8906	0.018308	0.015851	0.021144
255.0992	227.6682	285.8352	0.018318	0.015856	0.021161
256.4736	228.9195	287.3442	0.018334	0.015864	0.021187
257.2988	229.6706	288.2506	0.018343	0.015869	0.021203
258.4181	230.6889	289.4804	0.018356	0.015875	0.021225
259.9057	232.0417	291.1156	0.018373	0.015883	0.021253
261.413	233.4118	292.7733	0.01839	0.015892	0.021282
262.4432	234.3478	293.9068	0.018402	0.015897	0.021302
263.7318	235.5182	295.3252	0.018417	0.015904	0.021326
264.4318	236.1537	296.096	0.018425	0.015908	0.021339
265.2058	236.8563	296.9485	0.018433	0.015912	0.021354
266.1277	237.6929	297.9642	0.018444	0.015917	0.021371
267.4931	238.9314	299.4691	0.018459	0.015924	0.021397
268.8412	240.1536	300.9556	0.018474	0.015931	0.021422
269.6357	240.8737	301.8321	0.018483	0.015936	0.021437
271.152	242.2474	303.5055	0.0185	0.015944	0.021466
272.7438	243.6887	305.2631	0.018517	0.015952	0.021495
273.3735	244.2586	305.9587	0.018524	0.015955	0.021507
274.819	245.5665	307.556	0.01854	0.015962	0.021534
281.4692	251.575	314.9157	0.018612	0.015996	0.021656
282.8098	252.7845	316.4014	0.018626	0.016002	0.021681
284.2259	254.0616	317.9717	0.018641	0.016009	0.021706
285.3261	255.0532	319.1921	0.018653	0.016014	0.021726
286.1657	255.8098	320.1238	0.018662	0.016019	0.021741
287.7713	257.2559	321.9065	0.018679	0.016026	0.021771
288.4252	257.8446	322.6327	0.018686	0.016029	0.021782
289.902	259.1737	324.2736	0.018701	0.016036	0.021809
291.3052	260.4358	325.8334	0.018716	0.016043	0.021834
292.3348	261.3616	326.9786	0.018727	0.016048	0.021852
293.3463	262.2707	328.104	0.018737	0.016052	0.021871
294.4147	263.2305	329.2931	0.018748	0.016057	0.02189
296.0465	264.6959	331.1102	0.018765	0.016065	0.021919
297.2103	265.7404	332.4068	0.018777	0.01607	0.021939
298.0366	266.4819	333.3278	0.018785	0.016074	0.021954
298.6379	267.0212	333.9981	0.018791	0.016077	0.021964
299.3897	267.6954	334.8365	0.018799	0.01608	0.021978
300.7625	268.926	336.3679	0.018813	0.016086	0.022002
302.0611	270.0895	337.8172	0.018826	0.016092	0.022025
303.0403	270.9665	338.9105	0.018836	0.016096	0.022042
304.2462	272.0461	340.2575	0.018848	0.016102	0.022063
305.3397	273.0247	341.4795	0.018859	0.016107	0.022082
307	274.5097	343.3357	0.018876	0.016114	0.022111

Parametric Growth Curve: Time

```
System:  Compounding Machine

Model: Power-Law Process
Estimation Method: Maximum Likelihood


Parameter Estimates

                     Standard    95% Normal CI
Parameter  Estimate     Error     Lower    Upper
Shape       1.04169     0.070  0.912750  1.18885
Scale       79.3419    28.169   39.5638  159.114


Trend Tests

                MIL-Hdbk-189  Laplace’s  Anderson-Darling
Test Statistic        422.39      -0.01              0.29
P-Value                0.562      0.990             0.944
DF                       440
```

Mean Cumulative Function for Time

Nonparametric Growth Curve: Time

```
System:  Compounding Machine

Nonparametric Estimates


Table of Mean Cumulative Function

             Mean             95% Normal
       Cumulative  Standard       CI
 Time    Function     Error  Lower  Upper  System
   76           1         *      *      *       1
  148           2         *      *      *       1
  200           3         *      *      *       1
  282           4         *      *      *       1
  300           5         *      *      *       1
  456           6         *      *      *       1
  520           7         *      *      *       1
  700           8         *      *      *       1
  760           9         *      *      *       1
```

```
 820          10         *      *      *       1
 880          11         *      *      *       1
 956          12         *      *      *       1
 990          13         *      *      *       1
1050          14         *      *      *       1
1165          15         *      *      *       1
1280          16         *      *      *       1
1345          17         *      *      *       1
1420          18         *      *      *       1
1500          19         *      *      *       1
1628          20         *      *      *       1
1690          21         *      *      *       1
1750          22         *      *      *       1
1791          23         *      *      *       1
1843          24         *      *      *       1
1880          25         *      *      *       1
1950          26         *      *      *       1
2023          27         *      *      *       1
2099          28         *      *      *       1
2156          29         *      *      *       1
2214          30         *      *      *       1
2287          31         *      *      *       1
2330          32         *      *      *       1
2372          33         *      *      *       1
2459          34         *      *      *       1
2534          35         *      *      *       1
2677          36         *      *      *       1
2696          37         *      *      *       1
2778          38         *      *      *       1
2824          39         *      *      *       1
2880          40         *      *      *       1
2936          41         *      *      *       1
2990          42         *      *      *       1
3035          43         *      *      *       1
3077          44         *      *      *       1
3124          45         *      *      *       1
3190          46         *      *      *       1
3200          47         *      *      *       1
3265          48         *      *      *       1
3279          49         *      *      *       1
3308          50         *      *      *       1
3312          51         *      *      *       1
3400          52         *      *      *       1
3488          53         *      *      *       1
3565          54         *      *      *       1
3580          55         *      *      *       1
3634          56         *      *      *       1
3670          57         *      *      *       1
3742          58         *      *      *       1
3790          59         *      *      *       1
3810          60         *      *      *       1
```

