

# CHAPTER ONE

## INTRODUCTION

### 1.1 Background

In recent times non-ferrous metals and alloys have become so important that technological development without them is inconceivable. Among the most important non-ferrous metals is copper with its alloys. Copper excels among other non-ferrous metals because of its high electrical conductivity, high thermal conductivity, high corrosion resistance, good ductility, malleability, and reasonable tensile strength (Anyafulu, 2015). The ever-increasing demand by the electrical industry for the ever diminishing resource of copper has led to the search for cheaper materials to replace the now expensive copper alloys. Whilst the metallurgist has been perfecting more ductile mild steel, the engineer has been developing more efficient methods of forming metals so that copper alloys are now only used where high electrical conductivity or suitable formability coupled with good corrosion resistance are required (Anene, 2015). The copper-base alloys include bronzes and brasses, the former being copper-rich alloys containing either tin, aluminum, silicon or beryllium (Haggins, 2004). Aluminum bronzes belong to a family of copper based alloys containing from 2 to 15% Al {C60800 to C64200}, as the major alloying element. The aluminum confers on copper, solid-solution strengthening, work hardening, and corrosion resistance. The combination of strength and corrosion resistance makes bronzes very important engineering materials, especially for highly stressed components in corrosive environments. They are available in both cast and wrought forms and are readily weldable into fabricated components such as pipes and pressure vessels. The strength can be greater than that of carbon steels and corrosion resistance better than most stainless steels (Anup, 2014). Aluminum bronzes give a combination of chemo- mechanical properties which supersedes those of many other alloy series, making them

preferred particularly for demanding applications (Vin, 2002). Aluminum bronzes are distinguished by high strength even at elevated temperatures, and also by good corrosion and wear resistance. The features which make aluminum bronzes the first choice and sometimes the only logical choice for demanding applications are: excellent strength, excellent corrosion resistance especially in seawater; resistance to fatigue at high temperature, resistance to creep which makes the alloys irreplaceable at elevated temperatures. Other features include oxidation resistance on exposure to elevated temperatures and in oxidizing environments; ease of casting and fabrication (William, 2010). According to ISO 428 specifications, most categories of aluminum bronzes contain 4-10 wt% aluminum in addition to other alloying elements such as iron, nickel, manganese and silicon in varying proportions. The relatively higher strength of aluminum bronzes compared to other copper alloys makes it suitable for the production of forgings, plates, sheets, extruded rods and sections (Pisarek, 2007). The various applications of aluminum bronzes reflect fully their versatility as engineering materials (Nwaeju, 2016). Copper- commercial binary alloys of aluminum bronze usually contain about 8% aluminum but the best combination of properties can be obtained in the range of 9 to 11% aluminum (William, 2010).  $\text{CuAl}_{15}$  bronzes are used for cold forming. It is supplied in the form of sheets, strips, barbed wire and pipes. In the soft state this alloy can reach the tensile strength of 380MPa, ductility of 40% and hardness of 70 to 110HB. Alloys containing 9-12% aluminum with addition of up to 6% each of iron and nickel represent the most important group of commercial aluminum bronzes. The common alloys, which normally contain 3-6% each of these two elements, have been fully investigated in view of their excellent combination of mechanical properties and corrosion resistance. The improvement in mechanical properties of metals and their alloys increases the reliability and service life of the structure in which they are used. These properties can be improved by a

combination of metallurgical, manufacturing and design measures (Callister, 2006). Findings have shown that aluminum bronzes are fast replacing contemporary steel materials for some specific applications especially in components for marine/sub-sea application (Cenoz, 2010). The consumption of aluminum bronzes has increased in the USA and western countries due to resistance to rusting in marine environment as well as resistance to corrosion in marine and highly aggressive environments respectively. In the construction of basic oxygen and electric arc furnace hoods, roof and side vents, aluminum bronzes have been identified as a viable alternative to carbon steels. Aluminum bronzes have been found to be as much as five times more durable than carbon steels (Lawrence & Vimod, 2006). Manganese - nickel –aluminum- bronze (Aqualloy) was found to be more efficient than stainless steel in making propellers. At high temperatures, the aluminum bronzes have very good plastic properties in deformation conditions in which  $\beta$ -phase predominates. This property makes it particularly suitable for propellers, pump impellers, castings and turbine runners, giving them long service lives and optimum operating efficiency (Labanowski & Olkowski, 2011). Despite these desirable properties, aluminum bronze exhibits deficient responses in certain application and the need to overcome obvious performance limitations is imperative to meet modern emerging technologies. The plastic formability of aluminum bronze is determined by the structure, which depends on the composition, the temperature and strain rate (Gronostajski, 2001).

