

CHAPTER ONE: GENERAL INTRODUCTION

1.0 Introduction

The analysis of contingency tables has been studied over a long period, the beginning of which is clearly marked by Pearson (1900)'s influential article at the turn of the century. Many authors such as Fisher (1922), Bartlett (1935), Goodman (1964), Neyman (1949), Cochran (1954), Roy and Kastenbaum (1956), Fienberg (1970), Haberman (1974), Plackett (1974) and Kritzer (1978) and others, made contributions to the contingency table literature. Applications of cross-classified data analyses can be found in among others Cox (1970), Aitkin (1979, 1980), Abrahamse, Iammerts and van Bueren (1979, 1980a.), Pannekoek (1980), Willekens (1980) and Fingleton (1981)

Contingency tables (Agresti, 1996) are formed when a population is cross-classified according to a series of categories (or factors). Each cell count of the contingency tables gives the number of units observed under a particular cross-classification. Additionally, q-contingency table is a contingency table created by cross-classification of more than two categorical variables. A categorical variable is one for which the measurement scale consists of a set of categories (Everitt, 2002). Categorical data are data consisting of counts of observation falling in different categories. Categorical data in contingency tables are collected in many investigations and to understand the type of structures prevailing in q-dimensional data appropriate log linear models are fitted.

Let X , Y and Z be three categorical variables, with I , J and K levels respectively. When we cross-classify objects according to these variables, there are N possible combinations of classifications (IJK). The variables X , Y , Z have a joint distribution. We display the distribution in an $I \times J \times K$ contingency table. The cells of the table represent the IJK possible outcomes and a cell probability is denoted by p_{ijk} , where p_{ijk} is the probability that variable X is in category i , variable Y in category j and variable Z in k . A table containing frequency counts is called a contingency table and p_{ijk} represents the joint probability distribution of X , Y and Z . The marginal distribution of a variable is the sum over the remaining variables, denoted by p_{i++} for the marginal distribution of X :

$$p_{i++} = \sum_{j=1}^J \sum_{k=1}^K p_{ijk}$$

Suppose we have observed counts $\{n_{ijk}, i \in \{1, \dots, I\}, j \in \{1, \dots, J\}, k \in \{1, \dots, K\}\}$ in the $IJK=N$ cells of the contingency table. It is assumed that the observed data has been generated by a multinomial model with probabilities p_{ijk} , meaning that the probability having observed counts n_{ijk} given that the total number of counts is fixed is given by

$$\left(\frac{n!}{\prod_{ijk} n_{ijk}!} \right) \prod_{ijk} p_{ijk}^{n_{ijk}} \quad \text{where } \sum_{ijk} n_{ijk} = n$$

With known distribution of the cells counts, we can estimate the cell probabilities by maximizing the log-likelihood function

$$l = \sum_{ijk} n_{ijk} \log(p_{ijk}) \quad (1.1)$$

It is easily to maximize (1.1) subject to the constraint $\sum p_{ijk} = 1$. The maximum is achieved at $p_{ijk} = \hat{p}_{ijk} = \frac{n_{ijk}}{n}$ is the vector of sample proportions.

In 20th century, the analysis of categorical data was very largely confined to its presentation as a series of two-way tables together with the accompanying values of some measure of association (Upton, 1980). This analysis of categorical data is open to criticism as result of the intrinsic lack of interpretability of many measures of association (Goodman and Kruskal, 1954) and because the collapse of q -way table to a two-way table may result in a misleading picture of the association present in the data (Simpson, 1951). The two-way table remains a favorite method of presenting data, and rightly so, since it provides easily understood information. However, analysis of such data is now performed using log-linear techniques.

Typically, Log linear models are used to model the observed cell count where the log of expected cell count is proportional to a linear combination involving number of model parameters. Each interaction term between two or more factors is associated with a set of model parameters and if these model parameters are non-zero it indicates that there exists an association between factors or variables. Every unique combination of variables (factors) defines its own log linear models.

A family of log linear models is often referred to as an exponentially family. If we consider n independent, identically distributed discrete random variables Y_1, \dots, Y_n with a common point probability: (Dobson, A.J)

$$f(y | \theta_1, \dots, \theta_k) = P(Y = y | \theta_1, \dots, \theta_k), \quad (1.2)$$

which depends on k real valued parameters $\theta_1, \dots, \theta_k$. The model:

$$f(y_1, \dots, y_n | \theta_1, \dots, \theta_k) = \prod_{i=1}^n f(y_i | \theta_1, \dots, \theta_k) \quad (1.3)$$

is called log linear model or is said to form an exponential family, if the logarithm of (1.3) has the functional form:

$$\ln f(y | \theta_1, \dots, \theta_k) = \sum_{j=1}^m g_j(y) \varphi_j(\theta_1, \dots, \theta_k) + h(y) - k(\theta_1, \dots, \theta_k) \quad (1.4)$$

$$\text{where } k(\theta_1, \dots, \theta_k) = \ln \left\{ \sum_y \exp \left(\sum_j g_j(y) \varphi_j(\theta_1, \dots, \theta_k) + h(y) \right) \right\} \quad (1.5)$$

Substitute (1.5) in (1.4), we have

$$\begin{aligned} \ln f(y | \theta_1, \dots, \theta_k) &= \sum_{j=1}^m g_j(y) \varphi_j(\theta_1, \dots, \theta_k) + h(y) - \\ &\quad \ln \left\{ \sum_y \exp \left(\sum_j g_j(y) \varphi_j(\bar{\theta}) + h(y) \right) \right\} \end{aligned} \quad (1.5.1)$$

where $\bar{\theta} = \theta_1, \dots, \theta_k$

let $\eta(y; \theta) = \sum_{j=1}^m g_j(y) \varphi_j(\bar{\theta}) + h(y)$ substitute in (1.6), we have

$$\ln f = \ln \left(\frac{e^{\eta(y; \bar{\theta})}}{\sum_y e^{\eta(y; \bar{\theta})}} \right)$$

$$\Rightarrow f = \frac{e^{\eta(y; \bar{\theta})}}{\sum_y e^{\eta(y; \bar{\theta})}}$$

$$\Rightarrow f = e^{\eta(y; \bar{\theta})} \quad \left(\text{where } \sum_y f(y | \bar{\theta}) = 1 \right)$$

$$f \in [0, 1] \forall \eta(y; \bar{\theta})$$

Thus, f is exponential p.m.f

The exponential family covers a large number of distributions namely Bernoulli, Binomial, Multinomial, Poisson, Normal, Gamma and Inverse Gaussian distribution. The Binomial, Multinomial and Poisson are mostly used for modeling the frequency of occurrence of a particular value of both nominal and ordinal variables. These three distributions belong to a broader class of exponential family. The family has the unifying property that the logarithm of the frequencies of occurrence of a particular value of the variable may be expressed as linear function of the parameters of the distribution. Such models for log frequencies are called log linear models. The log –linear model is a specialized case of generalized linear model and is commonly used for analyzing contingency tables that involves more than two variables, although it can be used to analyze two-way contingency tables (Knoke and Burke 1980). A log –linear model is applied to natural logarithm of expected frequencies (Jibasen and Lawal, 2004). Response observations in log linear modeling are counts having Poisson, Multinomial or Product Multinomial distribution (Lawal, 2003) .

Log-linear models are mostly used to determine whether there are any significant relationships in q-way contingency tables that have three or more categorical variables and/ or to determine if the distribution of the counts among the cells of a contingency table can be explained by the simpler, underlying structure unsaturated model (reduced model). The saturated model contains all the variables being analyzed and all possible interactions between the variables (Chan, 2003). The variables investigated by log linear models are all response variables. In other words there is no distinction made between response and explanatory variables. Indeed, the log-linear model is a special case of generalized linear models (Mc Cullagh and Nelder, 1989). For cell frequencies in contingency tables, the generalized linear models assume a Poisson distribution as a link function. Thus the log linear model are given by

$$\log m = X\mu$$

where X is a $q \times r$ design matrix μ is an $r \times 1$ parameter vector and m is the vector of expected frequencies.

1.1.0 HISTORICAL BACKGROUND

1.1.1 CONTINGENCY TABLES AND LOG LINEAR MODELS

The contingency used in connection with $s \times t$ table of cross-classified categorical data originated from Pearson (1900). He defined contingency to be any measure of the total deviation from independent probability. The contingency used is refers to the table of counts. Pearson (1900) laid the groundwork on contingency tables when he proposed chi-square test for comparing observed and expected (theoretical) frequencies. Pearson viewed contingency tables from testing hypothesis of

independence in 2x2 tables (Pearson (1904), Yule (1911), Fisher (1934) and Yates (1934)) to testing interaction across a strata of 2x2 tables (Bartlett, 1935). Many discussions had emerged in the literature to build up the foundation of the statistical inference of the categorical data analysis. From 1960's until the 1980's, Fisher exact test repeatedly received criticism due to discrete nature of the test (Berkson (1978) and Yates (1984)). Although the arguments in favor of the unconditional tests essentially focused on using the unconditional exact tests in the recent decades, the reasons for preferred p-values and sensitivity of the unconditional tests have not been assured in theory. In this respect, a recent approach via information theory proved that the power analysis of unconditional tests is not suitable for testing independence, but simply for equal and unequal binomial rates. Consequently, the long term ambiguous criticism of the exact test was finally proved to be logically vacuous (Cheng et al, 2007). In retrospect, it is seen that noncentral hypergeometric distributions cannot determine power evaluations at arbitrary alternatives 2x2 tables (Fisher, 1962)

Bartlett (1935) addressed the topic of testing interaction and derived an estimate of the common odd ratio. Norton (1945), Simpson (1951) and Roy and Kastenbaum (1956) discussed interpretations of interactions and showed that Bartlett's test is a conditional maximum likelihood estimation given the table margins. For the same data, the celebrated CMH test for association (Cochran (1954) and Mantel and Haenszel (1959) has been applied extensively in the fields of biology, medicine, engineering, education, psychology and sociology. However, it was implicit that an inferential flaw lies in the estimating –equation design of the CMH score test (Woolf, 1955; Goodman, 1969; Mellenberg, 1982). In addition the probability at alternate interactions given the observed data, that is the power analysis at alternatives to the null has been discussed in the literature. A remedy to statistical inference was recently provided by an analysis of invariant information identities (Cheng et al., 2007).

Analysis of variance (Fisher, 1925) inspired discussions of the partitioning chi-squares within the contingency tables (Mood, 1950; Lancaster, 1951; Snedecor, 1958; Claringbold, 1961). It inspired in turn the development of log linear models (Kullback, 1959; Darroch, 1962; Lewis, 1962; Birch, 1964 and Goodman, 1964). Hierarchical log linear models were formulated to analyze contingency tables (Goodman, 1970; Bishop *et al.*, 1975) and have been widely used in the literature (Hagenaars, 1993; Christensen, 1997; Agresti, 2002). Also, authors that discussed categorical data analysis making use of log linear models among which are (Haberman, 1978, 1979; Hagenaars, 1990; Laird and Oliver, 1981).

1.1.2 GENERALISED LINEAR MODELS

Generalized linear models are class of models that generalized the linear models for regression and analysis of variance. They allow for more general mean structures and more general distributions than regression and analysis of variance. Generalized linear models were first introduced by Nelder and Weddeburn (1972) and were extensively treated by McCullagh and Nelder (1989). Generalized linear model provides a general framework for handling statistical models for normal and discrete data. Generalized linear models include log linear models as a special case. Other special cases among which are Poisson regression and logistic regression.

Generalized linear model (GLM) is based on exponential family. GLMs have three components:

- a. Random component.
- b. Systematic component.
- c. Link function, g , which describes the relationship between the random and systematic components.

1.1.3 EXPONENTIAL FAMILY

Exponential family comprises a set of flexible distribution of both continuous and discrete random variables. The members of this family have many important properties which merits discussing them in some general format. Some of the probability distributions that are specific members of this family include: Multinomial, Poisson, Binomial, Normal and Gamma. The general format for the distribution due to Fisher (1922) is given as

$$p(\mathbf{y}/\boldsymbol{\eta}) = h(\mathbf{y}) \exp\{\boldsymbol{\eta}^T \mathbf{T}(\mathbf{y}) - A(\boldsymbol{\eta})\} \quad (1.6)$$

where

$\boldsymbol{\eta}$ is the natural parameter (exponential parameter), $\boldsymbol{\eta}$ specifies all the parameters needed for that distribution.

$\mathbf{T}(\mathbf{y})$ is the sufficient statistic of the data

$A(\boldsymbol{\eta})$ is the log partition function (normalization factor, cumulant generating function) which ensures that $p(\mathbf{y}/\boldsymbol{\eta})$ remains a probability distribution .

$h(\mathbf{y})$ is the non-negative base measure (always equal to one)

1.1 STATEMENT OF THE PROBLEM

Birch (1963) in his pioneer work described the conditions for the existence of maximum likelihood estimate in hierarchical log-linear models. Justyna (2012) implemented maximum likelihood method in estimating best model fit of hierarchical log-linear model for three dimensional contingency table. However, his work did not address the estimation of parameters and best model fits of q -dimensional contingency table ($q=4,5,6$) in log-linear model. Justyna's work (2012) used maximum likelihood method which was based on direct estimates and his work did not address higher dimensional contingency table. Also, Brown (1976) proposed a procedure for screening effects or test significance of interaction of variables. His procedure involves many parameter models in testing for the significance of interaction of variables. This consequently takes time in testing for the significance of interaction of variables especially in higher-dimensional contingency table. In a nutshell, the problem of estimating the parameters and best fits of log linear models for q -dimensional contingency table as well as procedure for testing the significance of interaction of variables which involves many parameters has been identified.

In this work, we intend to propose a new method of estimation of the parameters of log linear models for q - dimensional contingency table. Also, the estimates of the parameters and best model fits of log linear model for q - dimensional contingency table shall be obtained by iterative proportional fitting procedure which Justyna (2012) did not use. Furthermore, a new approach shall be proposed for testing the significance of interaction of variables which will involve fewer parameter models, saves memory and time.

1.2 AIM AND OBJECTIVES OF THE STUDY

The aim of this research work is to propose a new method of estimating parameters and best fits of log linear model for contingency tables.

The specific objectives of this research work are:

- 1) to estimate the parameters of log linear model for q-dimensional contingency table.
- 2) to estimate the best model fit of q-dimensional contingency tables in log-linear model.
- 3) to develop a computer programs in R for q-dimensional contingency table.
- 4) to test the significance of interaction of variables.
- 5) to compare the proposed method with the existing methods.

1.3 SIGNIFICANCE OF THE STUDY

This research work will reveal the new method of estimation of parameters of log linear model for q-dimensional contingency table. Also, algorithms that will be developed based on iterative proportional fitting procedure which shall be implemented in R shall be used to analyze q-dimensional contingency table data. This work will also be of immense importance to any researcher and statistician who intend testing the significance of interaction of variables using new approach proposed.

1.4 SCOPE OF THE STUDY

The scope covers the new method of estimation of parameters of log linear model; estimation of parameters and best model fits of hierarchical log linear model for q-dimensional contingency table and a new approach for testing the significance of interaction of variables.

1.5 DEFINITION OF TERMS

1.5.1 For given set of possible points $(x_i : i = 1, 2, \dots, n)$ the function f defined by

$f(\theta | \cdot) = L(\theta_1, \theta_2, \dots, \theta_k | x_1, x_2, \dots, x_n) = \prod_{i=1}^n f(\theta_1, \dots, \theta_k | x_i)$ is called the likelihood function for the distribution $f(\theta_1, \dots, \theta_k | x_i)$. Where x_1, \dots, x_n are the independent observations that are identically distributed. The maximum likelihood estimators $\hat{\theta}_i$ are the values of the parameters $\theta_1, \theta_2, \dots, \theta_k$ that optimizes $f(\theta)$ or equivalently, $\log f(\theta)$ (Ingram et al., 2003)

1.5.2 $f(\cdot | x_{i_1, i_2}, \dots, x_{i_k}) = L(\cdot | x_{i_1, i_2}, \dots, x_{i_k}) = \prod_{i=1}^q f(\cdot; x_{i_i})$ is a likelihood function for the multinomial distribution $f(\cdot; x_{i_i})$. For every $t \in [q]$, if we let $\lambda(x_{i_i}) = X_{i_i}$, then X_{i_i} is a random variable that count the occurrences of event $x_{i_i} \in X$ (i.e $X_{i_i} = \alpha_{i_i} : i = 1, \dots, K$) then

$$f(\cdot | \alpha_{i_1, i_2, \dots, i_q}) = \prod_{i=1}^q f(\cdot; \alpha_{i_i}) \quad (\text{Johndrow et al., 2014})$$

1.5.3 Let $M_{(q)}(\mathbf{P}, \alpha^{(q)}, \mathbf{X}^{(q)})$ denote a description on q-dimensional finite multiset $M_{(q)}(\alpha^{(q)}, \mathbf{X}^{(q)})$ with corresponding multiplicity $\alpha_{i_1, i_2, \dots, i_q}$ and probability p_{i_1, i_2, \dots, i_q} for x_{i_1, i_2, \dots, i_q} for each $x_{i_1, i_2, \dots, i_q} \in \mathbf{X}^{(q)}$, then the probability that $x_{i_1, i_2, \dots, i_q} \in \mathbf{X}^{(q)}$ is selected exactly $\alpha_{i_1, i_2, \dots, i_q}$ times ($i_t \in [k_t]; t \in [q]; k_t \in \mathbb{N}$) in n-trials is

$$p(X_{i_1, i_2, \dots, i_q} = \alpha_{i_1, i_2, \dots, i_q} : i_t \in [k_t]; t \in [q]; k_t \in \mathbb{N}) = \frac{n!}{\prod_{i=1}^{k_1} \dots \prod_{i=q-1}^{k_q} \alpha_{i_1, i_2, \dots, i_q}!} \prod_{i=1}^{k_1} \dots \prod_{i=q-1}^{k_q} P_{i_1, i_2, \dots, i_q}^{\alpha_{i_1, i_2, \dots, i_q}} \quad (\text{Okoli et al., 2015})$$

1.5.4 For arbitrary but fixed $q \in \mathbb{N}$ (natural number), Let $[q] = \{1, 2, \dots, q\}$ denote the set of q categorical variables. Let $\alpha^{(q)}$ be a q-dimensional array ; that is $k_1 \times k_2 \times \dots \times k_q$ contingency table with cell counts (frequencies) $\alpha_{i_1, i_2, \dots, i_q}$ ($i_t \in [k_t]; t \in [q]; k_t \in \mathbb{N}$). For any $t \in \mathbb{N}$ such that $1 \leq t \leq q$, then subset variable $[t] \subset [q]$ generate $\alpha^{(q)}$ -t-dimensional sub array with cell counts (frequencies $\alpha_{i_1, i_2, \dots, i_t} = \alpha_{i_t}$ ($t \leq q$)). Let $\mu_{\{t\}(i_t)}$ denote the interaction among the variables in the index subset $[t]$ of the t-dimensional sub table of $\alpha^{(q)}$ that correspond to (i_t) cell . we shall assume that $[t] = \emptyset$ if $t = 0$, so that we define $\mu_{\emptyset(i_\emptyset)} = \mu$.

1.5.5 A log linear model is to be hierarchical if for every $t \in \mathbb{N}$ such that $1 < t < q$ ($[t] \subset [q]$) for which $\mu_{\{t\}(i_t)} = 0$, then we have $\mu_{\{s\}(i_s)} = 0$ for all $s \geq t$ ($[t] \subset [s]$)

1.5.6 let $\mathbf{S}_{q,t}^c$ be the set of strings of t-combinations of elements of $[q]$ and $\mathbf{P}([q]) = 2^{[q]}$ denote the saturated parameter effect or the power set of $[q]$. Thus, for any $t \in \mathbb{N}$ such that $1 < t < q$ ($[t] \subset [q]$) we define

$$\mathbf{P}([q] : 0 \leq n(\{j\}) \leq t) := \{\{j\} \in \mathbf{P}([q]) : 0 \leq n(j) \leq t\} ; \quad (1.7)$$

Where $\mathbf{j} \in \mathbf{S}_{q,t}^c$ for $t = 0, 1, 2, \dots, q$ ($q \leq t$) with $\{\mathbf{j}\} = \emptyset$ if $t = 0$ and $n(\{\mathbf{j}\})$ denote the length of the string $\mathbf{j} \in \mathbf{S}_{q,t}^c$ or the level of interaction or the cardinality of set $\{\mathbf{j}\} \in \mathbf{P}([q])$. Observe that

$(\mathbf{P}[q]: 0 \leq n(\{\mathbf{j}\}) \leq t)$ is a subclass of $\mathbf{P}([q])$, however,

$\mathbf{P}([q]: 0 \leq n(\{\mathbf{j}\}) \leq t)$ is equal to the power set $\mathbf{P}([q])$ if $q = t$, Also, by this, is clear to see that

$$(\mathbf{P}[q]: 0 \leq n(\{\mathbf{j}\}) \leq q) := \bigcup_{t=0}^q \mathbf{P}([q]: n(\{\mathbf{j}\}) = t) = \bigcup_{t=0}^q \{\mathbf{j}\}_t ; \quad (1.8)$$

From (1.8), notice that $\mathbf{P}([q]: n\{\mathbf{j}\} = t)$ is structurally equal to the set of elements of $\mathbf{S}_{q,t}^c$.

As a result of above definitions, we shall replace

$\mu_{\{t\}(i_t)}$ by $\mu_{\{\mathbf{j}\}(i_j)}$ such that $n(\{\mathbf{j}\}) = t$

Also, if $n(\{\mathbf{j}\}) = t$, then $\{\mathbf{j}\} \in \{\mathbf{j}\}_t$

For arbitrary but fixed $q \in \mathbb{N}$ and let $\mu_{\{\mathbf{j}\}(i_j)}$ be as define above such that $\mathbf{j} \in \mathbf{S}_{q,p}^c$

$(\{\mathbf{j}\} \in \mathbf{P}([q]): n(\{\mathbf{j}\}) = p)$ then

$$\sum_{p=0}^q \sum_{1 \leq j_1 < \dots < j_p \leq q} \mu_{\{\mathbf{j}\}_p(i_j)} = \sum_{\{\mathbf{j}\} \in \mathbf{P}([q]: 0 \leq n(\{\mathbf{j}\}) \leq q)} \mu_{\{\mathbf{j}\}(i_j)}$$

1.5.7 Let $\alpha^{(q)}$ be a q -dimensional $k_1 \times k_2 \times \dots \times k_q$ contingency table with cell counts (frequencies) $\alpha_{i_1, i_2, \dots, i_q}$ ($i_t \in [k_t]; t \in [q]; k_t \in \mathbb{N}$), then the saturated log linear model for the q -way table of variables indexed in $[q]$ is given by

$$\log_e \left(\mu_{\{\mathbf{j}\}_q} \right) = \sum_{\{\mathbf{j}\} \in \mathbf{P}([q]: 0 \leq n(\{\mathbf{j}\}) \leq q)} \mu_{\{\mathbf{j}\}(i_j)}$$

1.5.8 Hierarchical log linear model: This is a model such that whenever a higher –order interaction is included, lower-order interaction composed of these variables must also be included. For example model (AC) implies A, C and AC terms present in the model.

1.5.9 Saturated model (full model): This is a model that contains main effects and all the interaction of the variables, It is equally the model that fully explain all the relationship among variables.

1.6.0 Unsaturated model: This is a model that does not contain all the main effects and all the interaction of the variables.

1.6.1 Parsimonious Model: It is describes as a model that has fewer /simpler parameters that fits the observed data or counts.

1.6.2 Goodness of fit test: This test is used to test for the adequacy of the model fit. That is the model that most adequately fit to the data. This model must come from unsaturated, restricted or reduced model.

1.6.3 Effect: There are two types of effects: main effects and interactions. A model is specified by the effects it contains. For instance, model (A, C, AC) includes the main effects of A and B and the effect of the AC interaction.

1.6.4 Model of order k: This is a model that contains all k factor interactions but not higher order interactions. For example model (AB, AC, AD, BC, BD, and CD) describes the model of order 2 in four –dimensional contingency table while model: (ABC, ABD, ACD, and BCD) is a model of order 3.

1.7 ORGANISATION OF THE WORK

This work is divided into five chapters. Chapter one is the general overview of the work General introduction; Chapter two is on literature review. Chapter three is the methodology of the work. Chapter four is analysis of data and discussion of results . Chapter five is on summary, conclusion, findings, contribution to knowledge, and recommendation for further studies.

CHAPTER TWO: LITERATURE REVIEW

2.0 INTRODUCTION

This chapter reviews the relevant literature on the work.

2.1 REVIEW OF RELATED LITERATURE

Birch (1963) described conditions for existence of maximum likelihood estimate (MLE) of the cell mean vector in log-linear models. In his work, he considered hierarchical log-linear models and showed that under the assumption $n > 0$, the maximum likelihood estimate of m exists uniquely and satisfies $p_{M^n} = p_{M^m}$, where p_M is the projection matrix onto the log-linear subspace M .

Rinaldo (2006) proposed computational strategies for extended maximum likelihood estimation for classes of log-linear models under Poisson and product-multinomial sampling schemes. He also derived numerically efficient procedures for generating design matrices.

Ayinde et al (2006) simplified the modified log-likelihood ratio test for testing about goodness of fit of multinomial probabilities in one, two and multidimensional contingency tables. Accordingly, the simplified version of the statistic does not require calculating expected cell frequency before the appropriate test of significance can be done. This makes the technique computationally easier and faster than existing method. Numerical examples were considered to illustrate their usage. Results compare favorably with Pearson, Neyman and likelihood ratio-chi square statistics in the literature.

Brian (2006) presented exact testing of goodness of fit for log-linear model using the R-package exact log-lin test. The exact log-lin test evaluated model fit for Poisson log-linear model by conditioning on minimal sufficient statistics to remove nuisance parameters. A Monte Carlo algorithm was proposed to estimate p-values from the conditional distribution. The result showed sequentially rounded normal approximation estimates.

Darroch (1962) compares interactions in contingency tables and analysis of variance. The result showed that the comparison led to a new formulation of the hypothesis of no second order interaction in three –way contingency table.

Goodman (1971) provided both forward and backward stepwise procedures in obtaining suitable model that will suit the data for the hierarchical log linear model in contingency table. The results shown that backward stepwise procedure is more reliable to be used in obtaining suitable model that fit the data.

Shaffer (1973) described the log linear model as a frame work for analyzing effects in contingency tables formed by two or three variables of classification and he considered variables to have nominal categories. He further considered two test procedures via maximum likelihood estimation of expected cell frequencies and chi-square tests based on logarithm of adjusted cell frequencies for testing the parameter of the model.

Anderson (1974) presented a survey on the theory of contingency tables and showed that estimation and performance of asymptotic as well as exact tests are simple if only the decomposable models are considered.

Brown (1976) proposed screening effects and the procedure utilized two tests: marginal and partial associations. The test of marginal association of a k-factor interaction is defined as the test that k- factor interaction is insignificant in the marginal table formed by these k-factors. Thus the test of marginal association between A and B is obtained by fitting the model (A, B) to the marginal table of variables A and B alone. This value is equivalently obtained using the difference between the goodness of fit tests of models (A, B) and (AB) from the original contingency table. The goodness of fit statistic is the likelihood ratio statistic given as

$\chi_{ML}^2 = 2 \sum f \log_e (f/F)$. The test of partial association of a k-factor interaction is defined as the difference between the tests of fit of the models of order k with and without the k-factor of interest. Thus, the partial association of AB-interaction in the four –way table is the difference between the goodness of fit of models (AB, AC, AD, BC, BD,CD) and (AC, AD, BC, BD,CD). The χ_{ML}^2 has an additive property under partitioning.

This is, if M_1 and M_2 are two models, M_1 is nested in or subset of M_2 , the hypothesis that the additional terms added to M_1 to obtain M_2 are equal to zero is tested using the statistic

$$\chi_{ML}^2 (M_1 | M_2) = \chi_{ML}^2 (M_1) - \chi_{ML}^2 (M_2).$$

Benedetti and Brown (1978) proposed strategies in building log-linear model in contingency table using either stepwise method or standardized estimates of the parameters of the saturated model.

Marascuilo (1987) applied log linear model on categorical data of cross-tabulated contingency table and the results obtained showed the findings of hypothesis testing and model building for the categorical data.

Elliott (1988) presented a procedure for substantively interpreting higher order interaction. This procedure is based on the analogy between log-linear analysis and the linear model.

Green (1988) emphasized on log linear analysis of cross classification tables with ordinal variables. He also compared chi square of independence with log linear model of independence. The result showed that the log linear model is more adequate for the test.

Haberman (2000) proposed a model that includes log linear model for analysis of incomplete tables. By use of co-ordinates free methods of linear algebra and differential calculus, complete minimal statistics and likelihood equations were derived. The maximum likelihood is shown to be unique if exists, and necessary and sufficient conditions are given for its existence.

Xintae *et al.* (2001) proposed a method for examining the parameters of the fitted log linear model in finding all the significant association patterns among the items and technique is experimentally evaluated with data set and series of synthetic data sets. The result showed that the technique is effective in finding interesting associations among the items involved.

Lahiri and Dhar (2006) discussed the log linear for contingency table where the cells values represent the frequency counts that follow an extended negative multinomial distribution. These parameters of the model are estimated by maximum likelihood method. A practical application of the log-linear model had also been demonstrated and result shown that the reduced model is appropriate at 1% level of significance.

Onder and Adiguzel (2010) applied log linear analysis method to occupational fatalities in the underground coal mines of Turkish Hardcoal Enterprises. The accidents records were evaluated and the main factors affecting the accidents were defined. The results found showed that the mostly affected job group by the fatality accidents was the production workers and these workers were mostly exposed to roof collapses and methane explosions.

Olmus(2012) presented a modeling effort in estimating the relationships between driver's fault and carelessness and the traffic variables. The result of the analysis showed that the model regarding these variables was log-linear model. Also, the associations of the factors with the accident severity and the contributions of the various factors and interactions between these variables were assessed. The obtained results provided valuable information in regard to preventing undesired consequences of traffic accidents.

Justyna (2012) implemented maximum likelihood method in estimating the goodness of fits and best fit of log linear models for three dimensional contingency table. The results showed that the best model fit is homogeneous model which fits the observed data well. The model is showed as $([WE][WS][ES])$ as the best model fit to the data. This implies that the model has fewer parameters to explain the observed data without loss of information.

2.2 LOG LINEAR MODELING FOR THE g x h TABLES

If a sample of n observations is categorized to two categorical variables, one having g levels and the other having h levels, the resulting frequencies can be displayed in an g x h contingency table (Billingsley, 1991)

Table1: g x h contingency table

Level of X	Level of Y					Total
	1	2	.	.	h	
1	n_{11}	n_{12}	...		n_{1h}	n_{1+}
2	n_{21}	n_{22}	...		n_{2h}	n_{2+}
.
.
.
g	n_{g1}	n_{g2}	...		n_{gh}	n_{g+}
Total	n_{+1}	n_{+2}	...		n_{+h}	n

The saturated (full) model for 2x2 tables is

$$\log m_{ij} = \mu + \mu_{A(i)} + \mu_{B(j)} + \mu_{AB(ij)} \quad i=1, \dots, g, \quad j=1, \dots, h \quad \text{where A and B are variables.}$$

where $m_{ij} = n\pi_{ij}$ is the expected frequency in the (i, j) cell. The parameter μ is fixed by the sample size n and model has g+h+gh parameters $\mu_{A(i)}$, $\mu_{B(j)}$ and $\mu_{AB(ij)}$.

The sum to zero constraints

$$\sum_{i=1}^g \mu_{A(i)} = 0 \quad \sum_{j=1}^h \mu_{B(j)} = 0 \quad = \sum_{i=1}^g \mu_{AB(ij)} = \sum_{j=1}^h \mu_{AB(ij)} = 0$$

Implies $(g-1) + (h-1) + (g-1)(h-1) = gh-1$ parameters and zero degree of freedom for testing lack of fit. Denoting $\hat{m}_{ij} = \frac{n_{i+}n_{+j}}{n}$, the likelihood ratio statistic $G^2 = 2 \sum_{i=1}^g \sum_{j=1}^h n_{ij} \log \left(\frac{n_{ij}}{\hat{m}_{ij}} \right)$ test the null hypothesis $H_0 : \mu_{AB(ij)} = 0$, for $i=1, \dots, g-1, j=1, \dots, h-1$. Under the null hypothesis of independence, G^2 has an approximate chi-square distribution with $(g-1)(h-1)$ degree of freedom. If H_0 is true, the reduced model $\log m_{ij} = \mu + \mu_{A(i)} + \mu_{B(j)}$ is the model of independence of A and B. This model has $(g-1)(h-1)$ linearly independent μ parameters and $(g-1)(h-1)$ degree of freedom for lack of fit (Haberman, 1974).

Theorem: 2.1

G^2 has approximately χ^2 distribution with $(g-1)(h-1)$ degree of freedom

Proof: Wilks (1962)

$$\chi^2 = \sum_{ij} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

$$\text{But } p_{ij} = \frac{n_{ij}}{n}, \quad E_{ij} = np_{ij}$$

$$p_{i.} = \frac{n_{i.}}{n}; \quad p_{.j} = \frac{n_{.j}}{n}$$

$$E_{ij} = np_{i.}p_{.j}$$

$$\text{So, } E_{ij} = n \left(\frac{n_{i.}}{n} * \frac{n_{.j}}{n} \right) = \frac{n_{i.}n_{.j}}{n}$$

$$\text{So, } \chi^2 = \sum_{ij} \frac{\left(O_{ij} - \frac{n_{i.}n_{.j}}{n} \right)^2}{\frac{n_{i.}n_{.j}}{n}}$$

Consider Taylor's expansion of $x \log\left(\frac{x}{\alpha}\right)$ about $x = \alpha$

$$x \log\left(\frac{x}{\alpha}\right) = (x-\alpha) + \frac{(x-\alpha)^2}{2\alpha} + \frac{(x-\alpha)^3}{6\alpha} + \dots$$

Now put $x=n_{ij}$ and $\alpha = \frac{n_i \cdot n_j}{n}$ so that

$$n_{ij} \log\left(\frac{n_{ij} n}{n_i \cdot n_j}\right) = n_{ij} - \frac{n_i \cdot n_j}{n} + \frac{\left(n_{ij} - \frac{n_i \cdot n_j}{n}\right)^2}{2 \frac{n_i \cdot n_j}{n}} + \dots$$

Thus,

Summing through, we have

$$\sum_{ij} n_{ij} \log\left(\frac{n_{ij} n}{n_i \cdot n_j}\right) = n - n \sum_i \frac{n_i}{n} \sum_j \frac{n_j}{n} + \frac{1}{2} \sum_{ij} \frac{(O_{ij} - E_{ij})^2}{E_{ij}} + \dots; \quad \frac{1}{2} \chi^2$$

$$G^2; \quad \frac{1}{2} \chi^2$$

$$\text{Or } 2 \sum_{ij} n_{ij} \log\left(\frac{n_{ij} n}{n_i \cdot n_j}\right); \quad \chi^2$$

Thus, $G^2; \chi^2$

When testing contingency tables, degree of freedom of the resulting χ^2 distribution is given by

$$\text{d.f} = gh - 1 - (g-1) - (h-1)$$

$$= gh - g - h + 1$$

$$= (g-1)(h-1)$$

Log linear models can be used to analyze the association or relationship between two categorical variables (two- dimensional contingency tables), though they are mostly used to evaluate q-dimensional contingency tables that involves three or more categorical variables. In other words there is no distinction made between response and explanatory variables. Therefore log linear models demonstrate association between variables. They model the means of cell counts in contingency tables by describing the association patterns among a set of categorical variables. Log-linear methodology is appropriate when there is no clear distinction between response and explanatory variables. That is when all of the variables are observed simultaneously. The variables investigated by log-linear models are treated as response variables (Knoke and Burke 1980). The term log-linear comes from the form of the model; the natural logarithm of cell counts is modeled as a linear function of the effects of categorical variables and their interactions.

For example, suppose that we want to investigate relationships between three categorical variables, A, B and C, where A has I categories, B has J categories and C has K categories. Then the saturated log-linear model is Christensen (1997)

$$\log(m_{ijk}) = \mu + \mu_{A(i)} + \mu_{B(j)} + \mu_{C(k)} + \mu_{AB(ij)} + \mu_{AC(ik)} + \mu_{BC(jk)} + \mu_{ABC(ijk)} \dots \dots \dots (1)$$

For each combination of the levels $i = 1, 2, \dots, I$, $j = 1, 2, \dots, J$ and $k = 1, 2, \dots, K$ of the categorical variables A, B and C. In many situations, simpler models, containing a subset of the parameters from a saturated model may be adequate.

2.3 HIERARCHICAL APPROACH TO LOG-LINEAR MODELING FOR 3-WAY TABLE.

If there is no three-factor interaction or homogeneous association model, then the model is

$$\log(m_{ijk}) = \mu + \mu_{A(i)} + \mu_{B(j)} + \mu_{C(k)} + \mu_{AB(ij)} + \mu_{AC(ik)} + \mu_{BC(jk)} \quad (2)$$

and it implies that the conditional odds ratios between any two variables are the same at each level of the third variable.

If A and B are conditionally independent given C , then the following model describes the relationships between A, B and C .

$$\log(m_{ijk}) = \mu + \mu_{A(i)} + \mu_{B(j)} + \mu_{C(k)} + \mu_{AC(ik)} + \mu_{BC(jk)} \quad (3)$$

If A and C are associated but are independent of B respectively, then the following model describes the relationships between A, B and C .

$$\log(m_{ijk}) = \mu + \mu_{A(i)} + \mu_{B(j)} + \mu_{C(k)} + \mu_{AC(ik)} \quad (4)$$

If all the three categorical variables are mutually independent, then the following model will describes the relationship between A, B and C .

$$\log(m_{ijk}) = \mu + \mu_{A(i)} + \mu_{B(j)} + \mu_{C(k)} \quad (5)$$

Hierarchical log-linear models are models such that whenever a higher-order interactions is included, lower-order interactions composed of these variables must also be included. (Sharma, 1996). For example models above are all hierarchical models. The first order interaction $\mu_{AC(ik)}$ in model (4) forces the main effects $\mu_{A(i)}$ and $\mu_{C(k)}$ to be contained in the model as well. In hierarchical models, a symbol that lists the highest-order terms for each variable is assigned. These are called generators of the model. For example the symbol for hierarchical model in (3) is (AC, BC) . Non-hierarchical models should not be entertained because they do not provide statistical procedure for choosing among potential models (Jeansome, 2002).

2.4 Summary of hierarchy of models for 3-way contingency tables (Daniel and Wood, 1971)

3-way Association $\boxed{[ABC]}$

Homogeneous Association $\boxed{[AB, AC, BC]}$

Conditional Independence $\boxed{[AB,AC]} \boxed{[AB,BC]} \boxed{[AC,BC]}$

Joint Independence $\boxed{[AB,C]} \boxed{[AC,B]} \boxed{[BC,A]}$

Complete Independence $\boxed{[A, B, C]}$

Any model that lies below a given model is a special case of the more complex model(s).

Where A, B and C are variables (factors)

Given a three dimensional contingency table the model assume a sample size n distributed over

I J K = N cells. Under sampling , the probability that an observation (A,B,C) will fall into cell is then π_{ijk} for all $i= 1,2,\dots,I, j=1,2,\dots,J, k= 1,2,\dots,K$. The expected value m_{ijk} is $m_{ijk} = n\pi_{ijk}$.

Mutual independence of three variables I is equivalent to

$$\pi_{ijk} = \pi_{i++}\pi_{+j+}\pi_{++k} \quad \text{and} \quad m_{ijk} = n\pi_{i++}\pi_{+j+}\pi_{++k} ,$$

Thus, $\log m_{ijk} = \log(n) + \log(\pi_{i++}) + \log(\pi_{+j+}) + \log(\pi_{++k})$. (Jeffreys, 1961), which is equivalent to $\log m_{ijk} = \mu + \mu_{A(i)} + \mu_{B(j)} + \mu_{C(k)}$

Where

μ = the grand mean or overall mean

$\mu_{A(i)}$ = the main effect of the row variable A

$\mu_{B(j)}$ = the main effect of the column variable B

$\mu_{C(k)}$ = the main effect of the layer variable C

The parameters $[\mu_{A(i)}], [\mu_{B(j)}]$ and $[\mu_{C(k)}]$ satisfy

$$\sum_i \mu_{A(i)} = \sum_j \mu_{B(j)} = \sum_k \mu_{C(k)} = 0$$

2.5 FITTING THE LOG-LINEAR MODEL

Once a model has been chosen from a contingency of categorical variables, the expected frequencies are obtained. Fitting a log linear model is a process of deciding which associations are significantly different from zero; the corresponding interaction terms are included in the final model used to explain the observed frequencies. Terms which are excluded from the model go into the residual which reflects the overall lack of fit of the model. The goal of log-linear modeling is to find a fewer model (with few terms) which achieves a reasonable fit.

2. 6 MODEL SELECTION/ INFERENCE FOR LOG LINEAR MODELS

2.6.1 GOODNESS OF FIT OF THE LOG-LINEAR MODEL

After fitting the model, it is very important to make a decision about which particular model provides the best fit. Basically, we use the Pearson chi- square test statistic χ^2 or the likelihood ratio G^2 to test a model fit. According to Christensen (1997), the χ^2 and G^2 for 3-dimensional contingency table

are $\chi^2 = \sum_{ijk} \frac{(n_{ijk} - m_{ijk})^2}{m_{ijk}}$ and $G^2 = 2 \sum_{ijk} n_{ijk} \log \left(\frac{n_{ijk}}{m_{ijk}} \right)$ respectively. (6)

Both statistics are asymptotically distributed as chi-square, with n-p degrees of freedom, where n is the number of cells and p is the number of independent parameters fitted.

Larger G^2 values indicate that the model does not fit the data well and thus, the model should be rejected. But in finding the best fit of log-linear model to describe data, a low G^2 value is used. (Knoke and Burke, 1980). The best model fit to the data should have highest p-value and least likelihood ratio estimates among the models

2.6.2 Information Criteria Measures of Model Fit

The best known information criteria are the Akaike information criterion and (AIC) (Akaike 1987) and the Bayesian information criterion (BIC) (Raftery 1993). The definitions of these measures are given as follows:

The Akaike information criterion(AIC) is an estimator of the relative quantity of statistical models for a given set of data. Given a collection of models for the data, AIC estimates the quantity of each model, relative to each of the other models. Thus,AIC provides a means for model selection . Also, given a set of candidate models for the data, the preferred model is one with minimum AIC value.

$$AIC = G^2 - 2df \quad (7)$$

The Bayesian information criterion (BIC) is a criterion for model selection among a finite set of models; the model with the lowest BIC is preferred.

$$BIC = G^2 - (\log n) df \quad (8)$$

Where n and df are total sample size and degree of freedom respectively.

G^2 is the likelihood ratio.

2.7 Summary of Justyna's work (2012) and Brown's work (1976)

Justyna (2012) applied maximum likelihood method in estimating best model fit for three dimensional contingency table data. The maximum likelihood method applied has direct estimates for the fitted value and is only three dimensional contingency table that has direct estimation formula. His work did not handle higher-dimensional contingency table.

The direct estimation formula for three dimensional contingency table data is as follows:

MLE of expected values (\hat{m}_{ijk}) of mutual complete independence

$$\hat{m}_{ijk} = \frac{n_{i.} n_{.j} n_{..k}}{n_{...}^2}$$

MLES of expected values of partial independence models:

$$\hat{m}_{ijk} = \frac{n_{.jk} n_{i.}}{n_{...}}$$

$$\hat{m}_{ijk} = \frac{n_{i.k} n_{.j}}{n_{...}}$$

$$\hat{m}_{ijk} = \frac{n_{ij.} n_{..k}}{n_{...}}$$

MLES of expected values of conditional independence models:

$$\hat{m}_{ijk} = \frac{n_{ij.} n_{i.k}}{n_{i.}} ; \hat{m}_{ijk} = \frac{n_{.jk} n_{ij.}}{n_{.j}} ; \hat{m}_{ijk} = \frac{n_{i.k} n_{.jk}}{n_{..k}}$$

MLE of expected values of saturated model:

$$\hat{m}_{ijk} = n_{ijk}$$

This research work is based on estimation of parameters of log linear models for q-dimensional contingency table and the estimates of the parameters and best model fit of higher-dimensional contingency tables shall be found using new algorithms on iterative proportional fitting procedure.

Also, Brown (1976) proposed a procedure for screening effects or test significance of interaction of variables. His procedure involves many parameter models in testing for the significance of interaction of variables. This consequently takes time in testing for the significance of interaction of variables especially in higher-dimensional contingency table. A new approach is proposed.

Brown's Method is given as

$$\chi_{ML}^2 (M_1 | M_2) = \chi_{ML}^2 (M_1) - \chi_{ML}^2 (M_2) \quad (9)$$

2.9 ITERATIVE PROPORTIONAL FITTING PROCEDURE

The iterative proportional fitting procedure (IPFP) is used when reliable estimates for a desired cross-classification cannot be obtained directly, but estimates of the variable of interest are available at a higher level of aggregation. Also, Information on the relationship between the variables is available at the desired level of cross-classification. The iterative proportional fitting procedure (IPFP) was introduced by Deming and Stephan (1940) as a method for estimating cell frequencies or probabilities, p_{ij} in $r \times c$ table based on observations, where the marginal probabilities

$$p_{i.} = \sum_{j=1}^c p_{ij} \quad (i = 1, 2, \dots, r)$$

$$p_{.j} = \sum_{i=1}^r p_{ij} \quad (j = 1, 2, \dots, c)$$

are known and fixed. They proposed IPFP as a way of arriving at estimates which minimized

$$\sum_{i=1}^r \sum_{j=1}^c (n_{ij} - np_{ij})^2 / n_{ij} \text{ subject to the marginal totals, assuming } n_{ij} > 0.$$

The Deming and Stephan IPFP run as follows:

- i) Suppose that there are n_{ij} observations in the (i, j) cells, where

$$\sum_{i=1}^r \sum_{j=1}^c n_{ij} = n, \quad \text{and we take our initial values}$$

$$p_{ij}^{(0)} = n_{ij} / n \quad \forall i, j.$$

- ii) At the $(2m)$ th step ($m \geq 1$) we take

$$p_{ij}^{(2m-1)} = p_{ij}^{(2m-2)} \frac{p_{i.}}{p_{i.}^{(2m-2)}} \quad \forall i, j.$$

iii) At the $(2m+1)$ th step we take

$$p_{ij}^{(2m)} = p_{ij}^{(2m-1)} \frac{P_{.j}}{p_{.j}^{(2m-1)}} \quad \forall i, j.$$

iv) The iteration continued until difference between two successive set of values for the cell probabilities becomes sufficiently small, consequently the iteration stopped. Mathematically, this means the iteration has converged.

CHAPTER THREE: RESEARCH METHODOLOGY

3.0 INTRODUCTION

This chapter involves proposed method of estimation of parameters for q-dimensional contingency table, algorithms for estimating the parameters and best model fits of log linear models for q-dimensional contingency tables. Also, Iterative proportional fitting procedure will be used to estimate parameters and best model fits. Furthermore, proposed approach for testing the significance of interaction of variables will equally be discussed.

3.1 PROPOSED METHOD OF ESTIMATION OF PARAMETERS OF LOG LINEAR MODEL FOR q -dimensional CONTINGENCY TABLES

Theorem 3.1:

$$\hat{\mu}_{\{j_t\}(i_{j_t})} = \log_e \left(\frac{\sum_{i_{j_q} \setminus i_{j_t}} \alpha_{(i_{j_t}, +)}}{\mathbf{I}_{i_{j_q}} \setminus \mathbf{I}_{i_{j_t}}} \right) - \left(\sum_{p=0}^{t-1} \sum_{\{j_p\} \in \mathcal{P}(\{j_t\}; n(\{j_p\})=p)} \hat{\mu}_{\{j_p\}(i_{j_p})} \right); \quad (10)$$

$$\{j_t\} \in \mathcal{P}([q]; n(\{j_t\})=t) \quad t=1,2,\dots,q$$

Proof:

Let $f(\cdot | \alpha_{i_q})$ or $f(\cdot | x_{i_1, i_2, \dots, i_q})$ be the product- multinomial probability function associated with q-dimensional contingency table $\alpha^{(q)}$ in $M_{(q)}(p, \alpha^{(q)}, X^{(q)})$

$$\text{Then } f(\cdot | \alpha_{i_q}) = \binom{n}{\alpha^{(q)}} \mathbf{P}^{\alpha^{(q)}} \quad \text{or } f(\cdot | \alpha_{i_1, i_2, \dots, i_q}) = \left(\binom{n}{\alpha_{i_q}} \right) \prod_{i_q} \mathbf{P}_{i_q}^{\alpha_{i_q}}$$

Where $\alpha^{(q)} = (\alpha_{i_q})$ and $\mathbf{P}^{\alpha^{(q)}} = \prod_{i_q} \mathbf{p}_{i_q}^{\alpha_{i_q}} = \prod_{i_q=1}^{k_1} \dots \prod_{i_q=1}^{k_q} \mathbf{p}_{i_1, i_2, \dots, i_q}^{\alpha_{i_1, i_2, \dots, i_q}}$

Now, for arbitrary but fixed $q \in \mathbb{Y}$, let μ_{i_q} be the expected cell count with corresponding (expected) probability π_{i_q} , then we have $\mu_{i_q} = n\pi_{i_q}$ for $n = \sum_{i_q} \alpha_{i_q}$

$$\text{Where } \sum_{i_q} = \sum_{i_q=1}^{k_1} \dots \sum_{i_q=1}^{k_q}$$

Then we seek to solve the problem of

$$\text{Maximize } f(\cdot | \alpha_{i_1, i_2, \dots, i_q}) = \left(\binom{n}{\alpha_{i_q}} \right) \prod_{i_q} \pi_{i_q}^{\alpha_{i_q}}$$

$$\text{Subject to: } \mu_{i_q} = n\pi_{i_q} \quad (10a)$$

Equivalently, we consider;

$$\text{Maximize } \ln f(\cdot | \alpha_{i_1, i_2, \dots, i_q}) = \ln \left(\left(\binom{n}{\alpha_{i_q}} \right) \prod_{i_q} \pi_{i_q}^{\alpha_{i_q}} \right)$$

$$\text{Subject to: } \mu_{i_q} = n\pi_{i_q} \quad (10b)$$

Now, observe that

$$\begin{aligned}
\ln f(\cdot | \alpha_{i_1, i_2, \dots, i_q}) &= \ln \left\{ \binom{n}{\alpha_{i_q}} \right\} + \ln \left\{ \prod_{i_q} \pi_{i_q}^{\alpha_{i_q}} \right\} \\
&= \ln \left\{ \binom{n}{\alpha_{i_q}} \right\} + \sum_{i_q} \alpha_{i_q} \ln \pi_{i_q} = \ln \left\{ \binom{n}{\alpha_{i_q}} \right\} + \sum_{i_q} \alpha_{i_q} \ln \left(\frac{\mu_{i_q}}{n} \right) \\
&= \ln \left\{ \binom{n}{\alpha_{i_q}} \right\} + \sum_{i_q} \alpha_{i_q} (\ln \mu_{i_q} - \ln n) = \ln \left\{ \binom{n}{\alpha_{i_q}} \right\} + \sum_{i_q} \alpha_{i_q} \ln \mu_{i_q} - \ln n \sum_{i_q} \alpha_{i_q} \\
&= \ln \left\{ \binom{n}{\alpha_{i_q}} \right\} + \sum_{i_q} \alpha_{i_q} \ln \mu_{i_q} - n \ln n
\end{aligned}$$

from our definition, we have

$$\begin{aligned}
\ln f(\cdot | \alpha_{i_1, i_2, \dots, i_q}) &= \ln \left\{ \binom{n}{\alpha_{i_q}} \right\} + \sum_{i_q} \alpha_{i_q} \left(\sum_{\{j\} \in \mathcal{P}([q]: 0 \leq n(\{j\}) \leq q)} \mu_{\{j\}}(i_j) \right) - n \ln n \\
&= \ln \left\{ \binom{n}{\alpha_{i_q}} \right\} + \sum_{i_q} \alpha_{i_q} \left(\sum_{p=0}^q \sum_{j \in \mathcal{S}_{q,p}^c} \mu_{\{j\}}(i_j) \right) - n \ln n
\end{aligned}$$

Using the Constrain condition which implies $\sum_{i_q} \mu_{i_q} = n$. Now we construct appropriate

Lagrangian function G for the equivalent maximization problem, so that we have

$$\begin{aligned}
G(\mu_{\{j\}}(i_j); \lambda) &= \ln f(\mu_{\{j\}}(i_j) | \alpha_{i_1, i_2, \dots, i_q}) + \lambda \left(n - \sum_{i_q} \mu_{i_q} \right) \\
&= \ln \left\{ \binom{n}{\alpha_{i_q}} \right\} + \sum_{i_q} \alpha_{i_q} \left(\sum_{\{j\} \in \mathcal{P}([q]: 0 \leq n(\{j\}) \leq q)} \mu_{\{j\}}(i_j) \right) - n \ln n + \lambda \left(n - \sum_{i_q} \mu_{i_q} \right) \\
&= \ln \left\{ \binom{n}{\alpha_{i_q}} \right\} + \sum_{i_q} \alpha_{i_q} \left(\sum_{\{j\} \in \mathcal{P}([q]: 0 \leq n(\{j\}) \leq q)} \mu_{\{j\}}(i_j) \right) - n \ln n + \lambda \left(n - \sum_{i_q} \exp \left(\sum_{\{j\} \in \mathcal{P}([q]: 0 \leq n(\{j\}) \leq q)} \mu_{\{j\}}(i_j) \right) \right)
\end{aligned}$$

By optimality condition, we have that for $0 \leq t \leq q$ and $j \in \mathcal{S}_{q,t}^c$

$$\frac{\partial G}{\partial \mu_{\{j\}}[i_j]} = \sum_{i_q \setminus i_j} \alpha_{i_q} - \lambda \sum_{i_q \setminus i_j} \exp \left(\sum_{\{j\} \in \mathcal{P}([q]: 0 \leq n(\{j\}) \leq q)} \mu_{\{j\}}[i_j] \right) = 0 \quad (10c)$$

and

$$\frac{\partial G}{\partial \lambda} = n - \sum_{i_q} \exp \left(\sum_{\{j\} \in \mathcal{P}([q]; 0 \leq n(\{j\}) \leq q)} \mu_{\{j\}[i_j]} \right) = 0 \quad (10d)$$

Solving equation (10c) and (10d) we have that $\lambda = 1$, so that

$$\begin{aligned} & \sum_{i_q \setminus i_j} \exp \left(\sum_{\{j\} \in \mathcal{P}([q]; 0 \leq n(\{j\}) \leq q)} \hat{\mu}_{\{j\}[i_j]} \right) = \sum_{i_q \setminus i_j} \alpha_{i_q} \\ \Rightarrow & \sum_{i_q \setminus i_j} \exp \left(\sum_{\{j\} \in \mathcal{P}([q]; 0 \leq n(\{j\}) \leq t-1)} \hat{\mu}_{\{j\}[i_j]} \right) \exp \left(\sum_{\{j\} \in \mathcal{P}([q]; 0 \leq n(\{j\}) = t} \hat{\mu}_{\{j\}[i_j]} \right) \exp \left(\sum_{\{j\} \in \mathcal{P}([q]; t+1 \leq n(\{j\}) \leq q} \hat{\mu}_{\{j\}[i_j]} \right) = \sum_{i_q \setminus i_j} \alpha_{i_q} \\ \Rightarrow & \exp \left(\sum_{\{j\} \in \mathcal{P}([q]; 0 \leq n(\{j\}) = t} \hat{\mu}_{\{j\}[i_j]} \right) = \\ & \frac{\sum_{i_q \setminus i_j} \alpha_{i_q}}{\sum_{i_q \setminus i_j} \exp \left(\sum_{\{j\} \in \mathcal{P}([q]; 0 \leq n(\{j\}) \leq t-1} \hat{\mu}_{\{j\}}(i_j) \right) \exp \left(\sum_{\{j\} \in \mathcal{P}([q]; t+1 \leq n(\{j\}) \leq q} \hat{\mu}_{\{j\}}(i_j) \right)} \\ = & \frac{\sum_{i_q \setminus i_j} \alpha_{i_q}}{\sum_{i_q \setminus i_j} \exp \left(\sum_{(\{j\}_p; t+1 \leq p \leq q} \hat{\mu}_{\{j\}}(i_j) \right) \exp \left(\sum_{(\{j\}_p; 0 \leq p \leq t-1} \hat{\mu}_{\{j\}}(i_j) \right)} \end{aligned}$$

$$= \frac{\sum_{i_q \setminus i_j} \alpha_{i_q}}{\sum_{i_q \setminus i_j} \exp \left(\sum_{(\{j\}_p: 0 \leq p \leq q \setminus \{j\}_t: 0 \leq t \leq t)} \hat{\mu}_{\{j\}}(i_j) \right)} \exp \left(\sum_{(\{j\}_p: 0 \leq p \leq t-1)} \hat{\mu}_{\{j\}}(i_j) \right)$$

Hence we have

$$\hat{\mu}_{\{j\}[i_j]} \mid \left(\{j\} \in \mathcal{P}([q]: n(\{j\}) = t) \right)$$

$$= \log_e \left(\frac{\sum_{i_q \setminus i_j} \alpha_{(i_j, +)}}{\sum_{i_q \setminus i_j} \exp \left(\sum_{(\{j\}_p: 0 \leq p \leq q \setminus \{j\}_p: 0 \leq p \leq t)} \hat{\mu}_{\{j\}[i_j]} \right)} \right) - \left(\sum_{\{j\}_p: 0 \leq p \leq t-1} \hat{\mu}_{\{j\}[i_j]} \right)$$

Since, $n(\{j\}) = t$, we write;

$$\hat{\mu}_{\{j_t\}(i_{j_t})} = \log_e \left(\frac{\sum_{i_q \setminus i_{j_t}} \alpha_{(i_{j_t}, +)}}{\mathbf{I}_{i_q} \setminus \mathbf{I}_{j_t}} \right) - \left(\sum_{p=0}^{t-1} \sum_{\{j_p\} \in \mathcal{P}(\{j_t\}, n(\{j_p\}) = p)} \hat{\mu}_{\{j_p\}(i_{j_p})} \right);$$

$$\left(\{j_t\} \in \mathcal{P}([q]: n(\{j_t\}) = t) \text{ if } t = 1, 2, \dots, q \right)$$

and

$$\hat{\mu}_{\{j_t\}(i_{j_t})} = \log_e \left(\frac{\sum_{i_q \setminus i_{j_t}} \alpha_{i_{j_t}, +}}{\mathbf{I}_{i_q} \setminus \mathbf{I}_{j_t}} \right); \text{ if } t = 0$$

$$\text{where } \sum_{i_q \setminus i_{j_t}} \alpha_{(i_{j_t}, +)} = \sum_{i_q \setminus i_j} \alpha_{i_q}$$

3.1.1 Estimation of parameters for q- dimensional contingency table

For q=2 the saturated log linear model for 2-dimensional contingency table and its parameters effects is given by

$$\log m_{i_1 i_2} = \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{12(i_1 i_2)}$$

Where conventionally we take $\mu\phi(\phi) = \mu$ and

$$[[\underline{J}]] = 0 \text{ (i.e } [\underline{J}] = \phi \text{)};$$

$$\hat{\mu} = \log \left(\frac{\sum \alpha_{i_1 i_2}}{I_{i_1} I_{i_2}} \right)$$

$$[[\underline{J}]] = 1;$$

$$\hat{\mu}_{\{1\}(i_1)} = \log \left(\frac{\sum \alpha_{(i_1, +)}}{I_{i_2}} \right) - \hat{\mu}$$

$$\hat{\mu}_{\{2\}(i_2)} = \log \left(\frac{\sum \alpha_{(i_2, +)}}{I_{i_1}} \right) - \hat{\mu}$$