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3866          61         *      *      *       1
3900          62         *      *      *       1
3948          63         *      *      *       1
3999          64         *      *      *       1
4051          65         *      *      *       1
4078          66         *      *      *       1
4124          67         *      *      *       1
4180          68         *      *      *       1
4217          69         *      *      *       1
4290          70         *      *      *       1
4366          71         *      *      *       1
4409          72         *      *      *       1
4498          73         *      *      *       1
4580          74         *      *      *       1
4640          75         *      *      *       1
4681          76         *      *      *       1
4790          77         *      *      *       1
4865          78         *      *      *       1
4910          79         *      *      *       1
4990          80         *      *      *       1
5049          81         *      *      *       1
5124          82         *      *      *       1
5188          83         *      *      *       1
5245          84         *      *      *       1
5300          85         *      *      *       1
5378          86         *      *      *       1
5434          87         *      *      *       1
5500          88         *      *      *       1
5567          89         *      *      *       1
5620          90         *      *      *       1
5700          91         *      *      *       1
5788          92         *      *      *       1
5831          93         *      *      *       1
5890          94         *      *      *       1
5900          95         *      *      *       1
5966          96         *      *      *       1
6030          97         *      *      *       1
6099          98         *      *      *       1
6140          99         *      *      *       1
6199         100         *      *      *       1
6250         101         *      *      *       1
6303         102         *      *      *       1
6734         103         *      *      *       1
6780         104         *      *      *       1
6813         105         *      *      *       1
6877         106         *      *      *       1
6905         107         *      *      *       1
6955         108         *      *      *       1
6990         109         *      *      *       1
7024         110         *      *      *       1
7088         111         *      *      *       1
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 7148        112        *       *       *       1
 7198        113        *       *       *       1
 7250        114        *       *       *       1
 7320        115        *       *       *       1
 7390        116        *       *       *       1
 7450        117        *       *       *       1
 7520        118        *       *       *       1
 7590        119        *       *       *       1
 7660        120        *       *       *       1
 7700        121        *       *       *       1
 7843        122        *       *       *       1
 7890        123        *       *       *       1
 7934        124        *       *       *       1
 7988        125        *       *       *       1
 8045        126        *       *       *       1
 8124        127        *       *       *       1
 8200        128        *       *       *       1
 8280        129        *       *       *       1
 8300        130        *       *       *       1
 8365        131        *       *       *       1
 8459        132        *       *       *       1
 8499        133        *       *       *       1
 8544        134        *       *       *       1
 8580        135        *       *       *       1
 8694        136        *       *       *       1
 8740        137        *       *       *       1
 8788        138        *       *       *       1
 8834        139        *       *       *       1
 8878        140        *       *       *       1
 8950        141        *       *       *       1
 8990        142        *       *       *       1
 9032        143        *       *       *       1
 9080        144        *       *       *       1
 9140        145        *       *       *       1
 9200        146        *       *       *       1
 9288        147        *       *       *       1
 9324        148        *       *       *       1
 9399        149        *       *       *       1
 9452        150        *       *       *       1
 9500        151        *       *       *       1
 9522        152        *       *       *       1
 9589        153        *       *       *       1
 9642        154        *       *       *       1
 9728        155        *       *       *       1
 9780        156        *       *       *       1
 9800        157        *       *       *       1
 9860        158        *       *       *       1
 9920        159        *       *       *       1
 9977        160        *       *       *       1
10024        161        *       *       *       1
10080        162        *       *       *       1
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10154	163	*	*	*	1
10199	164	*	*	*	1
10268	165	*	*	*	1
10280	166	*	*	*	1
10330	167	*	*	*	1
10390	168	*	*	*	1
10420	169	*	*	*	1
10480	170	*	*	*	1
10500	171	*	*	*	1
10591	172	*	*	*	1
10641	173	*	*	*	1
10720	174	*	*	*	1
10889	175	*	*	*	1
10949	176	*	*	*	1
11000	177	*	*	*	1
11180	178	*	*	*	1
11250	179	*	*	*	1
11320	180	*	*	*	1
11482	181	*	*	*	1
11500	182	*	*	*	1
11555	183	*	*	*	1
11631	184	*	*	*	1
11689	185	*	*	*	1
11730	186	*	*	*	1
11788	187	*	*	*	1
11830	188	*	*	*	1
11900	189	*	*	*	1
12030	190	*	*	*	1
12099	191	*	*	*	1
12124	192	*	*	*	1
12200	193	*	*	*	1
12280	194	*	*	*	1
12356	195	*	*	*	1
12421	196	*	*	*	1
12500	197	*	*	*	1
12580	198	*	*	*	1
12600	199	*	*	*	1
12699	200	*	*	*	1
12758	201	*	*	*	1
12810	202	*	*	*	1
12889	203	*	*	*	1
12923	204	*	*	*	1
12990	205	*	*	*	1
13061	206	*	*	*	1
13110	207	*	*	*	1
13186	208	*	*	*	1
13239	209	*	*	*	1
13290	210	*	*	*	1
13360	211	*	*	*	1
13483	212	*	*	*	1
13548	213	*	*	*	1

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13629         214         *      *      *       1
13680         215         *      *      *       1
13742         216         *      *      *       1
13800         217         *      *      *       1
13869         218         *      *      *       1
13920         219         *      *      *       1
13990         220         *      *      *       1
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Table A6 Data for Compounding Machine

MCF1	CLMCF1	CLMCF2	ROCOF1	CLROCOF1	CLROCOF2
0.956163	0.460466	1.985483	0.013106	0.007184	0.023909
1.914468	1.008802	3.633206	0.013475	0.008075	0.022487
2.619802	1.437518	4.774453	0.013645	0.00851	0.02188
3.747219	2.153017	6.521848	0.013842	0.009032	0.021215
3.996701	2.315424	6.898785	0.013878	0.009128	0.021098
6.18197	3.786657	10.09248	0.014122	0.009807	0.020336
7.088323	4.417966	11.37273	0.0142	0.010028	0.020107
9.660967	6.261748	14.90547	0.014377	0.01054	0.01961
10.52508	6.895672	16.06475	0.014426	0.010685	0.019477
11.39204	7.538104	17.21633	0.014472	0.01082	0.019357
12.26165	8.188494	18.36089	0.014515	0.010946	0.019247
13.3667	9.022995	19.80147	0.014565	0.011094	0.019121
13.86226	9.399979	20.44285	0.014586	0.011157	0.019069
14.73851	10.07047	21.57037	0.014622	0.011263	0.018982
16.42375	11.37315	23.71721	0.014685	0.011451	0.018833
18.11594	12.69719	25.84724	0.014743	0.011622	0.018702
19.07524	13.45432	27.04447	0.014774	0.011712	0.018636
20.18453	14.33533	28.42037	0.014807	0.01181	0.018564
21.37047	15.2834	29.8819	0.014841	0.01191	0.018494
23.27341	16.81712	32.20834	0.014892	0.012057	0.018393
24.19742	17.56705	33.33029	0.014915	0.012124	0.018348
25.09297	18.29696	34.41321	0.014937	0.012187	0.018307
25.70567	18.79802	35.15166	0.014951	0.012228	0.018281
26.48359	19.4361	36.0865	0.014969	0.012279	0.018249
27.03768	19.89184	36.75054	0.014981	0.012314	0.018227
28.08718	20.75787	38.00435	0.015004	0.012378	0.018187
29.18333	21.66617	39.30859	0.015027	0.012442	0.018149
30.32628	22.61719	40.66303	0.01505	0.012506	0.018112
31.18463	23.33394	41.67669	0.015067	0.012552	0.018086
32.05902	24.06623	42.70633	0.015084	0.012598	0.01806
33.16089	24.99205	43.99976	0.015104	0.012653	0.018031
33.81062	25.5395	44.7604	0.015116	0.012684	0.018014
34.44573	26.07569	45.50249	0.015127	0.012714	0.017999
35.7628	27.19085	47.03707	0.01515	0.012774	0.017968
36.89977	28.15692	48.35731	0.015169	0.012823	0.017944
39.07144	30.01048	50.86814	0.015204	0.012911	0.017904
39.36035	30.25787	51.20115	0.015208	0.012922	0.017899

40.60821	31.3284	52.6368	0.015227	0.012969	0.017879
41.30891	31.93095	53.44112	0.015238	0.012994	0.017868
42.16257	32.66639	54.41931	0.01525	0.013025	0.017856
43.01692	33.40386	55.39646	0.015262	0.013054	0.017845
43.84141	34.11688	56.33777	0.015274	0.013081	0.017835
44.52895	34.71245	57.12151	0.015284	0.013103	0.017827
45.17104	35.26943	57.85246	0.015292	0.013123	0.01782
45.89001	35.89397	58.66982	0.015302	0.013145	0.017813
47.05354	36.90662	59.99021	0.015317	0.013179	0.017802
48.26423	37.96274	61.36111	0.015333	0.013214	0.017792
48.77032	38.40494	61.93329	0.015339	0.013227	0.017788
46.90038	36.77318	59.81657	0.015315	0.013175	0.017803
48.04959	37.77532	61.11828	0.01533	0.013208	0.017794
48.70896	38.35131	61.86395	0.015339	0.013226	0.017789
50.12092	39.58703	63.45782	0.015356	0.013263	0.017779
51.47298	40.77323	64.98056	0.015372	0.013297	0.017772
52.65719	41.81441	66.31159	0.015386	0.013325	0.017767
52.88801	42.01758	66.57074	0.015389	0.013331	0.017766
53.71928	42.74993	67.50331	0.015399	0.013349	0.017763
54.27375	43.23895	68.1247	0.015405	0.013362	0.017761
55.38337	44.21884	69.36676	0.015418	0.013386	0.017758
56.12361	44.87344	70.19429	0.015426	0.013401	0.017757
56.43216	45.14651	70.53898	0.015429	0.013407	0.017756
57.29645	45.91205	71.50374	0.015439	0.013424	0.017755
57.82146	46.37753	72.08924	0.015444	0.013434	0.017755
58.56297	47.03554	72.91553	0.015452	0.013448	0.017754
59.35123	47.73576	73.79308	0.01546	0.013463	0.017754
60.15539	48.45085	74.68745	0.015469	0.013477	0.017755
60.5731	48.82259	75.15169	0.015473	0.013484	0.017755
61.28502	49.45661	75.94239	0.01548	0.013496	0.017756
62.15215	50.22962	76.90462	0.015489	0.01351	0.017757
62.72535	50.74104	77.54017	0.015495	0.013519	0.017758
63.85685	51.75163	78.79362	0.015506	0.013537	0.017761
65.03572	52.80589	80.09796	0.015517	0.013554	0.017764
65.70309	53.40334	80.83569	0.015523	0.013563	0.017767
67.08524	54.64203	82.36206	0.015536	0.013582	0.017772
68.3597	55.78576	83.76776	0.015548	0.013598	0.017778
69.29283	56.6241	84.79599	0.015556	0.013609	0.017782
69.93076	57.19765	85.49847	0.015562	0.013617	0.017786
71.62786	58.72514	87.36548	0.015577	0.013636	0.017795
72.79652	59.77834	88.64972	0.015587	0.013648	0.017802
73.49808	60.41109	89.42012	0.015593	0.013655	0.017807
74.74595	61.53748	90.7895	0.015604	0.013667	0.017815
75.66679	62.36939	91.79925	0.015611	0.013675	0.017822
76.838	63.42834	93.08267	0.015621	0.013685	0.017831
77.838	64.3332	94.17773	0.015629	0.013693	0.017839
78.72906	65.14003	95.15294	0.015636	0.0137	0.017846
79.58923	65.91936	96.09387	0.015643	0.013706	0.017853
80.80976	67.02592	97.42822	0.015652	0.013715	0.017864
81.68648	67.82132	98.38619	0.015659	0.013721	0.017871