## **1.2 Statement of Problem**

- Aluminum bronzes have problem of self-annealing and embrittlement when slowly cooled or at a normal cooling rate. This is as a result of the formation of  $\alpha+\gamma_2$  phase which is very brittle. This also result in deterioration of corrosion resistance and may produce a coarse structure with great reduction in properties.

### **1.3 Aim and Objectives of this study**

The aim of this study is to enhance the structural sensitive properties of copper-10% aluminum alloy using carbide forming elements.

The objectives of this study include the following;

- To study the structure of copper 10% aluminum alloy and the structural sensitive properties.
- To study the effects of carbide forming elements on the structural sensitive properties of copper-10% aluminum alloys.
- To ascertain the element and composition that maximally enhanced the structural sensitive properties
- To correlate the physical and mechanical properties of copper-10% aluminum alloys.
- To design the experiment, develop mathematical equations for predicting the properties.
- To optimize the process parameters and identify if the modifying elements are significant factors that affected the process using Analysis of variance.

### **1.4 The Significance of this study**

This study will help to minimize the causes of failure in engineering designs and construction. It will develop veritable alloys that will have improved mechanical properties compared to the conventional ferrous (stainless steels) materials. The addition of these carbide forming elements will refine and modify intermetallic compound present in the alloys, the instability of beta phase and the formation of gamma phase which affects the alloy properties.



## **1.5 The Scope of this study**

This study is limited to the following:

- ✓ Melting and casting of copper-10% aluminum alloy.
- ✓ Additions of carbide forming elements (titanium, zirconium, manganese, vanadium, nickel, chromium, molybdenum and tungsten).
- ✓ The composition of the elements used is limited to 0.5 to 10% wt, at 0.5% wt intervals but Ni, V and Mn composition were limited to 0.5 to 5.0%
- ✓ Testing of prepared alloy samples for mechanical and physical properties.

## CHAPTER TWO

### LITERATURE REVIEW

#### 2.1 Copper

Copper is a chemical element with symbol Cu (from Latin: cuprum) and atomic number 29. It is a soft, malleable and ductile metal with very high thermal and electrical conductivity. A freshly exposed surface of pure copper has a reddish-orange color. It is used as a conductor of heat and electricity, as a building material and as a constituent of various metal alloys, such as sterling silver used in jewelry, cupronickel used to make marine hardware and coins and constantan used in strain gauges and thermocouples for temperature measurement. Copper is an important engineering metal and is widely used in the unalloyed condition as well as combined with other metals in the alloyed form. In the unalloyed form, copper has an extraordinary combination of properties for industrial applications (William et al, 2006). Some of these are high electrical and thermal conductivity, good corrosion resistance, ease of fabrication, medium tensile strength, controllable annealing properties, and general soldering and joining characteristics. Higher strengths are attained in a series of brasses and bronzes that are indispensable for many engineering applications. The melting and boiling points of copper are  $1084.64^{\circ}\text{C}$  and  $2562^{\circ}\text{C}$  respectively, while the heat of fusion and heat of vapourization are  $13.26\text{KJmol}^{-1}$  and  $300.4\text{KJmol}^{-1}$  respectively. Copper is used in making ornaments due to its corrosion resistance. It is used in building integrated circuits, chips and the printed circuit boards of computers. Copper is second to silver in its ability to conduct electricity with relative electrical conductivity and relative thermal conductivity of 100, while silver is 106 and 108 respectively. Copper is found as a pure metal in nature, and this was the first source of the metal to be used by humans, 8,000 BC. It was the first metal to be smelted from its

ore, 5,000 BC, the first metal to be cast into a shape in a mold, 4,000 BC and the first metal to be purposefully alloyed with another metal, tin, to create bronze, 3,500 BC (Kakani & Kakani, 2004). In the Roman era, copper was principally mined on Cyprus, the origin of the name of the metal, from *aes cyprium* (metal of Cyprus), later corrupted to *cuprum*, from which the words copper (English), *cuiivre* (French), *Koper* (Dutch) and *Kupfer* (German) are all derived. The commonly encountered compounds are copper(II) salts, which often impart blue or green colors to such minerals as azurite, malachite, and turquoise, and have been used widely and historically as pigments (Issac, 2010). The world's production of copper is increasing as the demand increases, since it has found application in many areas of technological evolution. USA, Chile, Canada, Zambia, Zaire, Peru are all producing reasonable quantities of copper in the world today (Joseph, 2001).