For q=3 the saturated log linear model for 3-dimensional contingency table and its parameters effects is given by

$$\log m_{i_1 i_2 i_3} = \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} + \mu_{12(i_1 i_2)} + \mu_{13(i_1 i_3)} + \mu_{23(i_2 i_3)} + \mu_{123(i_1 i_2 i_3)}$$

where conventionally we take $\mu\phi(\phi) = \mu$ and

$$[\underline{J}] = 0 \text{ (i.e } [\underline{J}] = \phi \text{)};$$

$$\hat{\mu} = \log \left(\frac{\sum_{i_1 i_2 i_3} \alpha_{i_1 i_2 i_3}}{I_{i_1} I_{i_2} I_{i_3}} \right)$$

$$[\underline{J}] = 1;$$

$$\hat{\mu}_{\{1\}(i_1)} = \log \left(\frac{\sum \alpha_{(i_1,+)}}{I_{i_2} I_{i_3}} \right) - \hat{\mu}$$

$$\hat{\mu}_{\{2\}(i_2)} = \log \left(\frac{\sum \alpha_{(i_2,+)}}{I_{i_1} I_{i_3}} \right) - \hat{\mu}$$

$$\hat{\mu}_{\{3\}(i_3)} = \log \left(\frac{\sum \alpha_{(i_3,+)}}{I_{i_1} I_{i_2}} \right) - \hat{\mu}$$

$$[\underline{J}] = 2;$$

$$\hat{\mu}_{\{12\}(i_2)} = \log \left(\frac{\sum_{i_3} \alpha_{(i_2, +)}}{I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)}$$

$$\hat{\mu}_{\{13\}(i_3)} = \log \left(\frac{\sum_{i_2} \alpha_{(i_3, +)}}{I_{i_2}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[3](i_3)}$$

$$\hat{\mu}_{\{23\}(i_3)} = \log \left(\frac{\sum_{i_1} \alpha_{(i_3, +)}}{I_{i_1}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)}$$

For $q=4$, the saturated log linear for 4-dimensional contingency table and its parameter effects is given as

$$\begin{aligned} \log m_{i_1 i_2 i_3 i_4} = & \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} + \mu_{4(i_4)} + \mu_{12(i_1 i_2)} + \mu_{13(i_1 i_3)} + \mu_{14(i_1 i_4)} + \mu_{23(i_2 i_3)} + \mu_{24(i_2 i_4)} + \mu_{34(i_3 i_4)} \\ & + \mu_{123(i_1 i_2 i_3)} + \mu_{124(i_1 i_2 i_4)} + \mu_{134(i_1 i_3 i_4)} + \mu_{234(i_2 i_3 i_4)} + \mu_{1234(i_1 i_2 i_3 i_4)} \end{aligned}$$

$$[[\underline{J}]] = 0 \text{ (i.e } [\underline{J}] = \phi \text{)};$$

$$\hat{\mu} = \log \left(\frac{\sum \alpha_{i_1 i_2 i_3 i_4}}{I_{i_1} I_{i_2} I_{i_3} I_{i_4}} \right)$$

$$[\underline{J}] = 1;$$

$$\hat{\mu}_{\{1\}(i_1)} = \log \left(\frac{\sum \alpha_{(i_1,+)}}{I_{i_2} I_{i_3} I_{i_4}} \right) - \hat{\mu}$$

$$\hat{\mu}_{\{2\}(i_2)} = \log \left(\frac{\sum \alpha_{(i_2,+)}}{I_{i_1} I_{i_3} I_{i_4}} \right) - \hat{\mu}$$

$$\hat{\mu}_{\{3\}(i_3)} = \log \left(\frac{\sum \alpha_{(i_3,+)}}{I_{i_1} I_{i_2} I_{i_4}} \right) - \hat{\mu}$$

$$\hat{\mu}_{\{4\}(i_4)} = \log \left(\frac{\sum \alpha_{(i_4,+)}}{I_{i_1} I_{i_2} I_{i_3}} \right) - \hat{\mu}$$

$$[\underline{J}] = 2;$$

$$\hat{\mu}_{\{12\}(i_2)} = \log \left(\frac{\sum_{i_3 i_4} \alpha_{(i_1 i_2, +)}}{I_{i_3} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)}$$

$$\hat{\mu}_{\{13\}(i_3)} = \log \left(\frac{\sum_{i_2 i_4} \alpha_{(i_1 i_3, +)}}{I_{i_2} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[3](i_3)}$$

$$\hat{\mu}_{\{14\}(i_4)} = \log \left(\frac{\sum_{i_2 i_3} \alpha_{(i_1 i_4, +)}}{I_{i_2} I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[4](i_4)}$$

$$\hat{\mu}_{\{24\}(i_4)} = \log \left(\frac{\sum_{i_2 i_3} \alpha_{(i_2 i_4, +)}}{I_{i_2} I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[4](i_4)}$$

$$\hat{\mu}_{\{23\}(i_3)} = \log \left(\frac{\sum_{i_2 i_4} \alpha_{(i_2 i_3, +)}}{I_{i_2} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)}$$

$$|[\underline{J}]| = 3;$$

$$\hat{\mu}_{\{123\}(i_2 i_3)} = \log \left(\frac{\sum_{i_4} \alpha_{(i_1 i_2 i_3, +)}}{I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[12](i_2)} - \hat{\mu}_{[13](i_3)} - \hat{\mu}_{[23](i_2 i_3)}$$

$$\hat{\mu}_{\{124\}(i_1 i_2 i_4)} = \log \left(\frac{\sum \alpha_{(i_1 i_2 i_4, +)}}{I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[12](i_1 i_2)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[24](i_2 i_4)}$$

$$\hat{\mu}_{\{134\}(i_1 i_3 i_4)} = \log \left(\frac{\sum \alpha_{(i_1 i_3 i_4, +)}}{I_{i_2}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[34](i_3 i_4)}$$

$$\hat{\mu}_{\{234\}(i_2 i_3 i_4)} = \log \left(\frac{\sum \alpha_{(i_2 i_3 i_4, +)}}{I_{i_1}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[23](i_2 i_3)} - \hat{\mu}_{[24](i_2 i_4)} - \hat{\mu}_{[34](i_3 i_4)}$$

For $q=5$, the saturated log linear for 5-dimensional contingency table and its parameter effects is given by

$$\begin{aligned} \log m_{i_1 i_2 i_3 i_4 i_5} = & \mu + \mu_{(i_1)} + \mu_{(i_2)} + \mu_{(i_3)} + \mu_{(i_4)} + \mu_{(i_5)} + \mu_{12(i_1 i_2)} + \mu_{13(i_1 i_3)} + \mu_{14(i_1 i_4)} + \mu_{15(i_1 i_5)} + \mu_{23(i_2 i_3)} + \mu_{24(i_2 i_4)} \\ & + \mu_{25(i_2 i_5)} + \mu_{34(i_3 i_4)} + \mu_{35(i_3 i_5)} + \mu_{45(i_4 i_5)} + \mu_{123(i_1 i_2 i_3)} + \mu_{124(i_1 i_2 i_4)} + \mu_{125(i_1 i_2 i_5)} + \mu_{134(i_1 i_3 i_4)} + \mu_{135(i_1 i_3 i_5)} \\ & + \mu_{145(i_1 i_4 i_5)} + \mu_{234(i_2 i_3 i_4)} + \mu_{235(i_2 i_3 i_5)} + \mu_{245(i_2 i_4 i_5)} + \mu_{345(i_3 i_4 i_5)} + \mu_{1234(i_1 i_2 i_3 i_4)} + \mu_{1235(i_1 i_2 i_3 i_5)} + \mu_{1245(i_1 i_2 i_4 i_5)} \\ & + \mu_{1345(i_1 i_3 i_4 i_5)} + \mu_{2345(i_2 i_3 i_4 i_5)} + \mu_{12345(i_1 i_2 i_3 i_4 i_5)} \end{aligned}$$

$$[\underline{j}] = 0 \text{ (i.e. } [j] = \phi \text{)};$$

$$\hat{\mu} = \log \left(\frac{\sum \alpha_{i_1 i_2 i_3 i_4 i_5}}{I_{i_1} I_{i_2} I_{i_3} I_{i_4} I_{i_5}} \right)$$

$$|\underline{j}| = 1;$$

$$\hat{\mu}_{\{1\}(i_1)} = \log \left(\frac{\sum \alpha_{(i_1,+)}}{I_{i_2} I_{i_3} I_{i_4} I_{i_5}} \right) - \hat{\mu} \quad \hat{\mu}_{\{2\}(i_2)} = \log \left(\frac{\sum \alpha_{(i_2,+)}}{I_{i_1} I_{i_3} I_{i_4} I_{i_5}} \right) - \hat{\mu}$$

$$\hat{\mu}_{\{3\}(i_3)} = \log \left(\frac{\sum \alpha_{(i_3,+)}}{I_{i_1} I_{i_2} I_{i_4} I_{i_5}} \right) - \hat{\mu} \quad \hat{\mu}_{\{4\}(i_4)} = \log \left(\frac{\sum \alpha_{(i_4,+)}}{I_{i_1} I_{i_2} I_{i_3} I_{i_5}} \right) - \hat{\mu}$$

$$\hat{\mu}_{\{5\}(i_5)} = \log \left(\frac{\sum \alpha_{(i_5,+)}}{I_{i_1} I_{i_2} I_{i_3} I_{i_4}} \right) - \hat{\mu}$$

$$|\underline{j}| = 2;$$

$$\hat{\mu}_{\{12\}(i_1 i_2)} = \log \left(\frac{\sum \alpha_{(i_1 i_2,+)}}{I_{i_3} I_{i_4} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)}$$

$$\hat{\mu}_{\{13\}(i_1 i_3)} = \log \left(\frac{\sum \alpha_{(i_1 i_3,+)}}{I_{i_2} I_{i_4} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[3](i_3)}$$

$$\hat{\mu}_{\{14\}(i_4)} = \log \left(\frac{\sum \alpha_{(i_4,+)}}{I_{i_2} I_{i_3} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[4](i_4)}$$

$$\hat{\mu}_{\{15\}(i_5)} = \log \left(\frac{\sum \alpha_{(i_5,+)}}{I_{i_2} I_{i_3} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[5](i_5)}$$

$$\hat{\mu}_{\{23\}(i_3)} = \log \left(\frac{\sum \alpha_{(i_3,+)}}{I_{i_1} I_{i_4} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)}$$

$$\hat{\mu}_{\{24\}(i_4)} = \log \left(\frac{\sum \alpha_{(i_4,+)}}{I_{i_1} I_{i_3} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[4](i_4)}$$

$$\hat{\mu}_{\{25\}(i_5)} = \log \left(\frac{\sum \alpha_{(i_5,+)}}{I_{i_1} I_{i_3} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[5](i_5)}$$

$$\hat{\mu}_{\{34\}(i_4)} = \log \left(\frac{\sum \alpha_{(i_4,+)}}{I_{i_1} I_{i_2} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)}$$

$$\hat{\mu}_{\{35\}(i_3 i_5)} = \log \left(\frac{\sum \alpha_{(i_3 i_5, +)}}{I_{i_1} I_{i_2} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[5](i_5)}$$

$$\hat{\mu}_{\{45\}(i_4 i_5)} = \log \left(\frac{\sum \alpha_{(i_4 i_5, +)}}{I_{i_1} I_{i_2} I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)}$$

$$[\underline{j}] = 3;$$

$$\hat{\mu}_{\{123\}(i_1 i_2 i_3)} = \log \left(\frac{\sum \alpha_{(i_1 i_2 i_3, +)}}{I_{i_4} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[12](i_1 i_2)} - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[23](i_2 i_3)}$$

$$\hat{\mu}_{\{124\}(i_1 i_2 i_4)} = \log \left(\frac{\sum \alpha_{(i_1 i_2 i_4, +)}}{I_{i_3} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[12](i_1 i_2)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[24](i_2 i_4)}$$

$$\hat{\mu}_{\{125\}(i_1 i_2 i_5)} = \log \left(\frac{\sum \alpha_{(i_1 i_2 i_5, +)}}{I_{i_3} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[12](i_1 i_2)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[25](i_2 i_5)}$$

$$\hat{\mu}_{\{134\}(i_1 i_3 i_4)} = \log \left(\frac{\sum \alpha_{(i_1 i_3 i_4, +)}}{I_{i_2} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[34](i_3 i_4)}$$

$$\hat{\mu}_{\{135\}(i_1 i_3 i_5)} = \log \left(\frac{\sum \alpha_{(i_1 i_3 i_5, +)}}{I_{i_2} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[35](i_3 i_5)}$$

$$\hat{\mu}_{\{145\}(i_1 i_4 i_5)} = \log \left(\frac{\sum \alpha_{(i_1 i_4 i_5, +)}}{I_{i_2} I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[45](i_4 i_5)}$$

$$\hat{\mu}_{\{134\}(i_1 i_3 i_4)} = \log \left(\frac{\sum \alpha_{(i_1 i_3 i_4, +)}}{I_{i_1} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[23](i_2 i_3)} - \hat{\mu}_{[24](i_2 i_4)} - \hat{\mu}_{[34](i_3 i_4)}$$

$$\hat{\mu}_{\{135\}(i_1 i_3 i_5)} = \log \left(\frac{\sum \alpha_{(i_1 i_3 i_5, +)}}{I_{i_1} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[23](i_2 i_3)} - \hat{\mu}_{[25](i_2 i_5)} - \hat{\mu}_{[35](i_3 i_5)}$$

$$\hat{\mu}_{\{245\}(i_2 i_4 i_5)} = \log \left(\frac{\sum \alpha_{(i_2 i_4 i_5, +)}}{I_{i_1} I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[24](i_2 i_4)} - \hat{\mu}_{[25](i_2 i_5)} - \hat{\mu}_{[45](i_4 i_5)}$$

$$\hat{\mu}_{\{345\}(i_3 i_4 i_5)} = \log \left(\frac{\sum \alpha_{(i_3 i_4 i_5, +)}}{I_{i_1} I_{i_2}} \right) - \hat{\mu} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[34](i_3 i_4)} - \hat{\mu}_{[35](i_3 i_5)} - \hat{\mu}_{[45](i_4 i_5)}$$

$$|\underline{j}| = 4;$$

$$\begin{aligned} \hat{\mu}_{\{1234\}(i_1 i_2 i_3 i_4)} &= \log \left(\frac{\sum \alpha_{(i_1 i_2 i_3 i_4, +)}}{I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[12](i_1 i_2)} - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[23](i_2 i_3)} \\ &\quad - \hat{\mu}_{[24](i_2 i_4)} - \hat{\mu}_{[34](i_3 i_4)} - \hat{\mu}_{[123](i_1 i_2 i_3)} - \hat{\mu}_{[124](i_1 i_2 i_4)} - \hat{\mu}_{[134](i_1 i_3 i_4)} - \hat{\mu}_{[234](i_2 i_3 i_4)} \end{aligned}$$

$$\begin{aligned} \hat{\mu}_{\{1235\}(i_1 i_2 i_3 i_5)} &= \log \left(\frac{\sum \alpha_{(i_1 i_2 i_3 i_5, +)}}{I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[12](i_1 i_2)} - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[23](i_2 i_3)} \\ &\quad - \hat{\mu}_{[25](i_2 i_5)} - \hat{\mu}_{[35](i_3 i_5)} - \hat{\mu}_{[123](i_1 i_2 i_3)} - \hat{\mu}_{[125](i_1 i_2 i_5)} - \hat{\mu}_{[135](i_1 i_3 i_5)} - \hat{\mu}_{[235](i_2 i_3 i_5)} \end{aligned}$$

$$\begin{aligned} \hat{\mu}_{\{2345\}(i_2 i_3 i_4 i_5)} &= \log \left(\frac{\sum \alpha_{(i_2 i_3 i_4 i_5, +)}}{I_{i_1}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[12](i_1 i_2)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[24](i_2 i_4)} \\ &\quad - \hat{\mu}_{[25](i_2 i_5)} - \hat{\mu}_{[45](i_4 i_5)} - \hat{\mu}_{[124](i_1 i_2 i_4)} - \hat{\mu}_{[125](i_1 i_2 i_5)} - \hat{\mu}_{[145](i_1 i_4 i_5)} - \hat{\mu}_{[245](i_2 i_4 i_5)} \end{aligned}$$

$$\begin{aligned} \hat{\mu}_{\{1345\}(i_1 i_3 i_4 i_5)} &= \log \left(\frac{\sum \alpha_{(i_1 i_3 i_4 i_5, +)}}{I_{i_2}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[34](i_3 i_4)} \\ &\quad - \hat{\mu}_{[35](i_3 i_5)} - \hat{\mu}_{[45](i_4 i_5)} - \hat{\mu}_{[134](i_1 i_3 i_4)} - \hat{\mu}_{[135](i_1 i_3 i_5)} - \hat{\mu}_{[145](i_1 i_4 i_5)} - \hat{\mu}_{[345](i_3 i_4 i_5)} \end{aligned}$$

$$\begin{aligned} \hat{\mu}_{\{2345\}(i_2 i_3 i_4 i_5)} &= \log \left(\frac{\sum \alpha_{(i_2 i_3 i_4 i_5, +)}}{I_{i_1}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[23](i_2 i_3)} - \hat{\mu}_{[24](i_2 i_4)} - \hat{\mu}_{[25](i_2 i_5)} - \hat{\mu}_{[34](i_3 i_4)} \\ &\quad - \hat{\mu}_{[35](i_3 i_5)} - \hat{\mu}_{[45](i_4 i_5)} - \hat{\mu}_{[234](i_2 i_3 i_4)} - \hat{\mu}_{[235](i_2 i_3 i_5)} - \hat{\mu}_{[245](i_2 i_4 i_5)} - \hat{\mu}_{[345](i_3 i_4 i_5)} \end{aligned}$$

For $q=6$, the saturated log linear for 6-dimensional contingency table and its parameter effects is given

$$\begin{aligned}
\log m_{i_1 i_2 i_3 i_4 i_5 i_6} = & \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} + \mu_{4(i_4)} + \mu_{5(i_5)} + \mu_{6(i_6)} + \mu_{12(i_1 i_2)} + \mu_{13(i_1 i_3)} + \mu_{14(i_1 i_4)} + \mu_{15(i_1 i_5)} + \mu_{16(i_1 i_6)} \\
& + \mu_{23(i_2 i_3)} + \mu_{24(i_2 i_4)} + \mu_{25(i_2 i_5)} + \mu_{26(i_2 i_6)} + \mu_{34(i_3 i_4)} + \mu_{35(i_3 i_5)} + \mu_{36(i_3 i_6)} + \mu_{45(i_4 i_5)} + \mu_{46(i_4 i_6)} + \mu_{56(i_5 i_6)} \\
& + \mu_{123(i_1 i_2 i_3)} + \mu_{124(i_1 i_2 i_4)} + \mu_{125(i_1 i_2 i_5)} + \mu_{126(i_1 i_2 i_6)} + \mu_{134(i_1 i_3 i_4)} + \mu_{135(i_1 i_3 i_5)} + \mu_{136(i_1 i_3 i_6)} + \mu_{145(i_1 i_4 i_5)} + \mu_{146(i_1 i_4 i_6)} \\
& + \mu_{156(i_1 i_5 i_6)} + \mu_{234(i_2 i_3 i_4)} + \mu_{235(i_2 i_3 i_5)} + \mu_{236(i_2 i_3 i_6)} + \mu_{245(i_2 i_4 i_5)} + \mu_{246(i_2 i_4 i_6)} + \mu_{256(i_2 i_5 i_6)} + \mu_{345(i_3 i_4 i_5)} + \mu_{346(i_3 i_4 i_6)} \\
& + \mu_{356(i_3 i_5 i_6)} + \mu_{456(i_4 i_5 i_6)} + \mu_{1234(i_1 i_2 i_3 i_4)} + \mu_{1235(i_1 i_2 i_3 i_5)} + \mu_{1236(i_1 i_2 i_3 i_6)} + \mu_{1245(i_1 i_2 i_4 i_5)} + \mu_{1246(i_1 i_2 i_4 i_6)} + \mu_{1256(i_1 i_2 i_5 i_6)} \\
& + \mu_{1345(i_1 i_3 i_4 i_5)} + \mu_{1346(i_1 i_3 i_4 i_6)} + \mu_{1356(i_1 i_3 i_5 i_6)} + \mu_{1456(i_1 i_4 i_5 i_6)} + \mu_{2345(i_2 i_3 i_4 i_5)} + \mu_{2346(i_2 i_3 i_4 i_6)} + \mu_{2356(i_2 i_3 i_5 i_6)} \\
& + \mu_{2456(i_2 i_4 i_5 i_6)} + \mu_{3456(i_3 i_4 i_5 i_6)} + \mu_{12345(i_1 i_2 i_3 i_4 i_5)} + \mu_{12346(i_1 i_2 i_3 i_4 i_6)} + \mu_{13456(i_1 i_3 i_4 i_5 i_6)} + \mu_{12456(i_1 i_2 i_4 i_5 i_6)} \\
& + \mu_{12356(i_1 i_2 i_3 i_5 i_6)} + \mu_{23456(i_2 i_3 i_4 i_5 i_6)} + \mu_{123456(i_1 i_2 i_3 i_4 i_5 i_6)}
\end{aligned}$$

$$[\underline{j}] = 0 \text{ (i.e. } [\underline{j}] = \phi \text{)};$$

$$\hat{\mu} = \log \left(\frac{\sum_{i_1 i_2 i_3 i_4 i_5 i_6} \alpha_{i_1 i_2 i_3 i_4 i_5 i_6}}{I_{i_1} I_{i_2} I_{i_3} I_{i_4} I_{i_5} I_{i_6}} \right)$$

$$[\underline{j}] = 1;$$

$$\hat{\mu}_{\{1\}(i_1)} = \log \left(\frac{\sum_{i_2 i_3 i_4 i_5 i_6} \alpha_{(i_1,+)}}{I_{i_2} I_{i_3} I_{i_4} I_{i_5} I_{i_6}} \right) - \hat{\mu} \quad \hat{\mu}_{\{2\}(i_2)} = \log \left(\frac{\sum_{i_1 i_3 i_4 i_5 i_6} \alpha_{(i_2,+)}}{I_{i_1} I_{i_3} I_{i_4} I_{i_5} I_{i_6}} \right) - \hat{\mu}$$

$$\hat{\mu}_{\{3\}(i_3)} = \log \left(\frac{\sum_{i_1 i_2 i_4 i_5 i_6} \alpha_{(i_3,+)}}{I_{i_1} I_{i_2} I_{i_4} I_{i_5} I_{i_6}} \right) - \hat{\mu} \quad \hat{\mu}_{\{4\}(i_4)} = \log \left(\frac{\sum_{i_1 i_2 i_3 i_5 i_6} \alpha_{(i_4,+)}}{I_{i_1} I_{i_2} I_{i_3} I_{i_5} I_{i_6}} \right) - \hat{\mu}$$

$$\hat{\mu}_{\{5\}(i_5)} = \log \left(\frac{\sum_{i_2 i_3 i_4 i_6} \alpha_{(i_5,+)}}{I_{i_1} I_{i_2} I_{i_3} I_{i_4} I_{i_6}} \right) - \hat{\mu} \quad \hat{\mu}_{\{6\}(i_6)} = \log \left(\frac{\sum_{i_1 i_2 i_3 i_4 i_5} \alpha_{(i_6,+)}}{I_{i_1} I_{i_2} I_{i_3} I_{i_4} I_{i_5}} \right) - \hat{\mu}$$

$$|\underline{j}| = 2;$$

$$\hat{\mu}_{\{12\}(i_{i_2})} = \log \left(\frac{\sum_{i_3 i_4 i_5 i_6} \alpha_{(i_{i_2},+)}}{I_{i_3} I_{i_4} I_{i_5} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)}$$

$$\hat{\mu}_{\{13\}(i_{i_3})} = \log \left(\frac{\sum_{i_2 i_4 i_5 i_6} \alpha_{(i_{i_3},+)}}{I_{i_2} I_{i_4} I_{i_5} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[3](i_3)}$$

$$\hat{\mu}_{\{14\}(i_{i_4})} = \log \left(\frac{\sum_{i_2 i_3 i_5 i_6} \alpha_{(i_{i_4},+)}}{I_{i_2} I_{i_3} I_{i_5} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[4](i_4)}$$

$$\hat{\mu}_{\{15\}(i_{i_5})} = \log \left(\frac{\sum_{i_2 i_3 i_4 i_6} \alpha_{(i_{i_5},+)}}{I_{i_2} I_{i_3} I_{i_4} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[5](i_5)}$$

$$\hat{\mu}_{\{16\}(i_{i_6})} = \log \left(\frac{\sum_{i_2 i_3 i_4 i_5} \alpha_{(i_{i_6},+)}}{I_{i_2} I_{i_3} I_{i_4} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[6](i_6)}$$

$$\hat{\mu}_{\{23\}(i_{i_3})} = \log \left(\frac{\sum_{i_1 i_4 i_5 i_6} \alpha_{(i_{i_3},+)}}{I_{i_1} I_{i_4} I_{i_5} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)}$$

$$\hat{\mu}_{\{24\}(i_2 i_4)} = \log \left(\frac{\sum \alpha_{(i_2 i_4, +)}}{I_{i_1} I_{i_3} I_{i_5} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[4](i_4)}$$

$$\hat{\mu}_{\{25\}(i_2 i_5)} = \log \left(\frac{\sum \alpha_{(i_2 i_5, +)}}{I_{i_1} I_{i_3} I_{i_4} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[5](i_5)}$$

$$\hat{\mu}_{\{26\}(i_2 i_6)} = \log \left(\frac{\sum \alpha_{(i_2 i_6, +)}}{I_{i_1} I_{i_3} I_{i_4} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[6](i_6)}$$

$$\hat{\mu}_{\{34\}(i_3 i_4)} = \log \left(\frac{\sum \alpha_{(i_3 i_4, +)}}{I_{i_1} I_{i_2} I_{i_5} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)}$$

$$\hat{\mu}_{\{35\}(i_3 i_5)} = \log \left(\frac{\sum \alpha_{(i_3 i_5, +)}}{I_{i_1} I_{i_2} I_{i_4} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[5](i_5)}$$

$$\hat{\mu}_{\{36\}(i_3 i_6)} = \log \left(\frac{\sum \alpha_{(i_3 i_6, +)}}{I_{i_1} I_{i_2} I_{i_4} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[6](i_6)}$$

$$\hat{\mu}_{\{45\}(i_4 i_5)} = \log \left(\frac{\sum \alpha_{(i_4 i_5, +)}}{I_{i_1} I_{i_2} I_{i_3} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)}$$

$$\hat{\mu}_{\{46\}(i_4 i_6)} = \log \left(\frac{\sum \alpha_{(i_4 i_6, +)}}{I_{i_1} I_{i_2} I_{i_3} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[6](i_6)}$$

$$\hat{\mu}_{\{56\}(i_5 i_6)} = \log \left(\frac{\sum \alpha_{(i_5 i_6, +)}}{I_{i_1} I_{i_2} I_{i_3} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[6](i_6)}$$

$$|\underline{j}| = 3;$$

$$\hat{\mu}_{\{123\}(i_2 i_3)} = \log \left(\frac{\sum \alpha_{(i_2 i_3, +)}}{I_{i_4} I_{i_5} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[12](i_2)} - \hat{\mu}_{[13](i_3)} - \hat{\mu}_{[23](i_2 i_3)}$$

$$\hat{\mu}_{\{124\}(i_2 i_4)} = \log \left(\frac{\sum \alpha_{(i_2 i_4, +)}}{I_{i_3} I_{i_5} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[12](i_2)} - \hat{\mu}_{[14](i_4)} - \hat{\mu}_{[24](i_2 i_4)}$$

$$\hat{\mu}_{\{125\}(i_2 i_5)} = \log \left(\frac{\sum \alpha_{(i_2 i_5, +)}}{I_{i_3} I_{i_4} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[12](i_2)} - \hat{\mu}_{[15](i_5)} - \hat{\mu}_{[25](i_2 i_5)}$$

$$\hat{\mu}_{\{126\}(i_2 i_6)} = \log \left(\frac{\sum \alpha_{(i_2 i_6, +)}}{I_{i_3} I_{i_4} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[12](i_2)} - \hat{\mu}_{[16](i_6)} - \hat{\mu}_{[26](i_2 i_6)}$$

$$\hat{\mu}_{\{134\}(i_1 i_3 i_4)} = \log \left(\frac{\sum \alpha_{(i_1 i_3 i_4, +)}}{I_{i_2} I_{i_5} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[34](i_3 i_4)}$$

$$\hat{\mu}_{\{135\}(i_1 i_3 i_5)} = \log \left(\frac{\sum \alpha_{(i_1 i_3 i_5, +)}}{I_{i_2} I_{i_4} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[35](i_3 i_5)}$$

$$\hat{\mu}_{\{136\}(i_1 i_3 i_6)} = \log \left(\frac{\sum \alpha_{(i_1 i_3 i_6, +)}}{I_{i_2} I_{i_4} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[16](i_1 i_6)} - \hat{\mu}_{[36](i_3 i_6)}$$

$$\hat{\mu}_{\{145\}(i_1 i_4 i_5)} = \log \left(\frac{\sum \alpha_{(i_1 i_4 i_5, +)}}{I_{i_2} I_{i_3} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[45](i_4 i_5)}$$

$$\hat{\mu}_{\{146\}(i_1 i_4 i_6)} = \log \left(\frac{\sum \alpha_{(i_1 i_4 i_6, +)}}{I_{i_2} I_{i_3} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[16](i_1 i_6)} - \hat{\mu}_{[46](i_4 i_6)}$$

$$\hat{\mu}_{\{156\}(i_1 i_5 i_6)} = \log \left(\frac{\sum \alpha_{(i_1 i_5 i_6, +)}}{I_{i_2} I_{i_3} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[16](i_1 i_6)} - \hat{\mu}_{[56](i_5 i_6)}$$

$$\hat{\mu}_{\{234\}(i_2 i_3 i_4)} = \log \left(\frac{\sum \alpha_{(i_2 i_3 i_4, +)}}{I_{i_1} I_{i_5} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[23](i_2 i_3)} - \hat{\mu}_{[24](i_2 i_4)} - \hat{\mu}_{[34](i_3 i_4)}$$

$$\hat{\mu}_{\{235\}(i_2 i_3 i_5)} = \log \left(\frac{\sum \alpha_{(i_2 i_3 i_5, +)}}{I_{i_1} I_{i_4} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[23](i_2 i_3)} - \hat{\mu}_{[25](i_2 i_5)} - \hat{\mu}_{[35](i_3 i_5)}$$

$$\hat{\mu}_{\{236\}(i_2 i_3 i_6)} = \log \left(\frac{\sum \alpha_{(i_2 i_3 i_6, +)}}{I_{i_1} I_{i_4} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[23](i_2 i_3)} - \hat{\mu}_{[26](i_2 i_6)} - \hat{\mu}_{[36](i_3 i_6)}$$

$$\hat{\mu}_{\{245\}(i_2 i_4 i_5)} = \log \left(\frac{\sum \alpha_{(i_2 i_4 i_5, +)}}{I_{i_1} I_{i_3} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[24](i_2 i_4)} - \hat{\mu}_{[25](i_2 i_5)} - \hat{\mu}_{[45](i_4 i_5)}$$

$$\hat{\mu}_{\{246\}(i_2 i_4 i_6)} = \log \left(\frac{\sum \alpha_{(i_2 i_4 i_6, +)}}{I_{i_1} I_{i_3} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[24](i_2 i_4)} - \hat{\mu}_{[26](i_2 i_6)} - \hat{\mu}_{[46](i_4 i_6)}$$

$$\hat{\mu}_{\{256\}(i_2 i_5 i_6)} = \log \left(\frac{\sum \alpha_{(i_2 i_5 i_6, +)}}{I_{i_1} I_{i_3} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[25](i_2 i_5)} - \hat{\mu}_{[26](i_2 i_6)} - \hat{\mu}_{[56](i_5 i_6)}$$

$$\hat{\mu}_{\{345\}(i_3 i_4 i_5)} = \log \left(\frac{\sum \alpha_{(i_3 i_4 i_5, +)}}{I_{i_1} I_{i_2} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[34](i_3 i_4)} - \hat{\mu}_{[35](i_3 i_5)} - \hat{\mu}_{[45](i_4 i_5)}$$

$$\hat{\mu}_{\{346\}(i_3 i_4 i_6)} = \log \left(\frac{\sum \alpha_{(i_3 i_4 i_6, +)}}{I_{i_1} I_{i_2} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[34](i_3 i_4)} - \hat{\mu}_{[36](i_3 i_6)} - \hat{\mu}_{[46](i_4 i_6)}$$

$$\hat{\mu}_{\{356\}(i_3 i_5 i_6)} = \log \left(\frac{\sum \alpha_{(i_3 i_5 i_6, +)}}{I_{i_1} I_{i_2} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[35](i_3 i_5)} - \hat{\mu}_{[36](i_3 i_6)} - \hat{\mu}_{[56](i_5 i_6)}$$

$$\hat{\mu}_{\{456\}(i_4 i_5 i_6)} = \log \left(\frac{\sum \alpha_{(i_4 i_5 i_6, +)}}{I_{i_1} I_{i_2} I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[45](i_4 i_5)} - \hat{\mu}_{[46](i_4 i_6)} - \hat{\mu}_{[56](i_5 i_6)}$$

$$\llbracket \underline{j} \rrbracket = 4;$$

$$\begin{aligned} \hat{\mu}_{\{1234\}(i_1 i_2 i_3 i_4)} &= \log \left(\frac{\sum \alpha_{(i_1 i_2 i_3 i_4, +)}}{I_{i_5} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[12](i_1 i_2)} - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[23](i_2 i_3)} \\ &\quad - \hat{\mu}_{[24](i_2 i_4)} - \hat{\mu}_{[34](i_3 i_4)} - \hat{\mu}_{[123](i_1 i_2 i_3)} - \hat{\mu}_{[124](i_1 i_2 i_4)} - \hat{\mu}_{[134](i_1 i_3 i_4)} - \hat{\mu}_{[234](i_2 i_3 i_4)} \end{aligned}$$

$$\begin{aligned} \hat{\mu}_{\{1235\}(i_1 i_2 i_3 i_5)} &= \log \left(\frac{\sum \alpha_{(i_1 i_2 i_3 i_5, +)}}{I_{i_4} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[12](i_1 i_2)} - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[23](i_2 i_3)} \\ &\quad - \hat{\mu}_{[25](i_2 i_5)} - \hat{\mu}_{[35](i_3 i_5)} - \hat{\mu}_{[123](i_1 i_2 i_3)} - \hat{\mu}_{[125](i_1 i_2 i_5)} - \hat{\mu}_{[135](i_1 i_3 i_5)} - \hat{\mu}_{[235](i_2 i_3 i_5)} \end{aligned}$$

$$\begin{aligned} \hat{\mu}_{\{1235\}(i_1 i_2 i_3 i_5)} &= \log \left(\frac{\sum \alpha_{(i_1 i_2 i_3 i_6, +)}}{I_{i_4} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[12](i_1 i_2)} - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[16](i_1 i_6)} - \hat{\mu}_{[23](i_2 i_3)} \\ &\quad - \hat{\mu}_{[26](i_2 i_6)} - \hat{\mu}_{[36](i_3 i_6)} - \hat{\mu}_{[123](i_1 i_2 i_3)} - \hat{\mu}_{[126](i_1 i_2 i_6)} - \hat{\mu}_{[136](i_1 i_3 i_6)} - \hat{\mu}_{[236](i_2 i_3 i_6)} \end{aligned}$$

$$\begin{aligned} \hat{\mu}_{\{2345\}(i_2 i_3 i_4 i_5)} &= \log \left(\frac{\sum \alpha_{(i_2 i_3 i_4 i_5, +)}}{I_{i_3} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[12](i_1 i_2)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[24](i_2 i_4)} \\ &\quad - \hat{\mu}_{[25](i_2 i_5)} - \hat{\mu}_{[45](i_4 i_5)} - \hat{\mu}_{[124](i_1 i_2 i_4)} - \hat{\mu}_{[125](i_1 i_2 i_5)} - \hat{\mu}_{[145](i_1 i_4 i_5)} - \hat{\mu}_{[245](i_2 i_4 i_5)} \end{aligned}$$

$$\hat{\mu}_{\{2346\}(i_2 i_3 i_4 i_6)} = \log \left(\frac{\sum_{i_3 i_5} \alpha_{(i_1 i_2 i_4 i_6, +)}}{I_{i_3} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[12](i_1 i_2)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[16](i_1 i_6)} - \hat{\mu}_{[24](i_2 i_4)} \\ - \hat{\mu}_{[26](i_2 i_6)} - \hat{\mu}_{[46](i_4 i_6)} - \hat{\mu}_{[124](i_1 i_2 i_4)} - \hat{\mu}_{[126](i_1 i_2 i_6)} - \hat{\mu}_{[145](i_1 i_4 i_6)} - \hat{\mu}_{[245](i_2 i_4 i_6)}$$

$$\hat{\mu}_{\{2356\}(i_2 i_3 i_5 i_6)} = \log \left(\frac{\sum_{i_3 i_4} \alpha_{(i_1 i_2 i_5 i_6, +)}}{I_{i_3} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[12](i_1 i_2)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[16](i_1 i_6)} - \hat{\mu}_{[25](i_2 i_5)} \\ - \hat{\mu}_{[26](i_2 i_6)} - \hat{\mu}_{[56](i_5 i_6)} - \hat{\mu}_{[124](i_1 i_2 i_5)} - \hat{\mu}_{[126](i_1 i_2 i_6)} - \hat{\mu}_{[145](i_1 i_5 i_6)} - \hat{\mu}_{[245](i_2 i_5 i_6)}$$

$$\hat{\mu}_{\{1345\}(i_1 i_3 i_4 i_5)} = \log \left(\frac{\sum_{i_2 i_6} \alpha_{(i_1 i_3 i_4 i_5, +)}}{I_{i_2} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[34](i_3 i_4)} \\ - \hat{\mu}_{[35](i_3 i_5)} - \hat{\mu}_{[45](i_4 i_5)} - \hat{\mu}_{[134](i_1 i_3 i_4)} - \hat{\mu}_{[135](i_1 i_3 i_5)} - \hat{\mu}_{[145](i_1 i_4 i_5)} - \hat{\mu}_{[345](i_3 i_4 i_5)}$$

$$\hat{\mu}_{\{1346\}(i_1 i_3 i_4 i_6)} = \log \left(\frac{\sum_{i_2 i_5} \alpha_{(i_1 i_3 i_4 i_6, +)}}{I_{i_2} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[16](i_1 i_6)} - \hat{\mu}_{[34](i_3 i_4)} \\ - \hat{\mu}_{[36](i_3 i_6)} - \hat{\mu}_{[46](i_4 i_6)} - \hat{\mu}_{[134](i_1 i_3 i_4)} - \hat{\mu}_{[136](i_1 i_3 i_6)} - \hat{\mu}_{[146](i_1 i_4 i_6)} - \hat{\mu}_{[346](i_3 i_4 i_6)}$$

$$\begin{aligned}\hat{\mu}_{\{1356\}(i_1 i_3 i_5 i_6)} &= \log \left(\frac{\sum \alpha_{(i_1 i_3 i_5 i_6, +)}}{I_{i_2} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[16](i_1 i_6)} - \hat{\mu}_{[35](i_3 i_5)} \\ &\quad - \hat{\mu}_{[36](i_3 i_6)} - \hat{\mu}_{[56](i_5 i_6)} - \hat{\mu}_{[135](i_1 i_3 i_5)} - \hat{\mu}_{[136](i_1 i_3 i_6)} - \hat{\mu}_{[156](i_1 i_5 i_6)} - \hat{\mu}_{[356](i_3 i_5 i_6)}\end{aligned}$$

$$\begin{aligned}\hat{\mu}_{\{1456\}(i_1 i_4 i_5 i_6)} &= \log \left(\frac{\sum \alpha_{(i_1 i_4 i_5 i_6, +)}}{I_{i_2} I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[16](i_1 i_6)} - \hat{\mu}_{[45](i_4 i_5)} \\ &\quad - \hat{\mu}_{[46](i_4 i_6)} - \hat{\mu}_{[56](i_5 i_6)} - \hat{\mu}_{[145](i_1 i_4 i_5)} - \hat{\mu}_{[146](i_1 i_4 i_6)} - \hat{\mu}_{[156](i_1 i_5 i_6)} - \hat{\mu}_{[456](i_4 i_5 i_6)}\end{aligned}$$

$$\begin{aligned}\hat{\mu}_{\{2345\}(i_2 i_3 i_4 i_5)} &= \log \left(\frac{\sum \alpha_{(i_2 i_3 i_4 i_5, +)}}{I_{i_1} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[23](i_2 i_3)} - \hat{\mu}_{[24](i_2 i_4)} - \hat{\mu}_{[25](i_2 i_5)} - \hat{\mu}_{[34](i_3 i_4)} \\ &\quad - \hat{\mu}_{[35](i_3 i_5)} - \hat{\mu}_{[45](i_4 i_5)} - \hat{\mu}_{[234](i_2 i_3 i_4)} - \hat{\mu}_{[235](i_2 i_3 i_5)} - \hat{\mu}_{[245](i_2 i_4 i_5)} - \hat{\mu}_{[345](i_3 i_4 i_5)}\end{aligned}$$

$$\begin{aligned}\hat{\mu}_{\{2345\}(i_2 i_3 i_4 i_5)} &= \log \left(\frac{\sum \alpha_{(i_2 i_3 i_4 i_5, +)}}{I_{i_1} I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[23](i_2 i_3)} - \hat{\mu}_{[24](i_2 i_4)} \\ &\quad - \hat{\mu}_{[25](i_2 i_5)} - \hat{\mu}_{[34](i_3 i_4)} - \hat{\mu}_{[35](i_3 i_5)} - \hat{\mu}_{[45](i_4 i_5)} - \hat{\mu}_{[234](i_2 i_3 i_4)} - \hat{\mu}_{[235](i_2 i_3 i_5)} - \hat{\mu}_{[245](i_2 i_4 i_5)} - \hat{\mu}_{[345](i_3 i_4 i_5)}\end{aligned}$$

$$\begin{aligned}\hat{\mu}_{\{2346\}(i_2 i_3 i_4 i_6)} &= \log \left(\frac{\sum \alpha_{(i_2 i_3 i_4 i_6, +)}}{I_{i_1} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[23](i_2 i_3)} - \hat{\mu}_{[24](i_2 i_4)} - \hat{\mu}_{[26](i_2 i_6)} - \hat{\mu}_{[34](i_3 i_4)} \\ &\quad - \hat{\mu}_{[36](i_3 i_6)} - \hat{\mu}_{[46](i_4 i_6)} - \hat{\mu}_{[234](i_2 i_3 i_4)} - \hat{\mu}_{[236](i_2 i_3 i_6)} - \hat{\mu}_{[246](i_2 i_4 i_6)} - \hat{\mu}_{[346](i_3 i_4 i_6)}\end{aligned}$$

$$\begin{aligned}\hat{\mu}_{\{2356\}(i_2 i_3 i_5 i_6)} &= \log \left(\frac{\sum \alpha_{(i_2 i_3 i_5 i_6, +)}}{I_{i_1} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[23](i_2 i_3)} - \hat{\mu}_{[25](i_2 i_5)} - \hat{\mu}_{[26](i_2 i_6)} - \hat{\mu}_{[35](i_3 i_5)} \\ &\quad - \hat{\mu}_{[36](i_3 i_6)} - \hat{\mu}_{[56](i_5 i_6)} - \hat{\mu}_{[235](i_2 i_3 i_5)} - \hat{\mu}_{[236](i_2 i_3 i_6)} - \hat{\mu}_{[256](i_2 i_5 i_6)} - \hat{\mu}_{[356](i_3 i_5 i_6)}\end{aligned}$$

$$\begin{aligned}\hat{\mu}_{\{1456\}(i_1 i_4 i_5 i_6)} &= \log \left(\frac{\sum \alpha_{(i_1 i_4 i_5 i_6, +)}}{I_{i_1} I_{i_2}} \right) - \hat{\mu} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[34](i_3 i_4)} - \hat{\mu}_{[35](i_3 i_5)} - \hat{\mu}_{[36](i_3 i_6)} - \hat{\mu}_{[45](i_4 i_5)} \\ &\quad - \hat{\mu}_{[46](i_4 i_6)} - \hat{\mu}_{[56](i_5 i_6)} - \hat{\mu}_{[345](i_3 i_4 i_5)} - \hat{\mu}_{[346](i_3 i_4 i_6)} - \hat{\mu}_{[356](i_3 i_5 i_6)} - \hat{\mu}_{[456](i_4 i_5 i_6)}\end{aligned}$$

$$\begin{aligned}\hat{\mu}_{\{12345\}(i_1 i_2 i_3 i_4 i_5)} &= \log \left(\frac{\sum \alpha_{(i_1 i_2 i_3 i_4 i_5, +)}}{I_{i_6}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[12](i_1 i_2)} - \hat{\mu}_{[13](i_1 i_3)} \\ &\quad - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[23](i_2 i_3)} - \hat{\mu}_{[24](i_2 i_4)} - \hat{\mu}_{[25](i_2 i_5)} - \hat{\mu}_{[34](i_3 i_4)} - \hat{\mu}_{[35](i_3 i_5)} - \hat{\mu}_{[45](i_4 i_5)} \\ &\quad - \hat{\mu}_{[123](i_1 i_2 i_3)} - \hat{\mu}_{[124](i_1 i_2 i_4)} - \hat{\mu}_{[125](i_1 i_2 i_5)} - \hat{\mu}_{[135](i_1 i_3 i_5)} - \hat{\mu}_{[234](i_2 i_3 i_4)} - \hat{\mu}_{[235](i_2 i_3 i_5)}\end{aligned}$$

$$\begin{aligned}\hat{\mu}_{\{12346\}(i_1 i_2 i_3 i_4 i_6)} &= \log \left(\frac{\sum \alpha_{(i_1 i_2 i_3 i_4 i_6, +)}}{I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[12](i_1 i_2)} \\ &\quad - \hat{\mu}_{[13](i_1 i_3)} - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[16](i_1 i_6)} - \hat{\mu}_{[23](i_2 i_3)} - \hat{\mu}_{[24](i_2 i_4)} - \hat{\mu}_{[26](i_2 i_6)} - \hat{\mu}_{[34](i_3 i_4)} - \hat{\mu}_{[36](i_3 i_6)} \\ &\quad - \hat{\mu}_{[46](i_4 i_6)} - \hat{\mu}_{[123](i_1 i_2 i_3)} - \hat{\mu}_{[124](i_1 i_2 i_4)} - \hat{\mu}_{[126](i_1 i_2 i_6)} - \hat{\mu}_{[136](i_1 i_3 i_6)} - \hat{\mu}_{[234](i_2 i_3 i_4)} - \hat{\mu}_{[236](i_2 i_3 i_6)}\end{aligned}$$

$$\begin{aligned} \hat{\mu}_{\{12356\}(i_1 i_2 i_3 i_5 i_6)} &= \log \left(\frac{\sum \alpha_{(i_1 i_2 i_3 i_5 i_6, +)}}{I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[12](i_1 i_2)} - \hat{\mu}_{[13](i_1 i_3)} \\ &\quad - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[16](i_1 i_6)} - \hat{\mu}_{[23](i_2 i_3)} - \hat{\mu}_{[25](i_2 i_5)} - \hat{\mu}_{[26](i_2 i_6)} - \hat{\mu}_{[35](i_3 i_5)} - \hat{\mu}_{[36](i_3 i_6)} - \hat{\mu}_{[56](i_5 i_6)} \\ &\quad - \hat{\mu}_{[123](i_1 i_2 i_3)} - \hat{\mu}_{[125](i_1 i_2 i_5)} - \hat{\mu}_{[126](i_1 i_2 i_6)} - \hat{\mu}_{[136](i_1 i_3 i_6)} - \hat{\mu}_{[235](i_2 i_3 i_5)} - \hat{\mu}_{[236](i_2 i_3 i_6)} \end{aligned}$$

$$\begin{aligned} \hat{\mu}_{\{12456\}(i_1 i_2 i_4 i_5 i_6)} &= \log \left(\frac{\sum \alpha_{(i_1 i_2 i_4 i_5 i_6, +)}}{I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[2](i_2)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[12](i_1 i_2)} \\ &\quad - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[16](i_1 i_6)} - \hat{\mu}_{[24](i_2 i_4)} - \hat{\mu}_{[25](i_2 i_5)} - \hat{\mu}_{[26](i_2 i_6)} - \hat{\mu}_{[35](i_3 i_5)} \\ &\quad - \hat{\mu}_{[36](i_3 i_6)} - \hat{\mu}_{[56](i_5 i_6)} - \hat{\mu}_{[124](i_1 i_2 i_4)} - \hat{\mu}_{[125](i_1 i_2 i_5)} - \hat{\mu}_{[126](i_1 i_2 i_6)} - \hat{\mu}_{[145](i_1 i_4 i_5)} \\ &\quad - \hat{\mu}_{[146](i_1 i_4 i_6)} - \hat{\mu}_{[245](i_2 i_4 i_5)} - \hat{\mu}_{[246](i_2 i_4 i_6)} - \hat{\mu}_{[456](i_4 i_5 i_6)} \end{aligned}$$

$$\begin{aligned} \hat{\mu}_{\{13456\}(i_1 i_3 i_4 i_5 i_6)} &= \log \left(\frac{\sum \alpha_{(i_1 i_3 i_4 i_5 i_6, +)}}{I_{i_2}} \right) - \hat{\mu} - \hat{\mu}_{[1](i_1)} - \hat{\mu}_{[3](i_3)} - \hat{\mu}_{[4](i_4)} - \hat{\mu}_{[5](i_5)} - \hat{\mu}_{[6](i_6)} - \hat{\mu}_{[13](i_1 i_3)} \\ &\quad - \hat{\mu}_{[14](i_1 i_4)} - \hat{\mu}_{[15](i_1 i_5)} - \hat{\mu}_{[16](i_1 i_6)} - \hat{\mu}_{[24](i_2 i_4)} - \hat{\mu}_{[34](i_3 i_4)} - \hat{\mu}_{[35](i_3 i_5)} - \hat{\mu}_{[36](i_3 i_6)} \\ &\quad - \hat{\mu}_{[45](i_4 i_5)} - \hat{\mu}_{[46](i_4 i_6)} - \hat{\mu}_{[56](i_5 i_6)} - \hat{\mu}_{[134](i_1 i_3 i_4)} - \hat{\mu}_{[135](i_1 i_3 i_5)} - \hat{\mu}_{[136](i_1 i_3 i_6)} \\ &\quad - \hat{\mu}_{[145](i_1 i_4 i_5)} - \hat{\mu}_{[146](i_1 i_4 i_6)} - \hat{\mu}_{[345](i_3 i_4 i_5)} - \hat{\mu}_{[346](i_3 i_4 i_6)} - \hat{\mu}_{[456](i_4 i_5 i_6)} \end{aligned}$$

3.2 THE ALGORITHMS FOR ESTIMATION OF PARAMETERS AND BEST MODEL FITS OF LOG LINEAR MODELS FOR q -dimensional CONTINGENCY TABLE USING THE ITERATIVE PROPORTIONAL FITTING PROCEDURE

- A) Given a set of observed data with variables say (2, 3 , ...,q)
- B) Assuming 2- dimensional contingency table of two variables with I_{i_1} I_{i_2} categories of level.
- C) Step 1: Estimating parameters of 2-dimension as follows

$$\hat{\mu} = \log \left[\frac{\sum \alpha_{i_1 i_2}}{I_{i_1} I_{i_2}} \right] \quad (3.3.1)$$

$$\hat{\mu}_{1(i_1)} = \log \left[\frac{\sum \alpha_{i_1 \cdot}}{I_{i_1}} \right] - \hat{\mu} \quad (3.3.2)$$

$$\hat{\mu}_{2(i_2)} = \log \left[\frac{\sum \alpha_{\cdot i_2}}{I_{i_2}} \right] - \hat{\mu} \quad (3.3.3)$$

D) Step 2: Using Iterative proportional fitting (IPF) we compute the parameter estimates iteratively for (3.3.1 to 3.3.3)

E) Step 3: Estimate expected frequencies for 2-dimension

$$m_{i_1 i_2} = e^{(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)})} \quad (3.3.5)$$

$$m_{i_1 i_2} = e^{(\hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{12(i_1 i_2)})} \quad (3.3.6)$$

F) Step 4 :We estimate expected counts estimates iteratively for (3.3.5 -3.3.6) using Iterative proportional method

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)}) \quad (3.3.7)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{12(i_1 i_2)}) \quad (3.3.8)$$

(G) Step 5: Model fits of (3.3.7 to 3.3.8) will be estimated as well iteratively using IPF

H) 3-dimension of three variables with categories of levels $I_{i_1}, I_{i_2}, I_{i_3}$

I) Step 6: calculate Parameters estimates as follows

$$\hat{\mu} = \log \left[\frac{\sum \alpha_{i_1 i_2 i_3}}{I_{i_1} I_{i_2} I_{i_3}} \right] \quad (3.3.9)$$

$$\hat{\mu}_{1(i_1)} = \log \left[\frac{\sum \alpha_{i_1..}}{I_{i_2} I_{i_3}} \right] \quad (3.4.0)$$

$$\hat{\mu}_{2(i_2)} = \log \left(\frac{\sum \alpha_{.i_2.}}{I_{i_1} I_{i_3}} \right) - \hat{\mu} \quad (3.4.1)$$

$$\hat{\mu}_{3(i_3)} = \log \left(\frac{\sum \alpha_{..i_3}}{I_{i_1} I_{i_2}} \right) - \hat{\mu} \quad (3.4.2)$$

$$\hat{\mu}_{12(i_1 i_2)} = \log \left(\frac{\sum \alpha_{i_1 i_2.}}{I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{2(i_2)} \quad (3.4.3)$$

$$\hat{\mu}_{13(i_1 i_3)} = \log \left(\frac{\sum \alpha_{i_1 .i_3}}{I_{i_2}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{3(i_3)} \quad (3.4.4)$$

J) Step 7: We compute the parameter estimates using IPF for (3.3.9 - 3.4.4)

K) step 8: We Estimate expected counts of 3-dimensional contingency table

$$(m_{i_1 i_2 i_3}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)}) \quad (3.4.5)$$

$$(m_{i_1 i_2 i_3}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{13(i_1 i_3)}) \quad (3.4.6)$$

$$(m_{i_1 i_2 i_3}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{12(i_1 i_2)}) \quad (3.4.7)$$

$$(m_{i_1 i_2 i_3}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{23(i_2 i_3)}) \quad (3.4.8)$$

$$(m_{i_1 i_2 i_3}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{12(i_1 i_2)} + \hat{\mu}_{13(i_1 i_3)}) \quad (3.4.9)$$

$$(m_{i_1 i_2 i_3}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{12(i_1 i_2)} + \hat{\mu}_{23(i_2 i_3)}) \quad (3.5.0)$$

$$(m_{i_1 i_2 i_3}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{13(i_1 i_3)} + \hat{\mu}_{23(i_2 i_3)}) \quad (3.5.1)$$

$$(m_{i_1 i_2 i_3}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{12(i_1 i_2)} + \hat{\mu}_{13(i_1 i_3)} + \hat{\mu}_{23(i_2 i_3)}) \quad (3.5.2)$$

$$(m_{i_1 i_2 i_3}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{12(i_1 i_2)} + \hat{\mu}_{13(i_1 i_3)} + \hat{\mu}_{23(i_2 i_3)} + \hat{\mu}_{123(i_1 i_2 i_3)}) \quad (3.5.3)$$

L) Step 9: We estimate the expected frequencies estimates iteratively for (3.4.5- 3.5.3) using IPF

M) Step 10: Model fits estimates will be iteratively done using IPF as shown in (3. 5.4 -3.7.4)

$$\text{model} = \text{logml}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)}) \quad (3.5.4)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{13(i_1 i_3)}) \quad (3.5.5)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{12(i_1 i_2)}) \quad (3.5.6)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{23(i_2 i_3)}) \quad (3.5.7)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{12(i_1 i_2)} + \hat{\mu}_{13(i_1 i_3)}) \quad (3.5.8)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{12(i_1 i_2)} + \hat{\mu}_{23(i_2 i_3)}) \quad (3.5.9)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{13(i_1 i_3)} + \hat{\mu}_{23(i_2 i_3)}) \quad (3.6.0)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{12(i_1 i_2)} + \hat{\mu}_{13(i_1 i_3)} + \hat{\mu}_{23(i_2 i_3)} + \hat{\mu}_{123(i_1 i_2 i_3)}) \quad (3.6.1)$$

N) Step 11: We calculate 4-dimensional estimating parameters as follows

$$\hat{\mu} = \log \left(\frac{\sum \alpha_{i_1 i_2 i_3 i_4}}{I_{i_1} I_{i_2} I_{i_3} I_{i_4}} \right) \quad (3.6.2)$$

$$\hat{\mu}_{1(i_1)} = \log \left(\frac{\sum \alpha_{i_1 \dots}}{I_{i_2} I_{i_3} I_{i_4}} \right) - \hat{\mu} \quad (3.6.2)$$

$$\hat{\mu}_{2(i_2)} = \log \left(\frac{\sum \alpha_{\dots i_2 \dots}}{I_{i_1} I_{i_3} I_{i_4}} \right) - \hat{\mu} \quad (3.6.3)$$

$$\hat{\mu}_{3(i_3)} = \log \left(\frac{\sum \alpha_{\dots i_3 \dots}}{I_{i_1} I_{i_2} I_{i_4}} \right) - \hat{\mu} \quad (3.6.4)$$

$$\hat{\mu}_{4(i_4)} = \log \left(\frac{\sum \alpha_{\dots i_4}}{I_{i_1} I_{i_2} I_{i_3}} \right) - \hat{\mu} \quad (3.6.5)$$

$$\hat{\mu}_{12(i_1 i_2)} = \log \left(\frac{\sum \alpha_{i_1 i_2 \dots}}{I_{i_3} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{2(i_2)} \quad (3.6.6)$$

$$\hat{\mu}_{13(i_1 i_3)} = \log \left(\frac{\sum \alpha_{i_1 i_3 \dots}}{I_{i_2} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{3(i_3)} \quad (3.6.7)$$

$$\hat{\mu}_{14(i_4)} = \log \left(\frac{\sum \alpha_{i_1 \dots i_4}}{I_{i_2}^{i_3}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{4(i_4)} \quad (3.6.8)$$

$$\hat{\mu}_{23(i_2 i_3)} = \log \left(\frac{\sum \alpha_{i_2 i_3 \cdot}}{I_{i_1} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{3(i_3)} \quad (3.6.9)$$

$$\hat{\mu}_{24(i_2 i_4)} = \log \left(\frac{\sum \alpha_{i_1 \dots i_4}}{I_{i_2} I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{4(i_4)} \quad (3.7.0)$$

$$\hat{\mu}_{123(i_2 i_3)} = \log \left(\frac{\sum \alpha_{i_1 i_2 i_3 \cdot}}{I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{3(i_3)} - \hat{\mu}_{12(i_2 i_3)} - \hat{\mu}_{13(i_2 i_3)} - \hat{\mu}_{23(i_2 i_3)} \quad (3.7.1)$$

$$\hat{\mu}_{124(i_2 i_4)} = \log \left(\frac{\sum \alpha_{i_1 i_2 \cdot i_4}}{I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{4(i_4)} - \hat{\mu}_{12(i_2 i_4)} - \hat{\mu}_{14(i_2 i_4)} - \hat{\mu}_{24(i_2 i_4)} \quad (3.7.1)$$

$$\hat{\mu}_{134(i_3 i_4)} = \log \left(\frac{\sum \alpha_{i_1 \cdot i_3 i_4}}{I_{i_2}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{3(i_3)} - \hat{\mu}_{4(i_4)} - \hat{\mu}_{13(i_3 i_4)} - \hat{\mu}_{14(i_3 i_4)} - \hat{\mu}_{34(i_3 i_4)} \quad (3.7.2)$$

$$\hat{\mu}_{234(i_2i_3i_4)} = \log \left(\frac{\sum_{i_1} \alpha_{.i_2i_3i_4}}{I_{i_1}} \right) - \hat{\mu} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{3(i_3)} - \hat{\mu}_{4(i_4)} - \hat{\mu}_{23(i_2i_3)} - \hat{\mu}_{24(i_2i_4)} - \hat{\mu}_{34(i_3i_4)} \quad (3.7.3)$$

O) Step 12: We compute the parameter estimates using IPF for (3.6.2 -3.7.3)