82.72025	68.75974	99.51522	0.015667	0.013727	0.017881
83.77022	69.71343	100.6614	0.015675	0.013734	0.017891
84.60116	70.46858	101.568	0.015681	0.013738	0.017899
85.85603	71.60963	102.9367	0.01569	0.013745	0.017911
87.23723	72.8664	104.4423	0.0157	0.013752	0.017925
87.91246	73.4811	105.1781	0.015705	0.013755	0.017932
88.83926	74.32514	106.1877	0.015712	0.013759	0.017942
88.99639	74.46827	106.3588	0.015713	0.01376	0.017943
90.03369	75.41341	107.4884	0.01572	0.013764	0.017955
91.04002	76.33072	108.5839	0.015727	0.013768	0.017965
92.12546	77.32057	109.7651	0.015735	0.013772	0.017977
92.77068	77.90915	110.4671	0.015739	0.013774	0.017985
93.69947	78.75666	111.4775	0.015745	0.013777	0.017995
94.50263	79.48974	112.3509	0.015751	0.013779	0.018004
95.33757	80.25204	113.2588	0.015756	0.013782	0.018014
102.1381	86.46741	120.6487	0.0158	0.013795	0.018096
102.865	87.13233	121.4383	0.015804	0.013796	0.018105
103.3865	87.60951	122.0048	0.015808	0.013797	0.018111
104.3984	88.53533	123.1038	0.015814	0.013798	0.018124
104.8413	88.94054	123.5847	0.015816	0.013798	0.01813
105.6322	89.66436	124.4437	0.015821	0.013799	0.01814
106.186	90.1712	125.0451	0.015824	0.0138	0.018147
106.7241	90.6637	125.6294	0.015828	0.0138	0.018154
107.7372	91.59111	126.7297	0.015834	0.0138	0.018167
108.6874	92.46096	127.7616	0.015839	0.013801	0.018179
109.4795	93.18612	128.6218	0.015844	0.013801	0.018189
110.3035	93.94056	129.5166	0.015849	0.013801	0.0182
111.4131	94.95655	130.7218	0.015855	0.013801	0.018214
112.5232	95.973	131.9274	0.015861	0.013801	0.018229
113.475	96.84459	132.9613	0.015867	0.013801	0.018242
114.5859	97.86182	134.1681	0.015873	0.013801	0.018256
115.6972	98.87945	135.3754	0.015879	0.0138	0.018271
116.809	99.89745	136.5834	0.015885	0.013799	0.018286
117.4444	100.4793	137.274	0.015888	0.013799	0.018294
119.7174	102.5604	139.7445	0.015901	0.013797	0.018325
120.4648	103.2447	140.557	0.015905	0.013796	0.018335
121.1647	103.8854	141.318	0.015908	0.013796	0.018345
122.0238	104.6719	142.2523	0.015913	0.013795	0.018356
122.931	105.5022	143.239	0.015918	0.013794	0.018369
124.1887	106.6533	144.6073	0.015924	0.013792	0.018386
125.3992	107.7609	145.9245	0.01593	0.01379	0.018402
126.6739	108.9271	147.312	0.015937	0.013788	0.01842
126.9926	109.2187	147.659	0.015938	0.013788	0.018424
128.0288	110.1665	148.7873	0.015943	0.013786	0.018438
129.5278	111.5373	150.4201	0.015951	0.013784	0.018459
130.1659	112.1208	151.1153	0.015954	0.013783	0.018467
130.8839	112.7772	151.8978	0.015958	0.013781	0.018477
131.4584	113.3023	152.524	0.01596	0.01378	0.018485
133.2784	114.9655	154.5084	0.015969	0.013777	0.01851
134.0131	115.6367	155.3097	0.015973	0.013775	0.01852

134.7798	116.3371	156.1463	0.015976	0.013774	0.018531
135.5148	117.0083	156.9484	0.01598	0.013772	0.018541
136.218	117.6504	157.716	0.015983	0.013771	0.018551
137.369	118.7011	158.9727	0.015988	0.013768	0.018566
138.0086	119.2849	159.6713	0.015991	0.013767	0.018575
138.6803	119.8978	160.4051	0.015995	0.013765	0.018585
139.4481	120.5983	161.2442	0.015998	0.013764	0.018595
140.4081	121.474	162.2936	0.016002	0.013761	0.018608
141.3684	122.3496	163.3436	0.016007	0.013759	0.018622
142.7773	123.6338	164.8849	0.016013	0.013756	0.018641
143.3538	124.1592	165.5159	0.016016	0.013754	0.018649
144.5552	125.2536	166.8311	0.016021	0.013751	0.018665
145.4044	126.027	167.7612	0.016025	0.013749	0.018677
146.1737	126.7274	168.604	0.016028	0.013747	0.018688
146.5263	127.0483	168.9905	0.01603	0.013746	0.018692
147.6005	128.0259	170.1679	0.016034	0.013744	0.018707
148.4504	128.7991	171.1	0.016038	0.013741	0.018719
149.8299	130.0536	172.6135	0.016044	0.013738	0.018738
150.6643	130.812	173.5294	0.016048	0.013736	0.018749
150.9853	131.1037	173.8818	0.016049	0.013735	0.018753
151.9483	131.9787	174.9395	0.016053	0.013732	0.018767
152.9116	132.8537	175.998	0.016057	0.013729	0.01878
153.827	133.6847	177.0041	0.016061	0.013727	0.018792
154.582	134.3699	177.8343	0.016064	0.013725	0.018802
155.4817	135.1862	178.824	0.016068	0.013722	0.018815
156.6709	136.2647	180.1329	0.016073	0.013719	0.018831
157.3942	136.9205	180.9293	0.016076	0.013717	0.018841
158.5036	137.9258	182.1514	0.01608	0.013713	0.018856
158.6965	138.1006	182.3641	0.016081	0.013713	0.018858
159.5007	138.829	183.2504	0.016084	0.013711	0.018869
160.4659	139.7029	184.3147	0.016088	0.013708	0.018882
160.9485	140.1398	184.8471	0.01609	0.013706	0.018889
161.9141	141.0134	185.9125	0.016094	0.013703	0.018902
162.2359	141.3046	186.2678	0.016095	0.013702	0.018906
163.7009	142.6292	187.8856	0.016101	0.013698	0.018926
164.506	143.3568	188.7753	0.016104	0.013696	0.018936
165.7784	144.5062	190.182	0.016109	0.013692	0.018953
168.5018	146.964	193.196	0.01612	0.013683	0.01899
169.4691	147.8361	194.2676	0.016123	0.01368	0.019003
170.2914	148.5773	195.179	0.016126	0.013678	0.019014
173.1952	151.192	198.4006	0.016137	0.013669	0.019052
174.325	152.2083	199.6553	0.016142	0.013665	0.019067
175.455	153.2242	200.9112	0.016146	0.013661	0.019082
178.0714	155.5741	203.822	0.016155	0.013653	0.019116
178.3622	155.8351	204.1458	0.016156	0.013652	0.01912
179.2509	156.6325	205.1356	0.01616	0.013649	0.019132
180.4792	157.7339	206.5045	0.016164	0.013645	0.019148
181.4168	158.5741	207.55	0.016167	0.013642	0.01916
182.0797	159.168	208.2896	0.01617	0.01364	0.019168
183.0177	160.0078	209.3365	0.016173	0.013637	0.019181