## **2.2 Aluminum**

Aluminum is a chemical element in the boron group with symbol Al and atomic number 13. It is silvery white and it is not soluble in water under normal circumstances. It is the third most abundant element after oxygen and silicon and the most abundant metal in the earth's crust. It makes up about 8% by weight of the earth's solid surface. Aluminum metal is so chemically reactive that native specimens are rare and limited to extreme reducing environments. Instead, it is found combined in over 270 different minerals (Taylor, 2004). The major ore of aluminum is bauxite. Aluminum is remarkable because of its low density and ability to resist corrosion due to the phenomenon of passivation. Structural components made from aluminium and its alloys are vital to the aerospace industry and are important in other areas of transportation and structural materials. The most useful compounds of aluminium at least on a weight basis are the oxides and sulphates. Despite its prevalence in the environment, aluminum salts are not known

to be used by any form of life. A fresh film of aluminum film serves as a good reflector (92%) of visible light and an excellent reflector (as much as 98%) of medium and far infrared radiation. The yield strength of pure aluminum is 7-11MPa, while aluminum alloys have yield strength ranging from 200MPa to 600MPa. Aluminum has about one third the density and stiffness of steel. It is easily machined, cast, drawn and extruded. Its corrosion resistance is excellent due to a thin surface layer of aluminum oxide that forms when the metal is exposed to air, effectively preventing further oxidation. This corrosion resistance is also often greatly reduced by aqueous salts, particularly in the presence of dissimilar metals (Donatus et al, 2012). The demand for aluminum grows rapidly because of its attributed unique combination of properties which makes it become one of the most versatile of engineering and construction materials (Mrowka, 2010). Aluminum is light in weight compared with copper and steel. Some of its alloys even have greater strengths than structured steel (Lee et al, 2002). Besides it has good electrical and thermal conductivities and high affectivity to both heat and light. An alloy is a mixture of either pure or fairly pure chemical elements, which forms an impure substance (admixture) that retains the characteristics of a metal. An alloy is distinct from an impure metal, such as wrought iron, in that, with an alloy, the added impurities are usually desirable and will typically have some useful benefit. When the alloy cools and solidifies (crystallizes), its mechanical properties will often be quite different from those of its individual constituents. A metal that is normally very soft and malleable, such as aluminium, can be altered by alloying it with another soft metal, like copper. Although both metals are very soft and ductile, the resulting aluminium alloy will be much harder and stronger. Adding a small amount of non-metallic carbon to iron produces an alloy called steel. Due to its very-high strength and toughness (which is much higher than pure iron), and its ability to be greatly altered by heat treatment, steel is one of the most

common alloys in modern use. By adding chromium to steel, its resistance to corrosion can be enhanced, creating stainless steel, while adding silicon will alter its electrical characteristics, producing silicon steel(Mrowka, 2010).

### **2.3 Copper-aluminum alloys (aluminum bronzes)**

Copper alloys are classified according to a designation system administered by the Copper Development Association (CDA). In this system the numbers C10100 to C79900 designate wrought alloys and the numbers from C80000 to C99900 designate casting alloys. Copper alloys are grouped as follows: unalloyed copper, brass which is copper-zinc alloy, copper-lead alloys and copper-zinc alloys with tin and aluminum additions known as alloy bronzes, Bronzes which are copper-tin alloys, copper- aluminum alloys, copper-silicon alloys, copper-beryllium alloys and copper-nickel based alloy (cupronickel and nickel silver(Cu-Ni-Zn)). Aluminum forms solid solution in copper ( $\alpha$  phase) up to 9.4% at 565°C. The microstructure of  $\alpha$  aluminum bronze consists of single  $\alpha$  phase solid solution with solid solubility of the  $\alpha$  phase increasing with decreasing temperature, above 9.5%Al, rapid quenching to room temperature produces martensitic transformation of metastable  $\beta'$  tetragonal structure(Mrowka, 2010). Aluminum bronzes have high strength, excellent corrosion and resistance to wear and fatigue. The tensile strength increases with increasing  $\beta$  phase while ductility decreases. Increasing aluminum content increases tensile strength of aluminum bronzes. The tensile strength of 10%aluminum varies from 300-480MPa.Aluminum bronze castings are produced by the recognized techniques of sand, shell, die, ceramic, investment, centrifugal and continuous casting. The size of casting ranges from tiny investment cast component to very large propellers weighing up to 70 tons.One of the very attractive characteristics of aluminum bronzes is that due to their short cooling range, they solidify compactly, as do pure metals, this means that, provided defects