P) Step 13: Estimates of expected counts of 4-dimensional data is done iteratively using IPF for (3.7.4- 3.9.9)

$$(m_{i_1i_2i_3i_4}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)}) \quad (3.7.4)$$

$$(m_{i_1i_2i_3i_4}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1i_4)}) \quad (3.7.5)$$

$$(m_{i_1i_2i_3i_4}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{12(i_1i_2)}) \quad (3.7.6)$$

$$(m_{i_1i_2i_3i_4}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{13(i_1i_3)}) \quad (3.7.7)$$

$$(m_{i_1i_2i_3i_4}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{23(i_2i_3)}) \quad (3.7.8)$$

$$(m_{i_1i_2i_3i_4}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{24(i_2i_4)}) \quad (3.7.9)$$

$$(m_{i_1i_2i_3i_4}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{34(i_3i_4)}) \quad (3.8.0)$$

$$(m_{i_1i_2i_3i_4}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{12(i_1i_2)} + \hat{\mu}_{14(i_1i_4)} + \hat{\mu}_{13(i_1i_3)}) \quad (3.8.1)$$

$$(m_{i_1i_2i_3i_4}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1i_4)} + \hat{\mu}_{13(i_1i_3)}) \quad (3.8.2)$$

$$(m_{i_1i_2i_3i_4}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1i_4)} + \hat{\mu}_{23(i_2i_3)}) \quad (3.8.3)$$

$$(m_{i_1i_2i_3i_4}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1i_4)} + \hat{\mu}_{24(i_2i_4)}) \quad (3.8.4)$$

$$(m_{i_1i_2i_3i_4}) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1i_4)} + \hat{\mu}_{12(i_1i_2)} + \hat{\mu}_{13(i_1i_3)}) \quad (3.8.5)$$

$$\left(m_{i_1 i_2 i_3 i_4}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{12(i_1 i_2)} + \hat{\mu}_{23(i_2 i_3)}\right) \quad (3.8.6)$$

$$\left(m_{i_1 i_2 i_3 i_4}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{12(i_1 i_2)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{24(i_2 i_4)}\right) \quad (3.8.7)$$

$$\left(m_{i_1 i_2 i_3 i_4}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{12(i_1 i_2)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{34(i_3 i_4)}\right) \quad (3.8.8)$$

$$\left(m_{i_1 i_2 i_3 i_4}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{123(i_1 i_2 i_3)}\right) \quad (3.8.9)$$

$$\left(m_{i_1 i_2 i_3 i_4}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{124(i_1 i_2 i_4)}\right) \quad (3.9.0)$$

$$\left(m_{i_1 i_2 i_3 i_4}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{134(i_1 i_3 i_4)}\right) \quad (3.9.1)$$

$$\left(m_{i_1 i_2 i_3 i_4}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{234(i_2 i_3 i_4)}\right) \quad (3.9.2)$$

$$\left(m_{i_1 i_2 i_3 i_4}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{123(i_1 i_2 i_3)} + \hat{\mu}_{124(i_1 i_2 i_4)}\right) \quad (3.9.3)$$

$$\left(m_{i_1 i_2 i_3 i_4}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{123(i_1 i_2 i_3)} + \hat{\mu}_{134(i_1 i_3 i_4)}\right) \quad (3.9.4)$$

$$\left(m_{i_1 i_2 i_3 i_4}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{123(i_1 i_2 i_3)} + \hat{\mu}_{234(i_2 i_3 i_4)}\right) \quad (3.9.5)$$

$$\left(m_{i_1 i_2 i_3 i_4}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{124(i_1 i_2 i_4)} + \hat{\mu}_{134(i_1 i_3 i_4)}\right) \quad (3.9.6)$$

$$\left(m_{i_1 i_2 i_3 i_4}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{124(i_1 i_2 i_4)} + \hat{\mu}_{234(i_2 i_3 i_4)}\right) \quad (3.9.7)$$

$$\left(m_{i_1 i_2 i_3 i_4}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{134(i_1 i_3 i_4)} + \hat{\mu}_{234(i_2 i_3 i_4)}\right) \quad (3.9.8)$$

$$\left(m_{ijkl}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{1234(i_1 i_2 i_3 i_4)}\right) \quad (3.9.9)$$

Q) Step 14: We compute model fits estimates for (4.0.0 - 4.2.5) iteratively using IPF

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{234(i_2 i_3 i_4)}) \quad (4.1.8)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{123(i_1 i_2 i_3)} + \hat{\mu}_{124(i_1 i_2 i_4)}) \quad (4.1.9)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{123(i_1 i_2 i_3)} + \hat{\mu}_{134(i_1 i_3 i_4)}) \quad (4.2.0)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{123(i_1 i_2 i_3)} + \hat{\mu}_{234(i_2 i_3 i_4)}) \quad (4.2.1)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{124(i_1 i_2 i_4)} + \hat{\mu}_{134(i_1 i_3 i_4)}) \quad (4.2.2)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{124(i_1 i_2 i_4)} + \hat{\mu}_{234(i_2 i_3 i_4)}) \quad (4.2.3)$$

$$\text{model} = \text{loglm}(\mu + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{134(i_1 i_3 i_4)} + \hat{\mu}_{234(i_2 i_3 i_4)}) \quad (4.2.4)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{1234(i_1 i_2 i_3 i_4)}) \quad (4.2.5)$$

R) Step 15: 5-dimensional estimating parameters are as follows

$$\hat{\mu} = \log \left(\frac{\sum \alpha_{i_1 i_2 i_3 i_4 i_5}}{\mathbf{I}_{i_1} \mathbf{I}_{i_2} \mathbf{I}_{i_3} \mathbf{I}_{i_4} \mathbf{I}_{i_5}} \right) \quad (4.2.6)$$

$$\hat{\mu}_{1(i_1)} = \log \left(\frac{\sum \alpha_{i_1 \dots}}{\mathbf{I}_{i_2} \mathbf{I}_{i_3} \mathbf{I}_{i_4} \mathbf{I}_{i_5}} \right) - \hat{\mu} \quad (4.2.7)$$

$$\hat{\mu}_{2(i_2)} = \log \left(\frac{\sum_{i_1 i_3 i_4 i_5} \alpha_{i_2 \dots}}{\mathbf{I}_{i_1} \mathbf{I}_{i_3} \mathbf{I}_{i_4} \mathbf{I}_{i_5}} \right) - \hat{\mu} \quad (4.2.8)$$

$$\hat{\mu}_{3(i_3)} = \log \left(\frac{\sum_{i_1 i_2 i_4 i_5} \alpha_{i_3 \dots}}{\mathbf{I}_{i_1} \mathbf{I}_{i_2} \mathbf{I}_{i_4} \mathbf{I}_{i_5}} \right) - \hat{\mu} \quad (4.2.9)$$

$$\hat{\mu}_{4(i_4)} = \log \left(\frac{\sum_{i_1 i_2 i_3 i_5} \alpha_{i_4 \dots}}{\mathbf{I}_{i_1} \mathbf{I}_{i_2} \mathbf{I}_{i_3} \mathbf{I}_{i_5}} \right) - \hat{\mu} \quad (4.3.0)$$

$$\hat{\mu}_{5(i_5)} = \log \left(\frac{\sum_{i_1 i_2 i_3 i_4} \alpha_{i_5 \dots}}{\mathbf{I}_{i_1} \mathbf{I}_{i_2} \mathbf{I}_{i_3} \mathbf{I}_{i_4}} \right) - \hat{\mu} \quad (4.3.1)$$

$$\hat{\mu}_{12(i_2)} = \log \left(\frac{\sum_{i_3 i_4 i_5} \alpha_{i_1 i_2 \dots}}{\mathbf{I}_{i_3} \mathbf{I}_{i_4} \mathbf{I}_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{2(i_2)} \quad (4.3.2)$$

$$\hat{\mu}_{13(i_3)} = \log \left(\frac{\sum_{i_1 i_4 i_5} \alpha_{i_1 i_3 \dots}}{\mathbf{I}_{i_2} \mathbf{I}_{i_4} \mathbf{I}_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{3(i_3)} \quad (4.3.3)$$

$$\hat{\mu}_{14(i_4)} = \log \left(\frac{\sum_{i_1 i_2 i_3 i_5} \alpha_{i_1 i_4 \dots}}{\mathbf{I}_{i_2} \mathbf{I}_{i_3} \mathbf{I}_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{4(i_4)} \quad (4.3.4)$$

$$\hat{\mu}_{15(i_1 i_5)} = \log \left(\frac{\sum \alpha_{i_1 \dots i_5}}{I_{i_2} I_{i_3} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{5(i_5)} \quad (4.3.5)$$

$$\hat{\mu}_{23(i_2 i_3)} = \log \left(\frac{\sum \alpha_{i_2 i_3 \dots}}{I_{i_1} I_{i_4} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{3(i_3)} \quad (4.3.6)$$

$$\hat{\mu}_{24(i_2 i_4)} = \log \left(\frac{\sum \alpha_{i_2 i_4 \dots}}{I_{i_1} I_{i_3} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{4(i_4)} \quad (4.3.7)$$

$$\hat{\mu}_{25(i_2 i_5)} = \log \left(\frac{\sum \alpha_{i_2 \dots i_5}}{I_{i_1} I_{i_3} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{5(i_5)} \quad (4.3.8)$$

$$\hat{\mu}_{34(i_3 i_4)} = \log \left(\frac{\sum \alpha_{\dots i_3 i_4 \dots}}{I_{i_1} I_{i_2} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{3(i_3)} - \hat{\mu}_{4(i_4)} \quad (4.3.9)$$

$$\hat{\mu}_{35(i_3 i_5)} = \log \left(\frac{\sum \alpha_{\dots i_3 \dots i_5}}{I_{i_1} I_{i_2} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{3(i_3)} - \hat{\mu}_{5(i_5)} \quad (4.4.0)$$

$$\hat{\mu}_{45(i_4 i_5)} = \log \left(\frac{\sum \alpha_{\dots i_4 i_5}}{I_{i_1} I_{i_2} I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{4(i_4)} - \hat{\mu}_{5(i_5)} \quad (4.4.1)$$

$$\hat{\mu}_{123(i_1 i_2 i_3)} = \log \left(\frac{\sum \alpha_{i_1 i_2 i_3 \cdot}}{I_{i_1} I_{i_2} I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{3(i_3)} - \hat{\mu}_{12(i_1 i_2)} - \hat{\mu}_{13(i_1 i_3)} - \hat{\mu}_{23(i_2 i_3)} \quad (4.4.2)$$

$$\hat{\mu}_{124(i_1 i_2 i_4)} = \log \left(\frac{\sum \alpha_{i_1 i_2 \cdot i_4}}{I_{i_1} I_{i_2} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{4(i_4)} - \hat{\mu}_{12(i_1 i_2)} - \hat{\mu}_{14(i_1 i_4)} - \hat{\mu}_{24(i_2 i_4)} \quad (4.4.3)$$

$$\hat{\mu}_{125(i_1 i_2 i_5)} = \log \left(\frac{\sum \alpha_{i_1 i_2 \cdot i_5}}{I_{i_1} I_{i_2} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{5(i_5)} - \hat{\mu}_{12(i_1 i_2)} - \hat{\mu}_{15(i_1 i_5)} - \hat{\mu}_{25(i_2 i_5)} \quad (4.4.4)$$

$$\hat{\mu}_{134(i_1 i_3 i_4)} = \log \left(\frac{\sum \alpha_{i_1 \cdot i_3 i_4}}{I_{i_1} I_{i_3} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{3(i_3)} - \hat{\mu}_{4(i_4)} - \hat{\mu}_{13(i_1 i_3)} - \hat{\mu}_{14(i_1 i_4)} - \hat{\mu}_{34(i_3 i_4)} \quad (4.4.5)$$

$$\hat{\mu}_{135(i_1 i_3 i_5)} = \log \left(\frac{\sum \alpha_{i_1 \cdot i_3 \cdot i_5}}{I_{i_1} I_{i_3} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{3(i_3)} - \hat{\mu}_{5(i_5)} - \hat{\mu}_{13(i_1 i_3)} - \hat{\mu}_{15(i_1 i_5)} - \hat{\mu}_{35(i_3 i_5)} \quad (4.4.6)$$

$$\hat{\mu}_{145(i_1 i_4 i_5)} = \log \left(\frac{\sum \alpha_{i_1 \cdot i_4 i_5}}{I_{i_1} I_{i_4} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{4(i_4)} - \hat{\mu}_{5(i_5)} - \hat{\mu}_{14(i_1 i_4)} - \hat{\mu}_{15(i_1 i_5)} - \hat{\mu}_{45(i_4 i_5)} \quad (4.4.7)$$

$$\hat{\mu}_{234(i_2 i_3 i_4)} = \log \left(\frac{\sum \alpha_{\cdot i_2 i_3 i_4}}{I_{i_2} I_{i_3} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{3(i_3)} - \hat{\mu}_{4(i_4)} - \hat{\mu}_{23(i_2 i_3)} - \hat{\mu}_{24(i_2 i_4)} - \hat{\mu}_{34(i_3 i_4)} \quad (4.4.8)$$

$$\hat{\mu}_{235(i_2 i_3 i_5)} = \log \left(\frac{\sum \alpha_{\cdot i_2 i_3 \cdot i_5}}{I_{i_2} I_{i_3} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{3(i_3)} - \hat{\mu}_{5(i_5)} - \hat{\mu}_{23(i_2 i_3)} - \hat{\mu}_{25(i_2 i_5)} - \hat{\mu}_{35(i_3 i_5)} \quad (4.4.9)$$

$$\hat{\mu}_{245(i_2 i_4 i_5)} = \log \left(\frac{\sum \alpha_{\cdot i_2 \cdot i_4 i_5}}{I_{i_2} I_{i_4} I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{4(i_4)} - \hat{\mu}_{5(i_5)} - \hat{\mu}_{24(i_2 i_4)} - \hat{\mu}_{25(i_2 i_5)} - \hat{\mu}_{45(i_4 i_5)} \quad (4.5.0)$$

$$\hat{\mu}_{345(i_3 i_4 i_5)} = \log \left(\frac{\sum \alpha_{\cdot i_3 i_4 i_5}}{I_{i_1} I_{i_2}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{3(i_3)} - \hat{\mu}_{4(i_4)} - \hat{\mu}_{34(i_3 i_4)} - \hat{\mu}_{35(i_3 i_5)} - \hat{\mu}_{45(i_4 i_5)} \quad (4.5.1)$$

$$\hat{\mu}_{1234(i_1 i_2 i_3 i_4)} = \log \left(\frac{\sum \alpha_{i_1 i_2 i_3 i_4 \cdot}}{I_{i_5}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{3(i_3)} - \hat{\mu}_{4(i_4)} - \hat{\mu}_{12(i_1 i_2)} - \hat{\mu}_{13(i_1 i_3)} - \hat{\mu}_{14(i_1 i_4)} - \hat{\mu}_{23(i_2 i_3)} - \hat{\mu}_{24(i_2 i_4)} - \hat{\mu}_{34(i_3 i_4)} \quad (4.5.2)$$

$$\hat{\mu}_{1235(i_1 i_2 i_3 i_5)} = \log \left(\frac{\sum \alpha_{i_1 i_2 i_3 i_5}}{I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{3(i_3)} - \hat{\mu}_{5(i_5)} - \hat{\mu}_{12(i_1 i_2)} - \hat{\mu}_{13(i_1 i_3)} - \hat{\mu}_{15(i_1 i_5)} - \hat{\mu}_{23(i_2 i_3)} - \hat{\mu}_{25(i_2 i_5)} - \hat{\mu}_{35(i_3 i_5)} \quad (4.5.3)$$

$$\hat{\mu}_{1245(i_1 i_2 i_4 i_5)} = \log \left(\frac{\sum \alpha_{i_1 i_2 i_4 i_5}}{I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{4(i_4)} - \hat{\mu}_{12(i_1 i_2)} - \hat{\mu}_{14(i_1 i_4)} - \hat{\mu}_{15(i_1 i_5)} - \hat{\mu}_{24(i_2 i_4)} - \hat{\mu}_{25(i_2 i_5)} - \hat{\mu}_{45(i_4 i_5)} \quad (4.5.4)$$

$$\hat{\mu}_{1345(i_1 i_3 i_4 i_5)} = \log \left(\frac{\sum \alpha_{i_1 i_3 i_4 i_5}}{I_{i_2}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{3(i_3)} - \hat{\mu}_{4(i_4)} - \hat{\mu}_{13(i_1 i_3)} - \hat{\mu}_{14(i_1 i_4)} - \hat{\mu}_{15(i_1 i_5)} - \hat{\mu}_{34(i_3 i_4)} - \hat{\mu}_{35(i_3 i_5)} - \hat{\mu}_{45(i_4 i_5)} \quad (4.5.5)$$

$$\hat{\mu}_{2345(i_2 i_3 i_4 i_5)} = \log \left(\frac{\sum \alpha_{i_2 i_3 i_4 i_5}}{I_{i_1}} \right) - \hat{\mu} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{3(i_3)} - \hat{\mu}_{4(i_4)} - \hat{\mu}_{5(i_5)} - \hat{\mu}_{23(i_2 i_3)} - \hat{\mu}_{24(i_2 i_4)} - \hat{\mu}_{25(i_2 i_5)} - \hat{\mu}_{34(i_3 i_4)} - \hat{\mu}_{35(i_3 i_5)} - \hat{\mu}_{45(i_4 i_5)} \quad (4.5.6)$$

S) Step 16: We compute the parameter estimates using IPF for (4.2.6 4.5.6)

T) Step 17: We compute estimates of expected frequencies of 5-dimensional contingency data for (4.5.7- 5.1.3) using IPF

$$\left(m_{i_1 i_2 i_3 i_4 i_5}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)}\right) \quad (4.5.7)$$

$$\left(m_{i_1 i_2 i_3 i_4 i_5}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{12(i_1 i_2)}\right) \quad (4.5.8)$$

$$\left(m_{i_1 i_2 i_3 i_4 i_5}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{13(i_1 i_3)}\right) \quad (4.5.9)$$

$$\left(m_{i_1 i_2 i_3 i_4 i_5}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{14(i_1 i_4)}\right) \quad (4.6.0)$$

$$\left(m_{i_1 i_2 i_3 i_4 i_5}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{15(i_1 i_5)}\right) \quad (4.6.1)$$

$$\left(m_{i_1 i_2 i_3 i_4 i_5}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{23(i_2 i_3)}\right) \quad (4.6.2)$$

$$\left(m_{i_1 i_2 i_3 i_4 i_5}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{24(i_2 i_4)}\right) \quad (4.6.3)$$

$$\left(m_{i_1 i_2 i_3 i_4 i_5}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{25(i_2 i_5)}\right) \quad (4.6.4)$$

$$\left(m_{i_1 i_2 i_3 i_4 i_5}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{34(i_3 i_4)}\right) \quad (4.6.5)$$

$$\left(m_{i_1 i_2 i_3 i_4 i_5}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{35(i_3 i_5)}\right) \quad (4.6.6)$$

$$\left(m_{i_1 i_2 i_3 i_4 i_5}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{45(i_4 i_5)}\right) \quad (4.6.7)$$

$$\left(m_{i_1 i_2 i_3 i_4 i_5}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{12(i_1 i_2)} + \hat{\mu}_{15(i_1 i_5)}\right) \quad (4.6.8)$$

$$\left(m_{i_1 i_2 i_3 i_4 i_5}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{13(i_1 i_3)} + \hat{\mu}_{15(i_1 i_5)}\right) \quad (4.6.9)$$

$$\left(m_{i_1 i_2 i_3 i_4 i_5}\right) = e\left(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{14(i_1 i_4)} + \hat{\mu}_{15(i_1 i_5)}\right) \quad (4.7.0)$$

$$\left(m_{i_1 i_2 i_3 i_4 i_5}\right) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{1245(i_1 i_2 i_4 i_5)} + \hat{\mu}_{2345(i_2 i_3 i_4 i_5)}) \quad (5.1.3)$$

$$\left(m_{i_1 i_2 i_3 i_4 i_5}\right) = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{1345(i_1 i_3 i_4 i_5)} + \hat{\mu}_{2345(i_2 i_3 i_4 i_5)}) \quad (5.1.4)$$

U) Step 18: We compute model fits estimates for (5.1.5 – 5.7.3) using IPF

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)}) \quad (5.1.5)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{12(i_1 i_2)}) \quad (5.1.6)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{13(i_1 i_3)}) \quad (5.1.7)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{14(i_1 i_4)}) \quad (5.1.8)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{15(i_1 i_5)}) \quad (5.1.9)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{23(i_2 i_3)}) \quad (5.2.0)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{24(i_2 i_4)}) \quad (5.2.1)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{25(i_2 i_5)}) \quad (5.2.2)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{34(i_3 i_4)}) \quad (5.2.3)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{35(i_3 i_5)}) \quad (5.2.4)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{45(i_4 i_5)}) \quad (5.2.5)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{12(i_1 i_2)} + \hat{\mu}_{15(i_1 i_5)}) \quad (5.2.6)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{13(i_1 i_3)} + \hat{\mu}_{15(i_1 i_5)}) \quad (5.2.7)$$

$$\text{model} = \log\text{lm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{1245(i_1i_2i_4i_5)} + \hat{\mu}_{2345(i_2i_3i_4i_5)}) \quad (5.7.2)$$

$$\text{model} = \log\text{lm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \hat{\mu}_{5(i_5)} + \hat{\mu}_{1345(i_1i_3i_4i_5)} + \hat{\mu}_{2345(i_2i_3i_4i_5)}) \quad (5.7.3)$$

V) Step 19: q-dimensional estimating parameters is given by

$$\hat{\mu} = \log \left(\frac{\sum_{i_1i_2i_3i_4 \dots} \alpha_{i_1i_2i_3i_4 \dots}}{\mathbf{I}_{i_1} \mathbf{I}_{i_2} \mathbf{I}_{i_3} \mathbf{I}_{i_4} * \dots * Qth} \right) \quad i_1, i_2, i_3, i_4, \dots, i_{qth} \quad (5.7.4)$$

$$\hat{\mu}_{1(i_1)} = \log \left(\frac{\sum_{i_2i_3i_4} \alpha_{i_1 \dots}}{\mathbf{I}_{i_2} \mathbf{I}_{i_3} \mathbf{I}_{i_4} * \dots * Qth} \right) - \hat{\mu} \quad (5.7.5)$$

$$\hat{\mu}_{2(i_2)} = \log \left(\frac{\sum_{i_1i_3i_4} \alpha_{i_2 \dots}}{\mathbf{I}_{i_1} \mathbf{I}_{i_3} \mathbf{I}_{i_4} * \dots * Qth} \right) - \hat{\mu} \quad (5.7.6)$$

$$\hat{\mu}_{4(i_4)} = \log \left(\frac{\sum_{i_1i_2i_3} \alpha_{\dots i_4}}{\mathbf{I}_{i_1} \mathbf{I}_{i_2} \mathbf{I}_{i_3} * \dots * Qth} \right) - \hat{\mu} \quad (5.7.7)$$

$$\hat{\mu}_{qth(qth)} = \log \left(\frac{\sum \alpha_{\dots, qth}}{\mathbf{I}_{i_1} \mathbf{I}_{i_2} \mathbf{I}_{i_3} \dots} \right) - \hat{\mu} \quad (5.7.8)$$

$$\hat{\mu}_{12(i_2)} = \log \left(\frac{\sum_{i_3 i_4} \alpha_{i_1 i_2 \dots}}{I_{i_3} I_{i_4} * \dots * Qth} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{2(i_2)} \quad (5.7.9)$$

$$\hat{\mu}_{13(i_3)} = \log \left(\frac{\sum_{i_2 i_4} \alpha_{i_1 \cdot i_3 \cdot}}{I_{i_2} I_{i_4} * \dots * Qth} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{3(i_3)} \quad (5.8.0)$$

$$\hat{\mu}_{14(i_4)} = \log \left(\frac{\sum_{i_2 i_3} \alpha_{i_1 \cdot i_4}}{I_{i_2} I_{i_3} * \dots * Qth} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{4(i_4)} \quad (5.8.1)$$

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$$\hat{\mu}_{1qth(i_1 q^{th})} = \log \left(\frac{\sum_{i_2 i_3 i_4} \alpha_{i_1 \dots qth}}{I_{i_2} I_{i_3} I_{i_4}} \right) - \hat{\mu} - \hat{\mu}_{1(i_1)} - \hat{\mu}_{qth(qth)} \quad (5.8.2)$$

$$\hat{\mu}_{23(i_2 i_3)} = \log \left(\frac{\sum_{i_1 i_4} \alpha_{\cdot i_2 i_3 \cdot}}{I_{i_1} I_{i_4} * \dots * Qth} \right) - \hat{\mu} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{3(i_3)} \quad (5.8.3)$$

$$\hat{\mu}_{24(i_2 i_4)} = \log \left(\frac{\sum_{i_1 i_3} \alpha_{\cdot i_2 \cdot i_4}}{I_{i_1} I_{i_3} * \dots * Qth} \right) - \hat{\mu} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{4(i_4)} \quad (5.8.4)$$

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$$\hat{\mu}_{2q^{th}(i_2q^{th})} = \log \left(\frac{\sum \alpha_{.i_2.q^{th}}}{I_{i_1} I_{i_3}} \right) - \hat{\mu} - \hat{\mu}_{2(i_2)} - \hat{\mu}_{q^{th}(q^{th})} \quad (5.8.5)$$

U) Step 20: Estimating of expected frequencies of q-dimensional contingency table

$$m_{i_1 i_2 i_3 i_4 \dots q^{th}} = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{q^{th}(q^{th})}) \quad (5.8.6)$$

$$m_{i_1 i_2 i_3 i_4 \dots q^{th}} = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{q^{th}(q^{th})}) + \hat{\mu}_{14(i_1 i_4)} \quad (5.8.7)$$

$$m_{i_1 i_2 i_3 i_4 \dots q^{th}} = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{q^{th}(q^{th})}) + \hat{\mu}_{12(i_1 i_2)} \quad (5.8.8)$$

$$m_{i_1 i_2 i_3 i_4 \dots q^{th}} = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{q^{th}(q^{th})}) + \hat{\mu}_{13(i_1 i_3)} \quad (5.8.9)$$

$$m_{i_1 i_2 i_3 i_4 \dots q^{th}} = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{q^{th}(q^{th})}) + \hat{\mu}_{23(i_2 i_3)} \quad (5.9.0)$$

$$m_{i_1 i_2 i_3 i_4 \dots q^{th}} = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{q^{th}(q^{th})}) + \hat{\mu}_{24(i_2 i_4)} \quad (5.9.1)$$

$$m_{i_1 i_2 i_3 i_4 \dots q^{th}} = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{q^{th}(q^{th})}) + \hat{\mu}_{34(i_3 i_4)} \quad (5.9.2)$$

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$$m_{i_1 i_2 i_3 i_4 \dots q^{th}} = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{q^{th}(q^{th})}) \quad (5.9.3)$$

$$m_{q^{th}(q^{th}+1) [i_1 i_2 i_3 i_4 \dots q^{th}]} = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{q^{th}(q^{th})} + \dots) \quad (5.9.4)$$

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$$m_{qth(i_1 i_2 i_3 i_4)} = e(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{qth(qth)} + \hat{\mu}_{12(i_1 i_2)} + \hat{\mu}_{13(i_1 i_3)} + \hat{\mu}_{23(i_2 i_3)} + \hat{\mu}_{24(i_2 i_4)} + \dots + \hat{\mu}_{(qth-1)qth[(qth-1),qth]}) \quad (5.9.5)$$

V) Step 21: estimating of model fits of q- dimensional contingency table

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{qth(qth)}) \quad (5.9.6)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{qth(qth)} + \hat{\mu}_{14(i_1 i_4)}) \quad (5.9.7)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{qth(qth)} + \hat{\mu}_{12(i_1 i_2)}) \quad (5.9.8)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{qth(qth)} + \hat{\mu}_{13(i_1 i_3)}) \quad (5.9.9)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{qth(qth)} + \hat{\mu}_{23(i_2 i_3)}) \quad (6.0.0)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{qth(qth)} + \hat{\mu}_{24(i_2 i_4)}) \quad (6.0.1)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{qth(qth)} + \hat{\mu}_{34(i_3 i_4)}) \quad (6.0.2)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{qth(qth)}) \quad (6.0.3)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{qth(qth)} + \dots) \quad (6.0.4)$$

$$\text{model} = \text{loglm}(\hat{\mu} + \hat{\mu}_{1(i_1)} + \hat{\mu}_{2(i_2)} + \hat{\mu}_{3(i_3)} + \hat{\mu}_{4(i_4)} + \dots + \hat{\mu}_{qth(qth)} + \hat{\mu}_{12(i_1 i_2)} + \hat{\mu}_{13(i_1 i_3)} + \hat{\mu}_{23(i_2 i_3)} + \hat{\mu}_{24(i_2 i_4)} + \dots + \hat{\mu}_{(qth-1)qth[(qth-1),qth]}) \quad (6.0.5)$$

3.3 STEPS OF ITERATIVE PROPORTIONAL FITTING PROCEDURE

The following steps are involved:

- 1) Start with initial estimates for the estimated expected cell counts. For example, set all $\hat{m}_{i_1 i_2 i_3 i_4}(0) = 1.0$, where $\hat{m}_{i_1 i_2 i_3 i_4}$ is the estimated expected frequency.
- 2) Adjust each cell entry by multiplying it by scaling factors. This moves the cells entries towards satisfaction of the marginal constraints specified by the model.
- 3) Iterate through the adjustment steps until the maximum difference b/w the marginal totals in the sample and estimated marginal totals reaches a certain minimal threshold by $\varepsilon = 0.1, 0.01$ or 0.0001

After each cycle, the estimates satisfy the constraints specified in the model, and estimated expected totals come closer to matching the observed totals. Thus the process converges. .

For instance, consider 4- factor model, without 4-factor interaction given as

$$\log_e (m_{i_1 i_2 i_3 i_4}) = \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} + \mu_{4(i_4)} + \mu_{12(i_1 i_2)} + \mu_{13(i_1 i_3)} + \mu_{14(i_1 i_4)} + \mu_{23(i_2 i_3)} + \mu_{24(i_2 i_4)} + \mu_{34(i_3 i_4)} + \mu_{123(i_1 i_2 i_3)} + \mu_{124(i_1 i_2 i_4)} + \mu_{134(i_1 i_3 i_4)} + \mu_{234(i_2 i_3 i_4)} \quad (11)$$

IPF algorithm will be used for estimating expected frequencies $(m_{i_1 i_2 i_3 i_4})$, which are inputs to parameter estimation in the fitted model. Estimates of parameters are functions of the logarithms of $m_{i_1 i_2 i_3 i_4}$ (Everitt, 1977).

Totals $\hat{m}_{i_1 i_2 i_3 .}$, $\hat{m}_{i_1 i_2 . i_4}$, $\hat{m}_{i_1 . i_3 i_4}$, and $\hat{m}_{. i_2 i_3 i_4}$ are characterized to be equal to the corresponding observed marginal totals $n_{i_1 i_2 i_3 .}$, $n_{i_1 i_2 . i_4}$, $n_{i_1 . i_3 i_4}$, and $n_{. i_2 i_3 i_4}$ respectively.

To start IPF procedure, we start initial values $\hat{m}_{i_1 i_2 i_3 i_4}(0) = 1$ and proceed by adjusting these proportionally to satisfy the first marginal constraint

$$(\hat{m}_{i_1 i_2 i_3 .} = n_{i_1 i_2 i_3 .}), \text{ calculated from:}$$

$$\hat{m}_{i_1 i_2 i_3 i_4} (1) = \frac{\hat{m}_{i_1 i_2 i_3 i_4}^{(0)} n_{i_1 i_2 i_3}}{\hat{m}_{i_1 i_2 i_3}^{(0)}} \quad (12)$$

Revise expected values $\hat{m}_{i_1 i_2 i_3 i_4} (1)$ to satisfy the second marginal's constraint $\hat{m}_{i_1 i_2 i_4} = n_{i_1 i_2 i_4}$, using

$$\hat{m}_{i_1 i_2 i_3 i_4} (2) = \frac{\hat{m}_{i_1 i_2 i_3 i_4} (1) n_{i_1 i_2 i_4}}{\hat{m}_{i_1 i_2 i_4} (1)} \quad (13)$$

Revise expected values $\hat{m}_{i_1 i_2 i_3 i_4} (2)$ to satisfy the marginal constraint $\hat{m}_{i_1 i_3 i_4} = n_{i_1 i_3 i_4}$, using

$$\hat{m}_{i_1 i_2 i_3 i_4} (3) = \frac{\hat{m}_{i_1 i_2 i_3 i_4} (2) n_{i_1 i_3 i_4}}{\hat{m}_{i_1 i_3 i_4} (2)} \quad (14)$$

Complete the cycle by adjusting $\hat{m}_{i_1 i_2 i_3 i_4} (3)$ to satisfy the fourth marginal constraint

$\hat{m}_{i_2 i_3 i_4} = n_{i_2 i_3 i_4}$, using

$$\hat{m}_{i_1 i_2 i_3 i_4} (4) = \frac{\hat{m}_{i_1 i_2 i_3 i_4} (3) n_{i_2 i_3 i_4}}{\hat{m}_{i_2 i_3 i_4} (3)} \quad (15)$$

This four-step is repeated until convergence to desired accuracy is attained. That is, the process is continued until differences between expected values is very small say 0.01 or 0.0001

For 3- dimensional table, for model of No-three factor interaction or pair- wise association [12][13][23], IPF algorithm for finding expected frequencies is as follows:

The $\hat{m}_{i_1 i_2 i_3}$'s of the model is characterized by fitted margins

$$\hat{m}_{i_1 i_2} = n_{i_1 i_2}, \quad \hat{m}_{i_1 i_3} = n_{i_1 i_3}, \quad \hat{m}_{i_2 i_3} = n_{i_2 i_3}$$

The IPF procedure start with initial value or guess as $\hat{m}_{i_1 i_2 i_3} (0) = 1$, the modifications are

$$\hat{m}_{i_1 i_2 i_3} (1) = \frac{\hat{m}_{i_1 i_2 i_3} (0) n_{i_1 i_2}}{\hat{m}_{i_1 i_2} (0)} \quad (16)$$

$$\hat{M}_{i_1 i_2 i_3} (2) = \frac{\hat{M}_{i_1 i_2 i_3} (1) n_{i_1 i_3}}{\hat{M}_{i_1 i_3} (1)} \quad (17)$$

$$\hat{M}_{i_1 i_2 i_3} (3) = \frac{\hat{M}_{i_1 i_2 i_3} (2) n_{i_2 i_3}}{\hat{M}_{i_2 i_3} (2)} \quad (18)$$

The second cycle is of the same form as the first cycle above but uses updated estimates

$$\hat{M}_{i_1 i_2 i_3} (4) = \frac{\hat{M}_{i_1 i_2 i_3} (3) n_{i_1 i_2}}{\hat{M}_{i_1 i_2} (3)} \quad (19)$$

$$\hat{M}_{i_1 i_2 i_3} (5) = \frac{\hat{M}_{i_1 i_2 i_3} (4) n_{i_1 i_3}}{\hat{M}_{i_1 i_3} (4)} \quad (20)$$

$$\hat{M}_{i_1 i_2 i_3} (6) = \frac{\hat{M}_{i_1 i_2 i_3} (5) n_{i_1 i_3}}{\hat{M}_{i_1 i_3} (5)} \quad (21)$$

The algorithm can be terminated at iteration h when hth estimates are close satisfying the likelihood equations

$$m_{ij+} = n_{ij+} \quad m_{+jk} = n_{+jk} \quad m_{i+k} = n_{i+k}$$

3.4 PROPOSED APPROACH FOR TESTING THE SIGNIFICANCE OF INTERACTION OF VARIABLES OF CONTINGENCY TABLE.

Proposed Approach is given as

$$\chi_{ML1}^2 - \chi_{ML2}^2 \quad (22)$$

CHAPTER FOUR: ANALYSIS OF DATA AND DISCUSSION OF RESULTS

4.0 INTRODUCTION

In this chapter, we shall present the application of algorithms to Justyna's data of 3-dimensional contingency table to determine best model fit using R. Moreso, implementation of the algorithms to real life data using computer program written in R for estimating parameters and best model fits to the data for q-dimensional contingency tables shall be presented and finally proposed approach in testing for significance of interaction of variables will also be discussed.

4.1 PRESENTATION OF DATA

Table 4.1: Justyna(2012) data for 3-dimensional contingency table

Sex	W.Hnd	Exercise Freq	Exercise Some	Exercise None
Male	Right	63	34	11
	Left	5	5	2
Female	Right	46	54	10
	Left	3	3	1

Data Survey: Responses of 237 Students at the University of Adelaide to a number of questions (Venables and Ripley [1999] shown in Load Mass). The 3-way table data that has three categorical variables: Sex (Male, Female), Writing Hand (Right, Left), Exercise (frequently, Some, None).

Table 4.2: Real life Data for 3-Dimensional contingency Table

Breed	Age	Chick loss	
		Yes	No
Broiler	1	55	67
	2	16	44
	3	8	45
Old layer	1	48	66
	2	20	52
	3	18	71

Source: Poultry diary of Sambo feeds, Awka.

Table 4.3: Real life Data for 4-Dimensional Contingency Table

			BSC Grade	BSC Grade	BSC grade	BSC Grade	BSC Grade
Age	Gender	State of Origin	Pass	Third Class	Lower Division	Upper Division	First class
Under 26 26&over	Male	Indigene	3	41	45	20	1
			5	35	17	4	1
Under 26 26& over	Male	Non-indigene	2	69	44	9	1
			7	54	29	3	1
Under 26 26& over	Female	Indigene	1	31	58	32	1
			2	31	21	7	2
Under26 26& over	Female	Non-Indigene	3	41	32	9	1
			4	37	11	3	1

Source: Retrieved files of graduated students from Statistics department, Nnamdi Azikiwe University, Awka.

The variables and their categories are:

Variable 1: BSC grade

1. Pass
2. Third Class
3. Lower Division
4. Upper Division
5. First Class

Variable 2: State of Origin

1. Indigene
2. Non-indigene

Variable 3: Gender

1. Male
2. Female

Variable 4: Age

1. Under 26
2. 26 and over

Table 4.4 Real life data for 5-Dimensional Contingency Table

Mode of admission	Age	Gender	State of Origin	BSC Grade				
				P	TC	LD	UD	FC
Jamb Pre Science	Under 26	Male	Indigene	7	14	61	22	1
	26 and over			12	8	45	19	1
	Under 26			4	10	52	31	1
	26 and over			2	4	28	39	1
Jamb Pre science	Under 26	Male	Non-indigene	1	6	17	28	1
	26 and over			4	9	14	22	2
	Under 26			8	11	17	19	1
	26 and over			2	4	21	15	1
Jamb Pre science	Under 26	Female	Indigene	4	11	78	42	2
	26 and over			12	8	63	31	2
	Under 26			1	2	94	45	1
	26 and over			1	1	81	28	1
Jamb Pre science	Under 26	Female	Non-indigene	2	7	33	32	1
	26 and over			4	2	24	21	1
	Under 26			1	1	27	17	2
	26 and over			3	6	22	11	1

Source: Degree Results of graduated students, Dept of Micro Biology, Anambra State University Uli.

4.3 PROPOSED APPROACH FOR TESTING THE SIGNIFICANCE OF INTERACTION OF VARIABLES

We present proposed approach to test for significance of interaction of variables. Consider the saturated log linear model for a four- way $(I_i \times I_{i_2} \times I_{i_3} \times I_{i_4})$ contingency table involving variables A, B, C, and D indexed numerically 1, 2, 3, and 4 respectively and is given by

$$\begin{aligned} \log m_{i_1 i_2 i_3 i_4} = & \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} + \mu_{4(i_4)} + \mu_{12(i_1 i_2)} + \mu_{13(i_1 i_3)} + \mu_{14(i_1 i_4)} + \mu_{23(i_2 i_3)} + \mu_{24(i_2 i_4)} + \mu_{34(i_3 i_4)} + \mu_{123(i_1 i_2 i_3)} \\ & + \mu_{124(i_1 i_2 i_4)} + \mu_{134(i_1 i_3 i_4)} + \mu_{234(i_2 i_3 i_4)} + \mu_{1234(i_1 i_2 i_3 i_4)} \end{aligned} \quad (23)$$

Where $\log m_{ijkl}$ is the natural logarithmic transform of the expected cell frequency, (or the expected cell probability). The μ -terms are parameters satisfying the ANOVA – type constraints.

$$\sum_{i_1} \mu_{1(i_1)} = \sum_{i_2} \mu_{2(i_2)} = \dots = \sum_{i_4} \mu_{1234(i_1 i_2 i_3 i_4)} = 0$$

The significance of the interaction of variables [12] can be tested by comparing these two models

$$\log m_{i_1 i_2 i_3 i_4} = \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} + \mu_{4(i_4)} \quad (24)$$

and

$$\log m_{i_1 i_2 i_3 i_4} = \mu + \mu_{3(i_3)} + \mu_{4(i_4)} + \mu_{12(i_1 i_2)} \quad (25)$$

Let χ_{ML1}^2 and χ_{ML2}^2 be the test statistics for equations (24) and (25) respectively.

$\chi_{ML1}^2 - \chi_{ML2}^2$ gives the test of significance of interaction for variables [12] because of the additive nature of the likelihood ratio statistic χ_{ML}^2

Similarly, the significance of the interaction of variables [24] can be tested by comparing these two models

$$\log m_{i_1 i_2 i_3 i_4} = \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} + \mu_{4(i_4)} \quad (26)$$

and

$$\log m_{i_1 i_2 i_3 i_4} = \mu + \mu_{1(i_1)} + \mu_{3(i_3)} + \mu_{24(i_2 i_4)} \quad (27)$$

Suppose the test statistics of the model given in equation (26) is χ_{ML1}^2 and that of equation (27) is χ_{ML2}^2 , then the significance of interaction of variables [24] is given by $\chi_{ML1}^2 - \chi_{ML2}^2$ with appropriate degrees freedom

More so, for 3-dimensional contingency table whose variables are indexed 1, 2, and 3.

The significance of the interaction of variables [13] can be tested by comparing these two models

$$\log m_{i_1 i_2 i_3} = \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} \quad (28)$$

and

$$\log m_{i_1 i_2 i_3} = \mu + \mu_{2(i_2)} + \mu_{13(i_1 i_3)} \quad (29)$$

Let χ_{ML1}^2 and χ_{ML2}^2 be the test statistics for equation (28) and (29) respectively.

$\chi_{ML1}^2 - \chi_{ML2}^2$ gives the test of significance of interaction of variables [13] with appropriate degrees of freedom.

Furthermore, consider the 5- dimensional table whose variables are indexed 1, 2, 3, 4 and 5

Suppose it is required to examine the interaction between variables 3 and 5. That is represented by [35],

The significance of the interaction of variables [35] can be tested by comparing these two models

$$\log m_{i_1 i_2 i_3 i_4 i_5} = \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} + \mu_{4(i_4)} + \mu_{5(i_5)} \quad (30)$$

and

$$\log m_{i_1 i_2 i_3 i_4 i_5} = \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{4(i_4)} + \mu_{35(i_3 i_5)} \quad (31)$$

Suppose the test statistics of the model given in equation (30) is χ_{ML1}^2 and that of equation (31) is χ_{ML2}^2 , then the significance of interaction of variables [35] is given by $\chi_{ML1}^2 - \chi_{ML2}^2$ with appropriate degrees freedom

4.3.1 BROWN'S METHOD

Suppose model M_1 is nested in model M_2 , the hypothesis that the additional terms added to M_1 to obtain M_2 are equal to zero is examined using the statistic

$$\chi_{ML}^2 \left(\frac{M_1}{M_2} \right) = \chi_{ML}^2 (M_1) - \chi_{ML}^2 (M_2) = \chi_{ML2}^2 - \chi_{ML1}^2 \quad (32)$$

Brown (1976) has shown that all interactions that are significant in at least one of the tests of marginal and partial associations form the base model. For further examination, using the method of backward stepping, eliminating those interactions terms that have non-significant effects are achieved using the value of the statistic $\chi_{ML}^2 \left(\frac{M_1}{M_2} \right)$. For instance, consider the 3-dimensional table whose variables are indexed 1, 2 and 3.

Suppose it is required to examine the interaction between variables 1 and 3. That is represented by [13]; Brown (1976) suggested that these two models

$$\log m_{i_1 i_2 i_3} = \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} + \mu_{12(i_1 i_2)} + \mu_{13(i_1 i_3)} + \mu_{23(i_2 i_3)} \quad (33)$$

and

$$\log m_{i_1 i_2 i_3} = \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} + \mu_{12(i_1 i_2)} + \mu_{23(i_2 i_3)} \quad (34)$$

are compared using the likelihood ratio statistic χ_{ML}^2 . Suppose the test statistics of the model in equation (33) is χ_{ML1}^2 and χ_{ML2}^2 for equation (34), then the significance of interaction of variables [13] is given by $\chi_{ML2}^2 - \chi_{ML1}^2$ with appropriate degrees freedom.

Also consider the 4- dimensional table whose variables are indexed 1, 2, 3 and 4

Suppose it is required to examine the interaction between variables 1 and 2. That is represented by [12], Brown suggested that these two models

$$\log m_{i_1 i_2 i_3 i_4} = \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} + \mu_{4(i_4)} + \mu_{12(i_1 i_2)} + \mu_{13(i_1 i_3)} + \mu_{14(i_1 i_4)} + \mu_{23(i_2 i_3)} + \mu_{24(i_2 i_4)} + \mu_{34(i_3 i_4)} \quad (35)$$

and

$$\log m_{i_1 i_2 i_3 i_4} = \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} + \mu_{4(i_4)} + \mu_{13(i_1 i_3)} + \mu_{14(i_1 i_4)} + \mu_{23(i_2 i_3)} + \mu_{24(i_2 i_4)} + \mu_{34(i_3 i_4)} \quad (36)$$

are compared using the likelihood ratio statistic χ_{ML}^2 . Suppose the test statistics of the model in equation (35) is χ_{ML1}^2 and that of equation (36) is χ_{ML2}^2 , then the significance of interaction of variables [12] is given by $\chi_{ML2}^2 - \chi_{ML1}^2$ with appropriate degrees of freedom.

Also significance of the interaction of variables [24] can be tested by comparing these two models

$$\log m_{i_1 i_2 i_3 i_4} = \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} + \mu_{4(i_4)} + \mu_{12(i_1 i_2)} + \mu_{13(i_1 i_3)} + \mu_{14(i_1 i_4)} + \mu_{23(i_2 i_3)} + \mu_{24(i_2 i_4)} + \mu_{34(i_3 i_4)} \quad (37)$$

and

$$\log m_{i_1 i_2 i_3 i_4} = \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} + \mu_{4(i_4)} + \mu_{12(i_1 i_2)} + \mu_{13(i_1 i_3)} + \mu_{14(i_1 i_4)} + \mu_{23(i_2 i_3)} + \mu_{34(i_3 i_4)} \quad (38)$$

The test statistics of the model in (37) is $\chi^2_{ML1}(M_2)$ and that of equation (38) is $\chi^2_{ML2}(M_1)$ then the significance of interaction of variables 24 is given by

$$\chi^2_{ML}\left(\frac{M_1}{M_2}\right) = \chi^2_{ML2}(M_1) - \chi^2_{ML1}(M_2)$$

Furthermore, consider the 5- dimensional table whose variables are indexed 1, 2, 3,4 and 5

Suppose it is required to examine the interaction between variables 3 and 5. That is represented by [35], Brown suggested that these two models to be tested:

$$\begin{aligned} \log m_{i_1 i_2 i_3 i_4 i_5} = & \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} + \mu_{4(i_4)} + \mu_{5(i_5)} + \mu_{12(i_1 i_2)} + \mu_{13(i_1 i_3)} + \mu_{14(i_1 i_4)} + \mu_{15(i_1 i_5)} + \mu_{23(i_2 i_3)} + \mu_{24(i_2 i_4)} \\ & + \mu_{25(i_2 i_5)} + \mu_{34(i_3 i_4)} + \mu_{35(i_3 i_5)} + \mu_{45(i_4 i_5)} \end{aligned} \quad (39)$$

and

$$\begin{aligned} \log m_{i_1 i_2 i_3 i_4 i_5} = & \mu + \mu_{1(i_1)} + \mu_{2(i_2)} + \mu_{3(i_3)} + \mu_{4(i_4)} + \mu_{5(i_5)} + \mu_{12(i_1 i_2)} + \mu_{13(i_1 i_3)} + \mu_{14(i_1 i_4)} + \mu_{15(i_1 i_5)} + \mu_{23(i_2 i_3)} + \mu_{24(i_2 i_4)} \\ & + \mu_{25(i_2 i_5)} + \mu_{34(i_3 i_4)} + \mu_{45(i_4 i_5)} \end{aligned} \quad (40)$$

Suppose the test statistics of the model in equation (39) is χ^2_{ML1} and that of equation (40) is

χ^2_{ML2} , then the significance of interaction of variables [35] is given by $\chi^2_{ML2} - \chi^2_{ML1}$ with appropriate degrees of freedom.

4.4 DATA ANALYSIS

Table 4.5 shows the Summary of the Results for Goodness and Best of fits for 3-dimensional contingency table by Justyna (2012)

Table 4.5: Summary of the Justyna's Results

Model	G^2	χ^2	Df	P-value
(S,W, E)	9.713843	10.06693	7	0.2053780
(E, SW)	9.167555	9.258123	6	0.1643719
(W,SE)	3.659013	4.097142	5	0.5210121
(S,WE)	8.082703	8.022221	5	0.1517362
(SW,SE,WE)	1.303964	1.348596	2	0.5994751
(SWE)	0.0	0.0	0	1

P-value for G^2

Discussion:

The best model fit to the data is (SW, SE, WE) which has highest p-value and least value of G^2 . This implies that the model has the simpler parameters to explain the observed data.

Table 4.6 Summary of algorithms in R for Justyna's data for 3-dimensional contingency table for estimating goodness of fits and best model fit.

Model	G^2	χ^2	Df	p-value
(S,W,E)	9.418498	9.563039	7	0.2239891
(E,SW)	8.106711	8.162490	6	0.2303892
(W,SE)	2.795338	3.059808	5	0.7314966
(S,WE)	7.607794	7.541177	5	0.2257682
(SW,SE,WE)	0.521116	0.530048	2	0.7706214
(SWE)	0.0	0.0	0	1

P-value for G^2

Discussion

The algorithms yielded better results with the Justyna's data with higher p-values and least likelihood estimates. The results confirmed the appropriateness and authenticity of the algorithms developed which will be used in estimating for q-dimensional contingency table.

The criteria for comparison of estimation of goodness of fits and best model fit are the higher p-values and least likelihood estimates. Also, the best fit model must have highest p-value and least likelihood estimates. Based on these criteria we compare the results of Justyna's work and our own results and the comparison is shown in the table below:

Table 4.6a: Summary of comparison of goodness of fits and best model fit of Justyna’s results and Our own results.

Justyna’s results

our own results

Model	G^2	χ^2	D.F	P-value	Model	G^2	χ^2	D.F	P-value
(S,W,E)	9.71384	10.06693	7	0.2053780	(S,W,E)	9.418498	9.563039	7	0.223989
(E,SW)	9.16756	9.258123	6	0.1643719	(E,SW)	8.106711	8.162490	6	0.230389
(W,SE)	3.65901	4.097142	5	0.5210121	(W,SE)	2.795338	3.059808	5	0.7314967
(S,WE)	8.08270	8.02221	5	0.1517363	(S,WE)	7.607794	7.541177	5	0.225768
(SW,SE,WE)	1.30396	1.348596	2	0.5994751	(SW,SE,WE)	0.530048	0.530048	2	0.770621
(SWE)	0.0000	0.0000	0	1.000000	(SWE)	0.000000	0.000000	0	1.000000

Table 4.7: Summary of the Results for Goodness of fits for 3-dimensional contingency table of real life data

Models	G^2	χ^2	d.f	p-value
[C, A, B]	37.14	36.34	7	0
[A,CB]	36.82	35.21	6	0
[B,CA]	8.28	8.18	5	0.14
[C,AB]	29.69	28.76	5	0
[CA,CB]	7.96	7.85	4	0.093
[CA,AB]	0.83	0.82	3	0.84
[CB,AB]	29.36	28.32	4	0
[CA,CB, AB]	0.83	0.81	2	0.66
[CAB]	0	0	0	1

p-value for G^2

Discussion: The best model fit that explained the observed data is [CA,AB]. This model means, the data provide sufficient evidence (likelihood ratio =0.83, d.f =2, P=0.84 or Pearson 0.82, d.f =2, P=0.84) that Breed and chick loss are independent given the Age, thus the association or relationship between the Breed and Chick loss is independent for each age.

Table 4.8: Summary of the Results for Goodness of fit for 4-dimensional contingency table of real life data

Model	G^2	χ^2	d.f	p-value
[B,S,G,A]	102.68	111.46	32	0
[S,G,BA]	62.30	65.53	28	0
[G,A,BS]	69.29	69.05	28	0
[S,A,BG]	92.66	94.96	28	0
[B,A,SG]	91.09	94.51	31	0
[B,G,SA]	99.30	103.93	31	0
[B,S,GA]	101.67	108.69	31	0
[G,BA,BS]	28.93	29.46	24	0.2225874
[S,BA,BG]	52.30	51.56	24	0.0007148
[BA,SG]	50.73	49.61	27	0.0037467
[G,BA,SA]	58.94	61.70	27	0.0003621
[S,BA,GA]	61.31	64.02	27	0
[BA,BS,BG]	18.93	19.58	20	0.525909
[BA,BS,SG]	17.36	17.51	23	0.791019
[G,BA,BS,SA]	28.49	28.79	23	0.1976995
[BA,BS,GA]	27.95	28.20	23	0.2178485
[A,BSG]	46.92	45.38	19	0.00366759
[G,BSA]	26.25	25.89	19	0.1235033
[S,BGA]	49.53	48.21	19	0.0001535
[B,SGA]	87.11	86.69	28	0
[BSG,BSA]	3.89	3.81	10	0.9529921
[BSG,BGA]	3.79	3.79	10	0.9562268
[BSG,SGA]	42.95	42.21	16	0.0002856
[BSA, SGA]	14.06	13.91	16	0.8945323
[BSGA]	0	0	0	1

P-value for G^2

Discussion:

The best model fit which has sufficient evidence to fit the data without loss of information in this 4-dimensional contingency table is [BSG, BGA]. This model has highest p-value and the least likelihood ratio estimate. This model implies that sex is independent of age given Bscgrade and gender.

Table 4.9: Summary of the Results for Goodness and best of fits for 5-dimensional contingency table of real life data

Model	G^2	χ^2	d.f	p-value
[B,S,G,A,M]	172.9839	176.3949	71	<0.001
[S,G,A,BM]	141.2813	139.1354	67	<0.001
[S,A,M,BG]	139.1010	141.8967	67	<0.001
[B,G,M,BA]	166.3272	171.4614	67	0
[B,G,M,SA]	172.9646	176.5658	70	0
[B,G,A,SM]	171.2029	174.1919	70	0
[B,S,M,GA]	172.9732	170.3752	70	0
[B,S,A,GM]	172.9719	176.5359	70	0
[B,S,G,AM]	172.8484	175.9629	70	0
[B,M,GA,BS]	127.9595	131.5539	63	<0.001
[S,A,BM,BG]	125.7791	134.4006	63	<0.001
[S,G,BM,BA]	153.0054	174.6249	63	<0.001
[A,M,BM,SG]	154.6406	163.1955	66	<0.001
[G,BM,SA]	159.6427	173.5571	66	0
[G,A,BM,SM]	157.8861	172.8167	66	0
[S,BM,GA]	159.6514	173.3912	66	0
[S,A,BM,GM]	159.6501	173.3092	66	0
[S,G,BM,AM]	159.5265	172.8867	66	0
[S,BM,BG,BA]	119.1225	132.3240	59	0
[A,BM,BG,SG]	120.7577	128.7868	62	0
[BM,BG,SA]	125.7598	134.5223	62	0
[A,BM,BG,SM]	124.0031	133.9196	62	0
[S,BM,BG,GA]	125.7684	134.4744	62	<0.001
[S,A,BM,BG,GM]	125.3519	134.4098	62	0
[S,BM,BG,AM]	125.6436	134.2637	62	0
[BM,BG,BS,BA]	87.41988	93.03104	55	0.003528851
[A,BM,BG,BS,SG]	91.75409	94.07371	58	0.003132519
[BM,BG,BS,SA]	94.05723	96.53581	58	0.001922623
[BM,BG,BS,SM,A]	93.04289	96.43197	58	0.002388153
[BM,BG,BS,GA]	94.0658	96.45887	58	0.001919054
[BM,BG,BS,GM,A]	93.64933	95.98648	58	0.002095519
[BM,BG,BS,AM]	93.94103	96.38938	58	0.001971264
[A,M,BSG]	104.3455	105.8493	58	0.000432178
[G,M,BSA]	131.8655	127.2042	58	0
[G,A,BSM]	100.7154	99.0451	58	0.0004329178
[S,A,BGA]	124.9574	123.9418	58	0
[S,A,BGM]	113.5894	125.3055	58	0
[S,G,BAM]	145.6230	155.5872	58	0
[B,M,SGA]	167.5483	167.5230	67	0

[B,A,SGM]	163.4812	161.3538	67	0
[B,G,SAM]	170.8800	174.4700	67	0
[M,BSGA]	83.86091	78.14037	39	0
[A,BSGM]	41.96151	40.78206	39	0.3437295
[G.BSAM]	83.35372	79.69283	39	0.000464587
[S,BGAM]	85.02937	87.22491	39	0
[B,SGAM]	161.2704	158.5060	60	0
[BSGA,BSGM]	21.47697	20.70749	20	0.3695523
[BSGA,BGAM]	43.93290	42.81167	20	0.00156376
[BSGA ,SGAM]	77.58305	72.17188	32	0
[BSGM,BSAM]	24.60369	23.95200	20	0.2170156
[BSGM,BGAM]	13.40143	13.44084	20	0.8595041
[BSGM,SGAM]	39.75069	38.61374	32	0.16305041
[BSAM,BGAM]	22.76011	22.69919	20	0.3007267
[BSAM,SGAM]	73.74410	70.67186	32	0
[BGAM,SGAM]	73.90678	76.54197	32	0
[BSGAM]	0	0	0	1

P-value for G^2

Discussion:

The best model fit which has sufficient evidence to fit the data without loss of information in this 4-dimensional contingency table is [BSGM, BGAM]. This model has highest p-value and the least likelihood ratio estimate. This model implies that state of origin and age is independent given Bscgrade and mode of admission.

Table 4.10: Summary Of Parameter Estimates Of Justyna's Data Using Algorithms Implemented in Written R-Program

PARAMETER	ESTIMATE
Constant	2.983153
[Exer =1]	0.3927261
[Exer =2]	0.1949003
[Exer =3]	-0.1913940
[W.Hnd =1]	0.6095821
[W.Hnd =2]	-1.8304740
[Sex =1]	0.0125788
[Sex =2]	-0.0127939
[Exer =1]*[W.Hnd =1]	0
[Exer =1]*[W.Hnd =2]	0
[Exer =2]*[W.Hnd =1]	0
[Exer =2]*[W.Hnd =2]	0
[Exer =3]*[W.Hnd =1]	0.0674639

[Exer =3]*[W.Hnd =2]	-0.0742724
[Exer =1]*[Sex =1]	0.0127390
[Exer =1]*[Sex =2]	-0.1591112
[Exer =2]*[Sex =1]	-0.0034462
[Exer =2]*[Sex =2]	0.0387145
[Exer =3]*[Sex =1]	-0.0499663
[Exer= 3]*[Sex =2]	0.4441796
[Sex =1]*[W.Hnd=1]	-0.0217954
[Sex =1]*[W.Hnd=2]	0.0218715
[Sex =2]*[W.Hnd=1]	0.2210336
[Sex =2]*[W.Hnd=2]	-0.2926426

Table 4.11: Summary of Parameter Estimates of 3-Dimensional Contingency table of Real life data

PARAMETER	ESTIMATE
Constant	3.7495054
[Chick =1]	-0.4353181
[Chick =2]	0.3022809
[Age =1]	0.3280334
[Age =2]	-0.2529965
[Age =3]	-0.1799714
[Breed =1]	-0.0816780
[Breed =2]	0.0755076
[Chick =1]*[Age =1]	0.2993624
[Chick =1]*[Age =2]	-0.1708177
[Chick =1]*[Age =3]	-0.5692655
[Chick =2]*[Age =1]	-0.1826164
[Chick =2]*[Age =2]	0.0724126
[Chick =2]*[Age =3]	0.1886294
[Chick =1]*[Breed =1]	0
[Chick =1]*[Breed =2]	0
[Chick =2]*[Breed =1]	-0.0188632
[Chick =2]*[Breed =2]	0.0158422
[Age =1]*[Breed =1]	0.1150145
[Age =1]*[Breed =2]	-0.1099937
[Age =2]*[Breed =1]	-0.0136322
[Age =2]*[Breed =2]	0.0115038
[Age =3]*[Breed =1]	-0.0136322
[Age =3]*[Breed =2]	0.0115038

Table 4.12: The summary of results for testing the significance of interaction of variables for q-dimensional contingency tables using proposed Approach

Models	proposed Approach $\chi^2_{ML} = \chi^2_{ML1} - \chi^2_{ML2}$	Vd.f	$\chi^2_{tab}(0.95, \text{Vd.f})$	Decision
3-way [CB] [C][A][B] χ^2_{ML1} And [A][CB] χ^2_{ML2} $\chi^2_{ML1}=37.14,$ d.f=7 $\chi^2_{ML2}=36.82,$ d.f=6	0.32	1	3.841	No significant interaction between Chick loss and Breed.
[AB] [C][A][B] χ^2_{ML1} and [C][AB] G_2^2 $\chi^2_{ML1}=37.14,$ d.f=7 $\chi^2_{ML2}=29.69,$ d.f=5	7.45	2	5.99	There is significant interaction between Age and Breed.
4-way [SA] [B][S][G][A] χ^2_{ML1} And [B][G][SA] χ^2_{ML2} $\chi^2_{ML1}=102.66,$ d.f=32 $\chi^2_{ML2}=99.30,$ d.f=31	3.36	1	3.841	No significant interaction.
[BS] [B][S][G][A]				

χ^2_{ML1} and $[G][A][BS]$ χ^2_{ML2} $\chi^2_{ML1}=102.66,$ d.f=32 $\chi^2_{ML2}=69.29,$ d.f=28	33.37	4	9.49	There is significant interaction between BSCgrade and State
5-way $[GM]$ $[B][S][G][A][M]$ χ^2_{ML1} And $[B][S][A][GM]$ χ^2_{ML2} $\chi^2_{ML1}=174.984$ d.f=71 $\chi^2_{ML2}=172.972,$ d.f=70	0.012	1	3.841	No significant interaction exist
$[AS]$ χ^2_{ML1} $[B][S][G][A][M]$ and $[B][G][M][AS]$ χ^2_{ML2} $\chi^2_{ML1}=172.984,$ d.f=71 $\chi^2_{ML2}=172.965,$ d.f=70	0.019	1	3.841	There is no significant interaction between Age and State of origin .