183.697	160.6158	210.095	0.016175	0.013635	0.019189
184.8294	161.6289	211.3602	0.016179	0.013631	0.019204
186.9332	163.5092	213.7129	0.016187	0.013624	0.019231
188.0503	164.5067	214.9632	0.016191	0.013621	0.019245
188.455	164.8681	215.4165	0.016192	0.013619	0.019251
189.6858	165.9661	216.7954	0.016196	0.013615	0.019266
190.9817	167.1215	218.2483	0.016201	0.013611	0.019283
192.2131	168.2187	219.63	0.016205	0.013607	0.019298
193.2665	169.1566	220.8128	0.016208	0.013604	0.019312
194.5471	170.2961	222.2517	0.016213	0.013599	0.019328
195.8443	171.4495	223.7102	0.016217	0.013595	0.019344
196.1687	171.7377	224.0751	0.016218	0.013594	0.019348
197.7745	173.1641	225.8827	0.016223	0.013589	0.019369
198.7318	174.0137	226.961	0.016226	0.013586	0.019381
199.5756	174.7623	227.9121	0.016229	0.013583	0.019391
200.8579	175.8991	229.3582	0.016233	0.013579	0.019407
201.4099	176.3882	229.981	0.016235	0.013577	0.019414
202.4978	177.3517	231.2092	0.016239	0.013573	0.019428
203.6508	178.3723	232.5118	0.016242	0.013569	0.019442
204.4468	179.0764	233.4115	0.016245	0.013567	0.019452
205.6815	180.1679	234.8081	0.016249	0.013563	0.019467
206.5428	180.9289	235.7829	0.016252	0.01356	0.019478
207.3717	181.6608	236.7215	0.016254	0.013557	0.019488
208.5096	182.6651	238.0108	0.016258	0.013553	0.019502
210.5097	184.4286	240.279	0.016264	0.013547	0.019526
211.567	185.36	241.4792	0.016267	0.013543	0.019539
212.8848	186.5201	242.9761	0.016271	0.013539	0.019555
213.7147	187.2502	243.9194	0.016274	0.013536	0.019565
214.7237	188.1374	245.0671	0.016277	0.013533	0.019577
215.6679	188.9671	246.1415	0.01628	0.01353	0.019589
216.7913	189.9536	247.4207	0.016283	0.013526	0.019602
217.6218	190.6826	248.3669	0.016286	0.013523	0.019612
218.7619	191.6826	249.6667	0.016289	0.01352	0.019626
220	192.7679	251.0792	0.016293	0.013516	0.01964

APPENDIX B

Lingo Computational Analysis

1. Lingo Model Codes

```
Model:
Data:
C = 5;
T = 36 ;
L = 1;
Enddata
Sets:
Component/1 .. C/: Lambda, Beta, Alpha, Failure_Cost, PM_Cost,
CRR_Cost;
Period/1 .. T/;
LinkComPer(Component, Period): X, XP, PM, CRR;
Endsets
Data:
Beta =;
Lamda =;
Alpha = 0.7 0.7 0.7 0.7 0.7;
Failure_Cost =;
PM_Cost = ;
RR_Cost = ;
Downtime_Cost = ;
Given_Reliability = 0.5;

Enddata

Min = @Sum(LinkComPer(i,j): (Failure_Cost (i) * Lambda (i) *
((XP(i,j)^Beta(i)) - (X(i,j)^Beta(i)))) + PM_Cost(i) * PM(i,j)
+ CRR_Cost(i) * CRR(i,j)) + @Sum(Period(j): Downtime_Cost);

@For(Component(i): X(i,l) = 0);
@For(LinkComPer(i,j): XP(i,j) = X(i,j) + (L));
@For(LinkComPer(i,j) | j #GE# 2: X(i,j) = ((1-PM(i,j-1)) * (l-
RR(i,j-
1)) * (XP(i,j-1)) + PM(i,j-1) * Alpha (i) * (XP(i,j-1))));

@For(LinkComPer(i,j): PM(i,j) + CRR(i,j) <= 1);
@For(LinkComPer(i,j): @BIN(PM));
@For(LinkComPer(i,j): @BIN(RR));
@Exp(@Sum(LinkComPer(i,j): (-Lambda (i) * ((XP(i,j)^Beta(i)) -
(X(i,j)^Beta(i)))))) >= Given_Reliability;
```

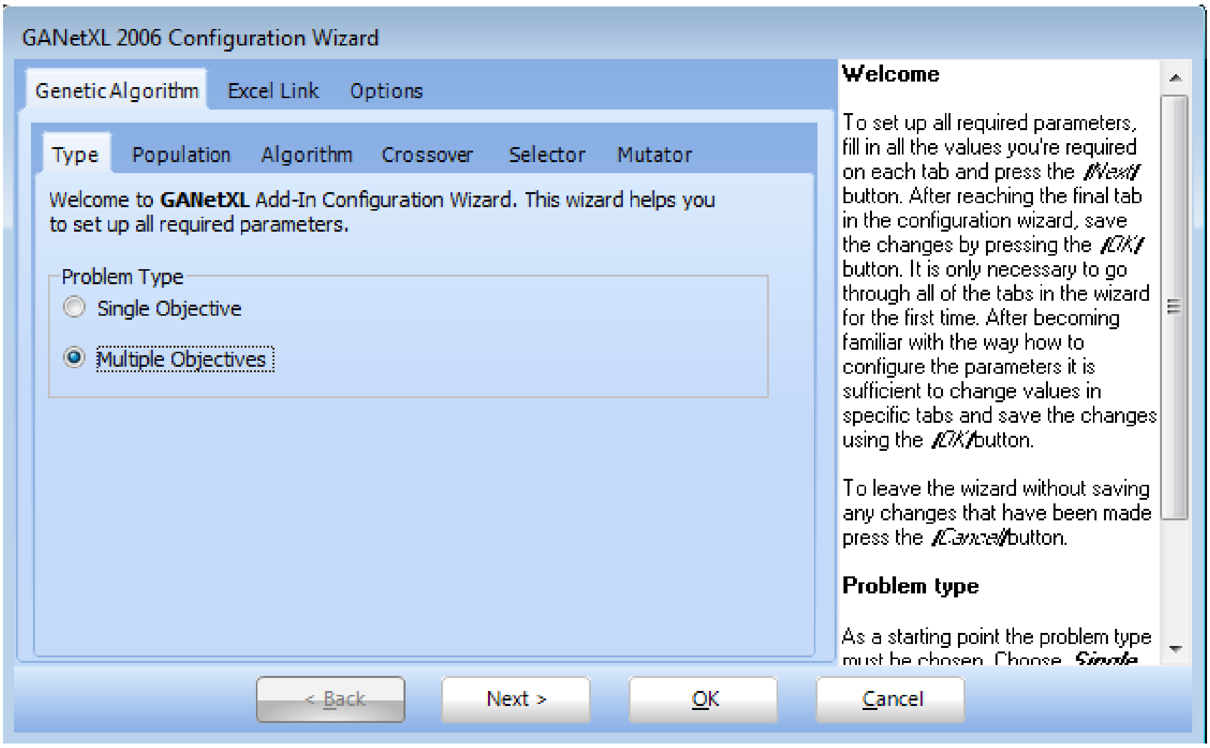
2. Computational Model Report

```
Global Optimum solution found.
Objective value:                              7593578.
Objective bound:                            -0.9194561E+17
Infeasibilities:                              0.000000
Extended solver steps:                            1774
Total solver iterations:                      84615945
Elapsed runtime seconds:                      34920.84

Model Class:                                     MINLP

Total variables:                   710
Nonlinear variables:               700
Integer variables:                 360

Total constraints:                 532
Nonlinear constraints:             177

Total nonzeros:                   2465
Nonlinear nonzeros:               1220
```

APPENDIX C

GAnetXL Optimization Window

Model Selection



Welcome to **GANetXL** Add-In Configuration Wizard. This wizard helps you to set up all required parameters.