are avoided, through design measures the alloys are inherently sound, more than alloys such as gun metals (tin bronze, UNSC90500) which may be porous unless cooled very rapidly. The short freezing range of the alloys means that adequate feeding is required as the metal solidifies. It is also essential to prevent the aluminum oxide dross on top of the liquid metal from becoming entrapped in the castings during pouring (Bukola et.al, 2013). Avoiding internal defects therefore requires a certain degree of precaution, although foundries with the required expertise routinely produce casting of very high integrity. Because aluminum bronze is often selected for critical applications, it is important that casting be well designed so as to achieve optimum results. Consultation with an experienced foundry man is therefore essential at a relatively early stage of design development. A leaflet giving guidance on the design of aluminum bronze castings is available from copper development association. It is helpful in the initial design work and gives a good basis for consultation between the designer and the foundry man (Iqbal et al, 2008). Some alloys occur naturally, such as electrum, which is an alloy that is native to Earth, consisting of silver and gold. Meteorites are sometimes made of naturally occurring alloys of iron and nickel, but are not native to the Earth. One of the first alloys made by humans was bronze, which is made by smelting the metals tin and copper. Bronze was an extremely useful alloy to the ancients, because it is much stronger and harder than either of its components. Steel was another common alloy. However, in ancient times, it could only be created as an accidental byproduct from the heating of iron ore in fires (smelting) during the manufacture of iron. Other ancient alloys include pewter, brass and pig iron. In the modern age, steel can be made in many forms. Carbon steel can be made by varying only the carbon content, producing soft alloys like mild steel or hard alloys like spring steel (Bukola et.al, 2013). Alloy steels can be made by adding other elements, such as molybdenum, vanadium or nickel, resulting to alloys such

as high-speed steel or tool steel. Small amounts of manganese are usually alloyed with most modern-steels because of its ability to remove unwanted impurities, like phosphorus, sulfur and oxygen, which can have detrimental effects on the alloy. However, most alloys were not produced until the 1900s, when various aluminium, titanium, nickel, and magnesium alloys were made. Some modern superalloys, such as incoloy, inconel, and hastelloy, may consist of a multitude of different components (Bukola et.al, 2013). When a molten metal is combined with another substance, there are two mechanisms that can cause an alloy to form. They are atom exchange and the interstitial mechanism. The relative size of each element in the mix plays a primary role in determining which mechanism will occur. When the atoms are relatively similar in size, the atom exchange method usually happens, where some of the atoms composing the metallic crystals are substituted with atoms of the other constituent. This is called a substitutional alloy. Examples of substitutional alloys include bronze and brass, in which some of the copper atoms are substituted with either tin or zinc atoms (Bradley, 1991). With the interstitial mechanism, one atom is usually much smaller than the other, so cannot successfully replace an atom in the crystals of the base metal. The smaller atoms become trapped in the spaces between the atoms in the crystal matrix, called the interstices. This is referred to as an interstitial alloy. Steel is an example of an interstitial alloy, because the very small carbon atoms fit into interstices of the iron matrix. Stainless steel is an example of a combination of interstitial and substitutional alloys, because the carbon atoms fit into the interstices, but some of the iron atoms are replaced with nickel and chromium atoms (Hong et al, 2009).

## **2.4 Structure of Copper – 10%Aluminum Binary Phase**

The binary phase diagram of Cu-Al is complex, but for the commercially important binary alloys the most important reaction is the eutectoid (phase transformation of one solid into two solids) which occurs at 565°C. The maximum solubility of aluminum in copper is 7.3% but it increases with temperature to 9.4% aluminum. Homogeneous alloy structure is formed by  $\alpha$ - solid solution crystals (substituted solid solution of aluminum in copper). The structure has similar properties as  $\alpha$  solid solution in brasses. It is a relatively soft and plastic phase. In the real alloys the absolutely equilibrium state does not occur (Daniel et al, 2002). In the case of aluminum content close to the solubility limit some portion of  $\beta$  phase will occur in the structure. The upper limit of aluminum in  $\alpha$  homogeneous structure alloy is dependent on the cooling rate and is in the range of 7.5 to 8.5%. Alloys with aluminum content in the 7.3 to 9.4% range solidify at eutectic reaction to ( $\alpha + \beta$ ) phase and close to the eutectic line they contain the primarily released phase  $\alpha$  or  $\beta$  and eutectic (After the change at lower temperature the eutectic disappears and its influence in the structure cannot be proved). By decreasing the temperature, the composition of  $\alpha$  and  $\beta$  crystals change according to the time the alloy starts to dissolve which is the solubility time.  $\beta$  phase is a disordered solid solution of electron compound  $\text{Cu}_3\text{Al}$  ( $e/a = 3/2$ ) with face centered cubic (FCC) lattice. It is a hard and brittle phase.  $\beta$  phase, from which the  $\alpha$  solid solution is created at lower temperatures is precipitated from liquid metal at aluminum content from 9.5 to 12% alloys, during the crystallization process (Labanowski et al, 2011). During a slower cooling rate, the  $\beta$ -phase is transformed at eutectoid temperature 565°C to the lamellar eutectoid ( $\alpha + \gamma_2$ ). For this reason the eutectoid reaction of  $\beta$ - phase is sometimes called “pearlitic transformation”.  $\gamma_2$ -phase solid solution is a hard and brittle electron compound  $\text{Cu}_9\text{Al}_4$  which has a complicated cubic lattice. After re-crystallization in the solid state, the slowly cooled alloys with aluminum content