Table 4.13: The summary of results for testing the significance of interaction of variables for q-dimensional contingency tables using Brown's method

Models	Brown's Method $\chi^2_{ML} = \chi^2_{ML2} - \chi^2_{ML1}$	Vd.f	$\chi^2_{tab}(0.95, Vd.f)$	Decision
3-way [CB] [CA][CB][AB] χ^2_{ML1} And [CA][AB] χ^2_{ML2} $\chi^2_{ML1} = 0.826$, d.f=2 $\chi^2_{ML2} = 0.830$, d.f=3	0.004	1	3.841	No significant interaction between Chick loss and Breed.
[AB] χ^2_{ML1} and [CA][CA][AB] [CA][CB] χ^2_{ML2} $\chi^2_{ML1} = 0.826$, d.f=2 $\chi^2_{ML2} = 7.96$, d.f=4	7.13	2	5.99	There is significant interaction between Age and Breed.
4-way [SA][BS][BG][BA][SG][SA][GA] χ^2_{ML1} And [BS][BG][BA][SG][GA] χ^2_{ML2} $\chi^2_{ML1} = 10.45$, d.f=17 $\chi^2_{ML2} = 10.85$, d.f=18	0.4	1	3.841	No significant interaction.
[BS] [BS][BG][BA][SG][SA][GA] χ^2_{ML1} and [BG][BA][SG][SA][GA] χ^2_{ML2} $\chi^2_{ML1} = 10.45$, d.f=17 $\chi^2_{ML2} = 37.64$, d.f=21	27.19	4	9.49	There is significant interaction between BSCgrade and State
5-way				

[GM] [BS][BG][BA][BM][SG][SA][SM] [GA][GM][AM] χ^2_{ML1} and [BS][BG][BA][BM][SG][SA][SM] [GA][AM] χ^2_{ML2} $\chi^2_{ML1}=83.509$, d.f=49 $\chi^2_{ML2}=83.996$, d.f=50	0.487	1	3.841	No significant interaction exist
[AS] [BS][BG][BA][BM][SG][SA][SM] [GA][GM][AM] χ^2_{ML1} and [BS][BG][BA][BM][SG][SM] [GA][GM][AM] χ^2_{ML2} $\chi^2_{ML1}=83.509$, d.f=49 $\chi^2_{ML2}=83.521$, d.f=50	0.012	1	3.841	There is no significant interaction between state and mode.

4.5 Results and Discussions

Log linear analysis is used to detect the varying associations and interactions between variables; determining the parsimonious model that fit the observed data and provide a systematic approach to the analysis of q -dimensional contingency tables. The hierarchical log linear analyses were used in this work and the analyses were carried out using algorithms implemented in R computer programs. The algorithms were also applied to real life data of three, four and five dimensional contingency tables and results were obtained.

For three - dimensional contingency table, the marginal and interaction terms were tested with likelihood-ratio statistic; it was observed that chick loss*breed are not significant ($p>0.05$). Other

interactions are statistically significant ($p < 0.05$). The goodness of fit models were produced and best model fit to the data has generating class: [CA,AB] . The best model fit in harmony with hierarchy principle is given as

$$\log m_{i_1 i_2 i_3} = \mu + \mu_{C(i_1)} + \mu_{A(i_2)} + \mu_{B(i_3)} + \mu_{CA(i_1 i_2)} + \mu_{AB(i_2 i_3)}$$

According to this model, it is observed that chick loss*breed of second order interaction terms is insignificant. The remaining model interaction terms chick loss *age and age *breed has been included in the model. This means that at 5% level of significance, the data provide sufficient evidence (likelihood ratio =0.83; d.f=2 ; P =0.84 or Pearson 0.82; d.f= 2; P =0.84) to fit the observed data and chick loss and breed are independent given the age, thus the association or relationship between the chick loss and breed is independent for each age. Chick loss is associated with age and age is associated with breed but breed is not associated with chick loss. This is an indication that chick loss is not a function of breed. This really showed that chick loss does not depend on breed. This implies that chick loss is not a matter of breed. The marginal and significance of interaction terms of 4-dimensional contingency table is also tested. The results indicated insignificance of all two- factor interactions except BSc grade *State of origin, State* Gender and BSc grade *age ($P < 0.05$). The result also showed insignificance of all three factor interactions but BSc grade* State *Gender and BSc grade *Gender *Age ($p < 0.05$). The best model obtained has generating class: [BSG, BGA]. The best model in harmony with hierarchy principle is

$$\begin{aligned} \log m_{i_1 i_2 i_3 i_4} = & \mu + \mu_{B(i_1)} + \mu_{S(i_2)} + \mu_{G(i_3)} + \mu_{A(i_4)} + \mu_{BS(i_1 i_2)} + \mu_{BG(i_1 i_3)} + \mu_{BA(i_1 i_4)} \\ & + \mu_{SG(i_2 i_3)} + \mu_{SA(i_2 i_4)} + \mu_{GA(i_3 i_4)} + \mu_{BSG(i_1 i_2 i_3)} + \mu_{BGA(i_1 i_3 i_4)} \end{aligned}$$

Again, at 5% level of significance, the data provide sufficient evidence

$$\left(\begin{array}{l} \text{likelihood ratio } G^2 = 3.79; \text{ d.f}=10; \text{ p- value} = 0.9562268 \\ \text{pearson } \chi^2 = 3.79; \text{ d.f}=10 ; \text{ p- value} = 0.9553035 \end{array} \right)$$

that state of origin is independent of age given BSc grade and gender, thus the relationship

between state of origin and gender is independent for each BSc grade and gender. BSc grade is associated with state of origin and BSc grade is also associated with age but BSc grade is not associated with gender. This is an indication that BSc grade is independent of gender. This implies that preference has not given to any gender in awarding BSc grade . Furthermore, the marginal and association of interaction terms of 5-dimensional contingency table were equally tested. The results showed that all two-factor interactions are insignificant except BSc grade *age and BSc grade * state (P<0.05). All three factor interactions are not significant but BSc grade*State *Mode of admission (P<0.05). Also all fourth- factor interactions are insignificant except [BSGM, BGAM]. The best model fit statistics are:

$$\text{Likelihood } (G^2) = 13.40143; \text{ d.f}=20; \text{ P-value} =0.8595041$$

$$\text{Pearson } (\chi^2) = 13.44084; \text{ d.f} =20; \text{ P-value} = 0.8595041$$

The best model with the hierarchy principle is given as

$$\begin{aligned} \log m_{i_2 i_3 i_4 i_5} = & \mu + \mu_{B(i_1)} + \mu_{S(i_2)} + \mu_{G(i_3)} + \mu_{A(i_4)} + \mu_{M(i_5)} + \mu_{BS(i_2)} + \mu_{BG(i_3)} + \mu_{BA(i_4)} \\ & + \mu_{BM(i_5)} + \mu_{SG(i_2 i_3)} + \mu_{SA(i_2 i_4)} + \mu_{SM(i_2 i_5)} + \mu_{GA(i_3 i_4)} + \mu_{GM(i_3 i_5)} + \mu_{AM(i_4 i_5)} \\ & + \mu_{BSA(i_1 i_2 i_4)} + \mu_{BSM(i_1 i_2 i_5)} + \mu_{BGA(i_1 i_3 i_4)} + \mu_{BGM(i_1 i_3 i_5)} + \mu_{BAM(i_1 i_4 i_5)} \\ & + \mu_{SGA(i_2 i_3 i_4)} + \mu_{SGM(i_2 i_3 i_5)} + \mu_{SAM(i_2 i_4 i_5)} + \mu_{GAM(i_3 i_4 i_5)} + \mu_{BSGM(i_1 i_2 i_3 i_5)} + \mu_{BGAM(i_1 i_3 i_4 i_5)} \end{aligned}$$

Also, at 5% level of significance, the data provide sufficient that state of origin and age is independent given BSc grade and mode of admission, thus the relationship between state of origin and age is independent for each BSc grade and mode of admission.

More so, BSc grade is associated with age and BSc grade is associated with state of origin but BSc grade is not associated with gender. This showed that preference has not given to any gender in awarding BSc grade. The models were tested for significance of interaction of variables using proposed method . With 3-dimensional case, the test to examine whether there exist an association or interaction between [CB] is tested by comparing two models. The results indicated that there is no significant interaction between chick loss and

breed. The test of significance of interaction of variables was also tested between age and breed [AB].

The result indicated that there is significant interaction between age and breed. The significance of interaction of variables was also tested for models of 4-dimensional and 5-dimensional contingency tables respectively. For 4-dimensional case, [SA] and [BS] were tested for significance of interaction of variables; the results showed that there is an association between BSC grade and state of origin but there is no significant interaction between state of origin and age. Also, models of 5-dimensional contingency table were equally examined for significance of interaction of variables. These models [GM] and [AS] being tested showed that gender and mode of admission is not significant so also age and state of origin has no significant interaction between them. The models of 3, 4 and 5-dimensional contingency tables being tested for significance of interaction of variables with proposed method were compared with Brown's method, the results showed that the proposed method requires lesser parameter models, saves computer memory and time. .

CHAPTER FIVE: Summary, Conclusion, Contribution to Knowledge and Recommendations

5.0 INTRODUCTION

This chapter gives the summary and conclusion of the study. Findings and contribution to knowledge are highlighted. Some recommendations for further studies were also made.

5.1 Summary and Conclusion

The problem of estimating the parameters and best fits of log linear models for q -dimensional contingency table ($q=4,5,6$) as well as procedure for testing the significance of interaction of variables which involves many parameters has been identified and considered in this research work. We proposed the new method of estimation of parameters of log linear models for q -dimensional contingency table. More so, we developed algorithms on the new method for estimation of the parameters and best model fits of log linear model for q -dimensional contingency table. In estimating these parameters and best model fits, a computer program in R was written for the implementation of the algorithms. Furthermore, the Iterative proportional fitting procedure algorithm is used to estimate the parameters and best fits of models of the log linear model. The proposed method compared favorably with the existing methods, and yielded higher p -values and least likelihood ratio estimates than the existing methods. The proposed method was also implemented on a computer program in R using real life data and the results showed the best model fits for three, four and five dimensional contingency tables as [CA, AB],[BSG, BGA]and[BSGM, BGAM] with their p -values 0.84, 0.96,0.86 and likelihood ratio estimates 0.83, 3.79 , 13.40 respectively. The models were subjected to the test for the significance of interaction of the variables using proposed approach. The proposed method involves fewer parameter models. This consequently makes the proposed method faster than the conventional Brown's method. Hence, proposed method requires lesser parameter models, saves computer memory and time than Brown's method.

5.2 Summary of the Findings

The following were the findings from this research work:

1. Parameter estimators of the log linear model for q-dimensional contingency have been found using proposed method.
2. In the estimation of goodness of fit of log-linear models with numerical data using the proposed algorithms, the results obtained have higher p-values and least likelihood estimates than the existing method.
3. The proposed method for testing the significance of interaction of variables involved fewer parameter models. This consequently makes the proposed method faster, saves computer memory and time than the conventional Brown's method.

5.3 Contribution to Knowledge

The following are the contribution to knowledge

1. New method for estimating the parameters of log linear models for q-dimensional contingency table is proposed.
2. A new algorithm for fitting the best fit models of hierarchical log – linear models for q- dimensional contingency table is proposed.
3. New method for testing the significance of interaction of variables of contingency table is also proposed.

5.4 Suggestions for Further Studies

- I) The work can be extended to hierarchical log linear model of ordered categories of variables.

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APPENDIX 1

Application of developed algorithms in R to Justyna's data in estimating parameter estimates, goodness and best of fits of log linear model for 3-dimensional contingency table is shown below:

```
> Ii1 = 3; Ii2 = 2; Ii3 = 2
> n=c (63, 34, 11, 5, 5, 2, 46, 54, 10, 3, 3, 1)
> n=array (n, dim=c (Ii1, Ii2, Ii3 ))
> dimnames(n)=list(Exercise=c("Frequency", "Some", "None"), W.Hnd=c("Right", "Left"),
Sex=c("Male", "Female"))
> n
, , Sex = Male
```

W.Hnd

Exercise	Right	Left
Frequency	63	5
Some	34	5
None	11	2

```
, , Sex = Female
```

W.Hnd

Exercise	Right	Left
Frequency	46	3
Some	54	3
None	10	1

```
> mu=log(sum(n)/( Ii1 * Ii2 * Ii3 ))
```

```

> mu_1=mu_2=mu_3=0
> mu_12=matrix(0, I_i1 , I_i2 ); mu_13=matrix(0, I_i1 , I_i3 ); mu_23=matrix(0, I_i2 , I_i3 )
> for (i1 in 1: I_i1){
+ mu_1 [i1] =log (sum (n [i1,,])/(I_i2 * I_i3 ))-mu
+ for (i2 in 1: I_i2){
+ mu_2 [i2] =log (sum (n[,i2,])/(I_i2 * I_i3 ))-mu
+ for (i3 in 1:I_i3){
+ mu_3[i3]=log(sum(n[, , i3 ])/( I_i1 * I_i2 ))-mu
+ }
+ }
+ }
> for (i2 in 1:I_i2){
+ for (i3 in 1:I_i3){
+ mu_13[i1 , i3]=log(sum(n[i1 , , i3 ])/I_i2 )-mu-mu_1[i1]-mu_3[i3]
+ }
+ }
> for (i1 in 1:I_i1){
+ for (i2 in 1:I_i2){
+ mu_12[i1 , i2]=log(sum(n[i1 , i2 , ])/I_i3 )-mu-mu_1[i1]-mu_2[i2]
+ }
+ }
> for (i2 in 1:I_i2){

```

```

+ for (i2 in 1:Ii) {
+ mu_23 [i2, i3] = log (sum (n[, i2, i3 ]) Ii) - mu - mu_2[i2] - mu_3[i3]
+ }
+ }
> mu; mu_1; mu_2; mu_3; mu_13; mu_12; mu_23
[1] 2.983153
[1] 0.3927261 0.1949003 -1.1913940
[1] 0.6095821 -1.8304740
[1] 0.01257878 -0.01273903
      [,1]      [,2]
[1,] 0.00000000 0.00000000
[2,] 0.00000000 0.00000000
[3,] 0.06746393 -0.07427235
      [,1]      [,2]
[1,] 0.012739026 -0.15911123
[2,] -0.003446299 0.03871451
[3,] -0.049966314 0.44417962
      [,1]      [,2]
[1,] -0.02179544 0.02187151
[2,] 0.22103607 -0.29264262
> m_1=m_2=m_3=m_4=array(0, dim=c(Ii, Ii2, Ii3))
> for(i1 in 1: Ii) {
+ for (i2 in 1: Ii2) {
+ for (i3 in 1: Ii3) {

```

```

+ m_1[i_1, i_2, i_3 ]=exp(mu+mu_1[i_1]+mu_2[i_2]+mu_3[i_3])
+ m_2[i_1, i_2 , i_3 ]=exp(mu+mu_1[i_1]+mu_2[i_2]+mu_3[i_3]+mu_13 [i_1 , i_3 ])
+ m_3[i_1,i_2 , i_3 ]=exp(mu+mu_1[i_1]+mu_2[i_2]+mu_3[i_3]+mu_12[i_1 , i_2 ])
+ m_4[i_1,i_2 , i_3 ]=exp(mu+mu_1[i_1]+mu_2[i_2]+mu_3[i_3]+mu_23[i_2, i_3 ])
+ }
+ }
+ }
> m_1; m_2; m_3; m_4
, , 1

```

```

      [,1] [,2]
[1,] 54.49127 4.7492389
[2,] 44.71078 3.8968114
[3,] 11.17770 0.9742029
, , 2

```

```

      [,1] [,2]
[1,] 53.12899 4.6305079
[2,] 43.59301 3.7993911
[3,] 10.89825 0.9498478
, , 1

```

[,1] [,2]

[1,] 54.49127 4.749239

[2,] 44.71078 3.896811

[3,] 11.95781 1.042194

, , 2

[,1] [,2]

[1,] 53.12899 4.6305079

[2,] 43.59301 3.7993911

[3,] 10.11814 0.8818565

, , 1

[,1] [,2]

[1,] 55.18987 4.8101266

[2,] 44.55696 3.8834049

[3,] 10.63291 0.9267216

, , 2

[,1] [,2]

[1,] 45.31379 3.949367

[2,] 45.31379 3.949367

[3,] 16.99267 1.481013

```
, , 1
```

```
    [,1] [,2]
```

```
[1,] 53.31646 5.924051
```

```
[2,] 43.74684 4.860759
```

```
[3,] 10.93671 1.215190
```

```
, , 2
```

```
    [,1] [,2]
```

```
[1,] 54.30380 3.4556962
```

```
[2,] 44.55696 2.8354430
```

```
[3,] 11.13924 0.7088608
```

```
> model1=loglm(~Exercise + W.Hnd + Sex, data=n)
```

```
> model2=loglm(~Exercise + W.Hnd + Sex + Exercise * Sex, data=n)
```

```
> model3=loglm(~Exercise + W.Hnd + Sex + Exercise * W.Hnd, data=n)
```

```
> model4=loglm(~Exercise + W.Hnd z+ Sex + W.Hnd * Sex , data=n)
```

```
> model1; model2; model3; model4
```

```
Call:
```

```
loglm(formula = ~Exercise + W.Hnd + Sex, data = n)
```

```
Statistics:
```

```
    X^2 df P(> X^2)
```

```
Likelihood Ratio 9.418498 7 0.2239891
```

```
Pearson      9.563039 7 0.2147170
```

```
Call:
```

```
loglm(formula = ~Exercise + W.Hnd + Sex + Exercise * Sex, data = n)
```

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 2.795378 5 0.7314966

Pearson 3.059808 5 0.6907656

Call:

loglm(formula = ~Exercise + W.Hnd + Sex + Exercise * W.Hnd, data = n)

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 7.607794 5 0.2257682

Pearson 7.541177 5 0.2288250

Call:

loglm(formula = ~Exercise + W.Hnd + Sex + W.Hnd * Sex, data = n)

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 8.106711 6 0.2303892

Pearson 8.162490 6 0.2264390

```
> m_5a=m_5b=m_5c=m_6=array(0, dim=c(Ii, Ii, Ii))
```

```
> for(i1 in 1: Ii){
```

```
+ for (i2 in 1: Ii){
```

```
+ for (i3 in 1: Ii){
```

```
+ m_5a[i1, i2, i3]=exp(mu+mu_1[i1]+mu_2[i2]+mu_3[i3]+mu_12[i1, i2]+mu_13[i1, i3])
```

```

+ m_5b[i_1, i_2, i_3]=exp(mu+mu_1 [i_1]+mu_2[i_2]+mu_3[i_3]+mu_12[i_1, i_2]+mu_23[i_2, i_3])
+ m_5c[i_1, i_2, i_3]=exp(mu+mu_1[i_1]+mu_2[i_2]+mu_3[i_3]+mu_13[i_1, i_3]+mu_23[i_2, i_3])
+ m_6[i_1, i_2, i_3]=exp(mu+mu_1[i_1]+mu_2[i_2]+mu_3[i_3]+mu_12[i_1, i_2 ]+mu_13[i_1, i_3]+mu_23[
i_2, i_3])
+ }
+ }
+ }
> m_5a; m_5b; m_5c; m_6
, , 1

```

```

      [,1] [,2]
[1,] 55.18987 4.050633
[2,] 44.55696 4.050633
[3,] 11.37500 1.625000
, , 2

```

```

      [,1] [,2]
[1,] 53.81013 3.949367
[2,] 43.44304 3.949367
[3,] 9.62500 1.375000
, , 1

```

```

      [,1] [,2]

```


[1,] 54.00000 5.052632

[2,] 43.59633 5.052632

[3,] 10.40367 1.894737

, , 2

[,1] [,2]

[1,] 55.00000 2.947368

[2,] 44.40367 2.947368

[3,] 10.59633 1.105263

, , 1

[,1] [,2]

[1,] 53.31646 5.924051

[2,] 43.74684 4.860759

[3,] 11.70000 1.300000

, , 2

[,1] [,2]

[1,] 54.30380 3.4556962

[2,] 44.55696 2.8354430

[3,] 10.34188 0.6581197

, , 1

[,1] [,2]

[1,] 54.00000 5.052632

[2,] 43.59633 5.052632

[3,] 11.12976 2.026974

, , 2

[,1] [,2]

[1,] 55.000000 2.947368

[2,] 44.403670 2.947368

[3,] 9.837832 1.026147

> model1=loglm(~Exercise + W.Hnd + Sex, data=n)

> model2=loglm(~Exercise + W.Hnd + Sex + Exercise * Sex, data=n)

> model3=loglm(~Exercise + W.Hnd + Sex + Exercise * W.Hnd, data=n)

> model4=loglm(~Exercise + W.Hnd + Sex + W.Hnd * Sex , data=n)

> model5a=loglm(~Exercise + W.Hnd + Sex + Exercise * W.Hnd + Exercise * Sex, data=n)

> model5b=loglm(~Exercise + W.Hnd + Sex + Exercise * W.Hnd + W.Hnd *Sex, data=n)

> model5c=loglm(~Exercise + W.Hnd + Sex + Exercise * Sex + W.Hnd *Sex, data=n)

> model6=loglm(~Exercise + W.Hnd + Sex + Exercise * W.Hnd + Exercise * Sex + W.Hnd *Sex, data=n)

> model5a; model5b; model5c; model6

Call:

loglm(formula = ~Exercise + W.Hnd + Sex + Exercise * W.Hnd +

Exercise * Sex, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 1.984674 3 0.5755938

Pearson 2.014797 3 0.5693413

Call:

loglm(formula = ~Exercise + W.Hnd + Sex + Exercise * W.Hnd +
W.Hnd * Sex, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 7.296007 4 0.1210483

Pearson 7.245559 4 0.1234668

Call:

loglm(formula = ~Exercise + W.Hnd + Sex + Exercise * Sex + W.Hnd *
Sex, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 1.483591 4 0.6295440

Pearson 1.536038 4 0.6202392

Call:

loglm(formula = ~Exercise + W.Hnd + Sex + Exercise * W.Hnd +
Exercise * Sex + W.Hnd * Sex, data = n)

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 0.5211162 2 0.7706214

Pearson 0.5304751 2 0.7670237

APPENDIX 2

COMPUTER PROGRAM IN R FOR ESTIMATION OF THE PARAMETERS, GOODNESS OF FITS AND BEST FITS OF LOG LINEAR MODEL.

IMPLEMENTATION OF ALGORITHMS FOR ESTIMATION OF PARAMETERS, GOODNESS AND BEST MODEL FITS OF LOG LINEAR MODEL FOR Q-DIMENSIONAL CONTINGENCY TABLE.

```
> I1=2; I2 = 3; I3 = 2
```

```
>n=c( 55, 67, 16, 44, 8, 45, 48, 66, 20, 52, 18, 71)
```

```
> n=array(n, dim=c(I1, I2 , I3 ))
```

```
> dimnames(n)=list(Chicksloss=c("Yes", "No"), Age=c(1, 2, 3), Breed=c("Broiler", "Old layer"))
```

```
> mu=log(sum(n)/( I1 * I2 * I3 ))
```

```
> mu_1=mu_2=mu_3=0
```

```
> mu_12=matrix(0, I1, I2 ); mu_13=matrix(0, I1, I3 ); mu_23=matrix(0, I2 , I3 )
```

```
> for (i1 in 1: I1){
```

```
+ mu_1[i1]=log(sum(n[i1 , , ])/( I2 * I3 ))-mu
```

```
+ for (i2 in 1: I2 ){
```

```

+ mu_2[i_2]=log(sum(n[,I_{i_2},,])/( I_{i_1} * I_{i_3} ))-mu
+ for (i_3 in 1: I_{i_2}){
+ mu_3[i_3 ]=log(sum(n[, , i_3 ])/( I_{i_1} * I_{i_3} ))-mu
+ }
+ }
+ }
> for (i_2 in 1: I_{i_3}){
+ for (i_3 in 1: I_{i_3}){
+ mu_13[i_1 , i_3 ]=log(sum(n[i_1 , , i_3 ])/I_{i_3} )-mu-mu_1[i_1 ]-mu_3[i_3 ]
+ }
+ }
> for (i_1 in 1: I_{i_1}){
+ for (i_3 in 1: I_{i_3}){
+ mu_12 [ i_1 , i_2 ]=log(sum(n[ i_1 , i_2 , , ])/ I_{i_3} )-mu-mu_1[ i_1 ]-mu_2[ i_2 ]
+ }
+ }
> for (i_2 in 1: I_{i_3}){
+ for (i_3 in 1: I_{i_3}){
+ mu_23[ i_2 , i_3 ]=log(sum(n[ , i_2 , i_3 ])/I)-mu-mu_2[ i_2 ]-mu_3[ i_3 ]
+ }
+ }
> m_1=m_2=m_3=m_4=array(0, dim=c ( i_1 , i_2 , i_3 ))

```

```

> for(i1 in 1: Ii1){
+ for (i2 in 1: Ii2){
+ for (i3 in 1: Ii3){
+ m_1[i1 ,i2 , i3 ]=exp(mu+mu_1[i1]+mu_2[i2]+mu_3[i3 ])
+ m_2[i1 ,i2 , i3 ]=exp(mu+mu_1[i1]+mu_2[i2]+mu_3[i3 ]+mu_13[i1 ,i3 ])
+ m_3[i1 ,i2 , i3 ]=exp(mu+mu_1[i1]+mu_2[i2]+mu_3[i3 ]+mu_12[i1 ,i2 ])
+ m_4[i1 ,i2 , i3 ]=exp(mu+mu_1[i1]+mu_2[i2]+mu_3[i3 ]+mu_23[i2 , i3 ])
+ }
+ }
+ }
> model1=loglm(~Chicksloss + Age + Breed, data=n)
> model2=loglm(~Chicksloss + Age+ Breed + Chicksloss*Breed, data=n)
> model3=loglm(~Chicksloss + Age+ Breed + Chicksloss*Age, data=n)
> model4=loglm(~Chicksloss + Age+ Breed + Age*Breed, data=n)
> model1; model2; model3; model4

> m_5a=m_5b=m_5c=m_6=array (0, dim=c(i1 ,i2 , i3 ))
> for(i1 in 1: Ii1){
+ for (i2 in 1: Ii2){
+ for (i3 in 1: Ii3){
+ m_5a[i1 ,i2 , i3 ]=exp(mu+mu_1[i1]+mu_2[i2]+mu_3[i3 ]+mu_12[i1 ,i2 ]+mu_13[i1 ,i3 ])
+ m_5b[i1 ,i2 , i3 ]=exp(mu+mu_1[i1]+mu_2[i2]+mu_3[i3 ]+mu_12[i1 ,i2 ]+mu_23[i2 ,i3 ])

```

```

+ m_5c[i1 , i2 , i3]=exp(mu+mu_1[i1]+mu_2[i2]+mu_3[i3]+mu_13[i1 , i3 ]+mu_23[i2 , i3 ])
+ m_6[i1 , i2 , i3]=exp(mu+mu_1[i1]+mu_2[i2]+mu_3[i3]+mu_12[i1 , i2 ]+mu_13[i1 , i3 ]+mu_23[
i2 , i3 ])
+ }
+ }
+ }
> model5a=loglm(~Chicksloss + Age+ Breed + Chicksloss*Age + Chicksloss *Breed, data=n)
> model5b=loglm(~Chicksloss + Age+ Breed + Chicksloss*Age + Age*Breed, data=n)
> model5c=loglm(~Chicksloss + Age+ Breed + Chicksloss* Breed + Age*Breed, data=n)
> model6=loglm(~Chicksloss + Age+ Breed + Chicksloss*Age + Chicksloss *Breed +
Age*Breed , data=n)
>model5a; model5b; model5c; model6

> I1 =5; I2 =2; I3 =2; I4 =2

> n=c(3, 41, 45, 20, 1, 5, 35, 17, 4, 1, 2, 69, 44, 9, 1, 7, 54, 29, 3, 1, 1, 31, 58, 32, 1, 2, 31, 21, 7,
2, 3, 41, 32, 9, 1, 4, 37, 11, 3, 1)

> n=c(3, 41, 45, 20, 1, 2, 69, 44, 9, 1, 1, 31, 58, 32, 1, 3, 41, 32, 9, 1, 5, 35, 17, 4, 1, 7, 54, 29, 3,
1, 2, 31, 21, 7, 2, 4, 37, 11, 3, 1)

> n=array(n, dim=c( I1 , I2 , I3 , I4 ))

> dimnames(n)=list(BScgrade=c("pass", "Third class", "Lower Division", "Upper Division",
"First Class"), State=c("Indigene", "Non-indigene"), Gender=c("Male", "Female"),
Age=c("Under 26", "26 and over"))

> mu=log(sum(n)/( I1 * I2 * I3 * I4 ))

> mu_1=mu_2=mu_3=mu_4=array(0, dim=c( I1 , I2 , I3 , I4 ))

> mu_12=matrix(0, I2 , I3 ); mu_13=matrix(0, I1 , I3 ); mu_14=matrix(0, I1 , I4 );
mu_23=matrix(0, I2 , I3 ); mu_24=matrix(0, I2 , I4 ); mu_34=matrix(0, I3 , I4 )

> for ( i1 in 1: I1 ){

```

```

+ for (i4 in 1: Ii4){
+ mu_14[i1,i4 ]=log(sum(n[i1, , , i4 ]Ii2 /* Ii3 )-mu-mu_1[i1 ]-mu_4[i4 ]
+ }
+ }
> for (i1 in 1: Ii1){
+ for (i2 in 1: Ii2){
+ mu_12[i1,i2 ]=log(sum(n[i1,i2, , ])/ Ii3 * Ii4 )-mu-mu_1[i1 ]-mu_2[i2 ]
+ }
+ }
> for (i1 in 1: Ii1){
+ for (i3 in 1:Ii3){
+ mu_13[i1,i3 ]=log(sum(n[i1, , i3 , ])/ Ii3 * Ii4 )-mu-mu_1[i1 ]-mu_3[i3 ]
+ }
+ }
> for (i2 in 1:Ii2){
+ for (i3 in 1: Ii3){
+ mu_23[i2,i3 ]=log(sum(n[,i2,i3 , ] Ii1 /* Ii4 )-mu-mu_2[i2 ]-mu_3[i3 ]
+ }
+ }
> for (i2 in 1: Ii2){
+ for (i4 in 1: Ii4){
+ mu_24[i2,i4 ]=log(sum(n[,i2, , i4 ])/ Ii1 * Ii3 )-mu-mu_2[i2 ]-mu_4[i4 ]

```



```

+ }
+ }
> for (i3 in 1:I3){
+ for (i4 in 1: I4){
+ mu_34[i3,i4 ]=log(sum(n[, ,i3 , i4 ]) I4 /*I3 )-mu-mu_3[i3]-mu_4[i4 ]
+ }
+ }
> m_1=m_2=m_3=m_4=m_5=m_6=m_7=array(0, dim=c(I1 ,I2 ,I3 ,I4 ))
> for(i1 in 1:I1){
+ for(i2 in 1:I2){
+ for(i3 in 1:I3){
+ for(i4 in 1: I4){
+ m_1[i1,i2,i3,i4 ]=exp(mu+mu_1[i1 ]+mu_2[i2 ]+mu_3[i3 ]+mu_4[i4 ])
+ m_2[i1,i2,i3,i4 ]=exp(mu+mu_1[i1 ]+mu_2[i2 ]+mu_3[i3 ]+ mu_4[i4 ]+mu_14[i1 ,i3 ])
+ m_3[i1,i2,i3,i4 ]=exp(mu+mu_1[i1 ]+mu_2[i2 ]+mu_3[i3 ]+ mu_4[i4 ]+mu_12[i1 ,i2 ])
+ m_4[i1,i2,i3,i4 ]=exp(mu+mu_1[i1 ]+mu_2[i2 ]+mu_3[i3 ]+ mu_4[i4 ]+mu_13[i1 ,i3 ])
+ m_5[i1,i2,i3,i4 ]=exp(mu+mu_1[i1 ]+mu_2[i2 ]+mu_3[i3 ]+ mu_4[i4 ]+mu_23[i2 ,i3 ])
+ m_6[i1,i2,i3,i4 ]=exp(mu+mu_1[i1 ]+mu_2[i2 ]+mu_3[i3 ]+ mu_4[i4 ]+mu_24[i2 ,i4 ])
+ m_7[i1,i2,i3,i4 ]=exp(mu+mu_1[i1 ]+mu_2[i2 ]+mu_3[i3 ]+ mu_4[i4 ]+mu_34[i3 ,i4 ])
+ }
+ }
+ }

```

```

+ }
> model1=loglm(~BScgrade+State+Gender+Age, data=n)
> model2=loglm(~BScgrade+State+Gender+Age+BScgrade*Age, data=n)
> model3=loglm(~BScgrade+State+Gender+Age+BScgrade*State, data=n)
> model4=loglm(~BScgrade+State+Gender+Age+BScgrade*Gender, data=n)
> model5=loglm(~BScgrade+State+Gender+Age+State*Gender, data=n)
> model6=loglm(~BScgrade+State+Gender+Age+State*Age, data=n)
> model7=loglm(~BScgrade+State+Gender+Age+Gender*Age, data=n)
> model1; model2; model3; model4; model5; model6; model7
> m_8a=m_8b=m_8c=m_8d=m_8e=array(0, dim=c(I1, I2, I3, I4))
> for(i1 in 1:I1){
+ for(i2 in 1:I2){
+ for(i3 in 1:I3){
+ for(i4 in 1: I4){

i1 = i, i2 = j, i3 = k, i4 = l, i5 = m
I1 = I, I2 = J, I3 = K, I4 = L, I5 = M

+ m_8a[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_12[i, j])
+ m_8b[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_13[i, k])
+ m_8c[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_23[j, k])
+ m_8d[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_24[j, l])
+ m_8e[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_34[k, l])
+ }
+ }
+ }

```

```

+ }
> model8a=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*State, data=n)
> model8b=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*Gender, data=n)
> model8c=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ State*Gender, data=n)
> model8d=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ State*Age, data=n)
> model8e=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ Gender*Age, data=n)
> model8a ; model8b; model8c ; model8d; model8e
> m_9a=m_9b=m_9c=m_9d=array(0, dim=c(I, J, K, L))
> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ m_9a[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_12[i,
j]+mu_13[i, k])
+ m_9b[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_12[i,
j]+mu_23[j, k])
+ m_9c[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+ mu_12[i,
j]+mu_24[j, l])
+ m_9d[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+ mu_12[i, j]+
mu_34[k, l])
+ }
+ }
+ }
+ }
> model9a=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*State+
BScgrade*Gender, data=n)

```

```

> model9b=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*State+
State*Gender, data=n)

> model9c=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*State+
State*Age, data=n)

> model9d=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*State+
Gender*Age, data=n)

> model9a ; model9b; model9c ; model9d

>mu_123=array(0, dim=c(I, J, K)); mu_124=array(0, dim=c(I, J, L)); mu_134= array(0, dim=c(I,
K, L)); mu_234= array(0, dim=c(J, K, L))

> for (i in 1:I){
+ for (j in 1:J){
+ for (k in 1:K){
+ mu_123[i, j, k]=log(sum(n[i, j, k, ])/L)-mu-mu_1[i]-mu_2[j]-mu_3[k]-mu_12[i, j]-mu_13[i, k]-
mu_23[j, k]
+ }
+ }
+ }

> for (i in 1:I){
+ for (j in 1:J){
+ for (l in 1:L){
+ mu_124[i, j, l]=log(sum(n[i, j, , l])/K)-mu-mu_1[i]-mu_2[j]-mu_4[l]-mu_12[i, j]-mu_14[i, l]-
mu_24[j, l]
+ }
+ }
+ }

> for (i in 1:I){
+ for (k in 1:K){

```

```

+ for (l in 1:L){
+ mu_134[i, k, l]=log(sum(n[i, , k, l])/J)-mu-mu_1[i]-mu_3[k]-mu_4[l]-mu_13[i, k]-mu_14[i, l]-
mu_34[k, l]
+ }
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ for (l in 1:L){
+ mu_234[j, k, l]=log(sum(n[, j, k, l])/I)-mu-mu_2[j]-mu_3[k]-mu_4[l]-mu_23[j, k]-mu_24[j, l]-
mu_34[k, l]
+ }
+ }
+ }
> m_10a=m_10b=m_10c=m_10d=array(0, dim=c(I, J, K, L))
> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ m_10a[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_123[i, j, k])
+ m_10b[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_124[i, j, l])
+ m_10c[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_134[i, k, l])
+ m_10d[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_234[j, k, l])
+ }
+ }
+ }

```

```

+ }
> model10a=loglm(~BScgrade+State+Gender+Age+ BScgrade*State *Gender, data=n)
> model10b=loglm(~BScgrade+State+Gender+Age+ BScgrade*State*Age, data=n)
> model10c=loglm(~BScgrade+State+Gender+Age+ BScgrade* Gender*Age, data=n)
> model10d=loglm(~BScgrade+State+Gender+Age+ State* Gender*Age, data=n)
> model10a ; model10b; model10c ; model10d
> m_11a=m_11b=m_11c=m_11d=m_11e=array(0, dim=c(I, J, K, L))
> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ m_11a[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_123[i, j, k]+mu_124[i, j,
l])
+ m_11b[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_123[i, j, k]+mu_134[i, k,
l])
+ m_11c[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_123[i, j, k]+mu_234[j, k,
l])
+ m_11d[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_124[i, j, l]+mu_234[j, k,
l])
+ m_11e[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_134[i, k, l]+mu_234[j, k,
l])
+ }
+ }
+ }
+ }
> model11a=loglm(~BScgrade+State+Gender+Age+BScgrade*State *Gender +
BScgrade*State*Age, data=n)

```

```

> model11b=loglm(~BScgrade+State+Gender+Age+ BScgrade*State *Gender+ BScgrade*
Gender*Age, data=n)

> model11c=loglm(~BScgrade+State+Gender+Age+ BScgrade*State *Gender+ State*
Gender*Age, data=n)

> model11d=loglm(~BScgrade+State+Gender+Age+ BScgrade*State*Age+ State*
Gender*Age, data=n)

> model11e=loglm(~BScgrade+State+Gender+Age+ BScgrade* Gender*Age+ State*
Gender*Age, data=n)

> model11a ; model11b; model11c ; model11d; model11e

> I=5; J=2; K=2; L=2; M=2

>n=c(7, 14, 61, 22,1, 1, 6, 17, 28, 1, 4, 11, 78, 42, 2, 2, 7, 33, 32, 1,12, 8, 45, 19, 1, 4, 9, 14, 22,
2, 12, 8, 63, 31, 2, 4, 2, 24, 21, 1, 4, 10, 52, 31, 1, 8, 11, 17, 19, 1, 1, 2, 94, 45, 1, 1, 1, 27, 17, 2,
2, 4, 28, 39, 1, 2, 4, 21, 15, 1, 1, 1, 81, 28, 1, 3, 6, 22, 11, 1)

> n=array(n, dim=c(I, J, K, L, M))

> dimnames(n)=list(BScgrade=c("pass", "Third class", "Lower Division", "Upper Division",
"First Class"), State=c("Indigene", "Non-indigene"), Gender=c("Male", "Female"),
Age=c("Under 26", "26 and over"), Mode=c("Jamb", "Pre science"))

> mu=log(sum(n)/(I*J*K*L*M))

> mu_1=mu_2=mu_3=mu_4=mu_5= array(0, dim=c(I, J, K, L, M))

> mu_12=matrix(0, I, J); mu_13=matrix(0, I, K); mu_14=matrix(0, I, L); mu_15=matrix(0, I,
M); mu_23=matrix(0, J, K); mu_24=matrix(0, J, L); mu_25=matrix(0, J, M); mu_34=matrix(0,
K, L); mu_35=matrix(0, K, M); mu_45=matrix(0, L, M)

> for (i in 1:I){
+ mu_1[i]=log(sum(n[i, , , ])/(J*K*L*M))-mu
+ for (j in 1:J){
+ mu_2[j]=log(sum(n [ , j, , ])/(I*K*L*M))-mu
+ for (k in 1:K){
+ mu_3[k]=log(sum(n[ , , k, ])/(I*J*L*M))-mu

```

```

+ for (l in 1:L){
+ mu_4[l]=log(sum(n[, , l, ])/(I*J*K*M))-mu
+ for (m in 1:M){
+ mu_5[m]=log(sum(n[, , , m])/(I*J*K*L))-mu
+ }
+ }
+ }
+ }
+ }
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ mu_12[i, j]=log(sum(n[i, j, , ])/K*L*M)-mu-mu_1[i]-mu_2[j]
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ mu_23[j, k]=log(sum(n[, j, k, ])/I*L*M)-mu-mu_2[j]-mu_3[k]
+ }
+ }
> for (j in 1:J){
+ for (l in 1:L){
+ mu_24[j, l]=log(sum(n[, j, , l])/I*K*M)-mu-mu_2[j]-mu_4[l]
+ }
+ }
> for (j in 1:J){

```



```

+ for (m in 1:M){
+ mu_25[j, m]=log(sum(n[,j, , m])/I*K*L)-mu-mu_2[j]-mu_5[m]
+ }
+ }
> for (i in 1:I){
+ for (k in 1:K){
+ mu_13[i, k]=log(sum(n[i, , k, ,])/J*L*M)-mu-mu_1[i]-mu_3[k]
+ }
+ }
> for (i in 1:I){
+ for (l in 1:L){
+ mu_14[i, l]=log(sum(n[i, , , l])/J*K*M)-mu-mu_1[i]-mu_4[l]
+ }
+ }
> for (i in 1:I){
+ for (m in 1:M){
+ mu_15[i, m]=log(sum(n[i, , , m])/J*K*L)-mu-mu_1[i]-mu_5[m]
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ mu_23[j, k]=log(sum(n[,j, k, ,])/I*L*M)-mu-mu_2[j]-mu_3[k]
+ }
+ }
> for (j in 1:J){

```

```

+ for (l in 1:L){
+ mu_24[j, l]=log(sum(n[,j, , l, ])/I*K*M)-mu-mu_2[j]-mu_4[l]
+ }
+ }
> for (j in 1:J){
+ for (m in 1:M){
+ mu_25[j, m]=log(sum(n[,j, , , m])/I*K*L)-mu-mu_2[j]-mu_5[m]
+ }
+ }
> for (k in 1:K){
+ for (l in 1:L){
+ mu_34[k, l]=log(sum(n[, , k, l, ])/I*J*M)-mu-mu_3[k]-mu_4[l]
+ }
+ }
> for (k in 1:K){
+ for (m in 1:M){
+ mu_35[k, m]=log(sum(n[, , k, , m])/I*J*L)-mu-mu_3[k]-mu_5[m]
+ }
+ }
> for (l in 1:L){
+ for (m in 1:M){
+ mu_45[l, m]=log(sum(n[, , , l, m])/I*J*K)-mu-mu_4[l]-mu_5[m]
+ }
+ }

```

```

> m_1=m_2=m_3=m_4=m_5=m_6=m_7=m_8=m_9=m_10=m_11=array(0, dim=c(I, J, K,
L,M))
> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ for(m in 1:M){
+ m_1[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_4[l]+mu_5[m])
+ m_2[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i, m])
+ m_3[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_12[i, j])
+ m_4[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_13[i, k])
+ m_5[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_14[i, l])
+ m_6[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+mu_23[j, k])
+ m_7[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+mu_24[j, l])
+ m_8[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_25[j, m])
+ m_9[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_34[k, l])
+ m_10[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_35[k, m])
+ m_11[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_45[l, m])
+ }
+ }
+ }
+ }
+ }
> model1=loglm(~BScgrade+State+Gender+Age+Mode, data=n)
> model2=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode, data=n)

```

```

> model3=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*State, data=n)
> model4=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Gender, data=n)
> model5=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Age, data=n)
> model6=loglm(~BScgrade+State+Gender+Age+Mode+State*Gender, data=n)
> model7=loglm(~BScgrade+State+Gender+Age+Mode+State*Age, data=n)
> model8=loglm(~BScgrade+State+Gender+Age+Mode+State*Mode, data=n)
> model9=loglm(~BScgrade+State+Gender+Age+Mode+Gender*Age, data=n)
> model10=loglm(~BScgrade+State+Gender+Age+Mode+Gender*Mode, data=n)
> model11=loglm(~BScgrade+State+Gender+Age+Mode+Age*Mode, data=n)

> model1; model2; model3; model4; model5; model6; model7; model8; model9; model10;
model11

> m_12a=m_12b=m_12c=m_12d=m_12e=m_12f=m_12g=m_12h=m_12i=array(0, dim=c(I, J,
K, L,M))
> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ for(m in 1:M){
+ m_12a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i,
m]+mu_12[i, j]
+ m_12b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i,
m]+mu_13[i, k])
+ m_12c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i, m]+
mu_14[i, l])
+ m_12d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+
mu_23[j, k])

```

```

+ m_12e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+
mu_24[j, l])

+ m_12f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_25[j, m])

+ m_12g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_34[k, l])

+ m_12h[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_35[k, m])

+ m_12i[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_45[l, m])

+ }

+ }

+ }

+ }

+ }

> model12a=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*State,
data=n)

>
model12b=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender,
data=n)

> model12c=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Age,
data=n)

> model12d=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+State*Gender,
data=n)

> model12e=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+State*Age,
data=n)

> model12f=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+State*Mode,
data=n)

> model12g=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+Gender*Age,
data=n)

```

```

> model12h=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+Gender*Mode,
data=n)

> model12i=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+Age*Mode,
data=n)

> model12a; model12b; model12c; model12d; model12e; model12f; model12g; model12h;
model12i

> m_13a=m_13b=m_13c=m_13d=m_13e=m_13f=m_13g=m_13h=array(0, dim=c(I, J, K, L,M))

> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ for(m in 1:M){
+ m_13a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i,
m]+mu_13[i, k]+mu_12[i, j])
+ m_13b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i, m]+
mu_13[i, k]+ mu_14[i, l])
+ m_13c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+mu_23[j, k])
+ m_13d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+mu_24[j, l])
+ m_13e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_25[j, m])
+ m_13f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_34[k, l])
+ m_13g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_35[k, m])
+ m_13h[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_45[l, m])
+ }
}
}
}
}

```

```
+ }  
+ }  
+ }  
+ }
```

```
>  
model13a=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+  
BScgrade*State, data=n)
```

```
>  
model13b=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+  
BScgrade*Age, data=n)
```

```
>  
model13c=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+  
State*Gender, data=n)
```

```
>  
model13d=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+  
State*Age, data=n)
```

```
>  
model13e=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+  
State*Mode, data=n)
```

```
>  
model13f=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+  
Gender*Age, data=n)
```

```
>model13g=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+  
+ Gender*Mode, data=n)
```

```
>model13h=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+  
+ Age*Mode, data=n)
```

```
> model13a; model13b; model13c; model13d; model13e; model13f; model13g; model13h
```

```

> m_14a=m_14b=m_14c=m_14d=m_14e=m_14f=m_14g=array(0, dim=c(I, J, K, L,M))
> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ for(m in 1:M){
+ m_14a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i,
m]+mu_13[i, k]+ mu_12[i, j]+ mu_14[i, l] )
+ m_14b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i,
m]+mu_13[i, k]+ mu_12[i, j]+ mu_23[j, k] )
+ m_14c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_24[j, l])
+ m_14d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_25[j, m])
+ m_14e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_34[k, l])
+ m_14f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_35[k, m])
+ m_14g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_45[l, m])
+ }
+ }
+ }
+ }
+ }
+ }
>
model14a=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ BScgrade*Age, data=n)

```



```

>
modell14b=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ State*Gender, data=n)

>
modell14c=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ State*Age, data=n)

>
modell14d=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ State*Mode, data=n)

>
modell14e=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ Gender*Age, data=n)

>
modell14f=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ Gender*Mode, data=n)

>
modell14g=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ Age*Mode, data=n)

> modell14a; modell14b; modell14c; modell14d; modell14e; modell14f; modell14g

> mu_123=array(0, dim=c(I, J, K)); mu_124=array(0, dim=c(I, J, L)); mu_125=array(0, dim=c(I,
J, M)); mu_134= array(0, dim=c(I, K, L)); mu_135=array(0, dim=c(I, K, M)); mu_145=array(0,
dim=c(I, L, M)); mu_234= array(0, dim=c(J, K, L)); mu_235=array(0, dim=c(J, K, M));
mu_245=array(0, dim=c(J, L, M)); mu_345=array(0, dim=c(K, L, M)); mu_1234=array(0,
dim=c(I, J, K, L)); mu_1235=array(0, dim=c(I, J, K, M)); mu_1245=array(0, dim=c(I, J, L, M));
mu_1345=array(0, dim=c(I, K, L, M)); mu_2345=array(0, dim=c(J, K, L, M))

> for (i in 1:I){
+ for (j in 1:J){
+ for (k in 1:K){
+ mu_123[i, j, k]=log(sum(n[i, j, k, , ])/L*M)-mu-mu_1[i]-mu_2[j]-mu_3[k]-mu_12[i, j]-
mu_13[i, k]-mu_23[j, k]
+ }
+ }

```

```

+ }
> for (i in 1:I){
+ for (j in 1:J){
+ for (l in 1:L){
+ mu_124[i, j, l]=log(sum(n[i, j, , l])/K*M)-mu-mu_1[i]-mu_2[j]-mu_4[l]-mu_12[i, j]-mu_14[i,
l]-mu_24[j, l]
+ }
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ for (m in 1:M){
+ mu_125[i, j, m]=log(sum(n[i, j, , m])/K*L)-mu-mu_1[i]-mu_2[j]-mu_5[m]-mu_12[i, j]-
mu_15[i, m]-mu_25[j, m]
+ }
+ }
+ }
> for (i in 1:I){
+ for (k in 1:K){
+ for (l in 1:L){
+ mu_134[i, k, l]=log(sum(n[i, , k, l])/J*M)-mu-mu_1[i]-mu_3[k]-mu_4[l]-mu_13[i, k]-
mu_14[i, l]-mu_34[k, l]
+ }
+ }
+ }
> for (i in 1:I){

```

```

+ for (k in 1:K){
+ for (m in 1:M){
+ mu_135[i, k, m]=log(sum(n[i, , k, , m])/J*L)-mu-mu_1[i]-mu_3[k]-mu_5[m]-mu_13[i, k]-
mu_15[i, m]-mu_35[k, m]
+ }
+ }
+ }
> for (i in 1:I){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_145[i, l, m]=log(sum(n[i, , , l, m])/J*L)-mu-mu_1[i]-mu_4[l]-mu_5[m]-mu_14[i, l]-
mu_15[i, m]-mu_45[l, m]
+ }
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ for (l in 1:L){
+ mu_234[j, k, l]=log(sum(n[, j, k, l, ])/I*M)-mu-mu_2[j]-mu_3[k]-mu_4[l]-mu_23[j, k]-
mu_24[j, l]-mu_34[k, l]
+ }
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ for (m in 1:M){

```

```

+ mu_235[j, k, m]=log(sum(n[, j, k, , m])/I*L)-mu-mu_2[j]-mu_3[k]-mu_5[m]-mu_23[j, k]-
mu_25[j, m]-mu_35[k, m]

+ }

+ }

+ }

> for (j in 1:J){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_245[j, l, m]=log(sum(n[, j, , l, m])/I)-mu-mu_2[j]-mu_4[l]-mu_5[m]-mu_24[j, l]-mu_25[j,
m]-mu_45[l, m]
+ }
+ }
+ }

> for (j in 1:J){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_345[k, l, m]=log(sum(n[, , k, l, m])/I*J)-mu-mu_3[k]-mu_4[l]-mu_5[m]-mu_34[k, l]-
mu_35[k, m]-mu_45[l, m]
+ }
+ }
+ }

> for (i in 1:I){
+ for (j in 1:J){
+ for (k in 1:K){
+ for (l in 1:L){
+ mu_1234[i, j, k, l]=log(sum(n[i, j, k, l, ])/M)-mu-mu_1[i]-mu_2[j]-mu_3[k]-mu_4[l]-mu_12[i,
j]-mu_13[i, k]-mu_14[i, l]-mu_123[i, j, K]-mu_134[i, k, l]-mu_234[j, k, l]

```

```

+ }
+ }
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_1245[i, j, l, m]=log(sum(n[i, j, , l, m])/K)-mu-mu_1[i]-mu_2[j]-mu_4[l]-mu_5[k]-
mu_12[i, j]-mu_14[i, l]-mu_15[i, m]-mu_24[j, l]-mu_25[j, m]-mu_45[l, m]-mu_124[i, j, l]-
mu_125[i, j, m]-mu_145[i, l, m]-mu_245[j, l, m]
+ }
+ }
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_2345[j, k, l, m]=log(sum(n[, j, k, l, , m])/I)-mu-mu_2[j]-mu_3[k]-mu_4[l]-mu_5[m]-
mu_23[j, k]-mu_24[j, l]-mu_25[j, m]-mu_34[k, l]-mu_35[k, m]-mu_45[l, m]-mu_234[j, k, l]-
mu_245[j, l, m]-mu_345[k, l, m]
+ }
+ }
+ }
+ }

```

```

> m_15a=m_15b=m_15c=m_15d=m_15e=m_15f=m_15g=m_15h=m_15i=m_15j=
m_16a=m_16b=m_16c=m_16d=m_16e =array(0, dim=c(I, J, K, L,M))

> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ for(m in 1:M){

+ m_15a[I , j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_123[i, j,
k])

+ m_15b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_124[i, j, l])

+ m_15c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_125[i, j,
m])

+ m_15d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_134[i, k,
l])

+ m_15e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_135[i, k,
m])

+ m_15f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_145[i, l,
m])

+ m_15g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_234[j, k,
l])

+ m_15h[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_235[j, k,
m])

+ m_15i[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_245[j, l,
m])

+ m_15j[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_345[k, l,
m])

+ m_16a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1234[i, j,
k, l])

+ m_16b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1235[i, j,
k, m])

```

```

+ m_16c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1245[i, j, l,
m])
+ m_16d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1345[i, k,
l, m])
+ m_16e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_2345[ j, k,
l, m])
+ }
+ }
+ } n
+ }
+ }
> model15a=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender, data=n)
> model15b=loglm(~BScgrade+State+Gender+Age+Mode + BScgrade*State*Age, data=n)
> model15c=loglm(~BScgrade+State+Gender+Age+Mode + BScgrade*State*Mode, data=n)
> model15d=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Gender*Age, data=n)
> model15e=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Gender*Mode, data=n)
> model15f=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Age*Mode, data=n)
> model15g=loglm(~BScgrade+State+Gender+Age+Mode+ State*Gender*Age, data=n)
> model15h=loglm(~BScgrade+State+Gender+Age+Mode + State*Gender*Mode, data=n)
> model15i=loglm(~BScgrade+State+Gender+Age+Mode + State*Age*Mode, data=n)
> model15j=loglm(~BScgrade+State+Gender+Age+Mode + Gender*Age*Mode, data=n)
> model16a=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Age,
data=n)
> model16b=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Mode,
data=n)
> model16c=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Age*Mode,
data=n)

```

```

> model16d=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Gender*Age*Mode,
data=n)

> model16e=loglm(~BScgrade+State+Gender+Age+Mode+ State*Gender*Age*Mode, data=n)

> model15a; model15b; model15c; model15d; model15e; model15f; model15g; model15h;
model15i; model16a; model16b; model16c; model16d; model16e

> m_17a=m_17b=m_17c=m_17d=m_17e=m_17f=m_17g=m_17h=m_17i=array(0, dim=c(I, J,
K, L,M))

> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ for(m in 1:M){

+ m_17a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1234[i, j,
k, l]+ mu_1235[i, j, k, m])

+ m_17b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1234[i, j,
k, l]+ mu_1345[i, k, l, m])

+ m_17c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1234[i, j,
k, l]+ mu_2345[j, k, l, m])

+ m_17d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1235[i, j,
k, m] + mu_1245[i, j, l, m])

+ m_17e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1235[i, j,
k, m] + mu_1345[i, k, l, m])

+ m_17f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1235[i, j, k,
m] + mu_2345[ j, k, l, m])

+ m_17g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1245[i, j, l,
m]+ mu_1345[i, k, l, m])

+ m_17h[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1245[i, j, l,
m]+ mu_2345[ j, k, l, m])

```



```

+ m_17i[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1345[i, k, l,
m] + mu_2345[ j, k, l, m])
+ }
+ }
+ }
+ }
+ }

> model17a=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Age+
BScgrade*State*Gender*Mode, data=n)

> model17b=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Age+
BScgrade*Gender*Age*Mode, data=n)

> model17c=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Age+
State*Gender*Age*Mode, data=n)

> model17d=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Mode+
BScgrade*State*Age*Mode, data=n)

> model17e=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Mode+
BScgrade*Gender*Age*Mode, data=n)

> model17f=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Mode+
State*Gender*Age*Mode, data=n)

> model17g=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Age*Mode+
BScgrade*Gender*Age*Mode, data=n)

> model17h=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Age*Mode+
State*Gender*Age*Mode, data=n)

> model17i=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Gender*Age*Mode+
State*Gender*Age*Mode, data=n)

> model18=loglm(~BScgrade+State+Gender+Age+Mode+
BScgrade*State*Gender*Age*Mode, data=n)

```

```
> model17a; model17b; model17c; model17d; model17e; model17f; model17g; model17h;
model17i; model18
```

```
R > I; J; K; L; ...; Qth
```

```
R > n=c(...)
```

```
R > n=array(n, dim=c(I, J, K, L, ..., Qth))
```

```
R > dimnames(n)=list(A=c("yes", "no"), B=c("yes", "no"), C=c("yes", "no"), D=c("yes", "no"),
... , nth=c("yes", "no"))
```

```
R > mu=log(sum(n)/(I*J*K*L*...*Qth))
```

```
> mu_1=mu_2=mu_3=mu_4=...=mu_nth=array(0, dim=c(I, J, K, L, Qth))
```

```
> mu_12=matrix(0, I, J); mu_13=matrix(0, I, K); mu_14=matrix(0, I, L); mu_1nth=matrix(0, I,
Nth); mu_23=matrix(0, J, K); mu_24=matrix(0, J, L); mu_2nth=matrix(0, J, Qth);
mu_34=matrix(0, K, L); mu_3nth=matrix(0, K, Qth); ...
```

```
>m_1=m_2=m_3=m_4=...=m_nth=m_(nth+1)a=m_(nth+1)b=m_(nth+1)c=m_(qth+1)d=m_(qth
+1)e=...=m_(qth+1)f=m_(qth+2)=array(0, dim=c(I, J, K, L,...,Qth))
```

```
> for (i in 1:I){
```

```
+ mu_1[i]=log(sum(n[i, , , ])/(J*K*L*...*Qth))-mu
```

```
+ for (j in 1:J){
```

```
+ mu_2[j]=log(sum(n[, j, , ])/(I*K*L*...*Qth))-mu
```

```
+ for (k in 1:K){
```

```
+ mu_3[k]=log(sum(n[, , k, ])/(I*J*L*...*Qth))-mu
```

```
+ for (l in 1:L){
```

```
+ mu_4[l]=log(sum(n[, , , l ])/(I*J*K*...*Qth))-mu
```

```
.
```

```
.
```

```
.
```

```
+ for (qth in 1:Qth){
```

```

+ mu_qth[nth]=log(sum(n[, , , qth])/(I*J*K*L*...))-mu
+ }
+ }
+ }
+ }
.
.
.
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ mu_12[i, j]=log(sum(n[i, j , , ])/K*L*...*Qth)-mu-mu_1[i]-mu_2[j]
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ mu_23[j, k]=log(sum(n[, j, k, , ])/I*L*...*Qth)-mu-mu_2[j]-mu_3[k]
+ }
+ }
> for (j in 1:J){
+ for (l in 1:L){
+ mu_24[j, l]=log(sum(n[, j, , l, ])/I*K*...*Nth)-mu-mu_2[j]-mu_4[l]
+ }
+ }
.