Problem Type

○ Single Objective

● Multiple Objectives

Welcome

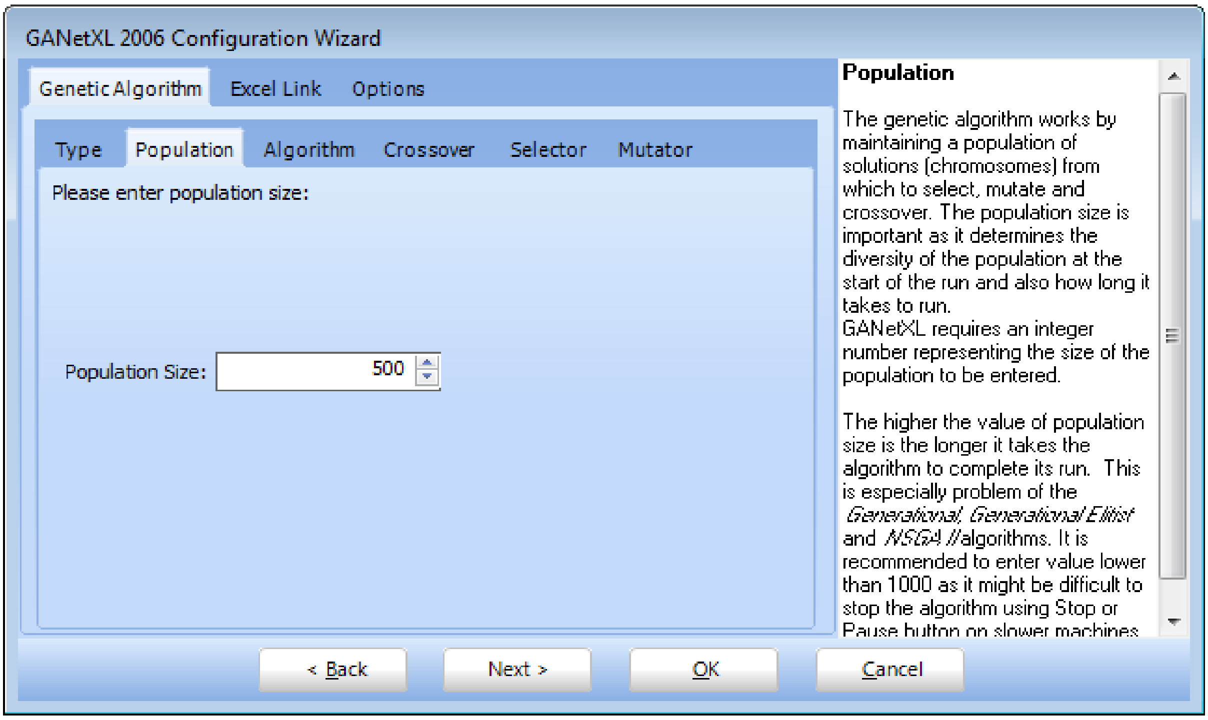
To set up all required parameters, fill in all the values you're required on each tab and press the *'Next'* button. After reaching the final tab in the configuration wizard, save the changes by pressing the *'OK'* button. It is only necessary to go through all of the tabs in the wizard for the first time. After becoming familiar with the way how to configure the parameters it is sufficient to change values in specific tabs and save the changes using the *'OK'* button.

To leave the wizard without saving any changes that have been made press the *'Cancel'* button.

Problem type

As a starting point the problem type must be chosen. Choose *'Single*

Fig C1: Model Selection



Please enter population size:

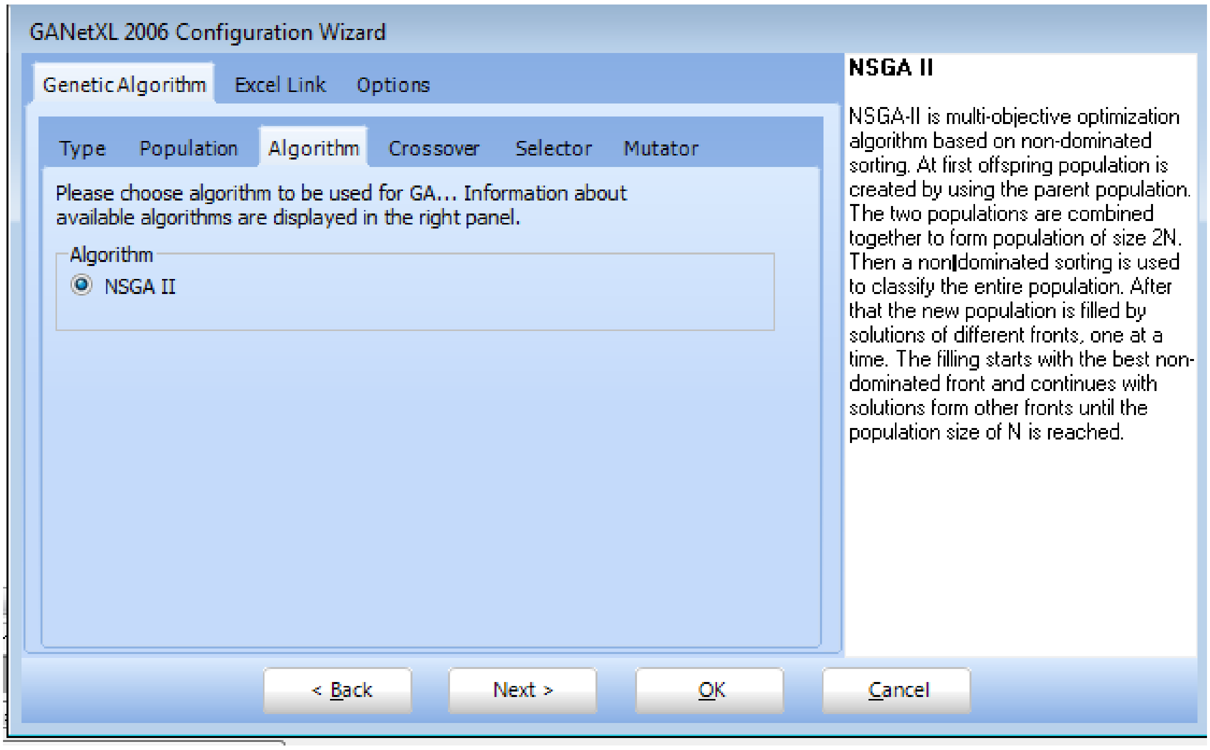
Population Size: 500

Population

The genetic algorithm works by maintaining a population of solutions (chromosomes) from which to select, mutate and crossover. The population size is important as it determines the diversity of the population at the start of the run and also how long it takes to run.
GANetXL requires an integer number representing the size of the population to be entered.

The higher the value of population size is the longer it takes the algorithm to complete its run. This is especially problem of the *Generational, Generational Elitist* and *NSGA II* algorithms. It is recommended to enter value lower than 1000 as it might be difficult to stop the algorithm using Stop or Pause button on slower machines