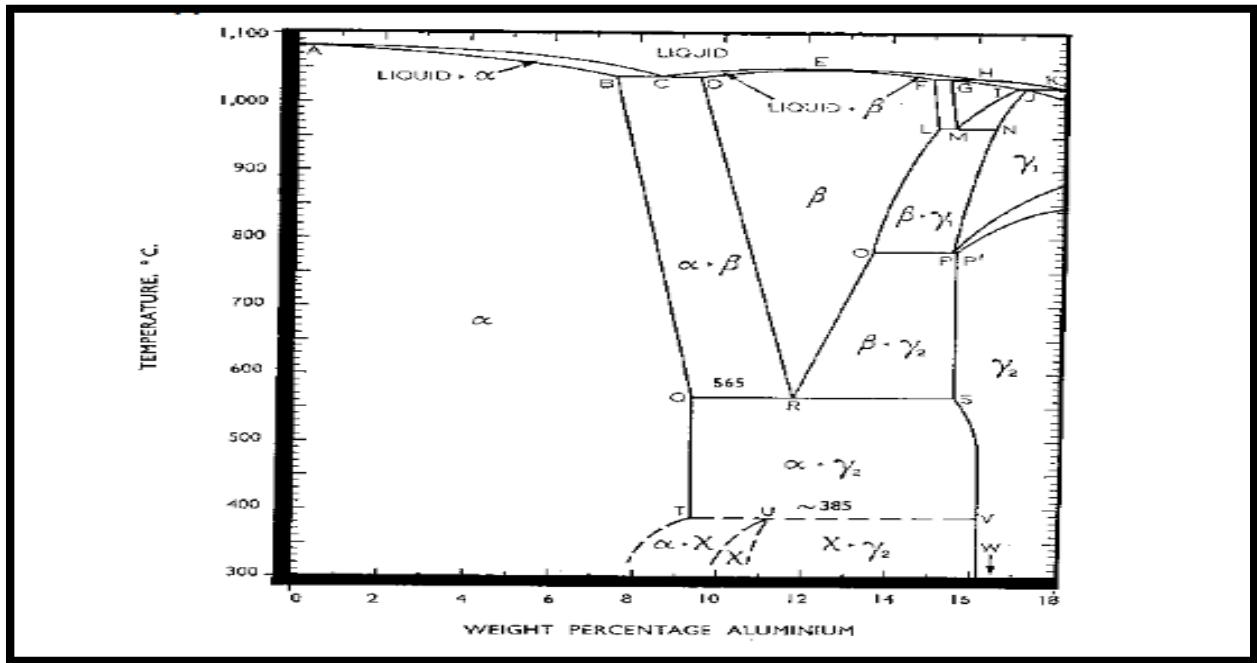


9.4 to 12% are heterogeneous. The structure consists of  $\alpha$ - solid solution crystal and the eutectoid ( $\alpha + \gamma_2$ ) phase. The simple aluminum bronzes containing only copper and aluminum have a single  $\alpha$ -phase structure containing up to about 8% aluminum. Above this level, a second phase ( $\beta$ -phase) is formed producing  $\alpha$ - $\beta$  phase alloy (Wharton et.al 2005). The gamma 2 phase is as a result of the transformation of  $\beta$ -phase and has higher aluminum content than the beta phase thereby showing a susceptibility to corrosion. Consequently if gamma 2 is formed as a continuing network, a higher rate of penetration of corrosion through the alloy can occur and the corrosion resistance is seriously affected. Small isolated areas of gamma 2 phase will result in localized superficial corrosion but this will not penetrate into the body of the material. The information concerning the formation of gamma 2 phase with resultant deterioration of corrosion resistance applies only to binary aluminum bronzes i.e. alloys of copper and aluminum, without additional alloying elements. The presence of iron in sufficient quantity suppresses the formation of gamma 2 and also refines the grain structure of the alloy. Consequently any gamma 2 that is present is more likely to be in a discontinuous form (Troeger & Starke, 2000). About 2% iron is generally sufficient to suppress the formation of gamma 2 phase with a diameter in sections of up to about 75mm, but this is not sufficient for heavier sections. Manganese additions will also suppress the breakdown of beta phase to alpha plus gamma 2 but at the same time modifies the beta phases making it more susceptible to corrosion. Manganese content must therefore be chosen to give the optimum balance between the two effects. In addition, in this research work, vanadium alloys are attractive candidate structural materials for breeding blanket of fusion reactors because of the low activation properties and high temperature strength (Muroga, 2005). Its addition is very effective in reducing  $\beta$ -grain size. Alloy containing 10 to 12% aluminum can be heat treated in similar method to steels. Martensitic transformation can be