```

```

.
.
> for (j in 1:J){
+ for (nth in 1:Qth){
+ mu_2nth[j, qth]=log(sum(n[,j, , m])/I*K*L)-mu-mu_2[j]-mu_qth[qth]
+ }
+ }
> for (i in 1:I){
+ for (k in 1:K){
+ mu_13[i, k]=log(sum(n[i, , k, ,])/J*L*...*Qth)-mu-mu_1[i]-mu_3[k]
+ }
+ }
> for (i in 1:I){
+ for (l in 1:L){
+ mu_14[i, l]=log(sum(n[i, , , l,])/J*K*...*Nth)-mu-mu_1[i]-mu_4[l]
+ }
+ }
.
.
.
> for (i in 1:I){
+ for (qth in 1:Qth){
+ mu_1qth[i, nth]=log(sum(n[i, , , qth])/J*K*L)-mu-mu_1[i]-mu_qth[qth]
+ }
+ }

```

```

> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
.
.
.
+ for(qth in 1:Qth){
+ m_1[i, j, k, l, ..., nth]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_4[l]+...+mu_nth[qth])
+ m_2[i, j, k, l,..., qth]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+...+mu_qth[nth]+mu_13[i,
k])
+      m_3[i,      j,      k,      l,...,      qth]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+
mu_4[l]+...+mu_qth[qth]+mu_12[i, j])
+      m_4[i,      j,      k,      l,...,qth]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+      mu_4[l]+...+
mu_qth[nth]+mu_23[j, k])
.
.
.
+ m_qth[i, j, k, l,...,nth]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ ...+mu_qth...[qth]
+ m_qth+1)a[i, j, k, l,...,qth]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_4[l]+...+mu_qth[qth]+...
+ m_(qth+1)b[i, j, k, l,...,qth]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_4[l]+...+mu_nth[qth]+...
.
.
.
+      m_(qth+...)[i,      j,      k,      l,      m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+
mu_4[l]+...+mu_nth[nth]+mu_12[i,      j]+mu_13[i,k]+mu_23[j,      k]+mu_24[l]+...+mu_(qth-
1)qth[(qth-1), qth])

```

+ }

+ }

+ }

+ }

.

.

.

+ }

>model1=loglm(~A+B+C+D+...+qth, data=n)

>model2=loglm(~A+B+C+D+...+qth+A*C, data=n)

>model3=loglm(~A+B+C+D+...+qth+A*B, data=n)

.

.

.

>model(qth+...)=loglm(~A+B+C+D+...+qth+A*B*C*D*..., data=n)

>model1; model2; model3; ...; model(qth+...)

APPENDIX 3

IMPLEMENTATION OF ALGORITHMS IN R FOR ESTIMATING PARAMETERS, EXPECTED VALUES AND MODEL FITS ESTIMATES OF REAL LIFE DATA FOR 3, 4 AND 5 DIMENSIONAL CONTINGENCY TABLES RESPECTIVELY

Results of 3 dimensional for parameter estimates, expected values and model estimates

```
> I=2; J= 3; K= 2
```

```
>n=c( 55, 67, 16, 44, 8, 45, 48, 66, 20 , 52, 18, 71)
```

```
> n=array(n, dim=c(I, J, K))
```

```
> dimnames(n)=list(Chicksloss=c("Yes", "No"), Age=c(1, 2, 3), Breed=c("Broiler", "Old layer"))
```

```
> n
```

```
, , Breed = Broiler
```

```
Age
```

```
Chicksloss 1 2 3
```

```
Yes 55 16 8
```

```
No 67 44 45
```

```
, , Breed = Old layer
```

```
Age
```

```
Chicksloss 1 2 3
```

```
Yes 48 20 18
```

```
No 66 52 71
```

```
> mu=log(sum(n)/(I*J*K))
```

```
> mu_1=mu_2=mu_3=0
```

```
> mu_12=matrix(0, I, J); mu_13=matrix(0, I, K); mu_23=matrix(0, J, K)
```

```

> for (i in 1:I){
+ mu_1[i]=log(sum(n[i, ])/(J*K))-mu
+ for (j in 1:J){
+ mu_2[j]=log(sum(n[,j, ])/(I*K))-mu
+ for (k in 1:K){
+ mu_3[k]=log(sum(n[, , k]/(I*J))-mu
+ }
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ mu_13[i, k]=log(sum(n[i, , k])/J)-mu-mu_1[i]-mu_3[k]
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ mu_12[i, j]=log(sum(n[i, j, ])/K)-mu-mu_1[i]-mu_2[j]
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ mu_23[j, k]=log(sum(n[, j, k])/I)-mu-mu_2[j]-mu_3[k]
+ }
+ }
> mu; mu_1; mu_2; mu_3; mu_13; mu_12; mu_23

```



```

[1] 3.749504
[1] -0.4353181 0.3022809
[1] 0.3280334 -0.2529965 -0.1799714
[1] -0.08167803 0.07550755
      [,1] [,2]
[1,] 0.0000000 0.0000000
[2,] -0.0188632 0.01584223
      [,1] [,2] [,3]
[1,] 0.2993624 -0.17081773 -0.5692653
[2,] -0.1826164 0.07241258 0.1886294
      [,1] [,2]
[1,] 0.11501445 -0.10999373
[2,] -0.01363215 0.01150382
[3,] -0.21070993 0.15044894
> m_1=m_2=m_3=m_4=array(0, dim=c(I, J, K))
> for(i in 1:I){
+ for (j in 1:J){
+ for (k in 1:K){
+ m_1[i, j, k]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k])
+ m_2[i, j, k]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_13[i, k])
+ m_3[i, j, k]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_12[i, k])
+ m_4[i, j, k]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_23[j, k])
+ }
+ }
+ }

```

```

> m_1; m_2; m_3; m_4
,, 1
      [,1] [,2] [,3]
[1,] 35.18224 19.67820 21.16897
[2,] 73.56286 41.14533 44.26240
,, 2
      [,1] [,2] [,3]
[1,] 41.1707 23.02768 24.77220
[2,] 86.0842 48.14879 51.79642
,, 1
      [,1] [,2] [,3]
[1,] 35.18224 19.67820 21.16897
[2,] 72.18824 40.37647 43.43529
,, 2
      [,1] [,2] [,3]
[1,] 41.17070 23.02768 24.77220
[2,] 87.45882 48.91765 52.62353
,, 1
      [,1] [,2] [,3]
[1,] 47.46078 26.54586 28.55691
[2,] 61.28431 34.27767 36.87446
,, 2
      [,1] [,2] [,3]
[1,] 34.70588 19.41176 20.88235

```

```
[2,] 92.54902 51.76471 55.68627
```

```
, , 1
```

```
    [,1] [,2] [,3]
```

```
[1,] 39.47059 19.41176 17.14706
```

```
[2,] 82.52941 40.58824 35.85294
```

```
, , 2
```

```
    [,1] [,2] [,3]
```

```
[1,] 36.88235 23.29412 28.79412
```

```
[2,] 77.11765 48.70588 60.20588
```

```
> model1=loglm(~Chicksloss + Age + Breed, data=n)
```

```
> model2=loglm(~Chicksloss + Age+ Breed + Chicksloss*Breed, data=n)
```

```
> model3=loglm(~Chicksloss + Age+ Breed + Chicksloss*Age, data=n)
```

```
> model4=loglm(~Chicksloss + Age+ Breed + Age*Breed, data=n)
```

```
> model1; model2; model3; model4
```

```
Call:
```

```
loglm(formula = ~Chicksloss + Age + Breed, data = n)
```

```
Statistics:
```

```
      X^2 df  P(> X^2)
```

```
Likelihood Ratio 37.13769 7 4.41719e-06
```

```
Pearson      36.33465 7 6.26767e-06
```

```
Call:
```

```
loglm(formula = ~Chicksloss + Age + Breed + Chicksloss * Breed,  
      data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 36.81976 6 1.909229e-06

Pearson 35.20502 6 3.932731e-06

Call:

```
loglm(formula = ~Chicksloss + Age + Breed + Chicksloss * Age,  
data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 8.280408 5 0.1414439

Pearson 8.181170 5 0.1465296

Call:

```
loglm(formula = ~Chicksloss + Age + Breed + Age * Breed, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 29.68738 5 1.699225e-05

Pearson 28.75595 5 2.588877e-05

```
> m_5a=m_5b=m_5c=m_6=array(0, dim=c(I, J, K))
```

```
> for(i in 1:I){
```

```
+ for (j in 1:J){
```

```
+ for (k in 1:K){
```

```
+ m_5a[i, j, k]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_12[i, j]+mu_13[i,k])
```

```
+ m_5b[i, j, k]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_12[i, j]+mu_23[j,k])
```

```

+ m_5c[i, j, k]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_13[i, k]+mu_23[j,k])
+ m_6[i, j, k]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_12[i, j]+mu_13[i,k]+mu_23[j, k])
+ }
+ }
+ }
> m_5a; m_5b; m_5c; m_6
,, 1

```

```

      [,1] [,2] [,3]
[1,] 47.46078 16.58824 11.98039
[2,] 60.13913 43.40870 52.45217

```

```

,, 2

```

```

      [,1] [,2] [,3]
[1,] 55.53922 19.41176 14.01961
[2,] 72.86087 52.59130 63.54783

```

```

,, 1

```

```

      [,1] [,2] [,3]
[1,] 53.24576 16.36364 9.704225
[2,] 68.75424 43.63636 43.295775

```

```

,, 2

```

```
      [,1] [,2] [,3]
[1,] 49.75424 19.63636 16.29577
[2,] 64.24576 52.36364 72.70423
```

```
., 1
```

```
      [,1] [,2] [,3]
[1,] 39.47059 19.41176 17.14706
[2,] 80.98723 39.82979 35.18298
```

```
., 2
```

```
      [,1] [,2] [,3]
[1,] 36.88235 23.29412 28.79412
[2,] 78.34909 49.48364 61.16727
```

```
., 1
```

```
      [,1] [,2] [,3]
[1,] 53.24576 16.36364 9.704225
[2,] 67.46947 42.82096 42.486732
```

```
., 2
```

```
      [,1] [,2] [,3]
[1,] 49.75424 19.63636 16.29577
[2,] 65.27166 53.19980 73.86519
```

```
> model5a=loglm(~Chicksloss + Age+ Breed + Chicksloss*Age + Chicksloss *Breed, data=n)
```

```
> model5b=loglm(~Chicksloss + Age+ Breed + Chicksloss*Age + Age*Breed, data=n)
```

```
> model5c=loglm(~Chicksloss + Age+ Breed + Chicksloss* Breed + Age*Breed, data=n)
```

```
> model6=loglm(~Chicksloss + Age+ Breed + Chicksloss*Age + Chicksloss *Breed +  
Age*Breed , data=n)
```

```
>model5a; model5b; model5c; model6
```

Call:

```
loglm(formula = ~Chicksloss + Age + Breed + Chicksloss * Age +  
Chicksloss * Breed, data = n)
```

Statistics:

```
X^2 df P(> X^2)
```

```
Likelihood Ratio 7.962476 4 0.09296247
```

```
Pearson 7.853555 4 0.09709239
```

Call:

```
loglm(formula = ~Chicksloss + Age + Breed + Chicksloss * Age +  
Age * Breed, data = n)
```

Statistics:

```
X^2 df P(> X^2)
```

```
Likelihood Ratio 0.8301007 3 0.8422546
```

```
Pearson 0.8172251 3 0.8453427
```

Call:

```
loglm(formula = ~Chicksloss + Age + Breed + Chicksloss * Breed +  
Age * Breed, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 29.36945 4 6.576301e-06

Pearson 28.32066 4 1.073878e-05

Call:

```
loglm(formula = ~Chicksloss + Age + Breed + Chicksloss * Age +  
Chicksloss * Breed + Age * Breed, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 0.8263105 2 0.6615596

Pearson 0.8145226 2 0.6654703

```
> model7=loglm(~Chicksloss + Age+ Breed + Chicksloss*Age *Breed, data=n); model7
```

Call:

```
loglm(formula = ~Chicksloss + Age + Breed + Chicksloss * Age *  
Breed, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 0 0 1

Pearson 0 0 1

Results of 4-dimensional for parameter estimates, expected values and model estimates

```
> I=5; J=2; K=2; L=2
```



```
> n=c(3, 41, 45, 20, 1, 2, 69, 44, 9, 1, 1, 31, 58, 32, 1, 3, 41, 32, 9, 1, 5, 35, 17, 4, 1, 7, 54, 29, 3, 1, 2, 31, 21, 7, 2, 4, 37, 11, 3, 1)
```

```
> n=array(n, dim=c(I, J, K, L))
```

```
> dimnames(n)=list(BScgrade=c("pass", "Third class", "Lower Division", "Upper Division", "First Class"), State=c("Indigene", "Non-indigene"), Gender=c("Male", "Female"), Age=c("Under 26", "26 and over"))
```

```
> n
```

```
, , Gender = Male, Age = Under 26
```

```
State
```

BScgrade	Indigene	Non-indigene
pass	3	2
Third class	41	69
Lower Division	45	44
Upper Division	20	9
First Class	1	1

```
, , Gender = Female, Age = Under 26
```

```
State
```

BScgrade	Indigene	Non-indigene
pass	1	3
Third class	31	41
Lower Division	58	32
Upper Division	32	9
First Class	1	1

```
, , Gender = Male, Age = 26 and over
```

BScgrade	State	
	Indigene	Non-indigene
pass	5	7
Third class	35	54
Lower Division	17	29
Upper Division	4	3
First Class	1	1

, , Gender = Female, Age = 26 and over

BScgrade	State	
	Indigene	Non-indigene
pass	2	4
Third class	31	37
Lower Division	21	11
Upper Division	7	3
First Class	2	1

```
> mu=log(sum(n)/(I*J*K*L))
```

```
> mu_1=mu_2=mu_3=mu_4=array(0, dim=c(I, J, K, L))
```

```
> mu_12=matrix(0, I, J); mu_13=matrix(0, I, K); mu_14=matrix(0, I, L); mu_23=matrix(0, J, K);
mu_24=matrix(0, J, L); mu_34=matrix(0, K, L)
```

```
> for (i in 1:I){
```

```
+ for (l in 1:L){
```

```
+ mu_14[i, l]=log(sum(n[i, , l])/J*K)-mu-mu_1[i]-mu_4[l]
```

```

+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ mu_12[i, j]=log(sum(n[i,j, , ])/K*L)-mu-mu_1[i]-mu_2[j]
+ }
+ }
> for (i in 1:I){
+ for (k in 1:K){
+ mu_13[i, k]=log(sum(n[i, , k, ])/J*L)-mu-mu_1[i]-mu_3[k]
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ mu_23[j, k]=log(sum(n[,j, k, ])/I*L)-mu-mu_2[j]-mu_3[k]
+ }
+ }
> for (j in 1:J){
+ for (l in 1:L){
+ mu_24[j, l]=log(sum(n[,j, , l])/I*K)-mu-mu_2[j]-mu_4[l]
+ }
+ }
> for (k in 1:K){
+ for (l in 1:L){
+ mu_34[k, l]=log(sum(n[, , k, l])/I*J)-mu-mu_3[k]-mu_4[l]

```

+ }

+ }

> mu; mu_1; mu_2; mu_3; mu_4; mu_12; mu_13; mu_14; mu_23; mu_24; mu_34

1] 2.888982

, , 1, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

[,1] [,2]

[1,] -0.4910866 -0.1163932

[2,] 2.0382718 2.4143230

[3,] 2.0597780 1.8646083

[4,] 1.2541528 0.2890719

[5,] -1.2795440 -1.5026875

[,1] [,2]

[1,] -0.05576856 -0.5863968

[2,] 2.40432292 2.0526605

[3,] 2.01629287 1.9150391

[4,] 0.69453703 1.0428437

[5,] -1.50268754 -1.2795440

[,1] [,2]

[1,] -0.6917573 0.001389854

[2,] 2.3150248 2.167263902

[3,] 2.2984039 1.467726923

[4,] 1.3595133 -0.055768560

[5,] -1.5026875 -1.279543991

[,1] [,2]

[1,] 1.342222 1.420474

[2,] 1.583799 1.150554

[,1] [,2]

[1,] 1.645766 1.023041

[2,] 1.546585 1.205363

[,1] [,2]

[1,] 1.654313 1.2445834

[2,] 1.537062 0.9738509

```

> m_1=m_2=m_3=m_4=m_5=m_6=m_7=array(0, dim=c(I, J, K, L))
> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ m_1[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_4[l])
+ m_2[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, k])
+ m_3[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_12[i, j])
+ m_4[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_13[i, k])
+ m_5[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_23[j, k])
+ m_6[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_24[j, k])
+ m_7[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_34[k, l])
+ }
+ }
+ }
+ }
, , 1, 1

```

[,1] [,2]

[1,] 17.975 17.975

[2,] 17.975 17.975

[3,] 17.975 17.975

[4,] 17.975 17.975

[5,] 17.975 17.975

, , 2, 1

[,1] [,2]

[1,] 17.975 17.975

[2,] 17.975 17.975

[3,] 17.975 17.975

[4,] 17.975 17.975

[5,] 17.975 17.975

, , 1, 2

[,1] [,2]

[1,] 17.975 17.975

[2,] 17.975 17.975

[3,] 17.975 17.975

[4,] 17.975 17.975

[5,] 17.975 17.975

, , 2, 2

[,1] [,2]

[1,] 17.975 17.975

[2,] 17.975 17.975

[3,] 17.975 17.975

[4,] 17.975 17.975

[5,] 17.975 17.975

, , 1, 1

[,1] [,2]

[1,] 9 9

[2,] 182 182

[3,] 179 179

[4,] 70 70

[5,] 4 4

, , 2, 1

[,1] [,2]

[1,] 18 18

[2,] 157 157

[3,] 78 78

[4,] 17 17

[5,] 5 5

, , 1, 2

[,1] [,2]

[1,] 9 9

[2,] 182 182

[3,] 179 179

[4,] 70 70

[5,] 4 4

, , 2, 2

[,1] [,2]

[1,] 18 18

[2,] 157 157

[3,] 78 78

[4,] 17 17

[5,] 5 5

, , 1, 1

[,1] [,2]

[1,] 11 16

[2,] 138 201

[3,] 141 116

[4,] 63 24

[5,] 5 4

, , 2, 1

[,1] [,2]

[1,] 11 16
[2,] 138 201
[3,] 141 116
[4,] 63 24
[5,] 5 4

, , 1, 2

[,1] [,2]

[1,] 11 16
[2,] 138 201
[3,] 141 116
[4,] 63 24
[5,] 5 4

, , 2, 2

[,1] [,2]

[1,] 11 16
[2,] 138 201
[3,] 141 116
[4,] 63 24
[5,] 5 4

, , 1, 1

[,1] [,2]

[1,] 17 17

[2,] 199 199

[3,] 135 135

[4,] 36 36

[5,] 4 4

, , 2, 1

[,1] [,2]

[1,] 10 10

[2,] 140 140

[3,] 122 122

[4,] 51 51

[5,] 5 5

, , 1, 2

[,1] [,2]

[1,] 17 17

[2,] 199 199

[3,] 135 135

[4,] 36 36

[5,] 4 4

, , 2, 2

[,1] [,2]

[1,] 10 10

[2,] 140 140

[3,] 122 122

[4,] 51 51

[5,] 5 5

, , 1, 1

[,1] [,2]

[1,] 68.8 87.6

[2,] 68.8 87.6

[3,] 68.8 87.6

[4,] 68.8 87.6

[5,] 68.8 87.6

, , 2, 1

[,1] [,2]

[1,] 74.4 56.8

[2,] 74.4 56.8

[3,] 74.4 56.8

[4,] 74.4 56.8

[5,] 74.4 56.8

, , 1, 2

[,1] [,2]

[1,] 68.8 87.6

[2,] 68.8 87.6

[3,] 68.8 87.6

[4,] 68.8 87.6

[5,] 68.8 87.6

, , 2, 2

[,1] [,2]

[1,] 74.4 56.8

[2,] 74.4 56.8

[3,] 74.4 56.8

[4,] 74.4 56.8

[5,] 74.4 56.8

, , 1, 1

[,1] [,2]

[1,] 93.2 84.4

[2,] 93.2 84.4

[3,] 93.2 84.4

[4,] 93.2 84.4

[5,] 93.2 84.4

, , 2, 1

[,1] [,2]

[1,] 50 60

[2,] 50 60

[3,] 50 60

[4,] 50 60

[5,] 50 60

, , 1, 2

[,1] [,2]

[1,] 93.2 84.4

[2,] 93.2 84.4

[3,] 93.2 84.4

[4,] 93.2 84.4

[5,] 93.2 84.4

, , 2, 2

[,1] [,2]

[1,] 50 60

[2,] 50 60

[3,] 50 60

[4,] 50 60

[5,] 50 60

, , 1, 1

[,1] [,2]

[1,] 94 94

[2,] 94 94

[3,] 94 94

[4,] 94 94

[5,] 94 94

, , 2, 1

[,1] [,2]

[1,] 83.6 83.6

[2,] 83.6 83.6

[3,] 83.6 83.6

[4,] 83.6 83.6

[5,] 83.6 83.6

, , 1, 2

[,1] [,2]

[1,] 62.4 62.4

[2,] 62.4 62.4

[3,] 62.4 62.4

[4,] 62.4 62.4

[5,] 62.4 62.4

, , 2, 2

[,1] [,2]

[1,] 47.6 47.6

[2,] 47.6 47.6

[3,] 47.6 47.6

[4,] 47.6 47.6

[5,] 47.6 47.6

```
> model1=loglm(~BScgrade+State+Gender+Age, data=n)
```

```
> model2=loglm(~BScgrade+State+Gender+Age+BScgrade*Age, data=n)
```

```
> model3=loglm(~BScgrade+State+Gender+Age+BScgrade*State, data=n)
```

```
> model4=loglm(~BScgrade+State+Gender+Age+BScgrade*Gender, data=n)
```

```
> model5=loglm(~BScgrade+State+Gender+Age+State*Gender, data=n)
```

```
> model6=loglm(~BScgrade+State+Gender+Age+State*Age, data=n)
```

```
> model7=loglm(~BScgrade+State+Gender+Age+Gender*Age, data=n)
```

```
> model1; model2; model3; model4; model5; model6; model7
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 102.6573 32 2.469969e-09

Pearson 111.4646 32 1.007785e-10

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *
```

```
Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 62.29832 28 2.054731e-04

Pearson 65.53400 28 7.637938e-05

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *
```

```
State, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 69.29393 28 2.334267e-05

Pearson 69.04752 28 2.525616e-05

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *
```

```
Gender, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 92.65951 28 7.576797e-09

Pearson 94.95682 28 3.274269e-09

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + State * Gender,  
       data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 91.08621 31 8.010669e-08

Pearson 94.50826 31 2.432876e-08

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + State * Age,  
       data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 99.30338 31 4.446626e-09

Pearson 103.93329 31 8.360928e-10

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Gender * Age,  
       data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 101.6678 31 1.900744e-09

Pearson 108.6935 31 1.458038e-10

```
> m_8a=m_8b=m_8c=m_8d=m_8e=array(0, dim=c(I, J, K, L))
```

```
> for(i in 1:I){
```

```

+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ m_8a[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_12[i, j])
+ m_8b[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_13[i, k])
+ m_8c[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_23[j, k])
+ m_8d[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_24[j, l])
+ m_8e[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_34[k, l])
+ }
+ }
+ }
+ }
> model8a=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*State, data=n)
> model8b=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*Gender, data=n)
> model8c=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ State*Gender, data=n)
> model8d=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ State*Age, data=n)
> model8e=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ Gender*Age, data=n)
> model8a ; model8b; model8c ; model8d; model8e

```

Call:

```

loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *
      Age + BScgrade * State, data = n)

```

Statistics:

X² df P(> X²)

Likelihood Ratio 28.93498 24 0.2225874

Pearson 29.45824 24 0.2033606

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
Age + BScgrade * Gender, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 52.30056 24 0.0007147718

Pearson 51.56515 24 0.0008912025

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
Age + State * Gender, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 50.72726 27 0.003746636

Pearson 49.60753 27 0.005049639

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
Age + State * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 58.94443 27 0.0003621419

Pearson 61.70485 27 0.0001569957

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
Age + Gender * Age, data = n)
```

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 61.30886 27 1.772439e-04

Pearson 64.02193 27 7.650879e-05

```
> m_9a=m_9b=m_9c=m_9d=array(0, dim=c(I, J, K, L))
> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ m_9a[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_12[i,
j]+mu_13[i, k])
+ m_9b[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_12[i,
j]+mu_23[j, k])
+ m_9c[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+ mu_12[i,
j]+mu_24[j, l])
+ m_9d[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+ mu_12[i, j]+
mu_34[k, l])
+ }
+ }
+ }
+ }
> m_9a; m_9b; m_9c; m_9d
, , 1, 1
```

[,1] [,2]

[1,] 5.208904e+00 7.576587e+00

[2,] 1.546913e+04 2.253112e+04

[3,] 1.054552e+04 8.675749e+03

[4,] 4.913640e+02 1.871863e+02

[5,] 2.476009e-01 1.980807e-01

, , 2, 1

[,1] [,2]

[1,] 3.064061e+00 4.456816e+00

[2,] 1.088280e+04 1.585104e+04

[3,] 9.530028e+03 7.840307e+03

[4,] 6.960989e+02 2.651805e+02

[5,] 3.095011e-01 2.476009e-01

, , 1, 2

[,1] [,2]

[1,] 1.041781e+01 1.515317e+01

[2,] 1.334425e+04 1.943618e+04

[3,] 4.595256e+03 3.780494e+03

[4,] 1.193312e+02 4.545952e+01

[5,] 3.095011e-01 2.476009e-01

, , 2, 2

[,1] [,2]

[1,] 6.1281219 8.913632e+00

[2,] 9387.9112738 1.367370e+04

[3,] 4152.7496271 3.416447e+03

[4,] 169.0525978 6.440099e+01

[5,] 0.3868764 3.095011e-01

, , 1, 1

[,1] [,2]

[1,] 21.080739 39.041707

[2,] 5348.119645 9918.220369

[3,] 5374.310867 5629.597281

[4,] 939.051108 455.486584

[5,] 4.258735 4.337967

, , 2, 1

[,1] [,2]

[1,] 22.796613 25.314714

[2,] 5783.431710 6430.992203

[3,] 5811.754775 3650.241159

[4,] 1015.485501 295.338333

[5,] 4.605376 2.812746

, , 1, 2

[,1] [,2]

[1,] 42.161478 78.083414

[2,] 4613.487826 8555.827461

[3,] 2341.878478 2453.120603

[4,] 228.055269 110.618170

[5,] 5.323419 5.422459

, , 2, 2

[,1] [,2]

[1,] 45.593227 50.629429

[2,] 4989.004277 5547.614153

[3,] 2532.496494 1590.607880

[4,] 246.617907 71.725024

[5,] 5.756721 3.515933

, , 1, 1

[,1] [,2]

[1,] 28.557048 37.615526

[2,] 7244.836496 9555.910949

[3,] 7280.316465 5423.949892

[4,] 1272.086676 438.847805

[5,] 5.769101 4.179503

, , 2, 1

[,1] [,2]

[1,] 28.557048 37.615526

[2,] 7244.836496 9555.910949

[3,] 7280.316465 5423.949892

[4,] 1272.086676 438.847805

[5,] 5.769101 4.179503

, , 1, 2

[,1] [,2]

[1,] 30.640609 53.481791

[2,] 3352.825455 5860.155795

[3,] 1701.946569 1680.219591

[4,] 165.737841 75.765870

[5,] 3.868764 3.714013

, , 2, 2

[,1] [,2]

[1,] 30.640609 53.481791

[2,] 3352.825455 5860.155795

[3,] 1701.946569 1680.219591

[4,] 165.737841 75.765870

[5,] 3.868764 3.714013

, , 1, 1

[,1] [,2]

[1,] 28.802173 41.894069

[2,] 7307.023934 10642.839208

[3,] 7342.808452 6040.892060

[4,] 1283.005875 488.764143

[5,] 5.818621 4.654897

, , 2, 1

[,1] [,2]

[1,] 25.615549 37.258981

[2,] 6498.587244 9465.333594

[3,] 6530.412623 5372.538044

[4,] 1141.056289 434.688110

[5,] 5.174858 4.139887

, , 1, 2

[,1] [,2]

[1,] 38.239480 55.621062

[2,] 4184.326168 6094.562027

[3,] 2124.029317 1747.428375

[4,] 206.840826 78.796505

[5,] 4.828217 3.862574

, , 2, 2

[,1] [,2]

[1,] 29.169860 42.428887

[2,] 3191.889833 4649.056931

[3,] 1620.253133 1332.974209

[4,] 157.782425 60.107590

[5,] 3.683063 2.946451

```
> model9a=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*State+
BScgrade*Gender, data=n)
```

```
> model9b=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*State+
State*Gender, data=n)
```

```
> model9c=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*State+
State*Age, data=n)
```

```
> model9d=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*State+
Gender*Age, data=n)
```

```
> model9a ; model9b; model9c ; model9d
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *
Age + BScgrade * State + BScgrade * Gender, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 18.93722 20 0.5259099

Pearson 19.58890 20 0.4838953

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
Age + BScgrade * State + State * Gender, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 17.36393 23 0.7910194

Pearson 17.50512 23 0.7837521

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
Age + BScgrade * State + State * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 28.49333 23 0.1976995

Pearson 28.78692 23 0.1874785

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
Age + BScgrade * State + Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 27.94553 23 0.2178485

Pearson 28.19695 23 0.2084256

```
>mu_123=array(0, dim=c(I, J, K)); mu_124=array(0, dim=c(I, J, L)); mu_134= array(0, dim=c(I,  
K, L)); mu_234= array(0, dim=c(J, K, L))
```

```

> for (i in 1:I){
+ for (j in 1:J){
+ for (k in 1:K){
+ mu_123[i, j, k]=log(sum(n[i, j, k, ])/L)-mu-mu_1[i]-mu_2[j]-mu_3[k]-mu_12[i, j]-mu_13[i, k]-
mu_23[j, k]
+ }
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ for (l in 1:L){
+ mu_124[i, j, l]=log(sum(n[i, j, , l])/K)-mu-mu_1[i]-mu_2[j]-mu_4[l]-mu_12[i, j]-mu_14[i, l]-
mu_24[j, l]
+ }
+ }
+ }
> for (i in 1:I){
+ for (k in 1:K){
+ for (l in 1:L){
+ mu_134[i, k, l]=log(sum(n[i, , k, l])/J)-mu-mu_1[i]-mu_3[k]-mu_4[l]-mu_13[i, k]-mu_14[i, l]-
mu_34[k, l]
+ }
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){

```

```

+ for (l in 1:L){
+ mu_234[j, k, l]=log(sum(n[, j, k, l])/I)-mu-mu_2[j]-mu_3[k]-mu_4[l]-mu_23[j, k]-mu_24[j, l]-
mu_34[k, l]
+ }
+ }
+ }
> mu_123; mu_124; mu_134; mu_234
, , 1

```

```

      [,1] [,2]
[1,] -2.298054 -2.796542
[2,] -5.036212 -5.172390
[3,] -4.873287 -4.756370
[4,] -3.694987 -3.664630
[5,] -1.448972 -1.467406

```

```

, , 2

```

```

      [,1] [,2]
[1,] -2.826507 -2.083983
[2,] -4.966401 -4.842958
[3,] -4.607972 -4.751131
[4,] -3.636038 -3.579693
[5,] -1.344903 -1.257305

```

, , 1

[,1] [,2]

[1,] -2.658757 -2.711126

[2,] -5.304525 -5.157582

[3,] -4.951348 -4.960993

[4,] -3.890317 -3.886928

[5,] -1.752516 -1.430192

, , 2

[,1] [,2]

[1,] -2.1695633 -2.274593

[2,] -4.6210511 -4.858219

[3,] -4.4950889 -4.430947

[4,] -3.4056592 -3.229036

[5,] -0.9474699 -1.312113

, , 1

[,1] [,2]

[1,] -2.879478 -2.454742

[2,] -5.255309 -5.210210

[3,] -5.062502 -4.832824

[4,] -3.923197 -3.807976

```
[5,] -1.537920 -1.643812
```

```
, , 2
```

```
    [,1] [,2]
```

```
[1,] -2.287427 -2.1792135
```

```
[2,] -4.909663 -4.5563967
```

```
[3,] -4.482091 -4.4730101
```

```
[4,] -3.519571 -3.2404700
```

```
[5,] -1.351334 -0.8982797
```

```
, , 1
```

```
    [,1] [,2]
```

```
[1,] -4.440240 -4.289537
```

```
[2,] -4.454804 -4.278274
```

```
, , 2
```

```
    [,1] [,2]
```

```
[1,] -3.981132 -3.772651
```

```
[2,] -3.988870 -3.802836
```

```
> m_10a=m_10b=m_10c=m_10d=array(0, dim=c(I, J, K, L))
```

```
> for(i in 1:I){
```

```
+ for(j in 1:J){
```

```

+ for(k in 1:K){
+ for(l in 1:L){
+ m_10a[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_123[i, j, k])
+ m_10b[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_124[i, j, l])
+ m_10c[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_134[i, k, l])
+ m_10d[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_234[j, k, l])
+ }
+ }
+ }
+ }
> m_10a; m_10b; m_10c; m_10d
, , 1, 1

```

```

      [,1] [,2]
[1,] 1.8056628 1.0968474
[2,] 0.1168072 0.1019362
[3,] 0.1374761 0.1545268
[4,] 0.4466388 0.4604051
[5,] 4.2207367 4.1436456

, , 2, 1

```

```

      [,1] [,2]
[1,] 1.0644673 2.2366932
[2,] 0.1252531 0.1417095

```

[3,] 0.1792470 0.1553385

[4,] 0.4737592 0.5012198

[5,] 4.6836562 5.1124417

, , 1, 2

[,1] [,2]

[1,] 1.8056628 1.0968474

[2,] 0.1168072 0.1019362

[3,] 0.1374761 0.1545268

[4,] 0.4466388 0.4604051

[5,] 4.2207367 4.1436456

, , 2, 2

[,1] [,2]

[1,] 1.0644673 2.2366932

[2,] 0.1252531 0.1417095

[3,] 0.1792470 0.1553385

[4,] 0.4737592 0.5012198

[5,] 4.6836562 5.1124417

, , 1, 1

[,1] [,2]

[1,] 1.25888363 1.1946530
[2,] 0.08931879 0.1034569
[3,] 0.12715278 0.1259322
[4,] 0.36738849 0.3686358
[5,] 3.11573698 4.3007507

, , 2, 1

[,1] [,2]

[1,] 1.25888363 1.1946530
[2,] 0.08931879 0.1034569
[3,] 0.12715278 0.1259322
[4,] 0.36738849 0.3686358
[5,] 3.11573698 4.3007507

, , 1, 2

[,1] [,2]

[1,] 2.0532392 1.8485264
[2,] 0.1769179 0.1395633
[3,] 0.2006673 0.2139601
[4,] 0.5964993 0.7117321
[5,] 6.9692805 4.8397781

, , 2, 2

[,1] [,2]

[1,] 2.0532392 1.8485264

[2,] 0.1769179 0.1395633

[3,] 0.2006673 0.2139601

[4,] 0.5964993 0.7117321

[5,] 6.9692805 4.8397781

, , 1, 1

[,1] [,2]

[1,] 1.00954904 1.00954904

[2,] 0.09382467 0.09382467

[3,] 0.11377637 0.11377637

[4,] 0.35550548 0.35550548

[5,] 3.86152509 3.86152509

, , 2, 1

[,1] [,2]

[1,] 1.54378887 1.54378887

[2,] 0.09815298 0.09815298

[3,] 0.14315287 0.14315287

[4,] 0.39892023 0.39892023

[5,] 3.47352496 3.47352496

, , 1, 2

[,1] [,2]

[1,] 1.8249540 1.8249540

[2,] 0.1325652 0.1325652

[3,] 0.2032926 0.2032926

[4,] 0.5322783 0.5322783

[5,] 4.6536328 4.6536328

, , 2, 2

[,1] [,2]

[1,] 2.0335202 2.0335202

[2,] 0.1887344 0.1887344

[3,] 0.2051471 0.2051471

[4,] 0.7036402 0.7036402

[5,] 7.3206728 7.3206728

, , 1, 1

[,1] [,2]

[1,] 0.2119811 0.2089163

[2,] 0.2119811 0.2089163

[3,] 0.2119811 0.2089163

[4,] 0.2119811 0.2089163

[5,] 0.2119811 0.2089163

, , 2, 1

[,1] [,2]

[1,] 0.2464601 0.2492517

[2,] 0.2464601 0.2492517

[3,] 0.2464601 0.2492517

[4,] 0.2464601 0.2492517

[5,] 0.2464601 0.2492517

, , 1, 2

[,1] [,2]

[1,] 0.3354945 0.3329083

[2,] 0.3354945 0.3329083

[3,] 0.3354945 0.3329083

[4,] 0.3354945 0.3329083

[5,] 0.3354945 0.3329083

, , 2, 2

[,1] [,2]

[1,] 0.4132638 0.4009758

[2,] 0.4132638 0.4009758

[3,] 0.4132638 0.4009758

[4,] 0.4132638 0.4009758

[5,] 0.4132638 0.4009758

```
> model10a=loglm(~BScgrade+State+Gender+Age+ BScgrade*State *Gender, data=n)
```

```
> model10b=loglm(~BScgrade+State+Gender+Age+ BScgrade*State*Age, data=n)
```

```
> model10c=loglm(~BScgrade+State+Gender+Age+ BScgrade* Gender*Age, data=n)
```

```
> model10d=loglm(~BScgrade+State+Gender+Age+ State* Gender*Age, data=n)
```

```
> model10a ; model10b; model10c ; model10d
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
      State * Gender, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 46.91952 19 0.0003667590

Pearson 45.38412 19 0.0006054104

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
      State * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 26.24509 19 0.1235033

Pearson 25.89426 19 0.1331625

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *
```

Gender * Age, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 49.53238 19 0.0001535136

Pearson 48.21890 19 0.0002384909

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + State * Gender *  
Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 87.11441 28 5.562476e-08

Pearson 86.69165 28 6.462981e-08

```
> m_11a=m_11b=m_11c=m_11d=m_11e=array(0, dim=c(I, J, K, L))
```

```
> for(i in 1:I){
```

```
+ for(j in 1:J){
```

```
+ for(k in 1:K){
```

```
+ for(l in 1:L){
```

```
+ m_11a[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_123[i, j, k]+mu_124[i, j,  
l])
```

```
+ m_11b[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_123[i, j, k]+mu_134[i, k,  
l])
```

```
+ m_11c[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_123[i, j, k]+mu_234[j, k,  
l])
```

```
+ m_11d[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_124[i, j, l]+mu_234[j, k,  
l])
```

```
+ m_11e[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_134[i, k, l]+mu_234[j, k,  
l])
```

+ }

+ }

+ }

+ }

> m_11a; m_11b; m_11c; m_11d; m_11e

, , 1, 1

[,1] [,2]

[1,] 0.1264600446 0.0728985800

[2,] 0.0005804217 0.0005867042

[3,] 0.0009724875 0.0010826090

[4,] 0.0091287875 0.0094421018

[5,] 0.7316108766 0.9914206882

, , 2, 1

[,1] [,2]

[1,] 0.0745502360 0.148654921

[2,] 0.0006223895 0.000815623

[3,] 0.0012679697 0.001088296

[4,] 0.0096830972 0.010279140

[5,] 0.8118520695 1.223217638

, , 1, 2

[,1] [,2]

[1,] 0.206256333 0.1127984028

[2,] 0.001149669 0.0007914633

[3,] 0.001534740 0.0018393639

[4,] 0.014821681 0.0182300449

[5,] 1.636467209 1.1156787477

, , 2, 2

[,1] [,2]

[1,] 0.121591435 0.230018715

[2,] 0.001232796 0.001100275

[3,] 0.002001058 0.001849026

[4,] 0.015721669 0.019846130

[5,] 1.815950709 1.376527582

, , 1, 1

[,1] [,2]

[1,] 0.1014133586 0.0616034058

[2,] 0.0006097023 0.0005320798

[3,] 0.0008701823 0.0009781081

[4,] 0.0088335213 0.0091057874

[5,] 0.9067305013 0.8901692136

, , 2, 1

[,1] [,2]

[1,] 0.0914221315 0.1920991442

[2,] 0.0006839478 0.0007738085

[3,] 0.0014275229 0.0012371153

[4,] 0.0105141656 0.0111235999

[5,] 0.9050791014 0.9879384558

, , 1, 2

[,1] [,2]

[1,] 0.1833241483 0.111360003

[2,] 0.0008614502 0.000751777

[3,] 0.0015548187 0.001747658

[4,] 0.0132259320 0.013633581

[5,] 1.0927265016 1.072768027

, , 2, 2

[,1] [,2]

[1,] 0.120423690 0.253038158

[2,] 0.001315135 0.001487925

[3,] 0.002045730 0.001772864

[4,] 0.018545536 0.019620494

[5,] 1.907511249 2.082142561

, , 1, 1

[,1] [,2]

[1,] 0.02129438 0.012748222

[2,] 0.00137752 0.001184764

[3,] 0.00162127 0.001796003

[4,] 0.00526726 0.005351105

[5,] 0.04977560 0.048159948

, , 2, 1

[,1] [,2]

[1,] 0.014595202 0.031015273

[2,] 0.001717379 0.001965025

[3,] 0.002457705 0.002154012

[4,] 0.006495842 0.006950201

[5,] 0.064218890 0.070892053

, , 1, 2

[,1] [,2]

[1,] 0.033701800 0.020314302

[2,] 0.002180149 0.001887923

[3,] 0.002565922 0.002861933

[4,] 0.008336292 0.008526991

[5,] 0.078777957 0.076742918

, , 2, 2

[,1] [,2]

[1,] 0.024473202 0.049894848

[2,] 0.002879697 0.003161172

[3,] 0.004121074 0.003465200

[4,] 0.010892212 0.011180918

[5,] 0.107682087 0.114045367

, , 1, 1

[,1] [,2]

[1,] 0.014846151 0.013884977

[2,] 0.001053346 0.001202439

[3,] 0.001499526 0.001463660

[4,] 0.004332652 0.004284507

[5,] 0.036744223 0.049985918

, , 2, 1

[,1] [,2]

[1,] 0.017260897 0.016565744

[2,] 0.001224674 0.001434593

[3,] 0.001743427 0.001746248

[4,] 0.005037364 0.005111715

[5,] 0.042720721 0.059636679

, , 1, 2

[,1] [,2]

[1,] 0.038322691 0.034235869

[2,] 0.003302086 0.002584799

[3,] 0.003745355 0.003962676

[4,] 0.011133363 0.013181725

[5,] 0.130078163 0.089635728

, , 2, 2

[,1] [,2]

[1,] 0.047206086 0.041235848

[2,] 0.004067526 0.003113296

[3,] 0.004613548 0.004772898

[4,] 0.013714133 0.015876904

[5,] 0.160230945 0.107962948

, , 1, 1

[,1] [,2]

[1,] 0.011905721 0.011733588

[2,] 0.001106485 0.001090487

[3,] 0.001341777 0.001322378

[4,] 0.004192515 0.004131899

[5,] 0.045539382 0.044880973

, , 2, 1

[,1] [,2]

[1,] 0.021167311 0.021407063

[2,] 0.001345802 0.001361046

[3,] 0.001962808 0.001985040

[4,] 0.005469704 0.005531657

[5,] 0.047626449 0.048165892

, , 1, 2

[,1] [,2]

[1,] 0.034061862 0.033799294

[2,] 0.002474263 0.002455190

[3,] 0.003794356 0.003765107

[4,] 0.009934710 0.009858128

[5,] 0.086857748 0.086188200

, , 2, 2

[,1] [,2]

[1,] 0.046752727 0.045362583

[2,] 0.004339198 0.004210176

[3,] 0.004716542 0.004576301

[4,] 0.016177414 0.015696396

[5,] 0.168309816 0.163305299

```
> model11a=loglm(~BScgrade+State+Gender+Age+BScgrade*State *Gender +  
BScgrade*State*Age, data=n)
```

```
> model11b=loglm(~BScgrade+State+Gender+Age+ BScgrade*State *Gender+ BScgrade*  
Gender*Age, data=n)
```

```
> model11c=loglm(~BScgrade+State+Gender+Age+ BScgrade*State *Gender+ State*  
Gender*Age, data=n)
```

```
> model11d=loglm(~BScgrade+State+Gender+Age+ BScgrade*State*Age+ State*  
Gender*Age, data=n)
```

```
> model11e=loglm(~BScgrade+State+Gender+Age+ BScgrade* Gender*Age+ State*  
Gender*Age, data=n)
```

```
> model11a ; model11b; model11c ; model11d; model11e
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
State * Gender + BScgrade * State * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 3.870686 10 0.9529921

Pearson 3.815083 10 0.9553035

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
State * Gender + BScgrade * Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 3.792395 10 0.9562268

Pearson 3.792102 10 0.9562386

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
State * Gender + State * Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 42.94772 16 0.0002845600

Pearson 42.21511 16 0.0003664753

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
State * Age + State * Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 14.05613 16 0.5945325

Pearson 13.90669 16 0.6056651

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
Gender * Age + State * Gender * Age, data = n)
```

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 34.97898 16 0.004000704

Pearson 34.38458 16 0.004820282

```
> model12=loglm(~BScgrade+State+Gender+Age+BScgrade*State *Gender *Age, data=n);  
model12
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
State * Gender * Age, data = n)
```

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 0 0 1

Pearson 0 0 1

Results of 5 dimensional for parameter estimates, expected values and model estimates

> I=5; J=2; K=2; L=2; M=2

> n=c(7, 14, 61, 22,1, 1, 6, 17, 28, 1, 4, 11, 78, 42, 2, 2, 7, 33, 32, 1,12, 8, 45, 19, 1, 4, 9, 14, 22, 2, 12, 8, 63, 31, 2, 4, 2, 24, 21, 1, 4, 10, 52, 31, 1, 8, 11, 17, 19, 1, 1, 2, 94, 45, 1, 1, 1, 27, 17, 2, 2, 4, 28, 39, 1, 2, 4, 21, 15, 1, 1, 1, 81, 28, 1, 3, 6, 22, 11, 1)

> n=array(n, dim=c(I, J, K, L, M))

> dimnames(n)=list(BScgrade=c("pass", "Third class", "Lower Division", "Upper Division", "First Class"), State=c("Indigene", "Non-indigene"), Gender=c("Male", "Female"), Age=c("Under 26", "26 and over"), Mode=c("Jamb", "Pre science"))

> n

, , Gender = Male, Age = Under 26, Mode = Jamb

BScgrade	State	
	Indigene	Non-indigene
pass	7	1
Third class	14	6
Lower Division	61	17
Upper Division	22	28
First Class	1	1

, , Gender = Female, Age = Under 26, Mode = Jamb

BScgrade	State	
	Indigene	Non-indigene
pass	4	2
Third class	11	7

Lower Division	78	33
Upper Division	42	32
First Class	2	1

, , Gender = Male, Age = 26 and over, Mode = Jamb

State

BScgrade	Indigene	Non-indigene
pass	12	4
Third class	8	9
Lower Division	45	14
Upper Division	19	22
First Class	1	2

, , Gender = Female, Age = 26 and over, Mode = Jamb

State

BScgrade	Indigene	Non-indigene
pass	12	4
Third class	8	2
Lower Division	63	24
Upper Division	31	21
First Class	2	1

, , Gender = Male, Age = Under 26, Mode = Pre science

State

BScgrade	Indigene	Non-indigene
pass	4	8
Third class	10	11
Lower Division	52	17
Upper Division	31	19
First Class	1	1

, , Gender = Female, Age = Under 26, Mode = Pre science

State

BScgrade	Indigene	Non-indigene
pass	1	1
Third class	2	1
Lower Division	94	27
Upper Division	45	17
First Class	1	2

, , Gender = Male, Age = 26 and over, Mode = Pre science

State

BScgrade	Indigene	Non-indigene
pass	2	2
Third class	4	4

Lower Division	28	21
Upper Division	39	15
First Class	1	1

, , Gender = Female, Age = 26 and over, Mode = Pre science

State

BScgrade	Indigene	Non-indigene
pass	1	3
Third class	1	6
Lower Division	81	22
Upper Division	28	11
First Class	1	1

```
> mu=log(sum(n)/(I*J*K*L*M))
```

```
> mu_1=mu_2=mu_3=mu_4=mu_5= array(0, dim=c(I, J, K, L, M))
```

```
> mu_12=matrix(0, I, J); mu_13=matrix(0, I, K); mu_14=matrix(0, I, L); mu_15=matrix(0, I, M); mu_23=matrix(0, J, K); mu_24=matrix(0, J, L); mu_25=matrix(0, J, M); mu_34=matrix(0, K, L); mu_35=matrix(0, K, M); mu_45=matrix(0, L, M)
```

```
> for (i in 1:I){
```

```
+ mu_1[i]=log(sum(n[i, , , ])/(J*K*L*M))-mu
```

```
+ for (j in 1:J){
```

```
+ mu_2[j]=log(sum(n [ , j, , ])/(I*K*L*M))-mu
```

```
+ for (k in 1:K){
```

```
+ mu_3[k]=log(sum(n[ , , k, ])/(I*J*L*M))-mu
```

```
+ for (l in 1:L){
```

```

+ mu_4[l]=log(sum(n[, , l, ])/(I*J*K*M))-mu
+ for (m in 1:M){
+ mu_5[m]=log(sum(n[, , , m])/(I*J*K*L))-mu
+ }
+ }
+ }
+ }
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ mu_12[i, j]=log(sum(n[i, j , , ])/K*L*M)-mu-mu_1[i]-mu_2[j]
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ mu_23[j, k]=log(sum(n[j, k , , ])/I*L*M)-mu-mu_2[j]-mu_3[k]
+ }
+ }
> for (j in 1:J){
+ for (l in 1:L){
+ mu_24[j, l]=log(sum(n[j, , l, ])/I*K*M)-mu-mu_2[j]-mu_4[l]
+ }
+ }
> for (j in 1:J){
+ for (m in 1:M){

```

```

+ mu_25[j, m]=log(sum(n[,j, , m])/I*K*L)-mu-mu_2[j]-mu_5[m]
+ }
+ }
> for (i in 1:I){
+ for (k in 1:K){
+ mu_13[i, k]=log(sum(n[i, , k, ,])/J*L*M)-mu-mu_1[i]-mu_3[k]
+ }
+ }
> for (i in 1:I){
+ for (l in 1:L){
+ mu_14[i, l]=log(sum(n[i, , , l])/J*K*M)-mu-mu_1[i]-mu_4[l]
+ }
+ }
> for (i in 1:I){
+ for (m in 1:M){
+ mu_15[i, m]=log(sum(n[i, , , m])/J*K*L)-mu-mu_1[i]-mu_5[m]
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ mu_23[j, k]=log(sum(n[,j, k, ,])/I*L*M)-mu-mu_2[j]-mu_3[k]
+ }
+ }
> for (j in 1:J){
+ for (l in 1:L){

```

```

+ mu_24[j, l]=log(sum(n[, , l, ])/I*K*M)-mu-mu_2[j]-mu_4[l]
+ }
+ }
> for (j in 1:J){
+ for (m in 1:M){
+ mu_25[j, m]=log(sum(n[, , , m])/I*K*L)-mu-mu_2[j]-mu_5[m]
+ }
+ }
> for (k in 1:K){
+ for (l in 1:L){
+ mu_34[k, l]=log(sum(n[, , k, l])/I*J*M)-mu-mu_3[k]-mu_4[l]
+ }
+ }
> for (k in 1:K){
+ for (m in 1:M){
+ mu_35[k, m]=log(sum(n[, , k, , m])/I*J*L)-mu-mu_3[k]-mu_5[m]
+ }
+ }
> for (l in 1:L){
+ for (m in 1:M){
+ mu_45[l, m]=log(sum(n[, , , l, m])/I*J*K)-mu-mu_4[l]-mu_5[m]
+ }
+ }
> mu; mu_1; mu_2; mu_3; mu_4; mu_5; mu_12; mu_13; mu_14; mu_15; mu_23; mu_24;
mu_25; mu_34; mu_35; mu_45

```

[1] 2.781146

, , 1, 1, 1

[,1] [,2]

[1,] -1.3342268 0

[2,] -0.9093436 0

[3,] 0.9639368 0

[4,] 0.4912708 0

[5,] -2.5580022 0

, , 2, 1, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 1, 1

[,1] [,2]

[1,] 0.2984680 0

[2,] -0.4273924 0

[3,] 0.0000000 0

[4,] 0.0000000 0

[5,] 0.0000000 0

, , 2, 1, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 1, 1

[,1] [,2]

[1,] -0.1331995 0

[2,] 0.1175248 0

[3,] 0.0000000 0

[4,] 0.0000000 0

[5,] 0.0000000 0

, , 2, 1, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 1, 1

[,1] [,2]

[1,] 0.1022573 0

[2,] -0.1139175 0

[3,] 0.0000000 0

[4,] 0.0000000 0

[5,] 0.0000000 0

, , 2, 1, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 1, 1

[,1] [,2]

[1,] 0.04320490 0

[2,] -0.04515619 0

[3,] 0.00000000 0

[4,] 0.00000000 0

[5,] 0.00000000 0

, , 2, 1, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

[,1] [,2]

[1,] 2.708960 2.892496

[2,] 2.583320 3.077379

[3,] 2.868197 2.540243

[4,] 2.671339 2.954068

[5,] 2.474121 3.199981

[,1] [,2]

[1,] 3.068307 2.460908

[2,] 3.144199 2.341406

[3,] 2.622528 2.875545

[4,] 2.826930 2.728156

[5,] 2.800428 2.750374

[,1] [,2]

[1,] 2.476175 3.049025

[2,] 2.846222 2.672932

[3,] 2.783344 2.759076

[4,] 2.782305 2.760395

[5,] 2.670331 2.886506

[,1] [,2]

[1,] 3.031665 2.382427

[2,] 2.952527 2.530063

[3,] 2.718990 2.828032

[4,] 2.757423 2.788897

[5,] 2.824694 2.712384

[,1] [,2]

[1,] 2.722086 2.810199

[2,] 2.869509 2.690073

[,1] [,2]

[1,] 2.775002 2.769585

[2,] 2.767583 2.778768

[,1] [,2]

[1,] 2.747608 2.799183

[2,] 2.822316 2.715283

[,1] [,2]

[1,] 2.769656 2.776217

[2,] 2.774865 2.769756

[,1] [,2]

[1,] 2.769285 2.776185

[2,] 2.775152 2.769781

```

    [,1] [,2]
[1,] 2.763751 2.782154
[2,] 2.783451 2.760586
> m_1=m_2=m_3=m_4=m_5=m_6=m_7=m_8=m_9=m_10=m_11=array(0, dim=c(I, J, K,
L,M))
> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ for(m in 1:M){
+ m_1[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_4[l]+mu_5[m])
+ m_2[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i, m])
+ m_3[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_12[i, j])
+ m_4[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_13[i, k])
+ m_5[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_14[i, l])
+ m_6[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+mu_23[j, k])
+ m_7[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+mu_24[j, l])
+ m_8[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_25[j, m])
+ m_9[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_34[k, l])
+ m_10[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_35[k, m])
+ m_11[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_45[l, m])
+ }
+ }
+ }
+ }

```

+ }

> m_1; m_2; m_3; m_4; m_5; m_6; m_7; m_8; m_9; m_10; m_11

, , 1, 1, 1

[,1] [,2]

[1,] 5.798793 2.8060824

[2,] 8.868742 4.2916555

[3,] 57.732098 27.9370266

[4,] 35.986625 17.4142175

[5,] 1.705527 0.8253184

, , 2, 1, 1

[,1] [,2]

[1,] 7.451192 3.605692

[2,] 11.395941 5.514587

[3,] 74.183191 35.897843

[4,] 46.241221 22.376499

[5,] 2.191527 1.060498

, , 1, 2, 1

[,1] [,2]

[1,] 4.671475 2.2605643

[2,] 7.144609 3.4573337

[3,] 46.508655 22.5059124

[4,] 28.990624 14.0287962

[5,] 1.373963 0.6648719

, , 2, 2, 1

[,1] [,2]

[1,] 6.002639 2.9047251

[2,] 9.180506 4.4425208

[3,] 59.761564 28.9191015

[4,] 37.251669 18.0263823

[5,] 1.765482 0.8543309

, , 1, 1, 2

[,1] [,2]

[1,] 5.308390 2.5687728

[2,] 8.118715 3.9287113

[3,] 52.849710 25.5743997

[4,] 32.943246 15.9415018

[5,] 1.561291 0.7555214

, , 2, 1, 2

[,1] [,2]

[1,] 6.821047 3.3007594
[2,] 10.432189 5.0482202
[3,] 67.909538 32.8619721
[4,] 42.330613 20.4841244
[5,] 2.006190 0.9708116

, , 1, 2, 2

[,1] [,2]
[1,] 4.276410 2.0693890
[2,] 6.540391 3.1649479
[3,] 42.575430 20.6025934
[4,] 26.538895 12.8423846
[5,] 1.257768 0.6086438

, , 2, 2, 2

[,1] [,2]
[1,] 5.494997 2.6590733
[2,] 8.404113 4.0668180
[3,] 54.707544 26.4734208
[4,] 34.101305 16.5018960
[5,] 1.616176 0.7820804

, , 1, 1, 1

[,1] [,2]

[1,] 120.2189 58.17490

[2,] 169.8746 82.20367

[3,] 875.5074 423.66506

[4,] 567.1197 274.43378

[5,] 28.7480 13.91139

, , 2, 1, 1

[,1] [,2]

[1,] 154.47600 74.75218

[2,] 218.28130 105.62808

[3,] 1124.98823 544.39086

[4,] 728.72372 352.63527

[5,] 36.93991 17.87552

, , 1, 2, 1

[,1] [,2]

[1,] 96.84769 46.86538

[2,] 136.85000 66.22282

[3,] 705.30384 341.30220

[4,] 456.86846 221.08232

[5,] 23.15923 11.20694

, , 2, 2, 1

[,1] [,2]

[1,] 124.44500 60.21994

[2,] 175.84619 85.09339

[3,] 906.28422 438.55823

[4,] 587.05575 284.08100

[5,] 29.75859 14.40042

, , 1, 1, 2

[,1] [,2]

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[4,] 539.11136 260.88033
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```
[5,] 25.55030 12.36400
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```
> model1=loglm(~BScgrade+State+Gender+Age+Mode, data=n)
```

```
> model2=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode, data=n)
```

```
> model3=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*State, data=n)
```

```
> model4=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Gender, data=n)
```

```
> model5=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Age, data=n)
```

```
> model6=loglm(~BScgrade+State+Gender+Age+Mode+State*Gender, data=n)
```

```
> model7=loglm(~BScgrade+State+Gender+Age+Mode+State*Age, data=n)
```

```

> model8=loglm(~BScgrade+State+Gender+Age+Mode+State*Mode, data=n)
> model9=loglm(~BScgrade+State+Gender+Age+Mode+Gender*Age, data=n)
> model10=loglm(~BScgrade+State+Gender+Age+Mode+Gender*Mode, data=n)
> model11=loglm(~BScgrade+State+Gender+Age+Mode+Age*Mode, data=n)
> model1; model2; model3; model4; model5; model6; model7; model8; model9; model10;
model11

```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode, data = n)
```

Statistics:

```
X^2 df P(> X^2)
```

```
Likelihood Ratio 172.9839 71 1.729357e-10
```

```
Pearson 176.3949 71 6.088841e-11
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Mode, data = n)
```

Statistics:

```
X^2 df P(> X^2)
```

```
Likelihood Ratio 159.6621 67 1.562371e-09
```

```
Pearson 173.3234 67 2.314782e-11
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
State, data = n)
```


Statistics:

X² df P(> X²)

Likelihood Ratio 141.2813 67 3.136906e-07

Pearson 139.1354 67 5.645593e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Gender, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 139.1010 67 5.698819e-07

Pearson 141.8967 67 2.646954e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 166.3272 67 2.054197e-10

Pearson 171.4614 67 4.159828e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *  
Gender, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 167.9625 70 5.003790e-10

Pearson 167.2945 70 6.118129e-10

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *  
Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 172.9646 70 1.092613e-10

Pearson 176.5658 70 3.592260e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *  
Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 171.2079 70 1.870341e-10

Pearson 174.1919 70 7.490208e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + Gender *  
Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 172.9732 70 1.089721e-10

Pearson 176.3752 70 3.811362e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + Gender *  
Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 172.9719 70 1.090137e-10

Pearson 176.5359 70 3.625722e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + Age *  
Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 172.8484 70 1.132279e-10

Pearson 175.9629 70 4.331746e-11

```
> m_12a=m_12b=m_12c=m_12d=m_12e=m_12f=m_12g=m_12h=m_12i=array(0, dim=c(I, J,  
K, L,M))
```

```
> for(i in 1:I){
```

```
+ for(j in 1:J){
```

```
+ for(k in 1:K){
```

```
+ for(l in 1:L){
```

```

+ for(m in 1:M){
+ m_12a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i,
m]+mu_12[i, j])
+ m_12b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i,
m]+mu_13[i, k])
+ m_12c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i, m]+
mu_14[i, l])
+ m_12d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+
mu_23[j, k])
+ m_12e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+
mu_24[j, l])
+ m_12f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_25[j, m])
+ m_12g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_34[k, l])
+ m_12h[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_35[k, m])
+ m_12i[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_45[l, m])
+ }
+ }
+ }
+ }
+ }
+ }
> m_12a; m_12b; m_12c; m_12d; m_12e; m_12f; m_12g; m_12h; m_12i
, , 1, 1, 1

```

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[1,] 1804.9258 1049.3754
[2,] 2249.3135 1783.9383
[3,] 15413.5198 5373.2390
[4,] 8200.1609 5264.6948
[5,] 341.2751 341.2751

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[1,] 2319.2497 1348.4010
[2,] 2890.2683 2292.2817
[3,] 19805.6910 6904.3743
[4,] 10536.8440 6764.8998
[5,] 438.5235 438.5235

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[5,] 379.9545 183.8631

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[4,] 11120.6749 5381.3841

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[5,] 488.2248 236.2559
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```
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```
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```
[2,] 1298.0862 628.1543
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```
[3,] 11383.2173 5508.4305
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```
[4,] 6823.2735 3301.8370
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```
[5,] 299.5584 144.9587
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```

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```
[2,] 1667.9833 807.1505
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```
[3,] 14626.9306 7078.0894
```

```
[4,] 8767.6046 4242.7144
```

```
[5,] 384.9192 186.2655
```

```
> model12a=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*State,  
data=n)
```

```
>  
model12b=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender,  
data=n)
```

```
> model12c=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Age,  
data=n)
```

```
> model12d=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+State*Gender,
data=n)
```

```
> model12e=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+State*Age,
data=n)
```

```
> model12f=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+State*Mode,
data=n)
```

```
> model12g=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+Gender*Age,
data=n)
```

```
> model12h=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+Gender*Mode,
data=n)
```

```
> model12i=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+Age*Mode,
data=n)
```

```
> model12a; model12b; model12c; model12d; model12e; model12f; model12g; model12h;
model12i
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      Mode + BScgrade * State, data = n)
```

Statistics:

```
X^2 df P(> X^2)
```

```
Likelihood Ratio 127.9595 63 2.519681e-06
```

```
Pearson 131.5539 63 9.507707e-07
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      Mode + BScgrade * Gender, data = n)
```

Statistics:

```
X^2 df P(> X^2)
```

Likelihood Ratio 125.7791 63 4.501963e-06

Pearson 134.4006 63 4.328119e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 153.0054 63 1.883332e-09

Pearson 174.6249 63 1.989187e-12

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + State * Gender, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 154.6406 66 4.478824e-09

Pearson 163.1955 66 3.365246e-10

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + State * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 159.6427 66 9.965438e-10

Pearson 173.5571 66 1.309208e-11

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Mode + State * Mode, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 157.8861 66 1.695212e-09

Pearson 172.8167 66 1.657330e-11

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Mode + Gender * Age, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 159.6514 66 9.939356e-10

Pearson 173.3912 66 1.380318e-11

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Mode + Gender * Mode, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 159.6501 66 9.943111e-10

Pearson 173.3093 66 1.416811e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
      Mode + Age * Mode, data = n)
```

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 159.5265 66 1.032304e-09

Pearson 172.8867 66 1.620848e-11

```
> m_13a=m_13b=m_13c=m_13d=m_13e=m_13f=m_13g=m_13h=array(0, dim=c(I, J, K, L,M))
```

```
> for(i in 1:I){
```

```
+ for(j in 1:J){
```

```
+ for(k in 1:K){
```

```
+ for(l in 1:L){
```

```
+ for(m in 1:M){
```

```
+ m_13a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i,  
m]+mu_13[i, k]+mu_12[i, j])
```

```
+ m_13b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i, m]+  
mu_13[i, k]+ mu_14[i, l])
```

```
+ m_13c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+  
mu_13[i, k]+mu_23[j, k])
```

```
+ m_13d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+  
mu_13[i, k]+mu_24[j, l])
```

```
+ m_13e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+  
mu_13[i, k]+ mu_25[j, m])
```

```
+ m_13f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+  
mu_13[i, k]+ mu_34[k, l])
```

```
+ m_13g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+  
mu_13[i, k]+ mu_35[k, m])
```

+ m_13h[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+ mu_13[i, k]+ mu_45[l, m])

+ }

+ }

+ }

+ }

+ }

> m_13a; m_13b; m_13c; m_13d; m_13e; m_13f; m_13g; m_13h

, , 1, 1, 1

[,1] [,2]

[1,] 38815.77 22567.31

[2,] 52186.52 41389.31

[3,] 212251.67 73992.12

[4,] 138529.52 88939.18

[5,] 5614.55 5614.55

, , 2, 1, 1

[,1] [,2]

[1,] 27171.039 15797.116

[2,] 30046.785 23830.209

[3,] 351255.704 122449.698

[4,] 161262.564 103534.331

[5,] 6862.228 6862.228

, , 1, 2, 1

[,1] [,2]

[1,] 31269.77 18180.10

[2,] 42041.17 33343.00

[3,] 170988.76 59607.63

[4,] 111598.60 71648.91

[5,] 4523.05 4523.05

, , 2, 2, 1

[,1] [,2]

[1,] 21888.837 12726.068

[2,] 24205.522 19197.483

[3,] 282969.630 98644.791

[4,] 129912.219 83406.678

[5,] 5528.172 5528.172

, , 1, 1, 2

[,1] [,2]

[1,] 18564.064 10793.060

[2,] 31311.913 24833.586

[3,] 216686.779 75538.220

[4,] 130868.897 84020.887

[5,] 4593.723 4593.723

, , 2, 1, 2

[,1] [,2]

[1,] 12994.84 7555.142

[2,] 18028.07 14298.125

[3,] 358595.38 125008.348

[4,] 152344.82 97808.930

[5,] 5614.55 5614.550

, , 1, 2, 2

[,1] [,2]

[1,] 14955.106 8694.829

[2,] 25224.702 20005.798

[3,] 174561.657 60853.167

[4,] 105427.251 67686.757

[5,] 3700.677 3700.677

, , 2, 2, 2

[,1] [,2]

[1,] 10468.57 6086.38
[2,] 14523.31 11518.49
[3,] 288882.43 100706.03
[4,] 122728.13 78794.33
[5,] 4523.05 4523.05

, , 1, 1, 1

[,1] [,2]

[1,] 30754.66 14882.43
[2,] 67878.97 32847.18
[3,] 194984.38 94354.51
[4,] 154786.97 74902.66
[5,] 6831.69 3305.91

, , 2, 1, 1

[,1] [,2]

[1,] 21528.265 10417.701
[2,] 39081.833 18912.013
[3,] 322680.030 156147.463
[4,] 180187.911 87194.380
[5,] 8349.844 4040.556

, , 1, 2, 1

[,1] [,2]

[1,] 43935.23 21260.61

[2,] 45982.53 22251.32

[3,] 153312.26 74189.03

[4,] 121993.12 59033.45

[5,] 6831.69 3305.91

, , 2, 2, 1

[,1] [,2]

[1,] 30754.664 14882.429

[2,] 26474.790 12811.364

[3,] 253716.752 122775.578

[4,] 142012.506 68720.994

[5,] 8349.844 4040.556

, , 1, 1, 2

[,1] [,2]

[1,] 14708.752 7117.684

[2,] 40727.384 19708.309

[3,] 199058.679 96326.096

[4,] 146227.324 70760.579

[5,] 5589.565 2704.835

, , 2, 1, 2

[,1] [,2]

[1,] 10296.13 4982.379

[2,] 23449.10 11347.208

[3,] 329422.60 159410.246

[4,] 170223.60 82372.571

[5,] 6831.69 3305.910

, , 1, 2, 2

[,1] [,2]

[1,] 21012.503 10168.119

[2,] 27589.518 13350.790

[3,] 156515.795 75739.253

[4,] 115246.959 55768.931

[5,] 5589.565 2704.835

, , 2, 2, 2

[,1] [,2]

[1,] 14708.75 7117.684

[2,] 15884.87 7686.818

[3,] 259018.30 125341.038

[4,] 134159.28 64920.755

[5,] 6831.69 3305.910

, , 1, 1, 1

[,1] [,2]

[1,] 39328.627 22054.451

[2,] 59954.781 33621.051

[3,] 183398.673 102845.112

[4,] 145741.006 81727.691

[5,] 7194.574 4034.526

, , 2, 1, 1

[,1] [,2]

[1,] 30065.871 12902.283

[2,] 37699.054 16177.941

[3,] 331463.284 142242.118

[4,] 185284.879 79512.015

[5,] 9603.338 4121.117

, , 1, 2, 1

[,1] [,2]

[1,] 31682.922 17766.942

[2,] 48299.236 27084.931
[3,] 147744.945 82851.447
[4,] 117408.139 65839.371
[5,] 5795.908 3250.192

, , 2, 2, 1

[,1] [,2]

[1,] 24220.897 10394.007
[2,] 30370.147 13032.858
[3,] 267024.967 114589.454
[4,] 149264.462 64054.435
[5,] 7736.395 3319.949

, , 1, 1, 2

[,1] [,2]

[1,] 18809.343 10547.781
[2,] 35972.869 20172.631
[3,] 187230.884 104994.115
[4,] 137681.596 77208.188
[5,] 5886.469 3300.976

, , 2, 1, 2

[,1] [,2]

[1,] 14379.330 6170.657

[2,] 22619.432 9706.764

[3,] 338389.382 145214.341

[4,] 175038.711 75115.037

[5,] 7857.277 3371.823

, , 1, 2, 2

[,1] [,2]

[1,] 15152.702 8497.233

[2,] 28979.542 16250.959

[3,] 150832.153 84582.671

[4,] 110915.523 62198.484

[5,] 4742.107 2659.248

, , 2, 2, 2

[,1] [,2]

[1,] 11583.907 4971.047

[2,] 18222.088 7819.715

[3,] 272604.593 116983.861

[4,] 141010.206 60512.254

[5,] 6329.778 2716.322

, , 1, 1, 1

[,1] [,2]

[1,] 41465.771 19917.306

[2,] 63212.765 30363.067

[3,] 193364.683 92879.102

[4,] 153660.673 73808.025

[5,] 7585.532 3643.568

, , 2, 1, 1

[,1] [,2]

[1,] 29026.040 13942.11

[2,] 36395.229 17481.77

[3,] 319999.593 153705.81

[4,] 178876.783 85920.11

[5,] 9271.206 4453.25

, , 1, 2, 1

[,1] [,2]

[1,] 33224.127 16225.737

[2,] 50648.737 24735.430

[3,] 154931.951 75664.441

[4,] 123119.421 60128.089

[5,] 6077.848 2968.252

, , 2, 2, 1

[,1] [,2]

[1,] 23256.889 11358.016

[2,] 29161.394 14241.611

[3,] 256397.189 125217.232

[4,] 143323.634 69995.263

[5,] 7428.481 3627.863

, , 1, 1, 2

[,1] [,2]

[1,] 19831.456 9525.668

[2,] 37927.659 18217.840

[3,] 197405.139 94819.860

[4,] 145163.309 69726.475

[5,] 6206.344 2981.101

, , 2, 1, 2

[,1] [,2]

[1,] 13882.019 6667.968

[2,] 21837.137 10489.060

[3,] 326686.152 156917.572

[4,] 168984.980 81168.769

[5,] 7585.532 3643.568

, , 1, 2, 2

[,1] [,2]

[1,] 15889.800 7760.135

[2,] 30389.242 14841.258

[3,] 158169.335 77245.489

[4,] 116310.974 56803.034

[5,] 4972.785 2428.569

, , 2, 2, 2

[,1] [,2]

[1,] 11122.860 5432.094

[2,] 17496.837 8544.967

[3,] 261754.743 127833.711

[4,] 135397.903 66124.557

[5,] 6077.848 2968.252

, , 1, 1, 1

[,1] [,2]

[1,] 40345.257 21037.820
[2,] 61504.590 32071.242
[3,] 188139.461 98104.324
[4,] 149508.358 77960.340
[5,] 7380.551 3848.549

, , 2, 1, 1

[,1] [,2]

[1,] 28241.680 14726.474
[2,] 35411.734 18465.261
[3,] 311352.363 162353.038
[4,] 174043.063 90753.832
[5,] 9020.673 4703.782

, , 1, 2, 1

[,1] [,2]

[1,] 32501.91 16947.95
[2,] 49547.75 25836.41
[3,] 151564.10 79032.29
[4,] 120443.10 62804.41
[5,] 5945.73 3100.37

, , 2, 2, 1

[,1] [,2]

[1,] 22751.339 11863.565

[2,] 28527.495 14875.511

[3,] 250823.722 130790.699

[4,] 140208.118 73110.779

[5,] 7267.004 3789.341

, , 1, 1, 2

[,1] [,2]

[1,] 20316.843 9040.281

[2,] 38855.962 17289.538

[3,] 202236.749 89988.249

[4,] 148716.269 66173.515

[5,] 6358.248 2829.197

, , 2, 1, 2

[,1] [,2]

[1,] 14221.790 6328.197

[2,] 22371.614 9954.582

[3,] 334681.993 148921.730

[4,] 173120.990 77032.759

[5,] 7771.192 3457.908

, , 1, 2, 2

[,1] [,2]

[1,] 16367.135 7282.800

[2,] 31302.145 13928.355

[3,] 162920.794 72494.030

[4,] 119804.994 53309.014

[5,] 5122.169 2279.185

, , 2, 2, 2

[,1] [,2]

[1,] 11456.994 5097.960

[2,] 18022.447 8019.356

[3,] 269617.941 119970.513

[4,] 139465.301 62057.160

[5,] 6260.429 2785.671

, , 1, 1, 1

[,1] [,2]

[1,] 41244.687 19958.636

[2,] 62875.732 30426.073

[3,] 192333.716 93071.833

[4,] 152841.397 73961.182

[5,] 7545.088 3651.129

, , 2, 1, 1

[,1] [,2]

[1,] 29022.067 14044.012

[2,] 36390.247 17609.533

[3,] 319955.790 154829.181

[4,] 178852.298 86548.066

[5,] 9269.936 4485.797

, , 1, 2, 1

[,1] [,2]

[1,] 33445.211 16184.407

[2,] 50985.770 24672.424

[3,] 155962.917 75471.711

[4,] 123938.697 59974.933

[5,] 6118.292 2960.691

, , 2, 2, 1

[,1] [,2]

[1,] 23260.862 11256.118

[2,] 29166.376 14113.844
[3,] 256440.993 124093.860
[4,] 143348.119 69367.308
[5,] 7429.751 3595.316

, , 1, 1, 2

[,1] [,2]

[1,] 19725.720 9545.435
[2,] 37725.439 18255.644
[3,] 196352.630 95016.617
[4,] 144389.338 69871.163
[5,] 6173.254 2987.287

, , 2, 1, 2

[,1] [,2]

[1,] 13880.119 6716.701
[2,] 21834.148 10565.720
[3,] 326641.433 158064.418
[4,] 168961.848 81761.998
[5,] 7584.493 3670.197

, , 1, 2, 2

[,1] [,2]

[1,] 15995.536 7740.368

[2,] 30591.462 14803.455

[3,] 159221.844 77048.731

[4,] 117084.944 56658.347

[5,] 5005.876 2422.383

, , 2, 2, 2

[,1] [,2]

[1,] 11124.760 5383.361

[2,] 17499.826 8468.306

[3,] 261799.461 126686.866

[4,] 135421.035 65531.328

[5,] 6078.887 2941.622

, , 1, 1, 1

[,1] [,2]

[1,] 41229.389 19951.233

[2,] 62852.411 30414.787

[3,] 192262.376 93037.311

[4,] 152784.705 73933.748

[5,] 7542.289 3649.774

, , 2, 1, 1

[,1] [,2]

[1,] 29030.401 14048.044

[2,] 36400.696 17614.590

[3,] 320047.669 154873.642

[4,] 178903.658 86572.919

[5,] 9272.598 4487.085

, , 1, 2, 1

[,1] [,2]

[1,] 33214.165 16072.602

[2,] 50633.550 24501.982

[3,] 154885.494 74950.337

[4,] 123082.504 59560.614

[5,] 6076.026 2940.238

, , 2, 2, 1

[,1] [,2]

[1,] 23386.728 11317.026

[2,] 29324.197 14190.215

[3,] 257828.612 124765.340

[4,] 144123.786 69742.660

[5,] 7469.953 3614.771

, , 1, 1, 2

[,1] [,2]

[1,] 19854.934 9607.962

[2,] 37972.560 18375.228

[3,] 197638.840 95639.025

[4,] 145335.163 70328.855

[5,] 6213.692 3006.855

, , 2, 1, 2

[,1] [,2]

[1,] 13809.73 6682.638

[2,] 21723.42 10512.137

[3,] 324984.91 157262.815

[4,] 168104.98 81347.353

[5,] 7546.03 3651.585

, , 1, 2, 2

[,1] [,2]

[1,] 15995.023 7740.121

[2,] 30590.482 14802.981

[3,] 159216.744 77046.264

[4,] 117081.194 56656.532

[5,] 5005.715 2422.306

, , 2, 2, 2

[,1] [,2]

[1,] 11125.039 5383.496

[2,] 17500.265 8468.519

[3,] 261806.029 126690.044

[4,] 135424.432 65532.972

[5,] 6079.039 2941.696

, , 1, 1, 1

[,1] [,2]

[1,] 41001.873 19841.136

[2,] 62505.573 30246.950

[3,] 191201.416 92523.904

[4,] 151941.595 73525.760

[5,] 7500.669 3629.634

, , 2, 1, 1

[,1] [,2]

[1,] 28701.311 13888.795
[2,] 35988.057 17414.910
[3,] 316419.599 153117.990
[4,] 176875.600 85591.526
[5,] 9167.484 4436.219

, , 1, 2, 1

[,1] [,2]

[1,] 33688.025 16301.907
[2,] 51355.930 24851.548
[3,] 157095.218 76019.640
[4,] 124838.499 60410.354
[5,] 6162.712 2982.186

, , 2, 2, 1

[,1] [,2]

[1,] 23581.618 11411.335
[2,] 29568.566 14308.467
[3,] 259977.184 125805.051
[4,] 145324.817 70323.848
[5,] 7532.203 3644.894

, , 1, 1, 2

[,1] [,2]

[1,] 19973.798 9665.481

[2,] 38199.888 18485.233

[3,] 198822.032 96211.581

[4,] 146205.231 70749.888

[5,] 6250.891 3024.856

, , 2, 1, 2

[,1] [,2]

[1,] 13981.658 6765.837

[2,] 21993.875 10643.013

[3,] 329030.970 159220.734

[4,] 170197.884 82360.126

[5,] 7639.978 3697.047

, , 1, 2, 2

[,1] [,2]

[1,] 15747.458 7620.322

[2,] 30117.013 14573.865

[3,] 156752.442 75853.768

[4,] 115269.052 55779.622

[5,] 4928.239 2384.814

, , 2, 2, 2

[,1] [,2]

[1,] 11023.221 5334.225

[2,] 17340.099 8391.013

[3,] 259409.924 125530.549

[4,] 134184.998 64933.200

[5,] 6023.403 2914.773

>

```
model13a=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State, data=n)
```

>

```
model13b=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*Age, data=n)
```

>

```
model13c=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
State*Gender, data=n)
```

>

```
model13d=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
State*Age, data=n)
```

>

```
model13e=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
State*Mode, data=n)
```

>

```
model13f=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
Gender*Age, data=n)
```

```
>model13g=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender
+ Gender*Mode, data=n)
```

```
>model13h=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender
+ Age*Mode, data=n)
```

```
> model13a; model13b; model13c; model13d; model13e; model13f; model13g; model13h
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      Mode + BScgrade * Gender + BScgrade * State, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 94.07656 59 0.002505307

Pearson 96.43592 59 0.001513309

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      Mode + BScgrade * Gender + BScgrade * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 119.1225 59 5.987478e-06

Pearson 132.3240 59 1.502593e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      Mode + BScgrade * Gender + State * Gender, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 120.7577 62 1.165599e-05

Pearson 128.7268 62 1.397857e-06

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + State * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 125.7598 62 3.119768e-06

Pearson 134.5223 62 2.793240e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + State * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 124.0031 62 4.981984e-06

Pearson 133.9196 62 3.310712e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 125.7684 62 3.112556e-06

Pearson 134.4744 62 2.831292e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + Gender * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 125.3519 62 3.479651e-06

Pearson 134.4898 62 2.818970e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 125.6436 62 3.218413e-06

Pearson 134.2637 62 3.004738e-07

```
> m_14a=m_14b=m_14c=m_14d=m_14e=m_14f=m_14g=array(0, dim=c(I, J, K, L,M))
```

```
> for(i in 1:I){
```

```
+ for(j in 1:J){
```

```
+ for(k in 1:K){
```

```
+ for(l in 1:L){
```

```
+ for(m in 1:M){
```

```

+ m_14a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i,
m]+mu_13[i, k]+ mu_12[i, j]+ mu_14[i, l] )

+ m_14b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i,
m]+mu_13[i, k]+ mu_12[i, j]+ mu_23[j, k] )

+ m_14c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_24[j, l])

+ m_14d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_25[j, m])

+ m_14e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_34[k, l])

+ m_14f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_35[k, m])

+ m_14g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_45[l, m])

+ }

+ }

+ }

+ }

+ }

+ }

> m_14a; m_14b; m_14c; m_14d; m_14e; m_14f; m_14g
, , 1, 1, 1

```

```

[1] [2]

```

```

[1,] 461740.0 268453.5

```

```

[2,] 898787.2 712831.2

```

```

[3,] 3432747.3 1196674.9

```

```

[4,] 2238113.2 1436920.9

```

```

[5,] 81100.8 81100.8

```

, , 2, 1, 1

[,1] [,2]

[1,] 323218.0 187917.4

[2,] 517483.6 410418.0

[3,] 5680860.3 1980379.6

[4,] 2605393.3 1672723.3

[5,] 99123.2 99123.2

, , 1, 2, 1

[,1] [,2]

[1,] 659628.6 383505.0

[2,] 608855.9 482885.7

[3,] 2699099.5 940921.1

[4,] 1763936.7 1132488.5

[5,] 81100.8 81100.8

, , 2, 2, 1

[,1] [,2]

[1,] 461740.0 268453.5

[2,] 350553.4 278025.1

[3,] 4466745.0 1557132.2

[4,] 2053403.2 1318332.8

[5,] 99123.2 99123.2

, , 1, 1, 2

[,1] [,2]

[1,] 220832.2 128390.8

[2,] 539272.3 427698.7

[3,] 3504476.4 1221680.0

[4,] 2114346.6 1357459.9

[5,] 66355.2 66355.2

, , 2, 1, 2

[,1] [,2]

[1,] 154582.5 89873.56

[2,] 310490.1 246250.79

[3,] 5799564.8 2021760.65

[4,] 2461316.3 1580222.51

[5,] 81100.8 81100.80

, , 1, 2, 2

[,1] [,2]

[1,] 315474.5 183415.4

[2,] 365313.5 289731.4
[3,] 2755498.6 960582.2
[4,] 1666391.8 1069862.4
[5,] 66355.2 66355.2

, , 2, 2, 2

[,1] [,2]
[1,] 220832.2 128390.8
[2,] 210332.0 166815.1
[3,] 4560080.0 1589669.3
[4,] 1939851.0 1245429.6
[5,] 81100.8 81100.8

, , 1, 1, 1

[,1] [,2]
[1,] 590466.54 397824.45
[2,] 793862.80 729625.35
[3,] 3228778.18 1304359.08
[4,] 2107314.76 1567851.24
[5,] 85408.69 98975.26

, , 2, 1, 1

[,1] [,2]

[1,] 451398.7 232735.1

[2,] 499174.1 351084.7

[3,] 5835491.6 1804021.5

[4,] 2679092.0 1525346.1

[5,] 114003.8 101099.5

, , 1, 2, 1

[,1] [,2]

[1,] 475676.54 320485.15

[2,] 639531.43 587782.10

[3,] 2601085.64 1050784.38

[4,] 1697640.98 1263052.19

[5,] 68804.76 79733.92

, , 2, 2, 1

[,1] [,2]

[1,] 363644.27 187490.06

[2,] 402131.90 282831.84

[3,] 4701039.40 1453309.66

[4,] 2158261.51 1228810.25

[5,] 91840.79 81445.21

, , 1, 1, 2

[,1] [,2]

[1,] 282397.04 190263.87

[2,] 476317.68 437775.21

[3,] 3296245.19 1331614.34

[4,] 1990781.23 1481149.79

[5,] 69879.83 80979.76

, , 2, 1, 2

[,1] [,2]

[1,] 215886.34 111308.1

[2,] 299504.48 210650.8

[3,] 5957427.26 1841717.5

[4,] 2530939.43 1440995.1

[5,] 93275.81 82717.8

, , 1, 2, 2

[,1] [,2]

[1,] 227497.5 153275.51

[2,] 383718.9 352669.26

[3,] 2655436.7 1072741.07

[4,] 1603762.2 1193205.98

[5,] 56294.8 65236.84

, , 2, 2, 2

[,1] [,2]

[1,] 173916.82 89669.16

[2,] 241279.14 169699.10

[3,] 4799270.07 1483677.33

[4,] 2038910.64 1160857.61

[5,] 75142.47 66636.99

, , 1, 1, 1

[,1] [,2]

[1,] 622552.90 359274.03

[2,] 837001.85 658922.39

[3,] 3404232.21 1177962.64

[4,] 2221827.69 1415921.60

[5,] 90049.85 89384.25

, , 2, 1, 1

[,1] [,2]

[1,] 435787.0 251491.8

[2,] 481910.2 379379.6

[3,] 5633670.6 1949412.7

[4,] 2586435.3 1648278.0

[5,] 110060.9 109247.4

, , 1, 2, 1

[,1] [,2]

[1,] 498815.68 292684.45

[2,] 670641.23 536794.53

[3,] 2727614.62 959633.36

[4,] 1780222.19 1153487.86

[5,] 72151.75 72817.35

, , 2, 2, 1

[,1] [,2]

[1,] 349170.98 204879.11

[2,] 386126.77 309063.52

[3,] 4513934.79 1588099.13

[4,] 2072361.21 1342778.17

[5,] 88185.47 88998.98

, , 1, 1, 2

[,1] [,2]

[1,] 297742.69 171826.71
[2,] 502201.11 395353.43
[3,] 3475365.42 1202576.79
[4,] 2098961.64 1337621.79
[5,] 73677.15 73132.57

, , 2, 1, 2

[,1] [,2]

[1,] 208419.88 120278.70
[2,] 289146.09 227627.73
[3,] 5751389.04 1990146.69
[4,] 2443406.63 1557128.95
[5,] 90049.85 89384.25

, , 1, 2, 2

[,1] [,2]

[1,] 238564.02 139979.52
[2,] 402384.74 322076.72
[3,] 2784609.56 979685.40
[4,] 1681776.72 1089700.51
[5,] 59033.25 59577.83

, , 2, 2, 2

[,1] [,2]

[1,] 166994.81 97985.66

[2,] 231676.06 185438.11

[3,] 4608255.82 1621283.29

[4,] 1957760.59 1268523.16

[5,] 72151.75 72817.35

, , 1, 1, 1

[,1] [,2]

[1,] 605729.89 379486.18

[2,] 814383.86 695992.25

[3,] 3312240.91 1244232.84

[4,] 2161788.07 1495578.96

[5,] 87616.47 94412.86

, , 2, 1, 1

[,1] [,2]

[1,] 424010.9 265640.3

[2,] 468887.7 400722.8

[3,] 5481434.0 2059083.4

[4,] 2516543.0 1741007.3

[5,] 107086.8 115393.5

, , 1, 2, 1

[,1] [,2]

[1,] 487972.61 305711.94

[2,] 656063.08 560687.46

[3,] 2668322.75 1002347.01

[4,] 1741524.38 1204830.05

[5,] 70583.34 76058.47

, , 2, 2, 1

[,1] [,2]

[1,] 341580.83 213998.36

[2,] 377733.29 322820.06

[3,] 4415812.54 1658786.04

[4,] 2027312.99 1402545.74

[5,] 86268.52 92960.36

, , 1, 1, 2

[,1] [,2]

[1,] 305030.13 163071.16

[2,] 514492.78 375207.93

[3,] 3560427.09 1141298.67

[4,] 2150335.00 1269462.36

[5,] 75480.45 69406.05

, , 2, 1, 2

[,1] [,2]

[1,] 213521.09 114149.81

[2,] 296223.12 216028.81

[3,] 5892157.78 1888737.41

[4,] 2503210.49 1477784.39

[5,] 92253.88 84829.62

, , 1, 2, 2

[,1] [,2]

[1,] 245730.56 131369.22

[2,] 414472.51 302265.41

[3,] 2868260.15 919423.82

[4,] 1732297.85 1022671.77

[5,] 60806.63 55913.13

, , 2, 2, 2

[,1] [,2]

[1,] 172011.39 91958.45

[2,] 238635.69 174031.60
[3,] 4746689.34 1521556.29
[4,] 2016572.37 1190494.84
[5,] 74319.21 68338.27

, , 1, 1, 1

[,1] [,2]

[1,] 619233.62 360019.55
[2,] 832539.19 660289.70
[3,] 3386081.79 1180407.00
[4,] 2209981.53 1418859.74
[5,] 89569.73 89569.73

, , 2, 1, 1

[,1] [,2]

[1,] 435727.4 253329.9
[2,] 481844.2 382152.3
[3,] 5632899.4 1963660.1
[4,] 2586081.3 1660324.5
[5,] 110045.9 110045.9

, , 1, 2, 1

[,1] [,2]

[1,] 502134.96 291938.93

[2,] 675103.89 535427.22

[3,] 2745765.04 957189.01

[4,] 1792068.35 1150549.72

[5,] 72631.87 72631.87

, , 2, 2, 1

[,1] [,2]

[1,] 349230.63 203041.06

[2,] 386192.74 306290.79

[3,] 4514705.96 1573851.68

[4,] 2072715.26 1330731.59

[5,] 88200.53 88200.53

, , 1, 1, 2

[,1] [,2]

[1,] 296155.21 172183.26

[2,] 499523.51 396173.82

[3,] 3456835.74 1205072.22

[4,] 2087770.57 1340397.45

[5,] 73284.33 73284.33

, , 2, 1, 2

[,1] [,2]

[1,] 208391.35 121157.76

[2,] 289106.51 229291.37

[3,] 5750601.76 2004691.85

[4,] 2443072.17 1568509.37

[5,] 90037.53 90037.53

, , 1, 2, 2

[,1] [,2]

[1,] 240151.50 139622.97

[2,] 405062.33 321256.33

[3,] 2803139.24 977189.97

[4,] 1692967.80 1086924.85

[5,] 59426.07 59426.07

, , 2, 2, 2

[,1] [,2]

[1,] 167023.34 97106.60

[2,] 231715.64 183774.47

[3,] 4609043.10 1606738.13

[4,] 1958095.06 1257142.74

[5,] 72164.07 72164.07

, , 1, 1, 1

[,1] [,2]

[1,] 619003.94 359886.01

[2,] 832230.38 660044.78

[3,] 3384825.82 1179969.16

[4,] 2209161.80 1418333.45

[5,] 89536.51 89536.51

, , 2, 1, 1

[,1] [,2]

[1,] 435852.5 253402.6

[2,] 481982.6 382262.0

[3,] 5634516.9 1964224.0

[4,] 2586823.9 1660801.3

[5,] 110077.5 110077.5

, , 1, 2, 1

[,1] [,2]

[1,] 498666.11 289922.16

[2,] 670440.14 531728.39

[3,] 2726796.75 950576.56

[4,] 1779688.39 1142601.49

[5,] 72130.11 72130.11

, , 2, 2, 1

[,1] [,2]

[1,] 351120.34 204139.73

[2,] 388282.45 307948.15

[3,] 4539135.33 1582367.89

[4,] 2083930.86 1337932.26

[5,] 88677.79 88677.79

, , 1, 1, 2

[,1] [,2]

[1,] 298095.18 173311.15

[2,] 502795.65 398768.96

[3,] 3479479.79 1212966.06

[4,] 2101446.54 1349177.74

[5,] 73764.38 73764.38

, , 2, 1, 2

[,1] [,2]

[1,] 207334.53 120543.33
[2,] 287640.35 228128.55
[3,] 5721438.36 1994525.32
[4,] 2430682.45 1560554.88
[5,] 89580.91 89580.91

, , 1, 2, 2

[,1] [,2]

[1,] 240143.81 139618.49
[2,] 405049.36 321246.04
[3,] 2803049.45 977158.67
[4,] 1692913.57 1086890.04
[5,] 59424.17 59424.17

, , 2, 2, 2

[,1] [,2]

[1,] 167027.53 97109.03
[2,] 231721.45 183779.08
[3,] 4609158.73 1606778.44
[4,] 1958144.19 1257174.28
[5,] 72165.88 72165.88

, , 1, 1, 1

[,1] [,2]

[1,] 615588.09 357900.05

[2,] 827637.89 656402.47

[3,] 3366147.37 1173457.75

[4,] 2196971.00 1410506.67

[5,] 89042.42 89042.42

, , 2, 1, 1

[,1] [,2]

[1,] 430911.7 250530.0

[2,] 476518.8 377928.7

[3,] 5570643.9 1941957.5

[4,] 2557499.6 1641974.4

[5,] 108829.6 108829.6

, , 1, 2, 1

[,1] [,2]

[1,] 505780.49 294058.42

[2,] 680005.19 539314.46

[3,] 2765699.46 964138.26

[4,] 1805078.88 1158902.78

[5,] 73159.18 73159.18

, , 2, 2, 1

[,1] [,2]

[1,] 354046.34 205840.90

[2,] 391518.14 310514.38

[3,] 4576961.46 1595554.29

[4,] 2101296.95 1349081.70

[5,] 89416.77 89416.77

, , 1, 1, 2

[,1] [,2]

[1,] 299879.77 174348.70

[2,] 505805.70 401156.24

[3,] 3500310.15 1220227.64

[4,] 2114027.13 1357254.77

[5,] 74205.98 74205.98

, , 2, 1, 2

[,1] [,2]

[1,] 209915.84 122044.09

[2,] 291221.46 230968.75

[3,] 5792670.14 2019357.12

[4,] 2460944.40 1579983.76

[5,] 90696.19 90696.19

, , 1, 2, 2

[,1] [,2]

[1,] 236426.95 137457.53

[2,] 398780.15 316273.91

[3,] 2759664.82 962034.55

[4,] 1666711.24 1070067.53

[5,] 58504.42 58504.42

, , 2, 2, 2

[,1] [,2]

[1,] 165498.86 96220.27

[2,] 229600.69 182097.10

[3,] 4566974.72 1592072.86

[4,] 1940222.83 1245668.35

[5,] 71505.41 71505.41

>

```
model14a=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ BScgrade*Age, data=n)
```

>

```
model14b=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ State*Gender, data=n)
```

```

>
model14c=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ State*Age, data=n)

>
model14d=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ State*Mode, data=n)

>
model14e=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ Gender*Age, data=n)

>
model14f=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ Gender*Mode, data=n)

>
model14g=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ Age*Mode, data=n)

> model14a; model14b; model14c; model14d; model14e; model14f; model14g

```

Call:

```

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      Mode + BScgrade * Gender + BScgrade * State + BScgrade *
      Age, data = n)

```

Statistics:

X² df P(> X²)

Likelihood Ratio 87.41988 55 0.003528881

Pearson 93.03104 55 0.001031502

Call:

```

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      Mode + BScgrade * Gender + BScgrade * State + State * Gender,

```

data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 91.75409 58 0.003132519

Pearson 94.07371 58 0.001915817

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *

Mode + BScgrade * Gender + BScgrade * State + State * Age,

data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 94.05723 58 0.001922623

Pearson 96.53581 58 0.001118463

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *

Mode + BScgrade * Gender + BScgrade * State + State * Mode,

data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 93.04289 58 0.002388163

Pearson 96.43197 58 0.001144516

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
  Mode + BScgrade * Gender + BScgrade * State + Gender * Age,  
  data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 94.06586 58 0.001919054

Pearson 96.45887 58 0.001137714

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
  Mode + BScgrade * Gender + BScgrade * State + Gender * Mode,  
  data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 93.64933 58 0.002098519

Pearson 95.98648 58 0.001262949

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
  Mode + BScgrade * Gender + BScgrade * State + Age * Mode,  
  data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 93.94103 58 0.001971264

Pearson 96.38938 58 0.001155369

```
> mu_123=array(0, dim=c(I, J, K)); mu_124=array(0, dim=c(I, J, L)); mu_125=array(0, dim=c(I,
J, M)); mu_134= array(0, dim=c(I, K, L)); mu_135=array(0, dim=c(I, K, M)); mu_145=array(0,
dim=c(I, L, M)); mu_234= array(0, dim=c(J, K, L)); mu_235=array(0, dim=c(J, K, M));
mu_245=array(0, dim=c(J, L, M)); mu_345=array(0, dim=c(K, L, M)); mu_1234=array(0,
dim=c(I, J, K, L)); mu_1235=array(0, dim=c(I, J, K, M)); mu_1245=array(0, dim=c(I, J, L, M));
mu_1345=array(0, dim=c(I, K, L, M)); mu_2345=array(0, dim=c(J, K, L, M))

> for (i in 1:I){
+ for (j in 1:J){
+ for (k in 1:K){
+ mu_123[i, j, k]=log(sum(n[i, j, k, , ])/L*M)-mu-mu_1[i]-mu_2[j]-mu_3[k]-mu_12[i, j]-
mu_13[i, k]-mu_23[j, k]
+ }
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ for (l in 1:L){
+ mu_124[i, j, l]=log(sum(n[i, j, , l, ])/K*M)-mu-mu_1[i]-mu_2[j]-mu_4[l]-mu_12[i, j]-mu_14[i,
l]-mu_24[j, l]
+ }
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ for (m in 1:M){
```

```

+ mu_125[i, j, m]=log(sum(n[i, j, , m])/K*L)-mu-mu_1[i]-mu_2[j]-mu_5[m]-mu_12[i, j]-
mu_15[i, m]-mu_25[j, m]

+ }

+ }

+ }

> for (i in 1:I){
+ for (k in 1:K){
+ for (l in 1:L){
+ mu_134[i, k, l]=log(sum(n[i, , k, l, ])/J*M)-mu-mu_1[i]-mu_3[k]-mu_4[l]-mu_13[i, k]-
mu_14[i, l]-mu_34[k, l]

+ }

+ }

+ }

> for (i in 1:I){
+ for (k in 1:K){
+ for (m in 1:M){
+ mu_135[i, k, m]=log(sum(n[i, , k, , m])/J*L)-mu-mu_1[i]-mu_3[k]-mu_5[m]-mu_13[i, k]-
mu_15[i, m]-mu_35[k, m]

+ }

+ }

+ }

> for (i in 1:I){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_145[i, l, m]=log(sum(n[i, , , l, m])/J*L)-mu-mu_1[i]-mu_4[l]-mu_5[m]-mu_14[i, l]-
mu_15[i, m]-mu_45[l, m]

+ }

```

```

+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ for (l in 1:L){
+ mu_234[j, k, l]=log(sum(n[, j, k, l, ])/I*M)-mu-mu_2[j]-mu_3[k]-mu_4[l]-mu_23[j, k]-
mu_24[j, l]-mu_34[k, l]
+ }
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ for (m in 1:M){
+ mu_235[j, k, m]=log(sum(n[, j, k, , m])/I*L)-mu-mu_2[j]-mu_3[k]-mu_5[m]-mu_23[j, k]-
mu_25[j, m]-mu_35[k, m]
+ }
+ }
+ }
> for (j in 1:J){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_245[j, l, m]=log(sum(n[, j, , l, m])/I)-mu-mu_2[j]-mu_4[l]-mu_5[m]-mu_24[j, l]-mu_25[j,
m]-mu_45[l, m]
+ }
+ }
+ }

```



```

> for (j in 1:J){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_345[k, l, m]=log(sum(n[, , k, l, m])/I*J)-mu-mu_3[k]-mu_4[l]-mu_5[m]-mu_34[k, l]-
mu_35[k, m]-mu_45[l, m]
+ }
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ for (k in 1:K){
+ for (l in 1:L){
+ mu_1234[i, j, k, l]=log(sum(n[i, j, k, l, ])/M)-mu-mu_1[i]-mu_2[j]-mu_3[k]-mu_4[l]-mu_12[i,
j]-mu_13[i, k]-mu_14[i, l]-mu_123[i, j, K]-mu_134[i, k,l]-mu_234[j, k, l]
+ }
+ }
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_1245[i, j, l, m]=log(sum(n[i, j, , l, m])/K)-mu-mu_1[i]-mu_2[j]-mu_4[l]-mu_5[k]-
mu_12[i, j]-mu_14[i, l]-mu_15[i, m]-mu_24[j, l]-mu_25[j, m]-mu_45[l, m]-mu_124[i, j, l]-
mu_125[i, j, m]-mu_145[i, l, m]-mu_245[j, l, m]
+ }
+ }

```

```

+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_2345[j, k, l, m]=log(sum(n[, j, k, l , m])/I)-mu-mu_2[j]-mu_3[k]-mu_4[l]-mu_5[m]-
mu_23[j, k]-mu_24[j, l]-mu_25[j, m]-mu_34[k, l]-mu_35[k, m]-mu_45[l, m]-mu_234[j, k, l]-
mu_245[j, l, m]-mu_345[k, l, m]
+ }
+ }
+ }
+ }
> mu_123; mu_124; mu_125; mu_134; mu_135; mu_145; mu_234; mu_235; mu_245; mu_345;
mu_1234; mu_1235; mu_1245; mu_1345; mu_2345
, , 1

```

```

      [,1] [,2]
[1,] -6.892666 -7.008589
[2,] -6.903157 -7.001099
[3,] -6.897415 -6.982663
[4,] -6.948510 -6.931515
[5,] -6.998753 -6.923031

```

```

, , 2

```

[,1] [,2]

[1,] -6.952608 -6.877944

[2,] -6.931678 -6.898204

[3,] -6.959274 -6.877637

[4,] -6.914497 -6.940397

[5,] -6.882071 -6.944267

, , 1

[,1] [,2]

[1,] -7.035193 -6.773132

[2,] -6.866154 -7.018975

[3,] -6.919861 -6.967822

[4,] -6.960145 -6.886890

[5,] -6.933885 -6.926466

, , 2

[,1] [,2]

[1,] -6.863203 -7.060949

[2,] -7.037667 -6.815048

[3,] -6.946593 -6.887410

[4,] -6.896112 -6.990231

[5,] -6.928468 -6.937651

, , 1

[,1] [,2]

[1,] -6.721476 -7.411313

[2,] -6.783358 -7.161783

[3,] -6.912162 -6.965107

[4,] -7.054260 -6.787307

[5,] -6.819479 -7.076509

, , 2

[,1] [,2]

[1,] -7.511359 -6.325520

[2,] -7.204466 -6.630936

[3,] -6.952542 -6.890184

[4,] -6.822302 -7.130982

[5,] -7.075849 -6.768806

, , 1

[,1] [,2]

[1,] -6.734383 -7.299208

[2,] -6.887365 -7.009555

[3,] -6.899235 -6.951881

[4,] -7.015195 -6.864870

[5,] -7.046322 -6.846737

, , 2

[,1] [,2]

[1,] -7.097619 -6.734483

[2,] -6.999158 -6.826290

[3,] -6.973655 -6.906042

[4,] -6.834964 -7.023471

[5,] -6.829740 -7.023949

, , 1

[,1] [,2]

[1,] -7.048127 -6.784331

[2,] -7.036901 -6.769413

[3,] -6.845910 -6.987233

[4,] -7.025200 -6.857595

[5,] -6.918118 -6.942334

, , 2

[,1] [,2]

[1,] -6.722894 -7.340644

[2,] -6.776598 -7.282836

[3,] -7.022787 -6.879163

[4,] -6.841682 -7.016498

[5,] -6.947491 -6.918614

, , 1

[,1] [,2]

[1,] -7.224915 -6.774611

[2,] -6.942179 -6.914163

[3,] -6.914883 -6.952280

[4,] -6.901077 -6.970373

[5,] -7.017945 -6.855323

, , 2

[,1] [,2]

[1,] -6.505719 -7.400442

[2,] -6.909288 -6.968260

[3,] -6.948689 -6.909822

[4,] -6.964375 -6.890621

[5,] -6.835677 -7.037253

, , 1

[,1] [,2]

[1,] -6.918501 -6.940963

[2,] -6.954502 -6.910163

, , 2

[,1] [,2]

[1,] -6.947772 -6.919706

[2,] -6.904126 -6.958371

, , 1

[,1] [,2]

[1,] -6.897865 -6.956204

[2,] -6.996768 -6.874128

, , 2

[,1] [,2]

[1,] -6.967509 -6.906173

[2,] -6.857546 -7.006691

, , 1

[,1] [,2]

[1,] -7.631937 -7.615678

[2,] -7.610274 -7.642284

, , 2

[,1] [,2]

[1,] -7.617695 -7.633586

[2,] -7.640922 -7.604408

, , 1

[,1] [,2]

[1,] 0.000000 0.000000

[2,] -6.917606 -6.948644

, , 2

[,1] [,2]

[1,] 0.000000 0.000000

[2,] -6.946589 -6.912586

, , 1, 1

[,1] [,2]

[1,] 5.663463 5.966454

[2,] 5.830493 5.719981

[3,] 5.845686 5.652837

[4,] 5.625933 6.010821

[5,] 5.732456 5.830653

, , 2, 1

[,1] [,2]

[1,] 5.818969 5.745004

[2,] 5.914110 5.596130

[3,] 5.837160 5.725387

[4,] 5.841736 5.705880

[5,] 5.760128 5.791524

, , 1, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 1

[,1] [,2]

[1,] 12.01262 11.59448

[2,] 11.85355 11.87372

[3,] 11.86263 11.90594

[4,] 11.88774 11.83717

[5,] 11.97449 11.72968

, , 2, 1

[,1] [,2]

[1,] 11.78327 12.05728

[2,] 11.90969 11.86532

[3,] 11.88431 11.83006

[4,] 11.85369 11.91588

[5,] 11.77591 11.98484

, , 1, 2

[,1] [,2]

[1,] 11.85987 11.65662

[2,] 11.84605 11.77169

[3,] 11.79268 11.74723

[4,] 11.77524 11.82112

[5,] 11.67122 11.87677

, , 2, 2

[,1] [,2]

[1,] 11.75798 11.86857

[2,] 11.63340 11.80774

[3,] 11.77161 11.81927

[4,] 11.79237 11.73724

[5,] 11.91025 11.65793

, , 1, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 1

[,1] [,2]

```
[1,] -2.044305 4.811871
```

```
[2,] -2.202487 4.935612
```

```
, , 2, 1
```

```
    [,1] [,2]
```

```
[1,] -2.047271 4.846003
```

```
[2,] -2.080589 4.871024
```

```
, , 1, 2
```

```
    [,1] [,2]
```

```
[1,] -2.116057 4.893230
```

```
[2,] -1.946688 4.731318
```

```
, , 2, 2
```

```
    [,1] [,2]
```

```
[1,] -2.115199 4.857783
```

```
[2,] -2.077733 4.830675
```

```
> m_15a=m_15b=m_15c=m_15d=m_15e=m_15f=m_15g=m_15h=m_15i=m_15j=  
m_16a=m_16b=m_16c=m_16d=m_16e =array(0, dim=c(I, J, K, L,M))
```

```
> for(i in 1:I){
```

```
+ for(j in 1:J){
```

```

+ for(k in 1:K){
+ for(l in 1:L){
+ for(m in 1:M){
+ m_15a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_123[i, j, k])
+ m_15b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_124[i, j, l])
+ m_15c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_125[i, j,
m])
+ m_15d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_134[i, k,
l])
+ m_15e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_135[i, k,
m])
+ m_15f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_145[i, l,
m])
+ m_15g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_234[j, k,
l])
+ m_15h[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_235[j, k,
m])
+ m_15i[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_245[j, l,
m])
+ m_15j[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_345[k, l,
m])
+ m_16a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1234[i, j,
k, l])
+ m_16b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1235[i, j,
k, m])
+ m_16c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1245[i, j, l,
m])
+ m_16d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1345[i, k,
l, m])

```

+ m_16e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_2345[j, k,
l, m])

+ }

+ }

+ }

+ }

+ }

> m_15a; m_15b; m_15c; m_15d; m_15e; m_15f; m_15g; m_15h; m_15i; m_15j; m_16a;
m_16b; m_16c; m_16d; m_16e

, , 1, 1, 1

[,1] [,2]

[1,] 0.005886959 0.0025369325

[2,] 0.008909613 0.0039091829

[3,] 0.058332150 0.0259207772

[4,] 0.034549498 0.0170053452

[5,] 0.001557181 0.0008128068

, , 2, 1, 1

[,1] [,2]

[1,] 0.007124373 0.003714800

[2,] 0.011126551 0.005567509

[3,] 0.070458121 0.036995458

[4,] 0.045930543 0.021657874

[5,] 0.002248544 0.001022476

, , 1, 2, 1

[,1] [,2]

[1,] 0.004742501 0.0020437386

[2,] 0.007177535 0.0031492158

[3,] 0.046992054 0.0208816331

[4,] 0.027832883 0.0136994110

[5,] 0.001254456 0.0006547926

, , 2, 2, 1

[,1] [,2]

[1,] 0.005739355 0.0029926225

[2,] 0.008963487 0.0044851543

[3,] 0.056760668 0.0298033343

[4,] 0.037001389 0.0174474624

[5,] 0.001811414 0.0008237008

, , 1, 1, 2

[,1] [,2]

[1,] 0.005389100 0.0023223848

[2,] 0.008156130 0.0035785843

[3,] 0.053399016 0.0237286640

[4,] 0.031627657 0.0155672077

[5,] 0.001425491 0.0007440679

, , 2, 1, 2

[,1] [,2]

[1,] 0.006521867 0.0034006408

[2,] 0.010185582 0.0050966664

[3,] 0.064499496 0.0338667624

[4,] 0.042046209 0.0198262737

[5,] 0.002058385 0.0009360053

, , 1, 2, 2

[,1] [,2]

[1,] 0.004341429 0.0018709002

[2,] 0.006570532 0.0028828875

[3,] 0.043017948 0.0191156789

[4,] 0.025479063 0.0125408554

[5,] 0.001148367 0.0005994169

, , 2, 2, 2

[,1] [,2]

[1,] 0.005253979 0.0027395372

[2,] 0.008205448 0.0041058460
[3,] 0.051960434 0.0272828743
[4,] 0.033872191 0.0159719352
[5,] 0.001658224 0.0007540406

, , 1, 1, 1

[,1] [,2]

[1,] 0.005104955 0.0032104543
[2,] 0.009245478 0.0038399240
[3,] 0.057037424 0.0263083372
[4,] 0.034149822 0.0177813878
[5,] 0.001661539 0.0008100195

, , 2, 1, 1

[,1] [,2]

[1,] 0.006559642 0.004125292
[2,] 0.011880030 0.004934132
[3,] 0.073290566 0.033805049
[4,] 0.043881010 0.022848297
[5,] 0.002135005 0.001040839

, , 1, 2, 1

[,1] [,2]

[1,] 0.004884306 0.0019394818

[2,] 0.006274200 0.0037931794

[3,] 0.044736991 0.0229684969

[4,] 0.029330156 0.0129181890

[5,] 0.001345797 0.0006452894

, , 2, 2, 1

[,1] [,2]

[1,] 0.006276117 0.0024921483

[2,] 0.008062070 0.0048740677

[3,] 0.057485054 0.0295135022

[4,] 0.037687953 0.0165993013

[5,] 0.001729290 0.0008291683

, , 1, 1, 2

[,1] [,2]

[1,] 0.004673231 0.0029389471

[2,] 0.008463590 0.0035151827

[3,] 0.052213784 0.0240834481

[4,] 0.031261781 0.0162776206

[5,] 0.001521023 0.0007415163

, , 2, 1, 2

[,1] [,2]

[1,] 0.006004895 0.0037764170

[2,] 0.010875339 0.0045168542

[3,] 0.067092403 0.0309461652

[4,] 0.040170005 0.0209160222

[5,] 0.001954448 0.0009528157

, , 1, 2, 2

[,1] [,2]

[1,] 0.004471241 0.0017754603

[2,] 0.005743593 0.0034723912

[3,] 0.040953596 0.0210260572

[4,] 0.026849713 0.0118257012

[5,] 0.001231984 0.0005907174

, , 2, 2, 2

[,1] [,2]

[1,] 0.005745347 0.0022813880

[2,] 0.007380263 0.0044618691

[3,] 0.052623559 0.0270175532

[4,] 0.034500693 0.0151955028

[5,] 0.001583044 0.0007590458

, , 1, 1, 1

[,1] [,2]

[1,] 0.006986143 0.0016959303

[2,] 0.010043546 0.0033289100

[3,] 0.057478266 0.0263798596

[4,] 0.031082418 0.0196432695

[5,] 0.001862930 0.0006971603

, , 2, 1, 1

[,1] [,2]

[1,] 0.008976885 0.0021791954

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[3,] 6991010.5 3232677.1
[4,] 4282399.6 2169593.4
[5,] 182906.5 108707.8

, , 2, 1, 2

[,1] [,2]

[1,] 964997.2 381084.3
[2,] 1455621.7 653907.9
[3,] 8983139.1 4153847.0
[4,] 5502694.0 2787831.5
[5,] 235026.8 139684.7

, , 1, 2, 2

[,1] [,2]

[1,] 546390.4 295322.69
[2,] 737780.0 425014.75
[3,] 5514466.8 2798765.55
[4,] 3509492.6 1607185.01
[5,] 187135.8 70361.97

, , 2, 2, 2

[,1] [,2]

[1,] 702087.5 379476.59

[2,] 948014.6 546125.14

[3,] 7085845.9 3596289.90

[4,] 4509542.7 2065161.63

[5,] 240461.3 90412.02

, , 1, 1, 1

[,1] [,2]

[1,] 5.798793 2.8060824

[2,] 8.868742 4.2916555

[3,] 57.732098 27.9370266

[4,] 35.986625 17.4142175

[5,] 1.705527 0.8253184

, , 2, 1, 1

[,1] [,2]

[1,] 7.451192 3.605692

[2,] 11.395941 5.514587

[3,] 74.183191 35.897843

[4,] 46.241221 22.376499

[5,] 2.191527 1.060498

, , 1, 2, 1

[,1] [,2]

[1,] 4.671475 2.2605643

[2,] 7.144609 3.4573337

[3,] 46.508655 22.5059124

[4,] 28.990624 14.0287962

[5,] 1.373963 0.6648719

, , 2, 2, 1

[,1] [,2]

[1,] 6.002639 2.9047251

[2,] 9.180506 4.4425208

[3,] 59.761564 28.9191015

[4,] 37.251669 18.0263823

[5,] 1.765482 0.8543309

, , 1, 1, 2

[,1] [,2]

[1,] 5.308390 2.5687728

[2,] 8.118715 3.9287113

[3,] 52.849710 25.5743997

[4,] 32.943246 15.9415018

[5,] 1.561291 0.7555214

, , 2, 1, 2

[,1] [,2]

[1,] 6.821047 3.3007594

[2,] 10.432189 5.0482202

[3,] 67.909538 32.8619721

[4,] 42.330613 20.4841244

[5,] 2.006190 0.9708116

, , 1, 2, 2

[,1] [,2]

[1,] 4.276410 2.0693890

[2,] 6.540391 3.1649479

[3,] 42.575430 20.6025934

[4,] 26.538895 12.8423846

[5,] 1.257768 0.6086438

, , 2, 2, 2

[,1] [,2]

[1,] 5.494997 2.6590733

[2,] 8.404113 4.0668180

[3,] 54.707544 26.4734208

[4,] 34.101305 16.5018960

[5,] 1.616176 0.7820804

, , 1, 1, 1

[,1] [,2]

[1,] 0.7507708 0.31015058

[2,] 1.1482377 0.47434795

[3,] 7.4745861 3.08782268

[4,] 4.6591955 1.92475801

[5,] 0.2208150 0.09122076

, , 2, 1, 1

[,1] [,2]

[1,] 916.2097 501.7618

[2,] 1401.2619 767.4004

[3,] 9121.6761 4995.4815

[4,] 5685.8897 3113.8748

[5,] 269.4734 147.5770

, , 1, 2, 1

[,1] [,2]

[1,] 0.6030253 0.28224649

[2,] 0.9222740 0.43167111

[3,] 6.0036487 2.81001288

[4,] 3.7423039 1.75158853

[5,] 0.1773604 0.08301367

, , 2, 2, 1

[,1] [,2]

[1,] 763.7207 378.9341

[2,] 1168.0434 579.5462

[3,] 7603.5134 3772.6231

[4,] 4739.5608 2351.6203

[5,] 224.6237 111.4512

, , 1, 1, 2

[,1] [,2]

[1,] 0.6396922 0.3666823

[2,] 0.9783528 0.5608082

[3,] 6.3687005 3.6506459

[4,] 3.9698546 2.2755873

[5,] 0.1881448 0.1078477

, , 2, 1, 2

[,1] [,2]

[1,] 909.8165 374.4547

[2,] 1391.4841 572.6955

[3,] 9058.0262 3728.0274

[4,] 5646.2143 2323.8221

[5,] 267.5931 110.1337

, , 1, 2, 2

[,1] [,2]

[1,] 0.5157747 0.25911597

[2,] 0.7888318 0.39629501

[3,] 5.1349918 2.57972812

[4,] 3.2008368 1.60804323

[5,] 0.1516984 0.07621058

, , 2, 2, 2

[,1] [,2]

[1,] 707.4176 333.16987

[2,] 1081.9328 509.55393

[3,] 7042.9666 3317.00007

[4,] 4390.1505 2067.61304

[5,] 208.0640 97.99114

```

> model15a=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender, data=n)
> model15b=loglm(~BScgrade+State+Gender+Age+Mode + BScgrade*State*Age, data=n)
> model15c=loglm(~BScgrade+State+Gender+Age+Mode + BScgrade*State*Mode, data=n)
> model15d=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Gender*Age, data=n)
> model15e=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Gender*Mode, data=n)
> model15f=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Age*Mode, data=n)
> model15g=loglm(~BScgrade+State+Gender+Age+Mode+ State*Gender*Age, data=n)
> model15h=loglm(~BScgrade+State+Gender+Age+Mode + State*Gender*Mode, data=n)
> model15i=loglm(~BScgrade+State+Gender+Age+Mode + State*Age*Mode, data=n)
> model15j=loglm(~BScgrade+State+Gender+Age+Mode + Gender*Age*Mode, data=n)
> model16a=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Age,
data=n)
> model16b=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Mode,
data=n)
> model16c=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Age*Mode,
data=n)
> model16d=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Gender*Age*Mode,
data=n)
> model16e=loglm(~BScgrade+State+Gender+Age+Mode+ State*Gender*Age*Mode, data=n)
> model15a; model15b; model15c; model15d; model15e; model15f; model15g; model15h;
model15i; model16a; model16b; model16c; model16d; model16e

```

Call:

```

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      State * Gender, data = n)

```

Statistics:

X² df P(> X²)

Likelihood Ratio 104.3455 58 0.0001829902

Pearson 105.8493 58 0.0001269782

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 131.8656 58 1.118972e-07

Pearson 127.2042 58 4.308532e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 100.71154 58 0.0004329178