Fig C2: Population Size



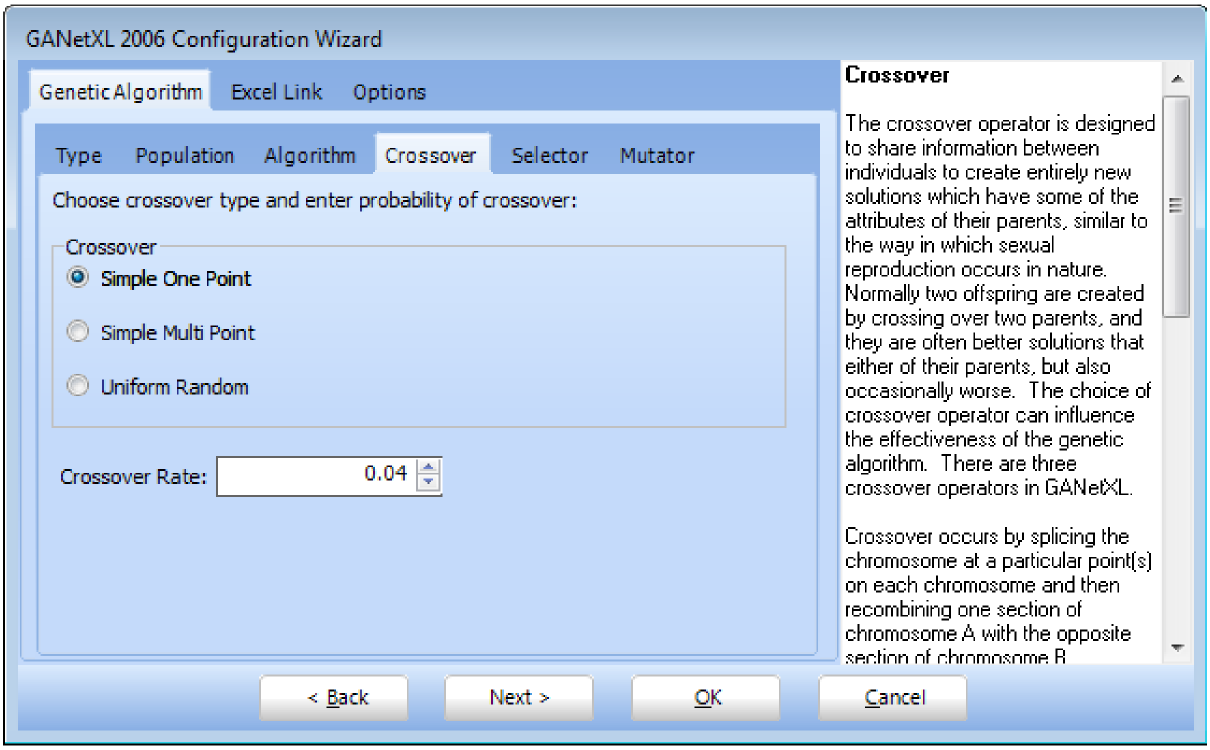
Please choose algorithm to be used for GA… Information about available algorithms are displayed in the right panel.

Algorithm
● NSGA II

NSGA II

NSGA-II is multi-objective optimization algorithm based on non-dominated sorting. At first offspring population is created by using the parent population. The two populations are combined together to form population of size 2N. Then a non-dominated sorting is used to classify the entire population. After that the new population is filled by solutions of different fronts, one at a time. The filling starts with the best non-dominated front and continues with solutions form other fronts until the population size of N is reached.

Fig C3: Algorithm



Choose crossover type and enter probability of crossover:

Crossover
● Simple One Point
○ Simple Multi Point
○ Uniform Random

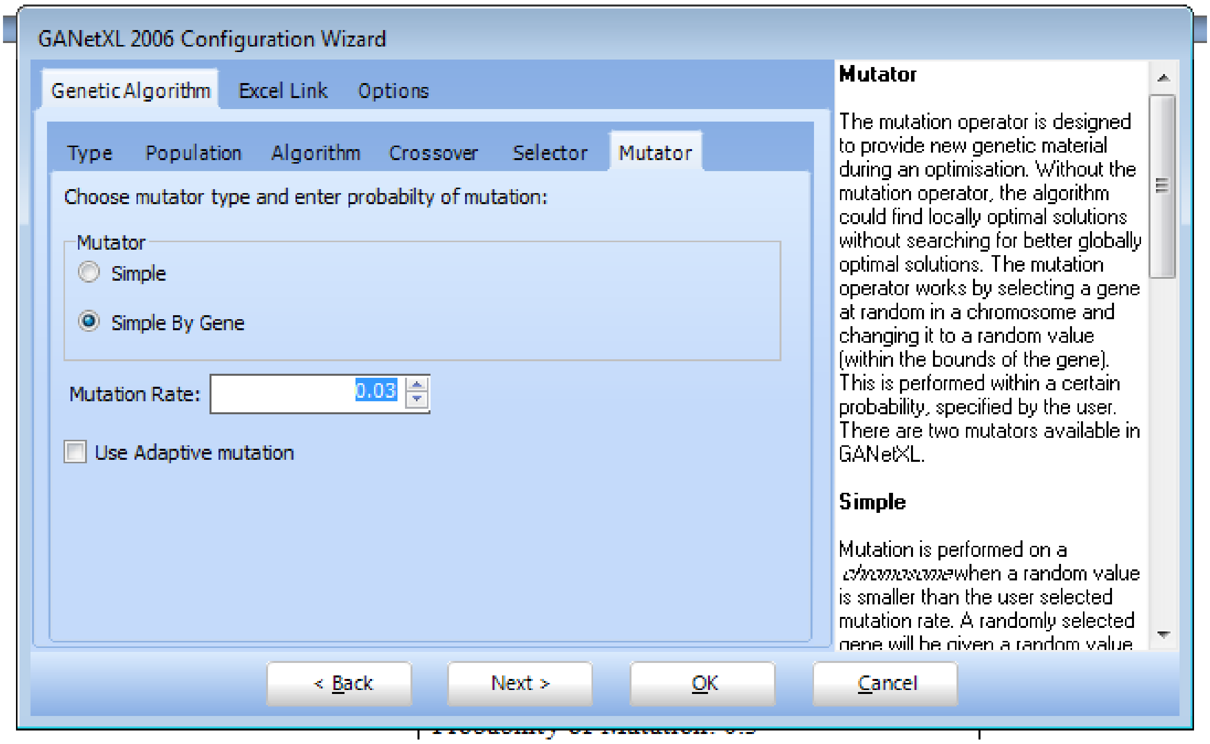
Crossover Rate: 0.04

Crossover

The crossover operator is designed to share information between individuals to create entirely new solutions which have some of the attributes of their parents, similar to the way in which sexual reproduction occurs in nature. Normally two offspring are created by crossing over two parents, and they are often better solutions that either of their parents, but also occasionally worse. The choice of crossover operator can influence the effectiveness of the genetic algorithm. There are three crossover operators in GANetXL.

Crossover occurs by splicing the chromosome at a particular point(s) on each chromosome and then recombining one section of chromosome A with the opposite section of chromosome B

Fig C4: Crossover



Choose mutator type and enter probabilty of mutation:

Mutator
○ Simple
● Simple By Gene

Mutation Rate: 0.03

☐ Use Adaptive mutation

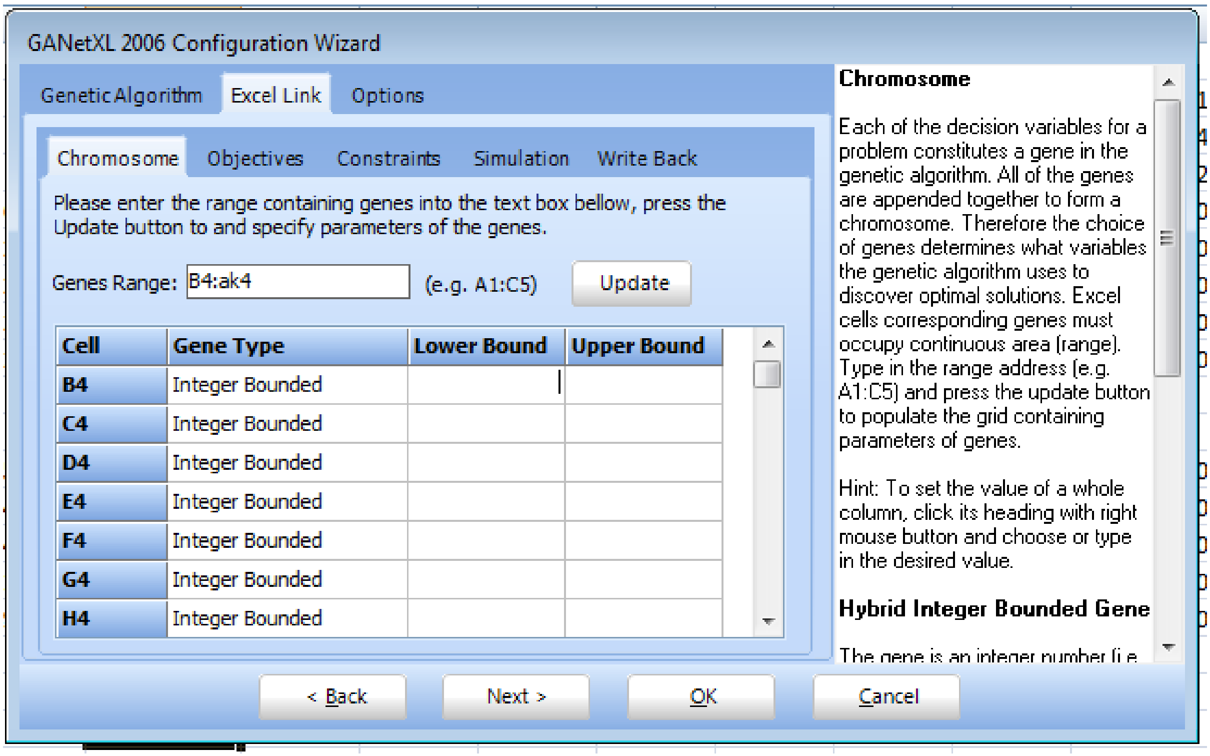
Mutator

The mutation operator is designed to provide new genetic material during an optimisation. Without the mutation operator, the algorithm could find locally optimal solutions without searching for better globally optimal solutions. The mutation operator works by selecting a gene at random in a chromosome and changing it to a random value (within the bounds of the gene). This is performed within a certain probability, specified by the user. There are two mutators available in GANetXL.