reached by fast cooling, in which case eutectoid transformation is limited and a microstructure with fine and hard needles of  $\beta$ , phase with a body centered cubic lattice will be produced/formed. In summary, aluminum bronze alloys can be categorized into three distinct series as can be seen from the binary equilibrium diagram of copper- aluminum system. They constitute the  $\alpha$  series, the  $\alpha + \beta$  series and  $\beta + \gamma_2$  series, possessing properties that can be obtained by suitable alloying addition and heat treatment which has opened up immense possibilities for their application in various engineering fields (Mokhtari et.al, 2012). This is also reported in the Al – 4%Cu system (Nnuka, 1991). The higher strength aluminum bronze alloy such as AB2 ( $\text{CuAl}_{10}\text{Fe}_5\text{Ni}_5$  – castings) and CA104 ( $\text{CuAl}_{10}\text{Fe}_5\text{Ni}_5$  – wrought) contain normally 10% aluminum with 5% each of iron and nickel. In these alloys the temperature range of 950°C to 750°C is maintained to produce alpha ( $\alpha$ ) plus kappa ( $\kappa$ ) phase. The alloy solidifies with an alpha – beta structure from which the kappa phase begins to precipitate as coarse particles (often in the form of resettes) at about 900°C. In marine environment, the requirements for marine component are, among others, high strength to weight ratio, good castability and tolerance to local working for repairing damage sustained during service which narrows the choice of alloy to aluminum bronze, to develop a ( $\alpha + \beta$ ) / ( $\alpha + \kappa$ ) phase aluminum bronze with a view of seeking replacement for conventionally used component that fail readily during service. The eutectoid structure of  $\alpha + \gamma_2$ , which has a lower electro-chemical potential corrodes at a higher rate and has therefore to be avoided (Pisarek, 2006). The most important aspect is the eutectoid transformation in which the phases are:  $\alpha$ -Al bronze having F.C.C lattice similar to iron in its working characteristics,  $\beta$ - Al bronze with B.C.C structure,  $\beta$ - Al bronze which at the eutectoid transformation of 525°C transforms to  $\beta + \gamma_2$ .

## **2.5 Aluminum bronze properties**

The mechanical properties of aluminum bronze depend primarily on aluminum content. Alloys with up to about 8% aluminum have a ductile single phase structure and are the most suitable for cold working into tube, sheet, strip and wire. As the aluminum content is increased to between 8% and 10% the alloys are progressively strengthened by a second harder phase which makes them more suitable for hot working and casting. Above 10%, even greater strength and hardness is developed for specialized wear resistant applications (Oh-shi et al, 2004). Other major alloying elements also modify the structure to increase strength and corrosion resistance. For example, iron improves the tensile strength and act as a grain refiner; nickel improves proof stress and corrosion resistance and has a beneficial stability effect on the metallurgical structure; manganese also performs a stability function.



**Fig: 2.1. Cu-Al phase diagram. (Source: Copper Development Association. (2000).Equilibrium Diagrams, CDA Publication No. 83.)**

Two further alloy types complete the range of commercial alloys: silicon up to about 2 % with aluminum up to about 6% from a range of alloys known as aluminum silicon bronze; these have a higher strength than normal single – phase aluminum bronze but are cast and hot – worked more readily, have a similarly low magnetic permeability and excellent resistance to shock loading. Silicon also improves machinability. The alloys are available in wrought and cast forms (Oluwayomi et al, 2014).

Manganese (about 13%) is the major addition in a series of manganese aluminum bronzes with aluminum levels of 8 – 9%. Their foundry properties are better than the aluminum bronzes and they have good resistance to impingement and cavitation as well as being heat treatable. They have excellent welding properties (Daroonparvar et al, 2011).