Pearson 99.04581 58 0.0006355605

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 124.9574 58 8.149994e-07

Pearson 123.9418 58 1.084171e-06

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Gender * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 113.5894 58 1.790516e-05

Pearson 125.3055 58 7.387566e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 145.6230 58 1.741845e-09

Pearson 155.5872 58 7.347278e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *  
Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 167.5483 67 1.408711e-10

Pearson 167.5230 67 1.419802e-10

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *
Gender * Mode, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 163.4812 67 4.916169e-10

Pearson 161.3538 67 9.381688e-10

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *
Age * Mode, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 170.88 67 4.991418e-11

Pearson 174.47 67 1.610523e-11

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
State * Gender * Age, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 83.86091 39 4.016366e-05

Pearson 78.14037 39 1.999306e-04

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 41.96151 39 0.3437295

Pearson 40.78206 39 0.3920119

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 83.35372 39 0.0000464587

Pearson 79.69282 39 0.0001304115

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 85.02937 39 2.865165e-05

Pearson 87.22491 39 1.506013e-05

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *
      Gender * Age * Mode, data = n)
```

Statistics:

```
X^2 df P(> X^2)
```

```
Likelihood Ratio 161.2704 60 3.253775e-11
```

```
Pearson 158.5060 60 7.924161e-11
```

```
> m_17a=m_17b=m_17c=m_17d=m_17e=m_17f=m_17g=m_17h=m_17i=array(0, dim=c(I, J,
K, L,M))
> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ for(m in 1:M){
+ m_17a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1234[i, j,
k, l]+ mu_1235[i, j, k, m])
+ m_17b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1234[i, j,
k, l]+ mu_1345[i, k, l, m])
+ m_17c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1234[i, j,
k, l]+ mu_2345[j, k, l, m])
+ m_17d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1235[i, j,
k, m] + mu_1245[i, j, l, m])
+ m_17e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1235[i, j,
k, m] + mu_1345[i, k, l, m])
+ m_17f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1235[i, j, k,
m] + mu_2345[ j, k, l, m])
+ m_17g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1245[i, j, l,
m]+ mu_1345[i, k, l, m])
```

+ m_17h[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1245[i, j, l, m]+ mu_2345[j, k, l, m])

+ m_17i[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1345[i, k, l, m] + mu_2345[j, k, l, m])

+ }

+ }

+ }

+ }

+ }

> m_17a; m_17b; m_17c; m_17d; m_17e; m_17f; m_17g; m_17h; m_17i

, , 1, 1, 1

[,1] [,2]

[1,] 1670.8925 1094.7089

[2,] 3020.0423 1308.5225

[3,] 19960.2757 7964.8211

[4,] 9987.4061 7101.8310

[5,] 526.5418 281.0877

, , 2, 1, 1

[,1] [,2]

[1,] 2508.2552 1127.2285

[2,] 4219.0588 1485.5299

[3,] 25430.3167 11004.5499

[4,] 15924.3995 6727.0433

[5,] 695.5666 347.3253

, , 1, 1, 2

[,1] [,2]

[1,] 1529.5856 1002.1297

[2,] 2764.6382 1197.8611

[3,] 18272.2405 7291.2383

[4,] 9142.7739 6501.2311

[5,] 482.0123 257.3162

, , 2, 1, 2

[,1] [,2]

[1,] 2296.1327 1031.8991

[2,] 3862.2542 1359.8991

[3,] 23279.6816 10073.8980

[4,] 14577.6773 6158.1390

[5,] 636.7427 317.9521

, , 1, 1, 1

[,1] [,2]

[1,] 1670.8925 1094.7089

[2,] 3020.0423 1308.5225

[3,] 19960.2757 7964.8211

[4,] 9987.4061 7101.8310

[5,] 526.5418 281.0877

, , 2, 1, 1

[,1] [,2]

[1,] 2508.2552 1127.2285

[2,] 4219.0588 1485.5299

[3,] 25430.3167 11004.5499

[4,] 15924.3995 6727.0433

[5,] 695.5666 347.3253

, , 1, 1, 2

[,1] [,2]

[1,] 1529.5856 1002.1297

[2,] 2764.6382 1197.8611

[3,] 18272.2405 7291.2383

[4,] 9142.7739 6501.2311

[5,] 482.0123 257.3162

, , 2, 1, 2

[,1] [,2]

[1,] 2296.1327 1031.8991
[2,] 3862.2542 1359.8991
[3,] 23279.6816 10073.8980
[4,] 14577.6773 6158.1390
[5,] 636.7427 317.9521

, , 1, 1, 1

[,1] [,2]

[1,] 216.33079 120.99595
[2,] 391.00548 144.62833
[3,] 2584.26083 880.33546
[4,] 1293.07145 784.95093
[5,] 68.17147 31.06805

, , 2, 1, 1

[,1] [,2]

[1,] 308418.81 156863.17
[2,] 518781.79 206723.77
[3,] 3126949.78 1531374.06
[4,] 1958087.99 936123.67
[5,] 85527.91 48333.19

, , 1, 1, 2

[,1] [,2]

[1,] 184.32405 143.05010

[2,] 333.15514 170.99001

[3,] 2201.91231 1040.79587

[4,] 1101.75796 928.02541

[5,] 58.08531 36.73089

, , 2, 1, 2

[,1] [,2]

[1,] 306266.7 117063.83

[2,] 515161.8 154273.79

[3,] 3105130.3 1142833.66

[4,] 1944424.7 698610.27

[5,] 84931.1 36070.09

, , 1, 1, 1

[,1] [,2]

[1,] 955768.1 304453.7

[2,] 1246796.2 615626.7

[3,] 8190124.7 4138718.0

[4,] 5235062.3 2408368.2

[5,] 270591.6 102508.2

, , 2, 1, 1

[,1] [,2]

[1,] 1228119.7 391209.5

[2,] 1602077.9 791053.1

[3,] 10523947.8 5318069.5

[4,] 6726823.5 3094646.6

[5,] 347698.2 131718.5

, , 1, 2, 1

[,1] [,2]

[1,] 612156.3 389606.6

[2,] 1062405.5 491794.2

[3,] 6742572.1 3090485.4

[4,] 4076136.7 2099038.2

[5,] 178726.1 106582.8

, , 2, 2, 1

[,1] [,2]

[1,] 786593.8 500627.3

[2,] 1365144.0 631933.8

[3,] 8663906.8 3971137.0

[4,] 5237655.3 2697171.2

[5,] 229655.2 136954.2

, , 1, 1, 2

[,1] [,2]

[1,] 750996.4 296573.9

[2,] 1132818.6 508895.3

[3,] 6991010.5 3232677.1

[4,] 4282399.6 2169593.4

[5,] 182906.5 108707.8

, , 2, 1, 2

[,1] [,2]

[1,] 964997.2 381084.3

[2,] 1455621.7 653907.9

[3,] 8983139.1 4153847.0

[4,] 5502694.0 2787831.5

[5,] 235026.8 139684.7

, , 1, 2, 2

[,1] [,2]

[1,] 546390.4 295322.69
[2,] 737780.0 425014.75
[3,] 5514466.8 2798765.55
[4,] 3509492.6 1607185.01
[5,] 187135.8 70361.97

, , 2, 2, 2

[,1] [,2]

[1,] 702087.5 379476.59
[2,] 948014.6 546125.14
[3,] 7085845.9 3596289.90
[4,] 4509542.7 2065161.63
[5,] 240461.3 90412.02

, , 1, 1, 1

[,1] [,2]

[1,] 5.798793 2.8060824
[2,] 8.868742 4.2916555
[3,] 57.732098 27.9370266
[4,] 35.986625 17.4142175
[5,] 1.705527 0.8253184

, , 2, 1, 1

[,1] [,2]

[1,] 7.451192 3.605692

[2,] 11.395941 5.514587

[3,] 74.183191 35.897843

[4,] 46.241221 22.376499

[5,] 2.191527 1.060498

, , 1, 2, 1

[,1] [,2]

[1,] 4.671475 2.2605643

[2,] 7.144609 3.4573337

[3,] 46.508655 22.5059124

[4,] 28.990624 14.0287962

[5,] 1.373963 0.6648719

, , 2, 2, 1

[,1] [,2]

[1,] 6.002639 2.9047251

[2,] 9.180506 4.4425208

[3,] 59.761564 28.9191015

[4,] 37.251669 18.0263823

[5,] 1.765482 0.8543309

, , 1, 1, 2

[,1] [,2]

[1,] 5.308390 2.5687728

[2,] 8.118715 3.9287113

[3,] 52.849710 25.5743997

[4,] 32.943246 15.9415018

[5,] 1.561291 0.7555214

, , 2, 1, 2

[,1] [,2]

[1,] 6.821047 3.3007594

[2,] 10.432189 5.0482202

[3,] 67.909538 32.8619721

[4,] 42.330613 20.4841244

[5,] 2.006190 0.9708116

, , 1, 2, 2

[,1] [,2]

[1,] 4.276410 2.0693890

[2,] 6.540391 3.1649479

[3,] 42.575430 20.6025934

[4,] 26.538895 12.8423846

[5,] 1.257768 0.6086438

, , 2, 2, 2

[,1] [,2]

[1,] 5.494997 2.6590733

[2,] 8.404113 4.0668180

[3,] 54.707544 26.4734208

[4,] 34.101305 16.5018960

[5,] 1.616176 0.7820804

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[,1] [,2]

[1,] 0.7507708 0.31015058

[2,] 1.1482377 0.47434795

[3,] 7.4745861 3.08782268

[4,] 4.6591955 1.92475801

[5,] 0.2208150 0.09122076

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[,1] [,2]

[1,] 916.2097 501.7618

[2,] 1401.2619 767.4004

[3,] 9121.6761 4995.4815

[4,] 5685.8897 3113.8748

[5,] 269.4734 147.5770

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[,1] [,2]

[1,] 0.6030253 0.28224649

[2,] 0.9222740 0.43167111

[3,] 6.0036487 2.81001288

[4,] 3.7423039 1.75158853

[5,] 0.1773604 0.08301367

, , 2, 2, 1

[,1] [,2]

[1,] 763.7207 378.9341

[2,] 1168.0434 579.5462

[3,] 7603.5134 3772.6231

[4,] 4739.5608 2351.6203

[5,] 224.6237 111.4512

, , 1, 1, 2

[,1] [,2]

[1,] 0.6396922 0.3666823

[2,] 0.9783528 0.5608082

[3,] 6.3687005 3.6506459

[4,] 3.9698546 2.2755873

[5,] 0.1881448 0.1078477

, , 2, 1, 2

[,1] [,2]

[1,] 909.8165 374.4547

[2,] 1391.4841 572.6955

[3,] 9058.0262 3728.0274

[4,] 5646.2143 2323.8221

[5,] 267.5931 110.1337

, , 1, 2, 2

[,1] [,2]

[1,] 0.5157747 0.25911597

[2,] 0.7888318 0.39629501

[3,] 5.1349918 2.57972812

[4,] 3.2008368 1.60804323

[5,] 0.1516984 0.07621058

, , 2, 2, 2

[,1] [,2]

[1,] 707.4176 333.16987

[2,] 1081.9328 509.55393

[3,] 7042.9666 3317.00007

[4,] 4390.1505 2067.61304

[5,] 208.0640 97.99114

, , 1, 1, 1

[,1] [,2]

[1,] 955768.1 304453.7

[2,] 1246796.2 615626.7

[3,] 8190124.7 4138718.0

[4,] 5235062.3 2408368.2

[5,] 270591.6 102508.2

, , 2, 1, 1

[,1] [,2]

[1,] 1228119.7 391209.5

[2,] 1602077.9 791053.1

[3,] 10523947.8 5318069.5

[4,] 6726823.5 3094646.6

[5,] 347698.2 131718.5

, , 1, 2, 1

[,1] [,2]

[1,] 612156.3 389606.6

[2,] 1062405.5 491794.2

[3,] 6742572.1 3090485.4

[4,] 4076136.7 2099038.2

[5,] 178726.1 106582.8

, , 2, 2, 1

[,1] [,2]

[1,] 786593.8 500627.3

[2,] 1365144.0 631933.8

[3,] 8663906.8 3971137.0

[4,] 5237655.3 2697171.2

[5,] 229655.2 136954.2

, , 1, 1, 2

[,1] [,2]

[1,] 750996.4 296573.9

[2,] 1132818.6 508895.3

[3,] 6991010.5 3232677.1

[4,] 4282399.6 2169593.4

[5,] 182906.5 108707.8

, , 2, 1, 2

[,1] [,2]

[1,] 964997.2 381084.3

[2,] 1455621.7 653907.9

[3,] 8983139.1 4153847.0

[4,] 5502694.0 2787831.5

[5,] 235026.8 139684.7

, , 1, 2, 2

[,1] [,2]

[1,] 546390.4 295322.69

[2,] 737780.0 425014.75

[3,] 5514466.8 2798765.55

[4,] 3509492.6 1607185.01

[5,] 187135.8 70361.97

, , 2, 2, 2

[,1] [,2]

[1,] 702087.5 379476.59
[2,] 948014.6 546125.14
[3,] 7085845.9 3596289.90
[4,] 4509542.7 2065161.63
[5,] 240461.3 90412.02

, , 1, 1, 1

[,1] [,2]

[1,] 123743.49 33650.65
[2,] 161422.95 68043.97
[3,] 1060377.06 457444.08
[4,] 677784.55 266192.04
[5,] 35033.55 11330.03

, , 2, 1, 1

[,1] [,2]

[1,] 151011439 54440040
[2,] 196993899 110081581
[3,] 1294040362 740053329
[4,] 827140274 430645650
[5,] 42753491 18329725

, , 1, 2, 1

[,1] [,2]

[1,] 79021.24 48644.98

[2,] 137142.42 61403.78

[3,] 870376.37 385867.66

[4,] 526175.03 262078.88

[5,] 23071.17 13307.57

, , 2, 2, 1

[,1] [,2]

[1,] 100078978 65309014

[2,] 173688409 82438520

[3,] 1102316051 518052166

[4,] 666391232 351857763

[5,] 29219216 17866275

, , 1, 1, 2

[,1] [,2]

[1,] 90499.48 42334.77

[2,] 136511.29 72642.82

[3,] 842457.83 461452.06

[4,] 516054.31 309701.01

[5,] 22041.31 15517.62

, , 2, 1, 2

[,1] [,2]

[1,] 128714905 43232122

[2,] 194156227 74182605

[3,] 1198204433 471233296

[4,] 733969745 316265631

[5,] 31348746 15846538

, , 1, 2, 2

[,1] [,2]

[1,] 65899.75 36978.464

[2,] 88983.11 53217.693

[3,] 665095.85 350443.950

[4,] 423277.36 201241.674

[5,] 22570.32 8810.287

, , 2, 2, 2

[,1] [,2]

[1,] 90385689 47546703

[2,] 122045972 68427014

[3,] 912221101 450598883

[4,] 580551718 258755425

[5,] 30956621 11328218

, , 1, 1, 1

[,1] [,2]

[1,] 0.7507708 0.31015058

[2,] 1.1482377 0.47434795

[3,] 7.4745861 3.08782268

[4,] 4.6591955 1.92475801

[5,] 0.2208150 0.09122076

, , 2, 1, 1

[,1] [,2]

[1,] 916.2097 501.7618

[2,] 1401.2619 767.4004

[3,] 9121.6761 4995.4815

[4,] 5685.8897 3113.8748

[5,] 269.4734 147.5770

, , 1, 2, 1

[,1] [,2]

[1,] 0.6030253 0.28224649

[2,] 0.9222740 0.43167111

[3,] 6.0036487 2.81001288

[4,] 3.7423039 1.75158853

[5,] 0.1773604 0.08301367

, , 2, 2, 1

[,1] [,2]

[1,] 763.7207 378.9341

[2,] 1168.0434 579.5462

[3,] 7603.5134 3772.6231

[4,] 4739.5608 2351.6203

[5,] 224.6237 111.4512

, , 1, 1, 2

[,1] [,2]

[1,] 0.6396922 0.3666823

[2,] 0.9783528 0.5608082

[3,] 6.3687005 3.6506459

[4,] 3.9698546 2.2755873

[5,] 0.1881448 0.1078477

, , 2, 1, 2

[,1] [,2]

[1,] 909.8165 374.4547

[2,] 1391.4841 572.6955

[3,] 9058.0262 3728.0274

[4,] 5646.2143 2323.8221

[5,] 267.5931 110.1337

, , 1, 2, 2

[,1] [,2]

[1,] 0.5157747 0.25911597

[2,] 0.7888318 0.39629501

[3,] 5.1349918 2.57972812

[4,] 3.2008368 1.60804323

[5,] 0.1516984 0.07621058

, , 2, 2, 2

[,1] [,2]

[1,] 707.4176 333.16987

[2,] 1081.9328 509.55393

[3,] 7042.9666 3317.00007

[4,] 4390.1505 2067.61304

[5,] 208.0640 97.99114

```

> model17a=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender* Age+
BScgrade*State*Gender*Mode, data=n)

> model17b=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender* Age+
BScgrade*Gender*Age*Mode, data=n)

> model17c=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender* Age+
State*Gender*Age*Mode, data=n)

> model17d=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Mode+
BScgrade*State*Age*Mode, data=n)

> model17e=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Mode+
BScgrade*Gender*Age*Mode, data=n)

> model17f=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Mode+
State*Gender*Age*Mode, data=n)

> model17g=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Age*Mode+
BScgrade*Gender*Age*Mode, data=n)

> model17h=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Age*Mode+
State*Gender*Age*Mode, data=n)

> model17i=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Gender*Age*Mode+
State*Gender*Age*Mode, data=n)

> model18=loglm(~BScgrade+State+Gender+Age+Mode+
BScgrade*State*Gender*Age*Mode, data=n)

> model17a; model17b; model17c; model17d; model17e; model17f; model17g; model17h;
model17i; model18

```

Call:

```

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      State * Gender * Age + BScgrade * State * Gender * Mode,
      data = n)

```

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 21.47697 20 0.3695523

Pearson 20.70749 20 0.4145259

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Age + BScgrade * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 43.93290 20 0.001536376

Pearson 42.81162 20 0.002164770

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Age + State * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 77.58305 32 1.174378e-05

Pearson 72.17188 32 6.199844e-05

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Mode + BScgrade * State * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 24.60369 20 0.2170156

Pearson 23.95280 20 0.2444600

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Mode + BScgrade * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 13.40143 20 0.8595041

Pearson 13.44084 20 0.8576788

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Mode + State * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 39.75069 32 0.1630520

Pearson 38.61374 32 0.1954771

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Age * Mode + BScgrade * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 22.76011 20 0.3007267

Pearson 22.69919 20 0.3038052

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Age * Mode + State * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 73.74410 32 3.851597e-05

Pearson 70.67786 32 9.688644e-05

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Gender * Age * Mode + State * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 73.90678 32 3.665176e-05

Pearson 76.54197 32 1.626252e-05

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 0 0 1

Pearson 0 0 1

[4,] 0 0

[5,] 0 0

, , 2, 1, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2, 1

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 1, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2, 2

[,1] [,2]

[1,] 0 0

[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 1, 1

[,1] [,2]

[1,] 1917.159 927.7285

[2,] 2709.028 1310.9207

[3,] 13961.916 6756.2834

[4,] 9043.987 4376.4582

[5,] 458.451 221.8481

, , 2, 1, 1

[,1] [,2]

[1,] 2477.9601 1199.1048

[2,] 3501.4653 1694.3873

[3,] 18046.0137 8732.6112

[4,] 11689.5074 5656.6467

[5,] 592.5557 286.7425

, , 1, 2, 1

[,1] [,2]

[1,] 1544.4522 747.3729

[2,] 2182.3782 1056.0704

[3,] 11247.6413 5442.8241

[4,] 7285.7856 3525.6503

[5,] 369.3255 178.7196

, , 2, 2, 1

[,1] [,2]

[1,] 1996.2308 965.9921

[2,] 2820.7609 1364.9889

[3,] 14537.7677 7034.9427

[4,] 9417.0017 4556.9629

[5,] 477.3595 230.9981

, , 1, 1, 2

[,1] [,2]

[1,] 923.2506 446.7684

[2,] 1636.6715 791.9985

[3,] 14352.3497 6945.2175

[4,] 8603.0166 4163.0690

[5,] 377.6934 182.7689

, , 2, 1, 2

[,1] [,2]

[1,] 1178.7627 570.4128

[2,] 2089.6248 1011.1863

[3,] 18324.4022 8867.3257

[4,] 10983.9253 5315.2098

[5,] 482.2211 233.3507

, , 1, 2, 2

[,1] [,2]

[1,] 743.7655 359.9141

[2,] 1318.4934 638.0296

[3,] 11562.1726 5595.0284

[4,] 6930.5421 3353.7451

[5,] 304.2677 147.2376

, , 2, 2, 2

[,1] [,2]

[1,] 949.6046 459.5213

[2,] 1683.3901 814.6060

[3,] 14762.0359 7143.4680

[4,] 8848.5888 4281.9033

[5,] 388.4746 187.9860

, , 1, 1, 1

[,1] [,2]

[1,] 1906.5792 922.6090

[2,] 2694.0792 1303.6866

[3,] 13884.8699 6719.0003

[4,] 8994.0799 4352.3076

[5,] 455.9211 220.6239

, , 2, 1, 1

[,1] [,2]

[1,] 2449.8699 1185.5117
[2,] 3461.7726 1675.1796
[3,] 17841.4435 8633.6181
[4,] 11556.9947 5592.5227
[5,] 585.8384 283.4919

, , 1, 2, 1

[,1] [,2]

[1,] 1566.4867 758.0355
[2,] 2213.5138 1071.1371
[3,] 11408.1093 5520.4759
[4,] 7389.7305 3575.9501
[5,] 374.5946 181.2694

, , 2, 2, 1

[,1] [,2]

[1,] 2012.8660 974.0421
[2,] 2844.2672 1376.3638
[3,] 14658.9157 7093.5673
[4,] 9495.4768 4594.9376
[5,] 481.3375 232.9231

, , 1, 1, 2

[,1] [,2]

[1,] 928.7777 449.4430

[2,] 1646.4696 796.7399

[3,] 14438.2720 6986.7960

[4,] 8654.5197 4187.9917

[5,] 379.9545 183.8631

, , 2, 1, 2

[,1] [,2]

[1,] 1193.4383 577.5144

[2,] 2115.6406 1023.7755

[3,] 18552.5406 8977.7237

[4,] 11120.6749 5381.3841

[5,] 488.2248 236.2559

, , 1, 2, 2

[,1] [,2]

[1,] 732.2537 354.3435

[2,] 1298.0862 628.1543

[3,] 11383.2173 5508.4305

[4,] 6823.2735 3301.8370

[5,] 299.5584 144.9587

, , 2, 2, 2

[,1] [,2]

[1,] 940.9137 455.3157

[2,] 1667.9833 807.1505

[3,] 14626.9306 7078.0894

[4,] 8767.6046 4242.7144

[5,] 384.9192 186.2655

```
> model12a=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*State,
data=n)
```

```
>
```

```
model12b=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender,
data=n)
```

```
> model12c=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Age,
data=n)
```

```
> model12d=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+State*Gender,
data=n)
```

```
> model12e=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+State*Age,
data=n)
```

```
> model12f=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+State*Mode,
data=n)
```

```
> model12g=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+Gender*Age,
data=n)
```

```
> model12h=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+Gender*Mode,
data=n)
```

```
> model12i=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+Age*Mode,
data=n)
```

```
> model12a; model12b; model12c; model12d; model12e; model12f; model12g; model12h;
model12i
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      Mode + BScgrade * State, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 127.9595 63 2.519681e-06

Pearson 131.5539 63 9.507707e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      Mode + BScgrade * Gender, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 125.7791 63 4.501963e-06

Pearson 134.4006 63 4.328119e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      Mode + BScgrade * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 153.0054 63 1.883332e-09

Pearson 174.6249 63 1.989187e-12

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + State * Gender, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 154.6406 66 4.478824e-09

Pearson 163.1955 66 3.365246e-10

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + State * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 159.6427 66 9.965438e-10

Pearson 173.5571 66 1.309208e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + State * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 157.8861 66 1.695212e-09

Pearson 172.8167 66 1.657330e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 159.6514 66 9.939356e-10

Pearson 173.3912 66 1.380318e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + Gender * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 159.6501 66 9.943111e-10

Pearson 173.3093 66 1.416811e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 159.5265 66 1.032304e-09

Pearson 172.8867 66 1.620848e-11

```
> m_13a=m_13b=m_13c=m_13d=m_13e=m_13f=m_13g=m_13h=array(0, dim=c(I, J, K, L,M))
```

```

> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ for(m in 1:M){
+ m_13a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i,
m]+mu_13[i, k]+mu_12[i, j])
+ m_13b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i, m]+
mu_13[i, k]+ mu_14[i, l])
+ m_13c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+mu_23[j, k])
+ m_13d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+mu_24[j, l])
+ m_13e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_25[j, m])
+ m_13f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_34[k, l])
+ m_13g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_35[k, m])
+ m_13h[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_45[l, m])
+ }
+ }
+ }
+ }
+ }

> m_13a; m_13b; m_13c; m_13d; m_13e; m_13f; m_13g; m_13h

```

, , 1, 1, 1

[,1] [,2]

[1,] 38815.77 22567.31

[2,] 52186.52 41389.31

[3,] 212251.67 73992.12

[4,] 138529.52 88939.18

[5,] 5614.55 5614.55

, , 2, 1, 1

[,1] [,2]

[1,] 27171.039 15797.116

[2,] 30046.785 23830.209

[3,] 351255.704 122449.698

[4,] 161262.564 103534.331

[5,] 6862.228 6862.228

, , 1, 2, 1

[,1] [,2]

[1,] 31269.77 18180.10

[2,] 42041.17 33343.00

[3,] 170988.76 59607.63

[4,] 111598.60 71648.91

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[5,] 6023.403 2914.773

```

>
model13a=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State, data=n)

>
model13b=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*Age, data=n)

>
model13c=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
State*Gender, data=n)

>
model13d=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
State*Age, data=n)

>
model13e=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
State*Mode, data=n)

>
model13f=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
Gender*Age, data=n)

>model13g=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender
+ Gender*Mode, data=n)

>model13h=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender
+ Age*Mode, data=n)

> model13a; model13b; model13c; model13d; model13e; model13f; model13g; model13h

```

Call:

```

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      Mode + BScgrade * Gender + BScgrade * State, data = n)

```

Statistics:

X² df P(> X²)

Likelihood Ratio 94.07656 59 0.002505307

Pearson 96.43592 59 0.001513309

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + BScgrade * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 119.1225 59 5.987478e-06

Pearson 132.3240 59 1.502593e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + State * Gender, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 120.7577 62 1.165599e-05

Pearson 128.7268 62 1.397857e-06

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + State * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 125.7598 62 3.119768e-06

Pearson 134.5223 62 2.793240e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + State * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 124.0031 62 4.981984e-06

Pearson 133.9196 62 3.310712e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 125.7684 62 3.112556e-06

Pearson 134.4744 62 2.831292e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + Gender * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 125.3519 62 3.479651e-06

Pearson 134.4898 62 2.818970e-07

Call:


```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      Mode + BScgrade * Gender + Age * Mode, data = n)
```

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 125.6436 62 3.218413e-06

Pearson 134.2637 62 3.004738e-07

```
> m_14a=m_14b=m_14c=m_14d=m_14e=m_14f=m_14g=array(0, dim=c(I, J, K, L,M))
> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ for(m in 1:M){
+ m_14a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i,
m]+mu_13[i, k]+ mu_12[i, j]+ mu_14[i, l] )
+ m_14b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i,
m]+mu_13[i, k]+ mu_12[i, j]+ mu_23[j, k] )
+ m_14c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_24[j, l])
+ m_14d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_25[j, m])
+ m_14e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_34[k, l])
+ m_14f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_35[k, m])
+ m_14g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_45[l, m])
```

+ }

+ }

+ }

+ }

+ }

> m_14a; m_14b; m_14c; m_14d; m_14e; m_14f; m_14g

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[3,] 5799564.8 2021760.65

[4,] 2461316.3 1580222.51

[5,] 81100.8 81100.80

, , 1, 2, 2

[,1] [,2]

[1,] 315474.5 183415.4

[2,] 365313.5 289731.4

[3,] 2755498.6 960582.2

[4,] 1666391.8 1069862.4

[5,] 66355.2 66355.2

, , 2, 2, 2

[,1] [,2]

[1,] 220832.2 128390.8

[2,] 210332.0 166815.1

[3,] 4560080.0 1589669.3

[4,] 1939851.0 1245429.6

[5,] 81100.8 81100.8

, , 1, 1, 1

[,1] [,2]

[1,] 590466.54 397824.45

[2,] 793862.80 729625.35

[3,] 3228778.18 1304359.08

[4,] 2107314.76 1567851.24

[5,] 85408.69 98975.26

, , 2, 1, 1

[,1] [,2]

[1,] 451398.7 232735.1

[2,] 499174.1 351084.7

[3,] 5835491.6 1804021.5

[4,] 2679092.0 1525346.1

[5,] 114003.8 101099.5

, , 1, 2, 1

[,1] [,2]

[1,] 475676.54 320485.15

[2,] 639531.43 587782.10
[3,] 2601085.64 1050784.38
[4,] 1697640.98 1263052.19
[5,] 68804.76 79733.92

, , 2, 2, 1

[,1] [,2]

[1,] 363644.27 187490.06
[2,] 402131.90 282831.84
[3,] 4701039.40 1453309.66
[4,] 2158261.51 1228810.25
[5,] 91840.79 81445.21

, , 1, 1, 2

[,1] [,2]

[1,] 282397.04 190263.87
[2,] 476317.68 437775.21
[3,] 3296245.19 1331614.34
[4,] 1990781.23 1481149.79
[5,] 69879.83 80979.76

, , 2, 1, 2

[,1] [,2]

[1,] 215886.34 111308.1

[2,] 299504.48 210650.8

[3,] 5957427.26 1841717.5

[4,] 2530939.43 1440995.1

[5,] 93275.81 82717.8

, , 1, 2, 2

[,1] [,2]

[1,] 227497.5 153275.51

[2,] 383718.9 352669.26

[3,] 2655436.7 1072741.07

[4,] 1603762.2 1193205.98

[5,] 56294.8 65236.84

, , 2, 2, 2

[,1] [,2]

[1,] 173916.82 89669.16

[2,] 241279.14 169699.10

[3,] 4799270.07 1483677.33

[4,] 2038910.64 1160857.61

[5,] 75142.47 66636.99

, , 1, 1, 1

[,1] [,2]

[1,] 622552.90 359274.03

[2,] 837001.85 658922.39

[3,] 3404232.21 1177962.64

[4,] 2221827.69 1415921.60

[5,] 90049.85 89384.25

, , 2, 1, 1

[,1] [,2]

[1,] 435787.0 251491.8

[2,] 481910.2 379379.6

[3,] 5633670.6 1949412.7

[4,] 2586435.3 1648278.0

[5,] 110060.9 109247.4

, , 1, 2, 1

[,1] [,2]

[1,] 498815.68 292684.45

[2,] 670641.23 536794.53

[3,] 2727614.62 959633.36

[4,] 1780222.19 1153487.86

[5,] 72151.75 72817.35

, , 2, 2, 1

[,1] [,2]

[1,] 349170.98 204879.11

[2,] 386126.77 309063.52

[3,] 4513934.79 1588099.13

[4,] 2072361.21 1342778.17

[5,] 88185.47 88998.98

, , 1, 1, 2

[,1] [,2]

[1,] 297742.69 171826.71

[2,] 502201.11 395353.43

[3,] 3475365.42 1202576.79

[4,] 2098961.64 1337621.79

[5,] 73677.15 73132.57

, , 2, 1, 2

[,1] [,2]

[1,] 208419.88 120278.70

[2,] 289146.09 227627.73

[3,] 5751389.04 1990146.69

[4,] 2443406.63 1557128.95

[5,] 90049.85 89384.25

, , 1, 2, 2

[,1] [,2]

[1,] 238564.02 139979.52

[2,] 402384.74 322076.72

[3,] 2784609.56 979685.40

[4,] 1681776.72 1089700.51

[5,] 59033.25 59577.83

, , 2, 2, 2

[,1] [,2]

[1,] 166994.81 97985.66

[2,] 231676.06 185438.11

[3,] 4608255.82 1621283.29

[4,] 1957760.59 1268523.16

[5,] 72151.75 72817.35

, , 1, 1, 1

[,1] [,2]

[1,] 605729.89 379486.18
[2,] 814383.86 695992.25
[3,] 3312240.91 1244232.84
[4,] 2161788.07 1495578.96
[5,] 87616.47 94412.86

, , 2, 1, 1

[,1] [,2]

[1,] 424010.9 265640.3
[2,] 468887.7 400722.8
[3,] 5481434.0 2059083.4
[4,] 2516543.0 1741007.3
[5,] 107086.8 115393.5

, , 1, 2, 1

[,1] [,2]

[1,] 487972.61 305711.94
[2,] 656063.08 560687.46
[3,] 2668322.75 1002347.01
[4,] 1741524.38 1204830.05
[5,] 70583.34 76058.47

, , 2, 2, 1

[,1] [,2]

[1,] 341580.83 213998.36

[2,] 377733.29 322820.06

[3,] 4415812.54 1658786.04

[4,] 2027312.99 1402545.74

[5,] 86268.52 92960.36

, , 1, 1, 2

[,1] [,2]

[1,] 305030.13 163071.16

[2,] 514492.78 375207.93

[3,] 3560427.09 1141298.67

[4,] 2150335.00 1269462.36

[5,] 75480.45 69406.05

, , 2, 1, 2

[,1] [,2]

[1,] 213521.09 114149.81

[2,] 296223.12 216028.81

[3,] 5892157.78 1888737.41

[4,] 2503210.49 1477784.39

[5,] 92253.88 84829.62

, , 1, 2, 2

[,1] [,2]

[1,] 245730.56 131369.22

[2,] 414472.51 302265.41

[3,] 2868260.15 919423.82

[4,] 1732297.85 1022671.77

[5,] 60806.63 55913.13

, , 2, 2, 2

[,1] [,2]

[1,] 172011.39 91958.45

[2,] 238635.69 174031.60

[3,] 4746689.34 1521556.29

[4,] 2016572.37 1190494.84

[5,] 74319.21 68338.27

, , 1, 1, 1

[,1] [,2]

[1,] 619233.62 360019.55

[2,] 832539.19 660289.70

[3,] 3386081.79 1180407.00

[4,] 2209981.53 1418859.74

[5,] 89569.73 89569.73

, , 2, 1, 1

[,1] [,2]

[1,] 435727.4 253329.9

[2,] 481844.2 382152.3

[3,] 5632899.4 1963660.1

[4,] 2586081.3 1660324.5

[5,] 110045.9 110045.9

, , 1, 2, 1

[,1] [,2]

[1,] 502134.96 291938.93

[2,] 675103.89 535427.22

[3,] 2745765.04 957189.01

[4,] 1792068.35 1150549.72

[5,] 72631.87 72631.87

, , 2, 2, 1

[,1] [,2]

[1,] 349230.63 203041.06

[2,] 386192.74 306290.79
[3,] 4514705.96 1573851.68
[4,] 2072715.26 1330731.59
[5,] 88200.53 88200.53

, , 1, 1, 2

[,1] [,2]

[1,] 296155.21 172183.26
[2,] 499523.51 396173.82
[3,] 3456835.74 1205072.22
[4,] 2087770.57 1340397.45
[5,] 73284.33 73284.33

, , 2, 1, 2

[,1] [,2]

[1,] 208391.35 121157.76
[2,] 289106.51 229291.37
[3,] 5750601.76 2004691.85
[4,] 2443072.17 1568509.37
[5,] 90037.53 90037.53

, , 1, 2, 2

[,1] [,2]

[1,] 240151.50 139622.97

[2,] 405062.33 321256.33

[3,] 2803139.24 977189.97

[4,] 1692967.80 1086924.85

[5,] 59426.07 59426.07

, , 2, 2, 2

[,1] [,2]

[1,] 167023.34 97106.60

[2,] 231715.64 183774.47

[3,] 4609043.10 1606738.13

[4,] 1958095.06 1257142.74

[5,] 72164.07 72164.07

, , 1, 1, 1

[,1] [,2]

[1,] 619003.94 359886.01

[2,] 832230.38 660044.78

[3,] 3384825.82 1179969.16

[4,] 2209161.80 1418333.45

[5,] 89536.51 89536.51

, , 2, 1, 1

[,1] [,2]

[1,] 435852.5 253402.6

[2,] 481982.6 382262.0

[3,] 5634516.9 1964224.0

[4,] 2586823.9 1660801.3

[5,] 110077.5 110077.5

, , 1, 2, 1

[,1] [,2]

[1,] 498666.11 289922.16

[2,] 670440.14 531728.39

[3,] 2726796.75 950576.56

[4,] 1779688.39 1142601.49

[5,] 72130.11 72130.11

, , 2, 2, 1

[,1] [,2]

[1,] 351120.34 204139.73

[2,] 388282.45 307948.15

[3,] 4539135.33 1582367.89

[4,] 2083930.86 1337932.26

[5,] 88677.79 88677.79

, , 1, 1, 2

[,1] [,2]

[1,] 298095.18 173311.15

[2,] 502795.65 398768.96

[3,] 3479479.79 1212966.06

[4,] 2101446.54 1349177.74

[5,] 73764.38 73764.38

, , 2, 1, 2

[,1] [,2]

[1,] 207334.53 120543.33

[2,] 287640.35 228128.55

[3,] 5721438.36 1994525.32

[4,] 2430682.45 1560554.88

[5,] 89580.91 89580.91

, , 1, 2, 2

[,1] [,2]

[1,] 240143.81 139618.49

[2,] 405049.36 321246.04

[3,] 2803049.45 977158.67

[4,] 1692913.57 1086890.04

[5,] 59424.17 59424.17

, , 2, 2, 2

[,1] [,2]

[1,] 167027.53 97109.03

[2,] 231721.45 183779.08

[3,] 4609158.73 1606778.44

[4,] 1958144.19 1257174.28

[5,] 72165.88 72165.88

, , 1, 1, 1

[,1] [,2]

[1,] 615588.09 357900.05

[2,] 827637.89 656402.47

[3,] 3366147.37 1173457.75

[4,] 2196971.00 1410506.67

[5,] 89042.42 89042.42

, , 2, 1, 1

[,1] [,2]

[1,] 430911.7 250530.0

[2,] 476518.8 377928.7

[3,] 5570643.9 1941957.5

[4,] 2557499.6 1641974.4

[5,] 108829.6 108829.6

, , 1, 2, 1

[,1] [,2]

[1,] 505780.49 294058.42

[2,] 680005.19 539314.46

[3,] 2765699.46 964138.26

[4,] 1805078.88 1158902.78

[5,] 73159.18 73159.18

, , 2, 2, 1

[,1] [,2]

[1,] 354046.34 205840.90

[2,] 391518.14 310514.38

[3,] 4576961.46 1595554.29

[4,] 2101296.95 1349081.70

[5,] 89416.77 89416.77

, , 1, 1, 2

[,1] [,2]

[1,] 299879.77 174348.70

[2,] 505805.70 401156.24

[3,] 3500310.15 1220227.64

[4,] 2114027.13 1357254.77

[5,] 74205.98 74205.98

, , 2, 1, 2

[,1] [,2]

[1,] 209915.84 122044.09

[2,] 291221.46 230968.75

[3,] 5792670.14 2019357.12

[4,] 2460944.40 1579983.76

[5,] 90696.19 90696.19

, , 1, 2, 2

[,1] [,2]

[1,] 236426.95 137457.53

[2,] 398780.15 316273.91

[3,] 2759664.82 962034.55

[4,] 1666711.24 1070067.53

[5,] 58504.42 58504.42

, , 2, 2, 2

[,1] [,2]

[1,] 165498.86 96220.27

[2,] 229600.69 182097.10

[3,] 4566974.72 1592072.86

[4,] 1940222.83 1245668.35

[5,] 71505.41 71505.41

>

```
model14a=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ BScgrade*Age, data=n)
```

>

```
model14b=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ State*Gender, data=n)
```

>

```
model14c=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ State*Age, data=n)
```

>

```
model14d=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ State*Mode, data=n)
```

>

```
model14e=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ Gender*Age, data=n)
```

>

```
model14f=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ Gender*Mode, data=n)
```

>

```
model14g=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ Age*Mode, data=n)
```

```
> model14a; model14b; model14c; model14d; model14e; model14f; model14g
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
  Mode + BScgrade * Gender + BScgrade * State + BScgrade *  
  Age, data = n)
```

Statistics:

```
      X^2 df  P(> X^2)
```

```
Likelihood Ratio 87.41988 55 0.003528881
```

```
Pearson      93.03104 55 0.001031502
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
  Mode + BScgrade * Gender + BScgrade * State + State * Gender,  
  data = n)
```

Statistics:

```
      X^2 df  P(> X^2)
```

```
Likelihood Ratio 91.75409 58 0.003132519
```

```
Pearson      94.07371 58 0.001915817
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
  Mode + BScgrade * Gender + BScgrade * State + State * Age,  
  data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 94.05723 58 0.001922623

Pearson 96.53581 58 0.001118463

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + BScgrade * State + State * Mode,  
data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 93.04289 58 0.002388163

Pearson 96.43197 58 0.001144516

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + BScgrade * State + Gender * Age,  
data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 94.06586 58 0.001919054

Pearson 96.45887 58 0.001137714

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + BScgrade * State + Gender * Mode,
```



```
data = n)
```

Statistics:

```
X^2 df P(> X^2)
```

```
Likelihood Ratio 93.64933 58 0.002098519
```

```
Pearson 95.98648 58 0.001262949
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
```

```
Mode + BScgrade * Gender + BScgrade * State + Age * Mode,
```

```
data = n)
```

Statistics:

```
X^2 df P(> X^2)
```

```
Likelihood Ratio 93.94103 58 0.001971264
```

```
Pearson 96.38938 58 0.001155369
```

```
> mu_123=array(0, dim=c(I, J, K)); mu_124=array(0, dim=c(I, J, L)); mu_125=array(0, dim=c(I, J, M)); mu_134= array(0, dim=c(I, K, L)); mu_135=array(0, dim=c(I, K, M)); mu_145=array(0, dim=c(I, L, M)); mu_234= array(0, dim=c(J, K, L)); mu_235=array(0, dim=c(J, K, M)); mu_245=array(0, dim=c(J, L, M)); mu_345=array(0, dim=c(K, L, M)); mu_1234=array(0, dim=c(I, J, K, L)); mu_1235=array(0, dim=c(I, J, K, M)); mu_1245=array(0, dim=c(I, J, L, M)); mu_1345=array(0, dim=c(I, K, L, M)); mu_2345=array(0, dim=c(J, K, L, M))
```

```
> for (i in 1:I){
```

```
+ for (j in 1:J){
```

```
+ for (k in 1:K){
```

```
+ mu_123[i, j, k]=log(sum(n[i, j, k, , ])/L*M)-mu-mu_1[i]-mu_2[j]-mu_3[k]-mu_12[i, j]-mu_13[i, k]-mu_23[j, k]
```

```

+ }
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ for (l in 1:L){
+ mu_124[i, j, l]=log(sum(n[i, j, , l])/K*M)-mu-mu_1[i]-mu_2[j]-mu_4[l]-mu_12[i, j]-mu_14[i,
l]-mu_24[j, l]
+ }
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ for (m in 1:M){
+ mu_125[i, j, m]=log(sum(n[i, j, , m])/K*L)-mu-mu_1[i]-mu_2[j]-mu_5[m]-mu_12[i, j]-
mu_15[i, m]-mu_25[j, m]
+ }
+ }
+ }
> for (i in 1:I){
+ for (k in 1:K){
+ for (l in 1:L){
+ mu_134[i, k, l]=log(sum(n[i, , k, l])/J*M)-mu-mu_1[i]-mu_3[k]-mu_4[l]-mu_13[i, k]-
mu_14[i, l]-mu_34[k, l]
+ }
+ }

```

```

+ }
> for (i in 1:I){
+ for (k in 1:K){
+ for (m in 1:M){
+ mu_135[i, k, m]=log(sum(n[i, , k, , m])/J*L)-mu-mu_1[i]-mu_3[k]-mu_5[m]-mu_13[i, k]-
mu_15[i, m]-mu_35[k, m]
+ }
+ }
+ }
> for (i in 1:I){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_145[i, l, m]=log(sum(n[i, , , l, m])/J*L)-mu-mu_1[i]-mu_4[l]-mu_5[m]-mu_14[i, l]-
mu_15[i, m]-mu_45[l, m]
+ }
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ for (l in 1:L){
+ mu_234[j, k, l]=log(sum(n[, j, k, l, ])/I*M)-mu-mu_2[j]-mu_3[k]-mu_4[l]-mu_23[j, k]-
mu_24[j, l]-mu_34[k, l]
+ }
+ }
+ }
> for (j in 1:J){

```

```

+ for (k in 1:K){
+ for (m in 1:M){
+ mu_235[j, k, m]=log(sum(n[, j, k, , m])/I*L)-mu-mu_2[j]-mu_3[k]-mu_5[m]-mu_23[j, k]-
mu_25[j, m]-mu_35[k, m]
+ }
+ }
+ }
> for (j in 1:J){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_245[j, l, m]=log(sum(n[, j, , l, m])/I)-mu-mu_2[j]-mu_4[l]-mu_5[m]-mu_24[j, l]-mu_25[j,
m]-mu_45[l, m]
+ }
+ }
+ }
> for (j in 1:J){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_345[k, l, m]=log(sum(n[, , k, l, m])/I*J)-mu-mu_3[k]-mu_4[l]-mu_5[m]-mu_34[k, l]-
mu_35[k, m]-mu_45[l, m]
+ }
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ for (k in 1:K){

```

```

+ for (l in 1:L){
+ mu_1234[i, j, k, l]=log(sum(n[i, j, k, l, ])/M)-mu-mu_1[i]-mu_2[j]-mu_3[k]-mu_4[l]-mu_12[i,
j]-mu_13[i, k]-mu_14[i, l]-mu_123[i, j, K]-mu_134[i, k,l]-mu_234[j, k, l]
+ }
+ }
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_1245[i, j, l, m]=log(sum(n[i, j, , l, m])/K)-mu-mu_1[i]-mu_2[j]-mu_4[l]-mu_5[k]-
mu_12[i, j]-mu_14[i, l]-mu_15[i, m]-mu_24[j, l]-mu_25[j, m]-mu_45[l, m]-mu_124[i, j, l]-
mu_125[i, j, m]-mu_145[i, l, m]-mu_245[j, l, m]
+ }
+ }
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_2345[j, k, l, m]=log(sum(n[, j, k, l, , m])/I)-mu-mu_2[j]-mu_3[k]-mu_4[l]-mu_5[m]-
mu_23[j, k]-mu_24[j, l]-mu_25[j, m]-mu_34[k, l]-mu_35[k, m]-mu_45[l, m]-mu_234[j, k, l]-
mu_245[j, l, m]-mu_345[k, l, m]
+ }
+ }

```

+ }

+ }

> mu_123; mu_124; mu_125; mu_134; mu_135; mu_145; mu_234; mu_235; mu_245; mu_345;
mu_1234; mu_1235; mu_1245; mu_1345; mu_2345

, , 1

[,1] [,2]

[1,] -6.892666 -7.008589

[2,] -6.903157 -7.001099

[3,] -6.897415 -6.982663

[4,] -6.948510 -6.931515

[5,] -6.998753 -6.923031

, , 2

[,1] [,2]

[1,] -6.952608 -6.877944

[2,] -6.931678 -6.898204

[3,] -6.959274 -6.877637

[4,] -6.914497 -6.940397

[5,] -6.882071 -6.944267

, , 1

[,1] [,2]

[1,] -7.035193 -6.773132

[2,] -6.866154 -7.018975

[3,] -6.919861 -6.967822

[4,] -6.960145 -6.886890

[5,] -6.933885 -6.926466

, , 2

[,1] [,2]

[1,] -6.863203 -7.060949

[2,] -7.037667 -6.815048

[3,] -6.946593 -6.887410

[4,] -6.896112 -6.990231

[5,] -6.928468 -6.937651

, , 1

[,1] [,2]

[1,] -6.721476 -7.411313

[2,] -6.783358 -7.161783

[3,] -6.912162 -6.965107

[4,] -7.054260 -6.787307

[5,] -6.819479 -7.076509

, , 2

[,1] [,2]

[1,] -7.511359 -6.325520

[2,] -7.204466 -6.630936

[3,] -6.952542 -6.890184

[4,] -6.822302 -7.130982

[5,] -7.075849 -6.768806

, , 1

[,1] [,2]

[1,] -6.734383 -7.299208

[2,] -6.887365 -7.009555

[3,] -6.899235 -6.951881

[4,] -7.015195 -6.864870

[5,] -7.046322 -6.846737

, , 2

[,1] [,2]

[1,] -7.097619 -6.734483

[2,] -6.999158 -6.826290

[3,] -6.973655 -6.906042

[4,] -6.834964 -7.023471

[5,] -6.829740 -7.023949

, , 1

[,1] [,2]

[1,] -7.048127 -6.784331

[2,] -7.036901 -6.769413

[3,] -6.845910 -6.987233

[4,] -7.025200 -6.857595

[5,] -6.918118 -6.942334

, , 2

[,1] [,2]

[1,] -6.722894 -7.340644

[2,] -6.776598 -7.282836

[3,] -7.022787 -6.879163

[4,] -6.841682 -7.016498

[5,] -6.947491 -6.918614

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[4,] -6.901077 -6.970373

[5,] -7.017945 -6.855323

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[2,] -6.909288 -6.968260

[3,] -6.948689 -6.909822

[4,] -6.964375 -6.890621

[5,] -6.835677 -7.037253

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[2,] -6.996768 -6.874128

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[2,] -6.857546 -7.006691

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[2,] -7.610274 -7.642284

, , 2

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[2,] -7.640922 -7.604408

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[2,] -6.917606 -6.948644

, , 2

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[1,] 0.000000 0.000000
[2,] -6.946589 -6.912586

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[2,] 5.830493 5.719981
[3,] 5.845686 5.652837
[4,] 5.625933 6.010821
[5,] 5.732456 5.830653

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[2,] 5.914110 5.596130
[3,] 5.837160 5.725387

[4,] 5.841736 5.705880

[5,] 5.760128 5.791524

, , 1, 1

[,1] [,2]

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[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

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[,1] [,2]

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[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

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[,1] [,2]

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[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

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[,1] [,2]

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[2,] 0 0

[3,] 0 0

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[5,] 0 0

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[5,] 11.67122 11.87677

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[2,] 11.63340 11.80774

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[4,] 11.79237 11.73724

[5,] 11.91025 11.65793

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[5,] 0 0

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[,1] [,2]

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[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 1, 2

[,1] [,2]

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[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

, , 2, 2

[,1] [,2]

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[2,] 0 0

[3,] 0 0

[4,] 0 0

[5,] 0 0

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[,1] [,2]

[1,] -2.044305 4.811871

[2,] -2.202487 4.935612

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[,1] [,2]

[1,] -2.047271 4.846003

[2,] -2.080589 4.871024

, , 1, 2

```

      [,1] [,2]
[1,] -2.116057 4.893230
[2,] -1.946688 4.731318

, , 2, 2

```

```

      [,1] [,2]
[1,] -2.115199 4.857783
[2,] -2.077733 4.830675

```

```

> m_15a=m_15b=m_15c=m_15d=m_15e=m_15f=m_15g=m_15h=m_15i=m_15j=
m_16a=m_16b=m_16c=m_16d=m_16e =array(0, dim=c(I, J, K, L,M))
> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ for(m in 1:M){
+ m_15a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_123[i, j, k])
+ m_15b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_124[i, j, l])
+ m_15c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_125[i, j,
m])
+ m_15d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_134[i, k,
l])
+ m_15e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_135[i, k,
m])

```

```

+ m_15f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_145[i, l,
m])
+ m_15g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_234[j, k,
l])
+ m_15h[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_235[j, k,
m])
+ m_15i[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_245[j, l,
m])
+ m_15j[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_345[k, l,
m])
+ m_16a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1234[i, j,
k, l])
+ m_16b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1235[i, j,
k, m])
+ m_16c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1245[i, j, l,
m])
+ m_16d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1345[i, k,
l, m])
+ m_16e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_2345[ j, k,
l, m])
+ }
+ }
+ }
+ }
+ }
> m_15a; m_15b; m_15c; m_15d; m_15e; m_15f; m_15g; m_15h; m_15i; m_15j; m_16a;
m_16b; m_16c; m_16d; m_16e
, , 1, 1, 1

```

[,1] [,2]

[1,] 0.005886959 0.0025369325

[2,] 0.008909613 0.0039091829

[3,] 0.058332150 0.0259207772

[4,] 0.034549498 0.0170053452

[5,] 0.001557181 0.0008128068

, , 2, 1, 1

[,1] [,2]

[1,] 0.007124373 0.003714800

[2,] 0.011126551 0.005567509

[3,] 0.070458121 0.036995458

[4,] 0.045930543 0.021657874

[5,] 0.002248544 0.001022476

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[,1] [,2]

[1,] 0.004742501 0.0020437386

[2,] 0.007177535 0.0031492158

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[4,] 0.027832883 0.0136994110

[5,] 0.001254456 0.0006547926

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[,1] [,2]

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[4,] 0.025479063 0.0125408554

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[,1] [,2]

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[5,] 0.001658224 0.0007540406

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[,1] [,2]

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[3,] 0.057037424 0.0263083372

[4,] 0.034149822 0.0177813878

[5,] 0.001661539 0.0008100195

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[,1] [,2]

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[,1] [,2]

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[5,] 0.001954448 0.0009528157

, , 1, 2, 2

[,1] [,2]

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[,1] [,2]

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[5,] 0.002393782 0.0008958201

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[5,] 208.0640 97.99114

> model15a=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender, data=n)

> model15b=loglm(~BScgrade+State+Gender+Age+Mode + BScgrade*State*Age, data=n)

> model15c=loglm(~BScgrade+State+Gender+Age+Mode + BScgrade*State*Mode, data=n)

> model15d=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Gender*Age, data=n)

> model15e=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Gender*Mode, data=n)

> model15f=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Age*Mode, data=n)

> model15g=loglm(~BScgrade+State+Gender+Age+Mode+ State*Gender*Age, data=n)

> model15h=loglm(~BScgrade+State+Gender+Age+Mode + State*Gender*Mode, data=n)

> model15i=loglm(~BScgrade+State+Gender+Age+Mode + State*Age*Mode, data=n)

> model15j=loglm(~BScgrade+State+Gender+Age+Mode + Gender*Age*Mode, data=n)

```

> model16a=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Age,
data=n)

> model16b=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Mode,
data=n)

> model16c=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Age*Mode,
data=n)

> model16d=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Gender*Age*Mode,
data=n)

> model16e=loglm(~BScgrade+State+Gender+Age+Mode+ State*Gender*Age*Mode, data=n)

> model15a; model15b; model15c; model15d; model15e; model15f; model15g; model15h;
model15i; model16a; model16b; model16c; model16d; model16e

```

Call:

```

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      State * Gender, data = n)

```

Statistics:

X² df P(> X²)

Likelihood Ratio 104.3455 58 0.0001829902

Pearson 105.8493 58 0.0001269782

Call:

```

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      State * Age, data = n)

```

Statistics:

X² df P(> X²)

Likelihood Ratio 131.8656 58 1.118972e-07

Pearson 127.2042 58 4.308532e-07

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
State * Mode, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 100.71154 58 0.0004329178

Pearson 99.04581 58 0.0006355605

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Gender * Age, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 124.9574 58 8.149994e-07

Pearson 123.9418 58 1.084171e-06

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Gender * Mode, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 113.5894 58 1.790516e-05

Pearson 125.3055 58 7.387566e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 145.6230 58 1.741845e-09

Pearson 155.5872 58 7.347278e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *  
Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 167.5483 67 1.408711e-10

Pearson 167.5230 67 1.419802e-10

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *  
Gender * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 163.4812 67 4.916169e-10

Pearson 161.3538 67 9.381688e-10

Call:


```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *  
Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 170.88 67 4.991418e-11

Pearson 174.47 67 1.610523e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 83.86091 39 4.016366e-05

Pearson 78.14037 39 1.999306e-04

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 41.96151 39 0.3437295

Pearson 40.78206 39 0.3920119

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
```

```
State * Age * Mode, data = n)
```

Statistics:

```
X^2 df P(> X^2)
```

```
Likelihood Ratio 83.35372 39 0.0000464587
```

```
Pearson 79.69282 39 0.0001304115
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
```

```
Gender * Age * Mode, data = n)
```

Statistics:

```
X^2 df P(> X^2)
```

```
Likelihood Ratio 85.02937 39 2.865165e-05
```

```
Pearson 87.22491 39 1.506013e-05
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *
```

```
Gender * Age * Mode, data = n)
```

Statistics:

```
X^2 df P(> X^2)
```

```
Likelihood Ratio 161.2704 60 3.253775e-11
```

```
Pearson 158.5060 60 7.924161e-11
```

```
> m_17a=m_17b=m_17c=m_17d=m_17e=m_17f=m_17g=m_17h=m_17i=array(0, dim=c(I, J,  
K, L,M))
```

```
> for(i in 1:I){
```

```

+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ for(m in 1:M){
+ m_17a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1234[i, j,
k, l]+ mu_1235[i, j, k, m])
+ m_17b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1234[i, j,
k, l]+ mu_1345[i, k, l, m])
+ m_17c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1234[i, j,
k, l]+ mu_2345[j, k, l, m])
+ m_17d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1235[i, j,
k, m] + mu_1245[i, j, l, m])
+ m_17e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1235[i, j,
k, m] + mu_1345[i, k, l, m])
+ m_17f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1235[i, j, k,
m] + mu_2345[ j, k, l, m])
+ m_17g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1245[i, j, l,
m]+ mu_1345[i, k, l, m])
+ m_17h[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1245[i, j, l,
m]+ mu_2345[ j, k, l, m])
+ m_17i[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1345[i, k, l,
m] + mu_2345[ j, k, l, m])
+ }
+ }
+ }
+ }
+ }
> m_17a; m_17b; m_17c; m_17d; m_17e; m_17f; m_17g; m_17h; m_17i

```

, , 1, 1, 1

[,1] [,2]

[1,] 1670.8925 1094.7089

[2,] 3020.0423 1308.5225

[3,] 19960.2757 7964.8211

[4,] 9987.4061 7101.8310

[5,] 526.5418 281.0877

, , 2, 1, 1

[,1] [,2]

[1,] 2508.2552 1127.2285

[2,] 4219.0588 1485.5299

[3,] 25430.3167 11004.5499

[4,] 15924.3995 6727.0433

[5,] 695.5666 347.3253

, , 1, 1, 2

[,1] [,2]

[1,] 1529.5856 1002.1297

[2,] 2764.6382 1197.8611

[3,] 18272.2405 7291.2383

[4,] 9142.7739 6501.2311

[5,] 482.0123 257.3162

, , 2, 1, 2

[,1] [,2]

[1,] 2296.1327 1031.8991

[2,] 3862.2542 1359.8991

[3,] 23279.6816 10073.8980

[4,] 14577.6773 6158.1390

[5,] 636.7427 317.9521

, , 1, 1, 1

[,1] [,2]

[1,] 1670.8925 1094.7089

[2,] 3020.0423 1308.5225

[3,] 19960.2757 7964.8211

[4,] 9987.4061 7101.8310

[5,] 526.5418 281.0877

, , 2, 1, 1

[,1] [,2]

[1,] 2508.2552 1127.2285

[2,] 4219.0588 1485.5299

[3,] 25430.3167 11004.5499

[4,] 15924.3995 6727.0433

[5,] 695.5666 347.3253

, , 1, 1, 2

[,1] [,2]

[1,] 1529.5856 1002.1297

[2,] 2764.6382 1197.8611

[3,] 18272.2405 7291.2383

[4,] 9142.7739 6501.2311

[5,] 482.0123 257.3162

, , 2, 1, 2

[,1] [,2]

[1,] 2296.1327 1031.8991

[2,] 3862.2542 1359.8991

[3,] 23279.6816 10073.8980

[4,] 14577.6773 6158.1390

[5,] 636.7427 317.9521

, , 1, 1, 1

[,1] [,2]

[1,] 216.33079 120.99595

[2,] 391.00548 144.62833

[3,] 2584.26083 880.33546

[4,] 1293.07145 784.95093

[5,] 68.17147 31.06805

, , 2, 1, 1

[,1] [,2]

[1,] 308418.81 156863.17

[2,] 518781.79 206723.77

[3,] 3126949.78 1531374.06

[4,] 1958087.99 936123.67

[5,] 85527.91 48333.19

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[,1] [,2]