Simple

Mutation is performed on a *chromosome* when a random value is smaller than the user selected mutation rate. A randomly selected gene will be given a random value

Fig C5: Mutation



Please enter the range containing genes into the text box bellow, press the Update button to and specify parameters of the genes.

Genes Range: B4:ak4 (e.g. A1:C5)

Cell	Gene Type	Lower Bound	Upper Bound
B4	Integer Bounded		
C4	Integer Bounded		
D4	Integer Bounded		
E4	Integer Bounded		
F4	Integer Bounded		
G4	Integer Bounded		
H4	Integer Bounded		

Chromosome

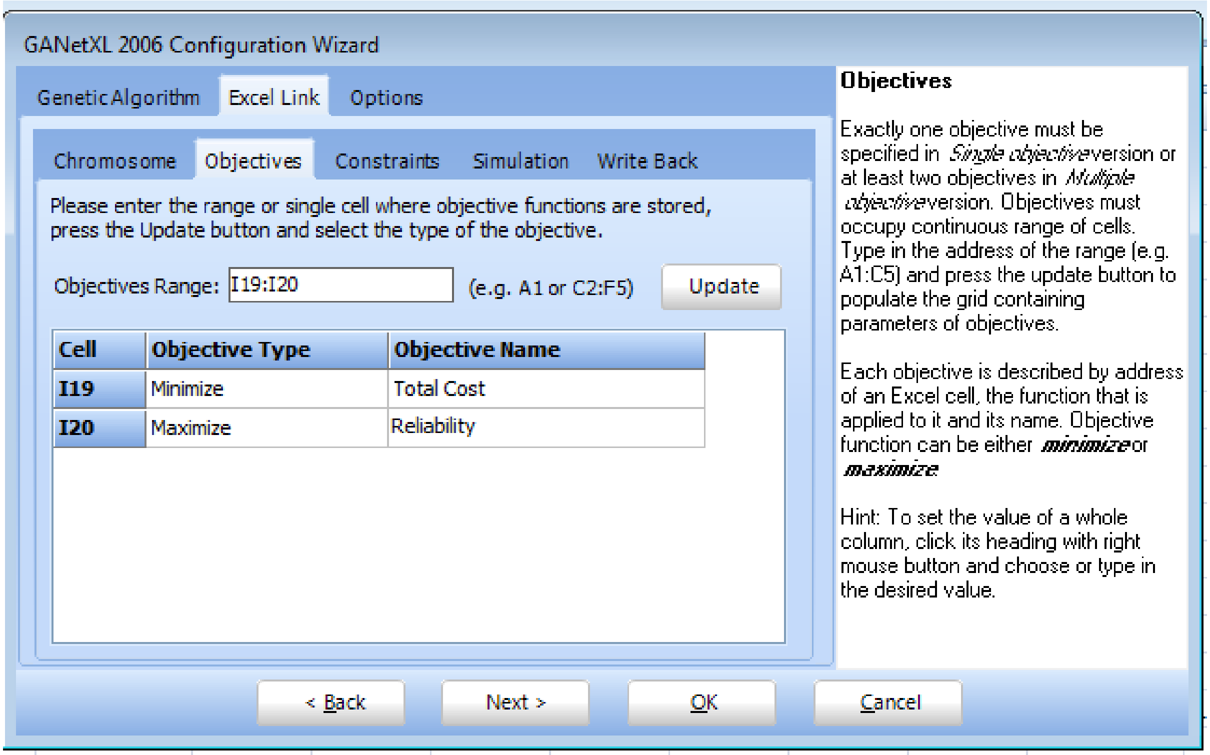
Each of the decision variables for a problem constitutes a gene in the genetic algorithm. All of the genes are appended together to form a chromosome. Therefore the choice of genes determines what variables the genetic algorithm uses to discover optimal solutions. Excel cells corresponding genes must occupy continuous area (range). Type in the range address (e.g. A1:C5) and press the update button to populate the grid containing parameters of genes.

Hint: To set the value of a whole column, click its heading with right mouse button and choose or type in the desired value.

Hybrid Integer Bounded Gene

The gene is an integer number (i.e.

Fig C6: Gene Range



Please enter the range or single cell where objective functions are stored, press the Update button and select the type of the objective.

Objectives Range: I19:I20 (e.g. A1 or C2:F5)

Cell	Objective Type	Objective Name
I19	Minimize	Total Cost
I20	Maximize	Reliability

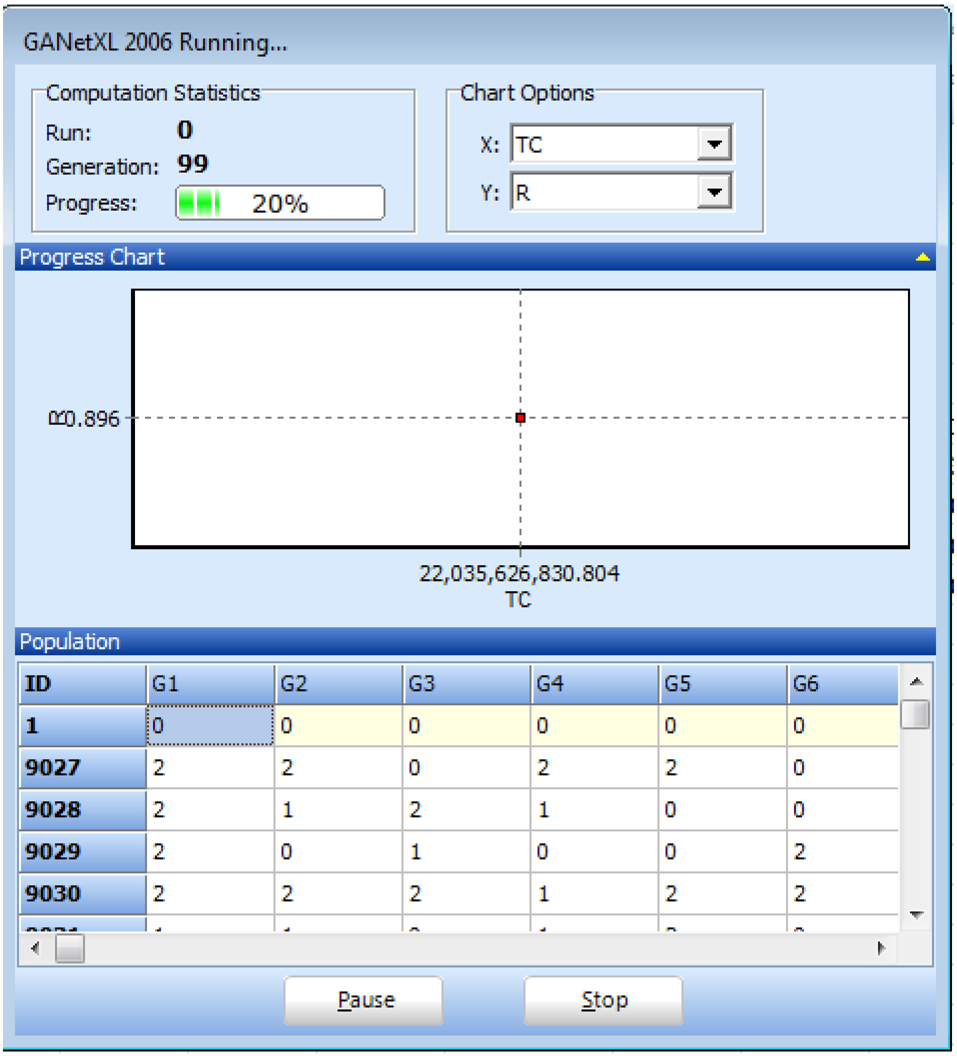
Objectives

Exactly one objective must be specified in *Single objective* version or at least two objectives in *Multiple objective* version. Objectives must occupy continuous range of cells. Type in the address of the range (e.g. A1:C5) and press the update button to populate the grid containing parameters of objectives.