The microstructure of the aluminum bronze with less than 11% aluminum consists of alpha solid solution and the iron and nickel rich kappa phase. The Kappa phase absorbs aluminum from the alpha solid solution preventing the formation of the beta phase unless the aluminum content is above 11%. The kappa phase increases the mechanical strength of the aluminum bronzes without decrease in ductility. The decrease in ductility of the aluminum bronzes occurs only when the beta phase forms. The beta phase is harder and more brittle than the alpha phase. Beta is formed if the material is quenched or fast cooled, which then transforms it into a martensite structure (Labanowski & Olkowski, 2009).

Tempering the martensite resulted in a structure of alpha with kappa precipitates. The tempered structure is very desirable and it has high strength and hardness.

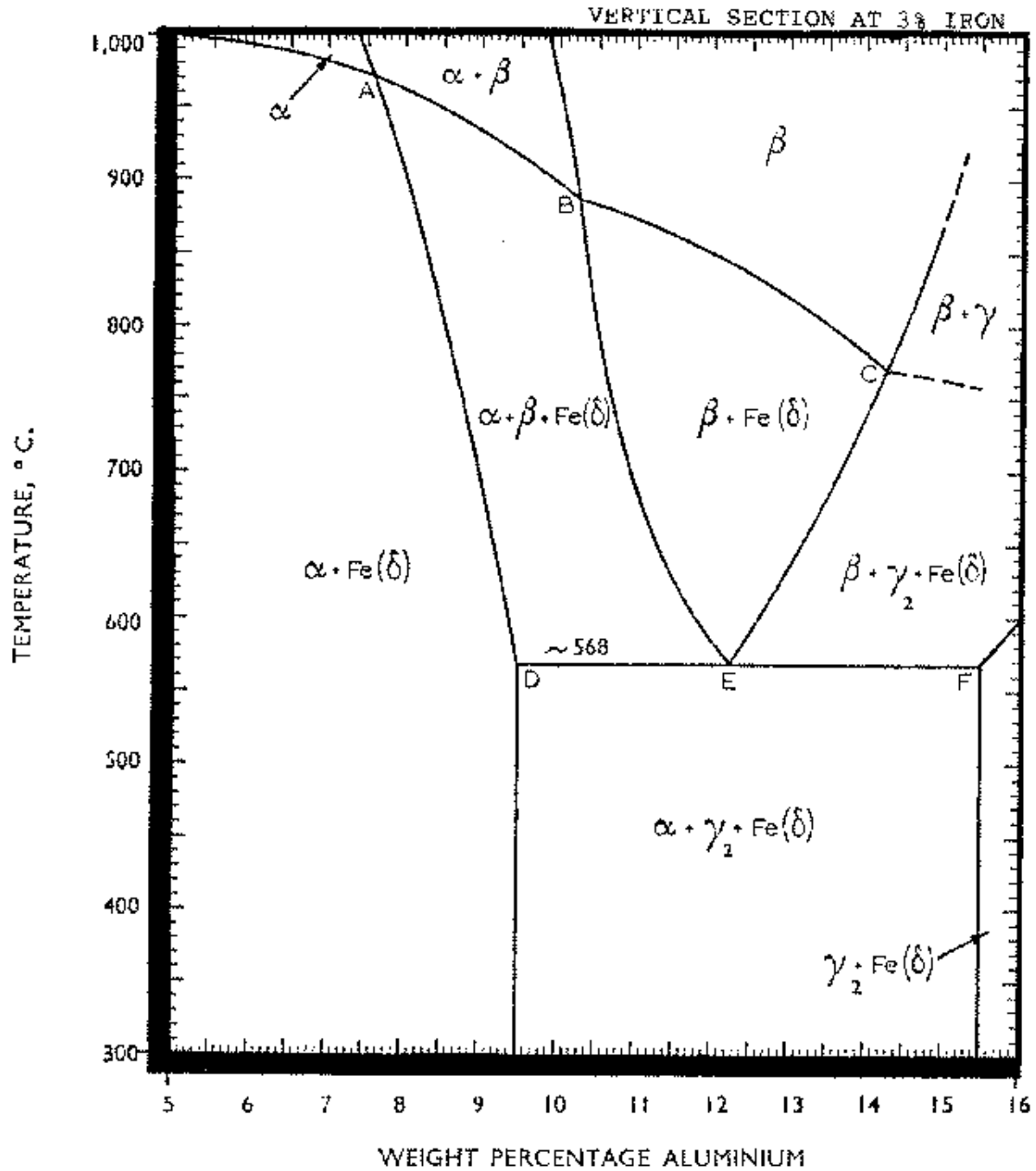
The slow cooled, as cast structures consist of alpha and kappa phases. Kappa is present in the lamellar form and finely divided in all the alpha areas. The additions of iron and nickel also suppress the formation of the gamma double prime phase

which has deleterious effects on the properties of aluminum copper alloys (Labanowski & Olkowski, 2014).