[1,] 184.32405 143.05010

[2,] 333.15514 170.99001

[3,] 2201.91231 1040.79587

[4,] 1101.75796 928.02541

[5,] 58.08531 36.73089

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[,1] [,2]

[1,] 306266.7 117063.83

[2,] 515161.8 154273.79

[3,] 3105130.3 1142833.66

[4,] 1944424.7 698610.27

[5,] 84931.1 36070.09

, , 1, 1, 1

[,1] [,2]

[1,] 955768.1 304453.7

[2,] 1246796.2 615626.7

[3,] 8190124.7 4138718.0

[4,] 5235062.3 2408368.2

[5,] 270591.6 102508.2

, , 2, 1, 1

[,1] [,2]

[1,] 1228119.7 391209.5

[2,] 1602077.9 791053.1

[3,] 10523947.8 5318069.5

[4,] 6726823.5 3094646.6

[5,] 347698.2 131718.5

, , 1, 2, 1

[,1] [,2]

[1,] 612156.3 389606.6

[2,] 1062405.5 491794.2

[3,] 6742572.1 3090485.4

[4,] 4076136.7 2099038.2

[5,] 178726.1 106582.8

, , 2, 2, 1

[,1] [,2]

[1,] 786593.8 500627.3

[2,] 1365144.0 631933.8

[3,] 8663906.8 3971137.0

[4,] 5237655.3 2697171.2

[5,] 229655.2 136954.2

, , 1, 1, 2

[,1] [,2]

[1,] 750996.4 296573.9

[2,] 1132818.6 508895.3

[3,] 6991010.5 3232677.1

[4,] 4282399.6 2169593.4

[5,] 182906.5 108707.8

, , 2, 1, 2

[,1] [,2]

[1,] 964997.2 381084.3

[2,] 1455621.7 653907.9

[3,] 8983139.1 4153847.0

[4,] 5502694.0 2787831.5

[5,] 235026.8 139684.7

, , 1, 2, 2

[,1] [,2]

[1,] 546390.4 295322.69

[2,] 737780.0 425014.75

[3,] 5514466.8 2798765.55

[4,] 3509492.6 1607185.01

[5,] 187135.8 70361.97

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[,1] [,2]

[1,] 702087.5 379476.59
[2,] 948014.6 546125.14
[3,] 7085845.9 3596289.90
[4,] 4509542.7 2065161.63
[5,] 240461.3 90412.02

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[,1] [,2]

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[2,] 8.868742 4.2916555
[3,] 57.732098 27.9370266
[4,] 35.986625 17.4142175
[5,] 1.705527 0.8253184

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[2,] 11.395941 5.514587
[3,] 74.183191 35.897843
[4,] 46.241221 22.376499
[5,] 2.191527 1.060498

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[,1] [,2]

[1,] 4.671475 2.2605643

[2,] 7.144609 3.4573337

[3,] 46.508655 22.5059124

[4,] 28.990624 14.0287962

[5,] 1.373963 0.6648719

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[,1] [,2]

[1,] 6.002639 2.9047251

[2,] 9.180506 4.4425208

[3,] 59.761564 28.9191015

[4,] 37.251669 18.0263823

[5,] 1.765482 0.8543309

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[1,] 5.308390 2.5687728

[2,] 8.118715 3.9287113

[3,] 52.849710 25.5743997

[4,] 32.943246 15.9415018

[5,] 1.561291 0.7555214

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[,1] [,2]

[1,] 6.821047 3.3007594

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[1,] 5.494997 2.6590733

[2,] 8.404113 4.0668180

[3,] 54.707544 26.4734208

[4,] 34.101305 16.5018960

[5,] 1.616176 0.7820804

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[,1] [,2]

[1,] 0.7507708 0.31015058

[2,] 1.1482377 0.47434795

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[4,] 4.6591955 1.92475801

[5,] 0.2208150 0.09122076

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[,1] [,2]

[1,] 916.2097 501.7618

[2,] 1401.2619 767.4004

[3,] 9121.6761 4995.4815

[4,] 5685.8897 3113.8748

[5,] 269.4734 147.5770

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[,1] [,2]

[1,] 0.6030253 0.28224649

[2,] 0.9222740 0.43167111

[3,] 6.0036487 2.81001288

[4,] 3.7423039 1.75158853

[5,] 0.1773604 0.08301367

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[,1] [,2]

[1,] 763.7207 378.9341

[2,] 1168.0434 579.5462

[3,] 7603.5134 3772.6231

[4,] 4739.5608 2351.6203

[5,] 224.6237 111.4512

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[,1] [,2]

[1,] 0.6396922 0.3666823

[2,] 0.9783528 0.5608082

[3,] 6.3687005 3.6506459

[4,] 3.9698546 2.2755873

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[,1] [,2]

[1,] 909.8165 374.4547

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[,1] [,2]

[1,] 0.5157747 0.25911597

[2,] 0.7888318 0.39629501

[3,] 5.1349918 2.57972812

[4,] 3.2008368 1.60804323

[5,] 0.1516984 0.07621058

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[,1] [,2]

[1,] 707.4176 333.16987

[2,] 1081.9328 509.55393

[3,] 7042.9666 3317.00007

[4,] 4390.1505 2067.61304

[5,] 208.0640 97.99114

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[,1] [,2]

[1,] 955768.1 304453.7

[2,] 1246796.2 615626.7

[3,] 8190124.7 4138718.0

[4,] 5235062.3 2408368.2

[5,] 270591.6 102508.2

, , 2, 1, 1

[,1] [,2]

[1,] 1228119.7 391209.5

[2,] 1602077.9 791053.1

[3,] 10523947.8 5318069.5

[4,] 6726823.5 3094646.6

[5,] 347698.2 131718.5

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[,1] [,2]

[1,] 612156.3 389606.6

[2,] 1062405.5 491794.2

[3,] 6742572.1 3090485.4

[4,] 4076136.7 2099038.2

[5,] 178726.1 106582.8

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[,1] [,2]

[1,] 786593.8 500627.3

[2,] 1365144.0 631933.8

[3,] 8663906.8 3971137.0

[4,] 5237655.3 2697171.2

[5,] 229655.2 136954.2

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[,1] [,2]

[1,] 750996.4 296573.9

[2,] 1132818.6 508895.3

[3,] 6991010.5 3232677.1

[4,] 4282399.6 2169593.4

[5,] 182906.5 108707.8

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[,1] [,2]

[1,] 964997.2 381084.3

[2,] 1455621.7 653907.9

[3,] 8983139.1 4153847.0

[4,] 5502694.0 2787831.5

[5,] 235026.8 139684.7

, , 1, 2, 2

[,1] [,2]

[1,] 546390.4 295322.69

[2,] 737780.0 425014.75

[3,] 5514466.8 2798765.55

[4,] 3509492.6 1607185.01

[5,] 187135.8 70361.97

, , 2, 2, 2

[,1] [,2]

[1,] 702087.5 379476.59

[2,] 948014.6 546125.14

[3,] 7085845.9 3596289.90

[4,] 4509542.7 2065161.63

[5,] 240461.3 90412.02

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[,1] [,2]

[1,] 123743.49 33650.65
[2,] 161422.95 68043.97
[3,] 1060377.06 457444.08
[4,] 677784.55 266192.04
[5,] 35033.55 11330.03

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[,1] [,2]

[1,] 151011439 54440040
[2,] 196993899 110081581
[3,] 1294040362 740053329
[4,] 827140274 430645650
[5,] 42753491 18329725

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[,1] [,2]

[1,] 79021.24 48644.98
[2,] 137142.42 61403.78
[3,] 870376.37 385867.66
[4,] 526175.03 262078.88
[5,] 23071.17 13307.57

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[,1] [,2]

[1,] 100078978 65309014

[2,] 173688409 82438520

[3,] 1102316051 518052166

[4,] 666391232 351857763

[5,] 29219216 17866275

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[,1] [,2]

[1,] 90499.48 42334.77

[2,] 136511.29 72642.82

[3,] 842457.83 461452.06

[4,] 516054.31 309701.01

[5,] 22041.31 15517.62

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[,1] [,2]

[1,] 128714905 43232122

[2,] 194156227 74182605

[3,] 1198204433 471233296

[4,] 733969745 316265631

[5,] 31348746 15846538

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[,1] [,2]

[1,] 65899.75 36978.464

[2,] 88983.11 53217.693

[3,] 665095.85 350443.950

[4,] 423277.36 201241.674

[5,] 22570.32 8810.287

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[,1] [,2]

[1,] 90385689 47546703

[2,] 122045972 68427014

[3,] 912221101 450598883

[4,] 580551718 258755425

[5,] 30956621 11328218

, , 1, 1, 1

[,1] [,2]

[1,] 0.7507708 0.31015058

[2,] 1.1482377 0.47434795

[3,] 7.4745861 3.08782268

[4,] 4.6591955 1.92475801

[5,] 0.2208150 0.09122076

, , 2, 1, 1

[,1] [,2]

[1,] 916.2097 501.7618

[2,] 1401.2619 767.4004

[3,] 9121.6761 4995.4815

[4,] 5685.8897 3113.8748

[5,] 269.4734 147.5770

, , 1, 2, 1

[,1] [,2]

[1,] 0.6030253 0.28224649

[2,] 0.9222740 0.43167111

[3,] 6.0036487 2.81001288

[4,] 3.7423039 1.75158853

[5,] 0.1773604 0.08301367

, , 2, 2, 1

[,1] [,2]

[1,] 763.7207 378.9341

[2,] 1168.0434 579.5462
[3,] 7603.5134 3772.6231
[4,] 4739.5608 2351.6203
[5,] 224.6237 111.4512

, , 1, 1, 2

[,1] [,2]

[1,] 0.6396922 0.3666823
[2,] 0.9783528 0.5608082
[3,] 6.3687005 3.6506459
[4,] 3.9698546 2.2755873
[5,] 0.1881448 0.1078477

, , 2, 1, 2

[,1] [,2]

[1,] 909.8165 374.4547
[2,] 1391.4841 572.6955
[3,] 9058.0262 3728.0274
[4,] 5646.2143 2323.8221
[5,] 267.5931 110.1337

, , 1, 2, 2

[,1] [,2]

[1,] 0.5157747 0.25911597

[2,] 0.7888318 0.39629501

[3,] 5.1349918 2.57972812

[4,] 3.2008368 1.60804323

[5,] 0.1516984 0.07621058

, , 2, 2, 2

[,1] [,2]

[1,] 707.4176 333.16987

[2,] 1081.9328 509.55393

[3,] 7042.9666 3317.00007

[4,] 4390.1505 2067.61304

[5,] 208.0640 97.99114

```
> model17a=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Age+
BScgrade*State*Gender*Mode, data=n)
```

```
> model17b=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Age+
BScgrade*Gender*Age*Mode, data=n)
```

```
> model17c=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Age+
State*Gender*Age*Mode, data=n)
```

```
> model17d=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Mode+
BScgrade*State*Age*Mode, data=n)
```

```
> model17e=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Mode+
BScgrade*Gender*Age*Mode, data=n)
```

```
> model17f=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Mode+
State*Gender*Age*Mode, data=n)
```

```
> model17g=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Age*Mode+
BScgrade*Gender*Age*Mode, data=n)
```

```
> model17h=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Age*Mode+
State*Gender*Age*Mode, data=n)
```

```
> model17i=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Gender*Age*Mode+
State*Gender*Age*Mode, data=n)
```

```
> model18=loglm(~BScgrade+State+Gender+Age+Mode+
BScgrade*State*Gender*Age*Mode, data=n)
```

```
> model17a; model17b; model17c; model17d; model17e; model17f; model17g; model17h;
model17i; model18
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
State * Gender * Age + BScgrade * State * Gender * Mode,
data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 21.47697 20 0.3695523

Pearson 20.70749 20 0.4145259

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
State * Gender * Age + BScgrade * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 43.93290 20 0.001536376

Pearson 42.81162 20 0.002164770

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Age + State * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 77.58305 32 1.174378e-05

Pearson 72.17188 32 6.199844e-05

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Mode + BScgrade * State * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 24.60369 20 0.2170156

Pearson 23.95280 20 0.2444600

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Mode + BScgrade * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 13.40143 20 0.8595041

Pearson 13.44084 20 0.8576788

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Mode + State * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 39.75069 32 0.1630520

Pearson 38.61374 32 0.1954771

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Age * Mode + BScgrade * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 22.76011 20 0.3007267

Pearson 22.69919 20 0.3038052

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Age * Mode + State * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 73.74410 32 3.851597e-05

Pearson 70.67786 32 9.688644e-05

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
```

Gender * Age * Mode + State * Gender * Age * Mode, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 73.90678 32 3.665176e-05

Pearson 76.54197 32 1.626252e-05

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
State * Gender * Age * Mode, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 0 0 1

Pearson 0 0 1

APPENDIX 4

IMPLEMENTATION OF ALGORITHMS USING REAL LIFE DATA FOR ESTIMATION OF GOODNESS OF FITS AND BEST MODEL FITS OF LOG LINEAR MODEL FOR 3, 4 AND 5 DIMENSIONAL CONTINGENCY TABLES.

3 - Dimensional Analysis and Results

```
> I=2; J= 3; K= 2
>n=c( 55, 67, 16, 44, 8, 45, 48, 66, 20, 52, 18, 71)
> n=array(n, dim=c(I, J, K))
> dimnames(n)=list(Chicksloss=c("Yes", "No"), Age=c(1, 2, 3), Breed=c("Broiler", "Old
layer"))
> mu=log(sum(n)/(I*J*K))
> mu_1=mu_2=mu_3=0
> mu_12=matrix(0, I, J); mu_13=matrix(0, I, K); mu_23=matrix(0, J, K)
> for (i in 1:I){
+ mu_1[i]=log(sum(n[i, ])/(J*K))-mu
```

```

+ for (j in 1:J){
+ mu_2[j]=log(sum(n[,j, ])/(I*K))-mu
+ for (k in 1:K){
+ mu_3[k]=log(sum(n[, , k])/(I*J))-mu
+ }
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ mu_13[i, k]=log(sum(n[i, , k])/J)-mu-mu_1[i]-mu_3[k]
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ mu_12[i, j]=log(sum(n[i, j, ])/K)-mu-mu_1[i]-mu_2[j]
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ mu_23[j, k]=log(sum(n[, j, k])/I)-mu-mu_2[j]-mu_3[k]
+ }
+ }
> m_1=m_2=m_3=m_4=array(0, dim=c(I, J, K))
> for(i in 1:I){
+ for (j in 1:J){

```

```

+ for (k in 1:K){
+ m_1[i, j, k]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k])
+ m_2[i, j, k]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_13[i, k])
+ m_3[i, j, k]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_12[i, k])
+ m_4[i, j, k]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_23[j, k])
+ }
+ }
+ }
> model1=loglm(~Chicksloss + Age + Breed, data=n)
> model2=loglm(~Chicksloss + Age+ Breed + Chicksloss*Breed, data=n)
> model3=loglm(~Chicksloss + Age+ Breed + Chicksloss*Age, data=n)
> model4=loglm(~Chicksloss + Age+ Breed + Age*Breed, data=n)
> model1; model2; model3; model4

```

Call:

```
loglm(formula = ~Chicksloss + Age + Breed, data = n)
```

Statistics:

```
X^2 df P(> X^2)
```

```
Likelihood Ratio 37.13769 7 4.41719e-06
```

```
Pearson 36.33465 7 6.26767e-06
```

Call:

```
loglm(formula = ~Chicksloss + Age + Breed + Chicksloss * Breed,
      data = n)
```

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 36.81976 6 1.909229e-06

Pearson 35.20502 6 3.932731e-06

Call:

```
loglm(formula = ~Chicksloss + Age + Breed + Chicksloss * Age,  
data = n)
```

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 8.280408 5 0.1414439

Pearson 8.181170 5 0.1465296

Call:

```
loglm(formula = ~Chicksloss + Age + Breed + Age * Breed, data = n)
```

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 29.68738 5 1.699225e-05

Pearson 28.75595 5 2.588877e-05

```
> m_5a=m_5b=m_5c=m_6=array(0, dim=c(I, J, K))
```

```
> for(i in 1:I){
```

```
+ for(j in 1:J){
```

```
+ for(k in 1:K){
```

```
+ m_5a[i, j, k]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_12[i, j]+mu_13[i,k])
```

```
+ m_5b[i, j, k]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_12[i, j]+mu_23[j,k])
```

```
+ m_5c[i, j, k]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_13[i, k]+mu_23[j,k])
```

```

+ m_6[i, j, k]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_12[i, j]+mu_13[i,k]+mu_23[j, k])
+ }
+ }
+ }
> model5a=loglm(~Chicksloss + Age+ Breed + Chicksloss*Age + Chicksloss *Breed, data=n)
> model5b=loglm(~Chicksloss + Age+ Breed + Chicksloss*Age + Age*Breed, data=n)
> model5c=loglm(~Chicksloss + Age+ Breed + Chicksloss* Breed + Age*Breed, data=n)
> model6=loglm(~Chicksloss + Age+ Breed + Chicksloss*Age + Chicksloss *Breed +
Age*Breed , data=n)
>model5a; model5b; model5c; model6

```

Call:

```

loglm(formula = ~Chicksloss + Age + Breed + Chicksloss * Age +
      Chicksloss * Breed, data = n)

```

Statistics:

X² df P(> X²)

Likelihood Ratio 7.962476 4 0.09296247

Pearson 7.853555 4 0.09709239

Call:

```

loglm(formula = ~Chicksloss + Age + Breed + Chicksloss * Age +
      Age * Breed, data = n)

```

Statistics:

X² df P(> X²)

Likelihood Ratio 0.8301007 3 0.8422546

Pearson 0.8172251 3 0.8453427

Call:

```
loglm(formula = ~Chicksloss + Age + Breed + Chicksloss * Breed +  
Age * Breed, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 29.36945 4 6.576301e-06

Pearson 28.32066 4 1.073878e-05

Call:

```
loglm(formula = ~Chicksloss + Age + Breed + Chicksloss * Age +  
Chicksloss * Breed + Age * Breed, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 0.8263105 2 0.6615596

Pearson 0.8145226 2 0.6654703

```
> model7=loglm(~Chicksloss + Age+ Breed + Chicksloss*Age *Breed, data=n); model7
```

Call:

```
loglm(formula = ~Chicksloss + Age + Breed + Chicksloss * Age *  
Breed, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 0 0 1

Pearson 0 0 1

4-Dimensional Analysis and Results

```

> I=5; J=2; K=2; L=2

> n=c(3, 41, 45, 20, 1, 5, 35, 17, 4, 1, 2, 69, 44, 9, 1, 7, 54, 29, 3, 1, 1, 31, 58, 32, 1, 2, 31, 21, 7,
2, 3, 41, 32, 9, 1, 4, 37, 11, 3, 1)

> n=c(3, 41, 45, 20, 1, 2, 69, 44, 9, 1, 1, 31, 58, 32, 1, 3, 41, 32, 9, 1, 5, 35, 17, 4, 1, 7, 54, 29, 3,
1, 2, 31, 21, 7, 2, 4, 37, 11, 3, 1)

> n=array(n, dim=c(I, J, K, L))

> dimnames(n)=list(BScgrade=c("pass", "Third class", "Lower Division", "Upper Division",
"First Class"), State=c("Indigene", "Non-indigene"), Gender=c("Male", "Female"),
Age=c("Under 26", "26 and over"))

> mu=log(sum(n)/(I*J*K*L))

> mu_1=mu_2=mu_3=mu_4=array(0, dim=c(I, J, K, L))

> mu_12=matrix(0, I, J); mu_13=matrix(0, I, K); mu_14=matrix(0, I, L); mu_23=matrix(0, J, K);
mu_24=matrix(0, J, L); mu_34=matrix(0, K, L)

> for (i in 1:I){
+ for (l in 1:L){
+ mu_14[i, l]=log(sum(n[i, , l])/J*K)-mu-mu_1[i]-mu_4[l]
+ }
+ }

> for (i in 1:I){
+ for (j in 1:J){
+ mu_12[i, j]=log(sum(n[i, j, , ])/K*L)-mu-mu_1[i]-mu_2[j]
+ }
+ }

> for (i in 1:I){
+ for (k in 1:K){
+ mu_13[i, k]=log(sum(n[i, , k, ])/J*L)-mu-mu_1[i]-mu_3[k]
+ }

```

```

+ }
> for (j in 1:J){
+ for (k in 1:K){
+ mu_23[j, k]=log(sum(n[,j, k, ])/I*L)-mu-mu_2[j]-mu_3[k]
+ }
+ }
> for (j in 1:J){
+ for (l in 1:L){
+ mu_24[j, l]=log(sum(n[,j, , l])/I*K)-mu-mu_2[j]-mu_4[l]
+ }
+ }
> for (k in 1:K){
+ for (l in 1:L){
+ mu_34[k, l]=log(sum(n[, , k, l])/I*J)-mu-mu_3[k]-mu_4[l]
+ }
+ }
> m_1=m_2=m_3=m_4=m_5=m_6=m_7=array(0, dim=c(I, J, K, L))
> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ m_1[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_4[l])
+ m_2[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, k])
+ m_3[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_12[i, j])
+ m_4[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_13[i, k])

```

```

+ m_5[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_23[j, k])
+ m_6[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_24[j, k])
+ m_7[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_34[k, l])
+ }
+ }
+ }
+ }
> model1=loglm(~BScgrade+State+Gender+Age, data=n)
> model2=loglm(~BScgrade+State+Gender+Age+BScgrade*Age, data=n)
> model3=loglm(~BScgrade+State+Gender+Age+BScgrade*State, data=n)
> model4=loglm(~BScgrade+State+Gender+Age+BScgrade*Gender, data=n)
> model5=loglm(~BScgrade+State+Gender+Age+State*Gender, data=n)
> model6=loglm(~BScgrade+State+Gender+Age+State*Age, data=n)
> model7=loglm(~BScgrade+State+Gender+Age+Gender*Age, data=n)
> model1; model2; model3; model4; model5; model6; model7

```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age, data = n)
```

Statistics:

```
X^2 df P(> X^2)
```

```
Likelihood Ratio 102.6573 32 2.469969e-09
```

```
Pearson 111.4646 32 1.007785e-10
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *
```

```
Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 62.29832 28 2.054731e-04

Pearson 65.53400 28 7.637938e-05

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
State, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 69.29393 28 2.334267e-05

Pearson 69.04752 28 2.525616e-05

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
Gender, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 92.65951 28 7.576797e-09

Pearson 94.95682 28 3.274269e-09

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + State * Gender,  
data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 91.08621 31 8.010669e-08

Pearson 94.50826 31 2.432876e-08

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + State * Age,  
      data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 99.30338 31 4.446626e-09

Pearson 103.93329 31 8.360928e-10

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Gender * Age,  
      data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 101.6678 31 1.900744e-09

Pearson 108.6935 31 1.458038e-10

```
> m_8a=m_8b=m_8c=m_8d=m_8e=array(0, dim=c(I, J, K, L))
```

```
> for(i in 1:I){
```

```
+ for(j in 1:J){
```

```
+ for(k in 1:K){
```

```
+ for(l in 1:L){
```

```
+ m_8a[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_12[i, j])
```

```
+ m_8b[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_13[i, k])
```

```
+ m_8c[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_23[j, k])
```

```
+ m_8d[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_24[j, l])
```

```
+ m_8e[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_34[k, l])
```

```
+ }
```

```
+ }
```



```

+ }
+ }
> model8a=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*State, data=n)
> model8b=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*Gender, data=n)
> model8c=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ State*Gender, data=n)
> model8d=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ State*Age, data=n)
> model8e=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ Gender*Age, data=n)
> model8a ; model8b; model8c ; model8d; model8e

```

Call:

```

loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *
      Age + BScgrade * State, data = n)

```

Statistics:

X² df P(> X²)

Likelihood Ratio 28.93498 24 0.2225874

Pearson 29.45824 24 0.2033606

Call:

```

loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *
      Age + BScgrade * Gender, data = n)

```

Statistics:

X² df P(> X²)

Likelihood Ratio 52.30056 24 0.0007147718

Pearson 51.56515 24 0.0008912025

Call:

```

loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *
      Age + State * Gender, data = n)

```

Statistics:

X² df P(> X²)

Likelihood Ratio 50.72726 27 0.003746636

Pearson 49.60753 27 0.005049639

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
Age + State * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 58.94443 27 0.0003621419

Pearson 61.70485 27 0.0001569957

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
Age + Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 61.30886 27 1.772439e-04

Pearson 64.02193 27 7.650879e-05

```
> m_9a=m_9b=m_9c=m_9d=array(0, dim=c(I, J, K, L))
```

```
> for(i in 1:I){
```

```
+ for(j in 1:J){
```

```
+ for(k in 1:K){
```

```
+ for(l in 1:L){
```

```
+ m_9a[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_12[i,  
j]+mu_13[i, k])
```

+ m_9b[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+mu_12[i, j]+mu_23[j, k])

+ m_9c[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+ mu_12[i, j]+mu_24[j, l])

+ m_9d[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_14[i, l]+ mu_12[i, j]+ mu_34[k, l])

+ }

+ }

+ }

+ }

> model9a=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*State+ BScgrade*Gender, data=n)

> model9b=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*State+ State*Gender, data=n)

> model9c=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*State+ State*Age, data=n)

> model9d=loglm(~BScgrade+State+Gender+Age+BScgrade*Age+ BScgrade*State+ Gender*Age, data=n)

> model9a ; model9b; model9c ; model9d

Call:

loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *
Age + BScgrade * State + BScgrade * Gender, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 18.93722 20 0.5259099

Pearson 19.58890 20 0.4838953

Call:

loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *

Age + BScgrade * State + State * Gender, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 17.36393 23 0.7910194

Pearson 17.50512 23 0.7837521

Call:

loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *

Age + BScgrade * State + State * Age, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 28.49333 23 0.1976995

Pearson 28.78692 23 0.1874785

Call:

loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *

Age + BScgrade * State + Gender * Age, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 27.94553 23 0.2178485

Pearson 28.19695 23 0.2084256

>mu_123=array(0, dim=c(I, J, K)); mu_124=array(0, dim=c(I, J, L)); mu_134= array(0, dim=c(I, K, L)); mu_234= array(0, dim=c(J, K, L))

> for (i in 1:I){

+ for (j in 1:J){

+ for (k in 1:K){

```

+ mu_123[i, j, k]=log(sum(n[i, j, k, ])/L)-mu-mu_1[i]-mu_2[j]-mu_3[k]-mu_12[i, j]-mu_13[i, k]-
mu_23[j, k]

+ }

+ }

+ }

> for (i in 1:I){
+ for (j in 1:J){
+ for (l in 1:L){

+ mu_124[i, j, l]=log(sum(n[i, j, , l])/K)-mu-mu_1[i]-mu_2[j]-mu_4[l]-mu_12[i, j]-mu_14[i, l]-
mu_24[j, l]

+ }

+ }

+ }

> for (i in 1:I){
+ for (k in 1:K){
+ for (l in 1:L){

+ mu_134[i, k, l]=log(sum(n[i, , k, l])/J)-mu-mu_1[i]-mu_3[k]-mu_4[l]-mu_13[i, k]-mu_14[i, l]-
mu_34[k, l]

+ }

+ }

+ }

> for (j in 1:J){
+ for (k in 1:K){
+ for (l in 1:L){

+ mu_234[j, k, l]=log(sum(n[, j, k, l])/I)-mu-mu_2[j]-mu_3[k]-mu_4[l]-mu_23[j, k]-mu_24[j, l]-
mu_34[k, l]

+ }

```

```

+ }
+ }
> m_10a=m_10b=m_10c=m_10d=array(0, dim=c(I, J, K, L))
> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ m_10a[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_123[i, j, k])
+ m_10b[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_124[i, j, l])
+ m_10c[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_134[i, k, l])
+ m_10d[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_234[j, k, l])
+ }
+ }
+ }
+ }
> model10a=loglm(~BScgrade+State+Gender+Age+ BScgrade*State *Gender, data=n)
> model10b=loglm(~BScgrade+State+Gender+Age+ BScgrade*State*Age, data=n)
> model10c=loglm(~BScgrade+State+Gender+Age+ BScgrade* Gender*Age, data=n)
> model10d=loglm(~BScgrade+State+Gender+Age+ State* Gender*Age, data=n)
> model10a ; model10b; model10c ; model10d

```

Call:

```

loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *
      State * Gender, data = n)

```

Statistics:

```

X^2 df P(> X^2)

```

Likelihood Ratio 46.91952 19 0.0003667590

Pearson 45.38412 19 0.0006054104

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
State * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 26.24509 19 0.1235033

Pearson 25.89426 19 0.1331625

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 49.53238 19 0.0001535136

Pearson 48.21890 19 0.0002384909

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + State * Gender *  
Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 87.11441 28 5.562476e-08

Pearson 86.69165 28 6.462981e-08

```
> m_11a=m_11b=m_11c=m_11d=m_11e=array(0, dim=c(I, J, K, L))
```

```
> for(i in 1:I){
```

```

+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ m_11a[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_123[i, j, k]+mu_124[i, j,
l])
+ m_11b[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_123[i, j, k]+mu_134[i, k,
l])
+ m_11c[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_123[i, j, k]+mu_234[j, k,
l])
+ m_11d[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_124[i, j, l]+mu_234[j, k,
l])
+ m_11e[i, j, k, l]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_134[i, k, l]+mu_234[j, k,
l])
+ }
+ }
+ }
+ }

> model11a=loglm(~BScgrade+State+Gender+Age+BScgrade*State *Gender +
BScgrade*State*Age, data=n)

> model11b=loglm(~BScgrade+State+Gender+Age+ BScgrade*State *Gender+ BScgrade*
Gender*Age, data=n)

> model11c=loglm(~BScgrade+State+Gender+Age+ BScgrade*State *Gender+ State*
Gender*Age, data=n)

> model11d=loglm(~BScgrade+State+Gender+Age+ BScgrade*State*Age+ State*
Gender*Age, data=n)

> model11e=loglm(~BScgrade+State+Gender+Age+ BScgrade* Gender*Age+ State*
Gender*Age, data=n)

> model11a ; model11b; model11c ; model11d; model11e

```

Call:


```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
State * Gender + BScgrade * State * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 3.870686 10 0.9529921

Pearson 3.815083 10 0.9553035

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
State * Gender + BScgrade * Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 3.792395 10 0.9562268

Pearson 3.792102 10 0.9562386

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
State * Gender + State * Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 42.94772 16 0.0002845600

Pearson 42.21511 16 0.0003664753

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *  
State * Age + State * Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 14.05613 16 0.5945325

Pearson 13.90669 16 0.6056651

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *
```

```
Gender * Age + State * Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 34.97898 16 0.004000704

Pearson 34.38458 16 0.004820282

```
> model12=loglm(~BScgrade+State+Gender+Age+BScgrade*State *Gender *Age, data=n);  
model12
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + BScgrade *
```

```
State * Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 0 0 1

Pearson 0 0 1

5-Dimensional Analysis and Results

```
> I=5; J=2; K=2; L=2; M=2
```

```
>n=c(7, 14, 61, 22,1, 1, 6, 17, 28, 1, 4, 11, 78, 42, 2, 2, 7, 33, 32, 1,12, 8, 45, 19, 1, 4, 9, 14, 22,  
2, 12, 8, 63, 31, 2, 4, 2, 24, 21, 1, 4, 10, 52, 31, 1, 8, 11, 17, 19, 1, 1, 2, 94, 45, 1, 1, 1, 27, 17, 2,  
2, 4, 28, 39, 1, 2, 4, 21, 15, 1, 1, 1, 81, 28, 1, 3, 6, 22, 11, 1)
```

```

> n=array(n, dim=c(I, J, K, L, M))

> dimnames(n)=list(BScgrade=c("pass", "Third class", "Lower Division", "Upper Division",
"First Class"), State=c("Indigene", "Non-indigene"), Gender=c("Male", "Female"),
Age=c("Under 26", "26 and over"), Mode=c("Jamb", "Pre science"))

> mu=log(sum(n)/(I*J*K*L*M))

> mu_1=mu_2=mu_3=mu_4=mu_5= array(0, dim=c(I, J, K, L, M))

> mu_12=matrix(0, I, J); mu_13=matrix(0, I, K); mu_14=matrix(0, I, L); mu_15=matrix(0, I,
M); mu_23=matrix(0, J, K); mu_24=matrix(0, J, L); mu_25=matrix(0, J, M); mu_34=matrix(0,
K, L); mu_35=matrix(0, K, M); mu_45=matrix(0, L, M)

> for (i in 1:I){
+ mu_1[i]=log(sum(n[i, , , ])/(J*K*L*M))-mu
+ for (j in 1:J){
+ mu_2[j]=log(sum(n [, j, , ])/(I*K*L*M))-mu
+ for (k in 1:K){
+ mu_3[k]=log(sum(n[, , k, ])/(I*J*L*M))-mu
+ for (l in 1:L){
+ mu_4[l]=log(sum(n[, , , l, ])/(I*J*K*M))-mu
+ for (m in 1:M){
+ mu_5[m]=log(sum(n[, , , , m ])/(I*J*K*L))-mu
+ }
+ }
+ }
+ }
+ }

> for (i in 1:I){
+ for (j in 1:J){
+ mu_12[i, j]=log(sum(n[i, j, , ])/K*L*M)-mu-mu_1[i]-mu_2[j]

```

```

+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ mu_23[j, k]=log(sum(n[,j, k, , ])/I*L*M)-mu-mu_2[j]-mu_3[k]
+ }
+ }
> for (j in 1:J){
+ for (l in 1:L){
+ mu_24[j, l]=log(sum(n[,j, , l, ])/I*K*M)-mu-mu_2[j]-mu_4[l]
+ }
+ }
> for (j in 1:J){
+ for (m in 1:M){
+ mu_25[j, m]=log(sum(n[,j, , , m])/I*K*L)-mu-mu_2[j]-mu_5[m]
+ }
+ }
> for (i in 1:I){
+ for (k in 1:K){
+ mu_13[i, k]=log(sum(n[i, , k, , ])/J*L*M)-mu-mu_1[i]-mu_3[k]
+ }
+ }
> for (i in 1:I){
+ for (l in 1:L){
+ mu_14[i, l]=log(sum(n[i, , , l, ])/J*K*M)-mu-mu_1[i]-mu_4[l]

```

```

+ }
+ }
> for (i in 1:I){
+ for (m in 1:M){
+ mu_15[i, m]=log(sum(n[i, , , m])/J*K*L)-mu-mu_1[i]-mu_5[m]
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ mu_23[j, k]=log(sum(n[j, k, , ])/I*L*M)-mu-mu_2[j]-mu_3[k]
+ }
+ }
> for (j in 1:J){
+ for (l in 1:L){
+ mu_24[j, l]=log(sum(n[j, , l, ])/I*K*M)-mu-mu_2[j]-mu_4[l]
+ }
+ }
> for (j in 1:J){
+ for (m in 1:M){
+ mu_25[j, m]=log(sum(n[j, , , m])/I*K*L)-mu-mu_2[j]-mu_5[m]
+ }
+ }
> for (k in 1:K){
+ for (l in 1:L){
+ mu_34[k, l]=log(sum(n[, , k, l])/I*J*M)-mu-mu_3[k]-mu_4[l]

```

```

+ }
+ }
> for (k in 1:K){
+ for (m in 1:M){
+ mu_35[k, m]=log(sum(n[, , k, , m])/I*J*L)-mu-mu_3[k]-mu_5[m]
+ }
+ }
> for (l in 1:L){
+ for (m in 1:M){
+ mu_45[l, m]=log(sum(n[, , , l, m])/I*J*K)-mu-mu_4[l]-mu_5[m]
+ }
+ }
> m_1=m_2=m_3=m_4=m_5=m_6=m_7=m_8=m_9=m_10=m_11=array(0, dim=c(I, J, K,
L,M))
> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ for(m in 1:M){
+ m_1[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+mu_4[l]+mu_5[m])
+ m_2[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i, m])
+ m_3[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_12[i, j])
+ m_4[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_13[i, k])
+ m_5[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_14[i, l])
+ m_6[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+mu_23[j, k])

```

```

+ m_7[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+mu_24[j, l])
+ m_8[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_25[j, m])
+ m_9[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_34[k, l])
+ m_10[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_35[k, m])
+ m_11[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_45[l, m])
+ }
+ }
+ }
+ }
+ }

```

```

> model1=loglm(~BScgrade+State+Gender+Age+Mode, data=n)
> model2=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode, data=n)
> model3=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*State, data=n)
> model4=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Gender, data=n)
> model5=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Age, data=n)
> model6=loglm(~BScgrade+State+Gender+Age+Mode+State*Gender, data=n)
> model7=loglm(~BScgrade+State+Gender+Age+Mode+State*Age, data=n)
> model8=loglm(~BScgrade+State+Gender+Age+Mode+State*Mode, data=n)
> model9=loglm(~BScgrade+State+Gender+Age+Mode+Gender*Age, data=n)
> model10=loglm(~BScgrade+State+Gender+Age+Mode+Gender*Mode, data=n)
> model11=loglm(~BScgrade+State+Gender+Age+Mode+Age*Mode, data=n)
> model1; model2; model3; model4; model5; model6; model7; model8; model9; model10;
model11

```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 172.9839 71 1.729357e-10

Pearson 176.3949 71 6.088841e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 159.6621 67 1.562371e-09

Pearson 173.3234 67 2.314782e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 141.2813 67 3.136906e-07

Pearson 139.1354 67 5.645593e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Gender, data = n)
```


Statistics:

X² df P(> X²)

Likelihood Ratio 139.1010 67 5.698819e-07

Pearson 141.8967 67 2.646954e-07

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Age, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 166.3272 67 2.054197e-10

Pearson 171.4614 67 4.159828e-11

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *
Gender, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 167.9625 70 5.003790e-10

Pearson 167.2945 70 6.118129e-10

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *
Age, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 172.9646 70 1.092613e-10

Pearson 176.5658 70 3.592260e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *  
Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 171.2079 70 1.870341e-10

Pearson 174.1919 70 7.490208e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + Gender *  
Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 172.9732 70 1.089721e-10

Pearson 176.3752 70 3.811362e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + Gender *  
Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 172.9719 70 1.090137e-10

Pearson 176.5359 70 3.625722e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + Age *
```

```
Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 172.8484 70 1.132279e-10

Pearson 175.9629 70 4.331746e-11

```
> m_12a=m_12b=m_12c=m_12d=m_12e=m_12f=m_12g=m_12h=m_12i=array(0, dim=c(I, J, K, L,M))
```

```
> for(i in 1:I){
```

```
+ for(j in 1:J){
```

```
+ for(k in 1:K){
```

```
+ for(l in 1:L){
```

```
+ for(m in 1:M){
```

```
+ m_12a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i, m]+mu_12[i, j])
```

```
+ m_12b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i, m]+mu_13[i, k])
```

```
+ m_12c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i, m]+mu_14[i, l])
```

```
+ m_12d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+mu_23[j, k])
```

```

+ m_12e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+
mu_24[j, l])

+ m_12f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_25[j, m])

+ m_12g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_34[k, l])

+ m_12h[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_35[k, m])

+ m_12i[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_45[l, m])

+ }

+ }

+ }

+ }

+ }

> model12a=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*State,
data=n)

>
model12b=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender,
data=n)

> model12c=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Age,
data=n)

> model12d=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+State*Gender,
data=n)

> model12e=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+State*Age,
data=n)

> model12f=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+State*Mode,
data=n)

> model12g=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+Gender*Age,
data=n)

```

```
> model12h=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+Gender*Mode,
data=n)
```

```
> model12i=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+Age*Mode,
data=n)
```

```
> model12a; model12b; model12c; model12d; model12e; model12f; model12g; model12h;
model12i
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Mode + BScgrade * State, data = n)
```

Statistics:

	X ²	df	P(> X ²)
--	----------------	----	----------------------

Likelihood Ratio	127.9595	63	2.519681e-06
------------------	----------	----	--------------

Pearson	131.5539	63	9.507707e-07
---------	----------	----	--------------

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Mode + BScgrade * Gender, data = n)
```

Statistics:

	X ²	df	P(> X ²)
--	----------------	----	----------------------

Likelihood Ratio	125.7791	63	4.501963e-06
------------------	----------	----	--------------

Pearson	134.4006	63	4.328119e-07
---------	----------	----	--------------

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Mode + BScgrade * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 153.0054 63 1.883332e-09

Pearson 174.6249 63 1.989187e-12

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Mode + State * Gender, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 154.6406 66 4.478824e-09

Pearson 163.1955 66 3.365246e-10

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Mode + State * Age, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 159.6427 66 9.965438e-10

Pearson 173.5571 66 1.309208e-11

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Mode + State * Mode, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 157.8861 66 1.695212e-09

Pearson 172.8167 66 1.657330e-11

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Mode + Gender * Age, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 159.6514 66 9.939356e-10

Pearson 173.3912 66 1.380318e-11

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Mode + Gender * Mode, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 159.6501 66 9.943111e-10

Pearson 173.3093 66 1.416811e-11

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Mode + Age * Mode, data = n)

Statistics:

X² df P(> X²)

Likelihood Ratio 159.5265 66 1.032304e-09

Pearson 172.8867 66 1.620848e-11

```
> m_13a=m_13b=m_13c=m_13d=m_13e=m_13f=m_13g=m_13h=array(0, dim=c(I, J, K, L,M))
> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ for(m in 1:M){
+ m_13a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i,
m]+mu_13[i, k]+mu_12[i, j])
+ m_13b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i, m]+
mu_13[i, k]+ mu_14[i, l])
+ m_13c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+mu_23[j, k])
+ m_13d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+mu_24[j, l])
+ m_13e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_25[j, m])
+ m_13f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_34[k, l])
+ m_13g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_35[k, m])
+ m_13h[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_45[l, m])
+ }
+ }
+ }
+ }
```


+ }

>

```
model13a=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State, data=n)
```

>

```
model13b=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*Age, data=n)
```

>

```
model13c=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
State*Gender, data=n)
```

>

```
model13d=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
State*Age, data=n)
```

>

```
model13e=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
State*Mode, data=n)
```

>

```
model13f=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
Gender*Age, data=n)
```

```
>model13g=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender
+ Gender*Mode, data=n)
```

```
>model13h=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender
+ Age*Mode, data=n)
```

```
> model13a; model13b; model13c; model13d; model13e; model13f; model13g; model13h
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
      Mode + BScgrade * Gender + BScgrade * State, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 94.07656 59 0.002505307

Pearson 96.43592 59 0.001513309

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + BScgrade * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 119.1225 59 5.987478e-06

Pearson 132.3240 59 1.502593e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + State * Gender, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 120.7577 62 1.165599e-05

Pearson 128.7268 62 1.397857e-06

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + State * Age, data = n)
```

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 125.7598 62 3.119768e-06

Pearson 134.5223 62 2.793240e-07

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Mode + BScgrade * Gender + State * Mode, data = n)

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 124.0031 62 4.981984e-06

Pearson 133.9196 62 3.310712e-07

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Mode + BScgrade * Gender + Gender * Age, data = n)

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 125.7684 62 3.112556e-06

Pearson 134.4744 62 2.831292e-07

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Mode + BScgrade * Gender + Gender * Mode, data = n)

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 125.3519 62 3.479651e-06

Pearson 134.4898 62 2.818970e-07

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
      Mode + BScgrade * Gender + Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 125.6436 62 3.218413e-06

Pearson 134.2637 62 3.004738e-07

```
> m_14a=m_14b=m_14c=m_14d=m_14e=m_14f=m_14g=array(0, dim=c(I, J, K, L,M))  
> for(i in 1:I){  
+ for(j in 1:J){  
+ for(k in 1:K){  
+ for(l in 1:L){  
+ for(m in 1:M){  
+ m_14a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i,  
m]+mu_13[i, k]+ mu_12[i, j]+ mu_14[i, l] )  
+ m_14b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i,  
m]+mu_13[i, k]+ mu_12[i, j]+ mu_23[j, k] )  
+ m_14c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_15[i, m]+  
mu_13[i, k]+ mu_12[i, j] + mu_24[j, l])  
+ m_14d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+ mu_5[m]+ mu_15[i, m]+  
mu_13[i, k]+ mu_12[i, j] + mu_25[j, m])  
+ m_14e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+  
mu_13[i, k]+ mu_12[i, j] + mu_34[k, l])
```

```

+ m_14f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_35[k, m])

+ m_14g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_15[i, m]+
mu_13[i, k]+ mu_12[i, j] + mu_45[l, m])

+ }

+ }

+ }

+ }

+ }

>
modell14a=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ BScgrade*Age, data=n)

>
modell14b=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ State*Gender, data=n)

>
modell14c=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ State*Age, data=n)

>
modell14d=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ State*Mode, data=n)

>
modell14e=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ Gender*Age, data=n)

>
modell14f=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ Gender*Mode, data=n)

>
modell14g=loglm(~BScgrade+State+Gender+Age+Mode+BScgrade*Mode+BScgrade*Gender+
BScgrade*State+ Age*Mode, data=n)

> modell14a; modell14b; modell14c; modell14d; modell14e; modell14f; modell14g

```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
  Mode + BScgrade * Gender + BScgrade * State + BScgrade *  
  Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 87.41988 55 0.003528881

Pearson 93.03104 55 0.001031502

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
  Mode + BScgrade * Gender + BScgrade * State + State * Gender,  
  data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 91.75409 58 0.003132519

Pearson 94.07371 58 0.001915817

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
  Mode + BScgrade * Gender + BScgrade * State + State * Age,  
  data = n)
```

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 94.05723 58 0.001922623

Pearson 96.53581 58 0.001118463

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + BScgrade * State + State * Mode,  
data = n)
```

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 93.04289 58 0.002388163

Pearson 96.43197 58 0.001144516

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + BScgrade * State + Gender * Age,  
data = n)
```

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 94.06586 58 0.001919054

Pearson 96.45887 58 0.001137714

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Mode + BScgrade * Gender + BScgrade * State + Gender * Mode,  
data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 93.64933 58 0.002098519

Pearson 95.98648 58 0.001262949

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
      Mode + BScgrade * Gender + BScgrade * State + Age * Mode,  
      data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 93.94103 58 0.001971264

Pearson 96.38938 58 0.001155369

```
> mu_123=array(0, dim=c(I, J, K)); mu_124=array(0, dim=c(I, J, L)); mu_125=array(0, dim=c(I,  
J, M)); mu_134= array(0, dim=c(I, K, L)); mu_135=array(0, dim=c(I, K, M)); mu_145=array(0,  
dim=c(I, L, M)); mu_234= array(0, dim=c(J, K, L)); mu_235=array(0, dim=c(J, K, M));  
mu_245=array(0, dim=c(J, L, M)); mu_345=array(0, dim=c(K, L, M)); mu_1234=array(0,  
dim=c(I, J, K, L)); mu_1235=array(0, dim=c(I, J, K, M)); mu_1245=array(0, dim=c(I, J, L, M));  
mu_1345=array(0, dim=c(I, K, L, M)); mu_2345=array(0, dim=c(J, K, L, M))
```

```
> for (i in 1:I){
```

```
+ for (j in 1:J){
```

```
+ for (k in 1:K){
```

```
+ mu_123[i, j, k]=log(sum(n[i, j, k, , ])/L*M)-mu_1[i]-mu_2[j]-mu_3[k]-mu_12[i, j]-  
mu_13[i, k]-mu_23[j, k]
```

```
+ }
```

```
+ }
```



```

+ }
> for (i in 1:I){
+ for (j in 1:J){
+ for (l in 1:L){
+ mu_124[i, j, l]=log(sum(n[i, j, , l])/K*M)-mu-mu_1[i]-mu_2[j]-mu_4[l]-mu_12[i, j]-mu_14[i,
l]-mu_24[j, l]
+ }
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ for (m in 1:M){
+ mu_125[i, j, m]=log(sum(n[i, j, , , m])/K*L)-mu-mu_1[i]-mu_2[j]-mu_5[m]-mu_12[i, j]-
mu_15[i, m]-mu_25[j, m]
+ }
+ }
+ }
> for (i in 1:I){
+ for (k in 1:K){
+ for (l in 1:L){
+ mu_134[i, k, l]=log(sum(n[i, , k, l, ])/J*M)-mu-mu_1[i]-mu_3[k]-mu_4[l]-mu_13[i, k]-
mu_14[i, l]-mu_34[k, l]
+ }
+ }
+ }
> for (i in 1:I){

```

```

+ for (k in 1:K){
+ for (m in 1:M){
+ mu_135[i, k, m]=log(sum(n[i, , k, , m])/J*L)-mu-mu_1[i]-mu_3[k]-mu_5[m]-mu_13[i, k]-
mu_15[i, m]-mu_35[k, m]
+ }
+ }
+ }
> for (i in 1:I){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_145[i, l, m]=log(sum(n[i, , , l, m])/J*L)-mu-mu_1[i]-mu_4[l]-mu_5[m]-mu_14[i, l]-
mu_15[i, m]-mu_45[l, m]
+ }
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ for (l in 1:L){
+ mu_234[j, k, l]=log(sum(n[, j, k, l, ])/I*M)-mu-mu_2[j]-mu_3[k]-mu_4[l]-mu_23[j, k]-
mu_24[j, l]-mu_34[k, l]
+ }
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ for (m in 1:M){

```

```

+ mu_235[j, k, m]=log(sum(n[, j, k, , m])/I*L)-mu-mu_2[j]-mu_3[k]-mu_5[m]-mu_23[j, k]-
mu_25[j, m]-mu_35[k, m]

+ }

+ }

+ }

> for (j in 1:J){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_245[j, l, m]=log(sum(n[, j, , l, m])/I)-mu-mu_2[j]-mu_4[l]-mu_5[m]-mu_24[j, l]-mu_25[j,
m]-mu_45[l, m]
+ }
+ }
+ }
> for (j in 1:J){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_345[k, l, m]=log(sum(n[, , k, l, m])/I*J)-mu-mu_3[k]-mu_4[l]-mu_5[m]-mu_34[k, l]-
mu_35[k, m]-mu_45[l, m]
+ }
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ for (k in 1:K){
+ for (l in 1:L){
+ mu_1234[i, j, k, l]=log(sum(n[i, j, k, l, ])/M)-mu-mu_1[i]-mu_2[j]-mu_3[k]-mu_4[l]-mu_12[i,
j]-mu_13[i, k]-mu_14[i, l]-mu_123[i, j, K]-mu_134[i, k, l]-mu_234[j, k, l]

```

```

+ }
+ }
+ }
+ }
> for (i in 1:I){
+ for (j in 1:J){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_1245[i, j, l, m]=log(sum(n[i, j, , l, m])/K)-mu-mu_1[i]-mu_2[j]-mu_4[l]-mu_5[k]-
mu_12[i, j]-mu_14[i, l]-mu_15[i, m]-mu_24[j, l]-mu_25[j, m]-mu_45[l, m]-mu_124[i, j, l]-
mu_125[i, j, m]-mu_145[i, l, m]-mu_245[j, l, m]
+ }
+ }
+ }
+ }
> for (j in 1:J){
+ for (k in 1:K){
+ for (l in 1:L){
+ for (m in 1:M){
+ mu_2345[j, k, l, m]=log(sum(n[, j, k, l, , m])/I)-mu-mu_2[j]-mu_3[k]-mu_4[l]-mu_5[m]-
mu_23[j, k]-mu_24[j, l]-mu_25[j, m]-mu_34[k, l]-mu_35[k, m]-mu_45[l, m]-mu_234[j, k, l]-
mu_245[j, l, m]-mu_345[k, l, m]
+ }
+ }
+ }
+ }

```

```

> m_15a=m_15b=m_15c=m_15d=m_15e=m_15f=m_15g=m_15h=m_15i=m_15j=
m_16a=m_16b=m_16c=m_16d=m_16e =array(0, dim=c(I, J, K, L,M))

> for(i in 1:I){
+ for(j in 1:J){
+ for(k in 1:K){
+ for(l in 1:L){
+ for(m in 1:M){
+ m_15a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+mu_123[i, j, k])
+ m_15b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_124[i, j, l])
+ m_15c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_125[i, j,
m])
+ m_15d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_134[i, k,
l])
+ m_15e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_135[i, k,
m])
+ m_15f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_145[i, l,
m])
+ m_15g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_234[j, k,
l])
+ m_15h[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_235[j, k,
m])
+ m_15i[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_245[j, l,
m])
+ m_15j[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_345[k, l,
m])
+ m_16a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1234[i, j,
k, l])
+ m_16b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1235[i, j,
k, m])

```

```

+ m_16c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1245[i, j, l,
m])
+ m_16d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1345[i, k,
l, m])
+ m_16e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_2345[ j, k,
l, m])
+ }
+ }
+ }
+ }
+ }
> model15a=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender, data=n)
> model15b=loglm(~BScgrade+State+Gender+Age+Mode + BScgrade*State*Age, data=n)
> model15c=loglm(~BScgrade+State+Gender+Age+Mode + BScgrade*State*Mode, data=n)
> model15d=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Gender*Age, data=n)
> model15e=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Gender*Mode, data=n)
> model15f=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Age*Mode, data=n)
> model15g=loglm(~BScgrade+State+Gender+Age+Mode+ State*Gender*Age, data=n)
> model15h=loglm(~BScgrade+State+Gender+Age+Mode + State*Gender*Mode, data=n)
> model15i=loglm(~BScgrade+State+Gender+Age+Mode + State*Age*Mode, data=n)
> model15j=loglm(~BScgrade+State+Gender+Age+Mode + Gender*Age*Mode, data=n)
> model16a=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Age,
data=n)
> model16b=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Mode,
data=n)
> model16c=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Age*Mode,
data=n)

```

```
> model16d=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Gender*Age*Mode,
data=n)
```

```
> model16e=loglm(~BScgrade+State+Gender+Age+Mode+ State*Gender*Age*Mode, data=n)
```

```
> model15a; model15b; model15c; model15d; model15e; model15f; model15g; model15h;
model15i; model16a; model16b; model16c; model16d; model16e
```

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
State * Gender, data = n)
```

Statistics:

	X ²	df	P(> X ²)
--	----------------	----	----------------------

Likelihood Ratio	104.3455	58	0.0001829902
------------------	----------	----	--------------

Pearson	105.8493	58	0.0001269782
---------	----------	----	--------------

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
State * Age, data = n)
```

Statistics:

	X ²	df	P(> X ²)
--	----------------	----	----------------------

Likelihood Ratio	131.8656	58	1.118972e-07
------------------	----------	----	--------------

Pearson	127.2042	58	4.308532e-07
---------	----------	----	--------------

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
State * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 100.71154 58 0.0004329178

Pearson 99.04581 58 0.0006355605

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 124.9574 58 8.149994e-07

Pearson 123.9418 58 1.084171e-06

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Gender * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 113.5894 58 1.790516e-05

Pearson 125.3055 58 7.387566e-07

Call:

```
qloglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 145.6230 58 1.741845e-09

Pearson 155.5872 58 7.347278e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *  
Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 167.5483 67 1.408711e-10

Pearson 167.5230 67 1.419802e-10

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *  
Gender * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 163.4812 67 4.916169e-10

Pearson 161.3538 67 9.381688e-10

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *  
Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 170.88 67 4.991418e-11

Pearson 174.47 67 1.610523e-11

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Age, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 83.86091 39 4.016366e-05

Pearson 78.14037 39 1.999306e-04

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 41.96151 39 0.3437295

Pearson 40.78206 39 0.3920119

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 83.35372 39 0.0000464587

Pearson 79.69282 39 0.0001304115

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 85.02937 39 2.865165e-05

Pearson 87.22491 39 1.506013e-05

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + State *  
Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 161.2704 60 3.253775e-11

Pearson 158.5060 60 7.924161e-11

```
> m_17a=m_17b=m_17c=m_17d=m_17e=m_17f=m_17g=m_17h=m_17i=array(0, dim=c(I, J,  
K, L,M))
```

```
> for(i in 1:I){
```

```
+ for(j in 1:J){
```

```
+ for(k in 1:K){
```

```
+ for(l in 1:L){
```

```
+ for(m in 1:M){
```

```
+ m_17a[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1234[i, j,  
k, l]+ mu_1235[i, j, k, m])
```

```

+ m_17b[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1234[i, j,
k, l]+ mu_1345[i, k, l, m])

+ m_17c[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1234[i, j,
k, l]+ mu_2345[j, k, l, m])

+ m_17d[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1235[i, j,
k, m] + mu_1245[i, j, l, m])

+ m_17e[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1235[i, j,
k, m] + mu_1345[i, k, l, m])

+ m_17f[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1235[i, j, k,
m] + mu_2345[ j, k, l, m])

+ m_17g[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1245[i, j, l,
m]+ mu_1345[i, k, l, m])

+ m_17h[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1245[i, j, l,
m]+ mu_2345[ j, k, l, m])

+ m_17i[i, j, k, l, m]=exp(mu+mu_1[i]+mu_2[j]+mu_3[k]+ mu_4[l]+mu_5[m]+ mu_1345[i, k, l,
m] + mu_2345[ j, k, l, m])

+ }

+ }

+ }

+ }

+ }

> model17a=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Age+
BScgrade*State*Gender*Mode, data=n)

> model17b=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Age+
BScgrade*Gender*Age*Mode, data=n)

> model17c=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Age+
State*Gender*Age*Mode, data=n)

> model17d=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Mode+
BScgrade*State*Age*Mode, data=n)

```

```

> model17e=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Mode+
BScgrade*Gender*Age*Mode, data=n)

> model17f=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Gender*Mode+
State*Gender*Age*Mode, data=n)

> model17g=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Age*Mode+
BScgrade*Gender*Age*Mode, data=n)

> model17h=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*State*Age*Mode+
State*Gender*Age*Mode, data=n)

> model17i=loglm(~BScgrade+State+Gender+Age+Mode+ BScgrade*Gender*Age*Mode+
State*Gender*Age*Mode, data=n)

> model18=loglm(~BScgrade+State+Gender+Age+Mode+
BScgrade*State*Gender*Age*Mode, data=n)

> model17a; model17b; model17c; model17d; model17e; model17f; model17g; model17h;
model17i; model18

```

Call:

```

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
State * Gender * Age + BScgrade * State * Gender * Mode,
data = n)

```

Statistics:

X² df P(> X²)

Likelihood Ratio 21.47697 20 0.3695523

Pearson 20.70749 20 0.4145259

Call:

```

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
State * Gender * Age + BScgrade * Gender * Age * Mode, data = n)

```

Statistics:

X² df P(> X²)

Likelihood Ratio 43.93290 20 0.001536376

Pearson 42.81162 20 0.002164770

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Age + State * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 77.58305 32 1.174378e-05

Pearson 72.17188 32 6.199844e-05

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Mode + BScgrade * State * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 24.60369 20 0.2170156

Pearson 23.95280 20 0.2444600

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Mode + BScgrade * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 13.40143 20 0.8595041

Pearson 13.44084 20 0.8576788

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Gender * Mode + State * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 39.75069 32 0.1630520

Pearson 38.61374 32 0.1954771

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Age * Mode + BScgrade * Gender * Age * Mode, data = n)
```

Statistics:

X² df P(> X²)

Likelihood Ratio 22.76011 20 0.3007267

Pearson 22.69919 20 0.3038052

Call:

```
loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *  
State * Age * Mode + State * Gender * Age * Mode, data = n)
```

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 73.74410 32 3.851597e-05

Pearson 70.67786 32 9.688644e-05

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
Gender * Age * Mode + State * Gender * Age * Mode, data = n)

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 73.90678 32 3.665176e-05

Pearson 76.54197 32 1.626252e-05

Call:

loglm(formula = ~BScgrade + State + Gender + Age + Mode + BScgrade *
State * Gender * Age * Mode, data = n)

Statistics:

X^2 df P(> X^2)

Likelihood Ratio 0 0 1

Pearson 0 0 1

Journal Publications from the Thesis

- (I) Okoli,C.N., Onyeagu, S.I. and Osuji, G.A. (2015). On the Estimation of Parameters and Best Model Fits of Log Linear Model for Contingency Table. Open Access Library Journal, U.S.A Vol. 2 :e1189, 1-11.
- (II) Okoli, C.N., Onyeagu, S.I. and Osuji, G.A. (2015). A Generalized Method for Estimating Parameters and Model of Best Fits of Log Linear Model. European Journal of Statistics and Probability. IJMSS, United Kingdom. Vol. 3, No.1, pp 1-12.
- (III) Okoli,C.N., Onyeagu, S.I. and Osuji, G.A. (2014). Proposed Generalized Method and Algorithms for the Estimation of Parameters and Best Fits of log Linear Model. Journal of Natural Sciences Research, IISTE, U.K, 4(24): 90-99.
- (IV) Okoli,C.N., Onyeagu, S.I .and Osuji, G.A.(2014). Information Criteria and Log Linear Models Selection for Contingency Tables. IISTE, U.K. 4(24):25-32.