Each objective is described by address of an Excel cell, the function that is applied to it and its name. Objective function can be either ***minimize*** or ***maximize***.

Hint: To set the value of a whole column, click its heading with right mouse button and choose or type in the desired value.

Fig C7: Objectives



Run: **0**
Generation: **99**
Progress: 20%

X: TC
Y: R

ID	G1	G2	G3	G4	G5	G6
1	0	0	0	0	0	0
9027	2	2	0	2	2	0
9028	2	1	2	1	0	0
9029	2	0	1	0	0	2
9030	2	2	2	1	2	2

Fig C8: Optimization Process

APPENDIX D

Genetic Algorithm Fitness Function Code for Matlab

```
N = 5;
T = 36;
generation_number = 500;
population_size = 1000;
p_selection = 0.20;
p_crossover = 0.40;
p_mutation = 0.30;
min = 0;
max = 2;
a = zeros(1,T*N);
initial_population = zeros(population_size,T*N+3);
for i = 1:1:population_size
for j = 1:1:T*N
a(j) = fix((max-min+1)*rand+min);
end
[Tcost,Reliability,fit1,reliability_schd,A,Aver_Rel] = Fitness(a);
initial_population(i,1:N*T) = a ;
initial_population(i,N*T+1:N*T+3) =[Tcost,Reliability,fit1];
end
population = initial_population;
for g = 1:1:generation_number
population_sorted = sortrows(population,N*T+3);
population_selected =population_sorted(1:fix(p_selection*population_size),:);
```

```
for i = 1:1:p_crossover*population_size

parent1 = population(fix((population_size)*rand+1),:);

parent2 = population(fix((population_size)*rand+1),:);

if parent1(:,N*T+3) ~= parent2(:,N*T+3)

offspring = NTpointcrossover(parent1,parent2);

elseif parent1(:,N*T+3) == parent2(:,N*T+3)

offspring = Tpointcrossover(parent1,parent2);

end

[Tcost,Reliability,fit1] = Fitness(offspring);

population_crossover(i,1:N*T) = offspring;

population_crossover(i,N*T+1:N*T+3) =[Tcost,Reliability,fit1];

end

for i = 1:1:p_mutation*population_size

individual = population(fix((population_size)*rand+1),:);

individual_mutated = Mutation(individual);

[Tcost,Reliability,fit1] =Fitness(individual_mutated);

population_mutation(i,1:N*T) = individual_mutated(:,1:N*T);

population_mutation(i,N*T+1:N*T+3) =[Tcost,Reliability,fit1];

end

population =[population_selected;population_crossover;population_mutation];

ss = sortrows(population,N*T+3);

solution_improvement(g,:) = ss(1:1,:);

end

last_population = sortrows(population,N*T+3);

final_solution = last_population(1:1,:);
```

```
PMR_Schedule = zeros(N,T);

for i = 1:1:N

for j = 1:1:T

PMR_Schedule(i,j) = final_solution(1,(i-1)*T+j);

end

end


N = 5;
T = 36;
J = 36;
L = T/J;
Lamda = [0.852 0.59 1.26 1.19 1.04];
Beta = [260.5 7.0 181.0 159.9 79.3];
Alpha = [0.7 0.7 0.7 0.7 0.7];

FailureCost = [450922 422670 450000 365910 560000];
MCost = [90188 32200 40000 40000 40000];
CRCost = [500000 490750 499000 310000 700000];

Downtime_Cost = 103200;
% Parameters of the multi-objective optimization model
% Weights of the objective functions in weighted method, W1+W2 = 1
%W1 = 0.0; W2 = 1.0;
%W1 = 0.1; W2 = 0.9;
%W1 = 0.2; W2 = 0.8;
%W1 = 0.3; W2 = 0.7;
%W1 = 0.4; W2 = 0.6;
W1 = 0.5; W2 = 0.5;
%W1 = 0.6; W2 = 0.4;
%W1 = 0.7; W2 = 0.3;
%W1 = 0.8; W2 = 0.2;
%W1 = 0.9; W2 = 0.1;
```

```
%W1 = 1.0; W2 = 0.0;
RR = 0.5;
A = zeros(N,T);
for i = 1:1:N
for j = 1:1:T
A(i,j) = a(1,(i-1)*T+j);
end
end
x = zeros(N,T);
for i = 1:1:N
for j = 1:1:T-1
if A(i,j) == 0
x(i,j+1) = x(i,j)+L;
elseif A(i,j) == 1
x(i,j+1) = Alpha(i)*(x(i,j)+L);
elseif A(i,j) == 2
x(i,j+1) = 0;
end
end
end
xp = x+L;
E = zeros(N,T);
for i = 1:1:N
for j = 1:1:T
E(i,j) = Lamda(i)*((xp(i,j)^Beta(i))-(0^Beta(i)));
end
end
Fcost = zeros(N,T);
for i = 1:1:N
for j = 1:1:T
Fcost(i,j) = Failure_Cost(i)*E(i,j);
end
end
cost = zeros (N,T);
```

```
for i = 1:1:N
for j = 1:1:T
if A(i,j)== 0
cost(i,j) = Fcost(i,j);
elseif A(i,j) == 1
cost(i,j) = Fcost(i,j)+M_Cost(i);
elseif A(i,j) == 2
cost(i,j) = Fcost(i,j)+CRCost(i);
end
end
end
Tcost = 0;
costm = zeros(1,T);
for j= 1:T
costm (j) = sum(cost(:,j));
if sum(A(:,j))>0
costm(j)= Opportunity_Cost+sum(cost(:,j));
Tcost = sum(costm);
end
end
Max_cost = 0;
xx = zeros(N,T);
costma = zeros (N,T);
for j = 1:1:T
for i = 1:1:N
xxp = xx+L;
costma (i,j) = (Failure_Cost(i)*(Lambda(i)*((xxp(i,j)^Beta(i))-(xx(i,j)^Beta(i)))))+CRCost(i);
end
end
costmaa = zeros(1,T);
for j= 1:T
costmaa(j)= Downtime_Cost+sum(costma(:,j));
Max_cost = sum(costmaa);
end
```

```
reliability_schd=zeros(N,T);
System_Reliability = zeros(1,T);
for j = 1:1:T
for i = 1:1:N
reliability_schd(i,j)= exp(-E(i,j));
end
System_Reliability(j) = prod(reliability_schd(:,j));
end
Reliability = prod(System_Reliability);
Aver_Rel = mean(System_Reliability);
fit1 = W1*(Tcost/Max_cost)+W2*(-Reliability);
```

APPENDIX E

Witness 14 Codes for Simulation

```
N = 5;
T = 36;
J = 36;
L = T/J;

t_initial = 1000000;
t_final = 0.01;
t_rate = 0.99;
min = 0;
max = 2;

a = zeros(1,T*N);
for j = 1:1:T*N
a(j) = fix((max-min+1)*rand+min);
end
[Tcost,Reliability,fit1,reliability_schd,A] = Fitness(a);
initial_solution(1,1:N*T) = a ;
initial_solution(1,N*T+1:N*T+3) = [Tcost,Reliability,fit1];
x = initial_solution;
t_current = t_initial;
i = 1;
while t_final <= t_current
% Transition procedure
y = Transition(x);
[Tcost,Reliability,fit1] = Fitness(y);
y(1,N*T+1:N*T+3) = [Tcost,Reliability,fit1];
if y(1,N*T+3) < x(1,N*T+3)
x = y;
elseif y(1,N*T+3) >= x(1,N*T+3)
if rand <= exp(-(y(1,N*T+3)-x(1,N*T+3))/t_current)
x = y;
```

```
end
end
ss = sortrows(solution_improvement,N*T+3);
final_solution = ss(1:1,:);
PMR_Schedule = zeros(N,T);
for i = 1:1:N
for j = 1:1:T
PMR_Schedule(i,j) = final_solution(1,(i-1)*T+j);
end
end
```