## **2.6 Structure of Some Alloying Element in Aluminum Bronze**

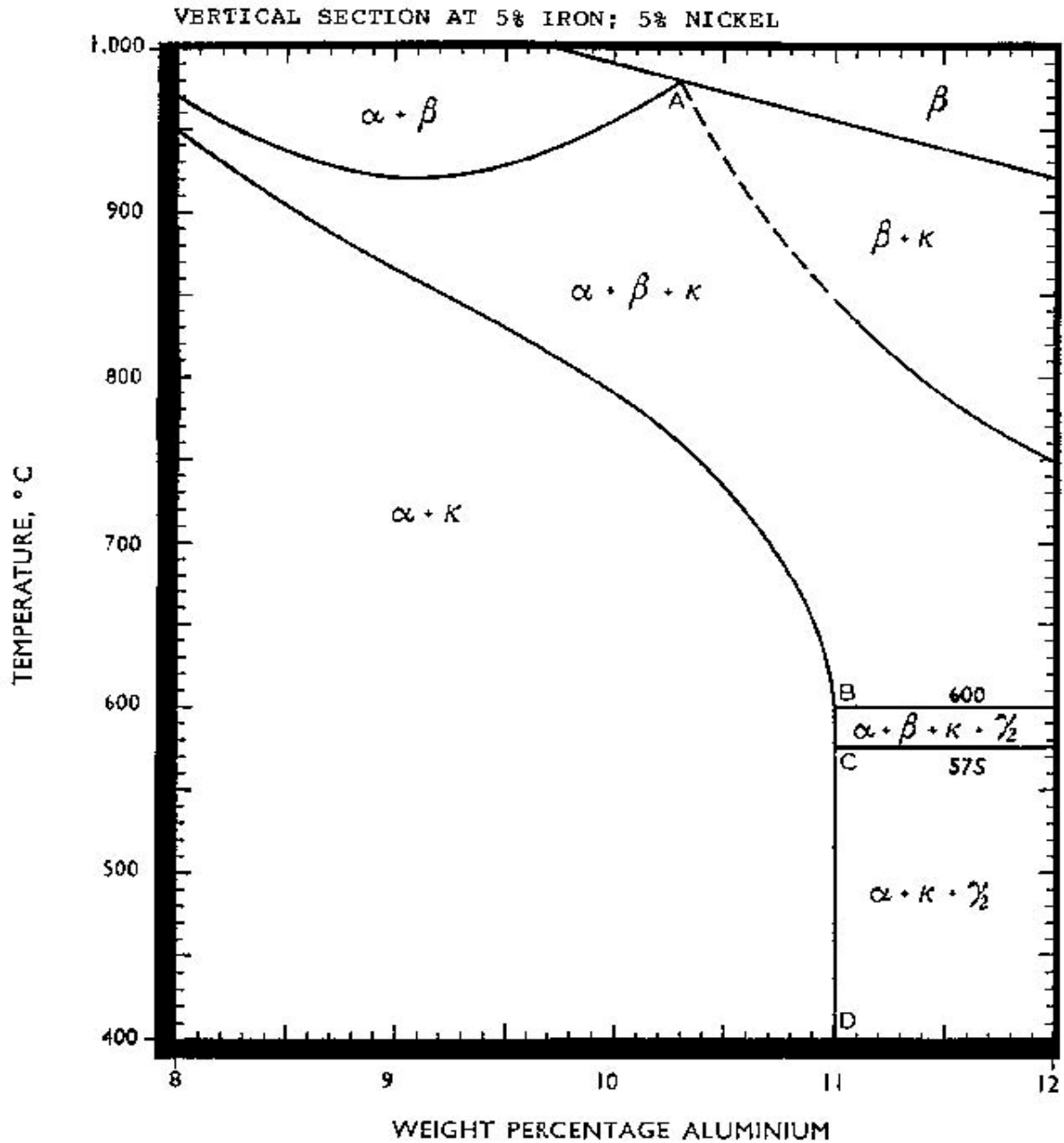
### **2.6.1 Copper-Aluminum-Iron**

The influence of 3% iron on the copper-aluminum system is shown Figure 2.2. Iron addition of this order only slightly modifies the binary diagram and particles of iron are precipitated throughout both the  $\alpha$  and  $\beta$  phases. The solubility of iron at high temperature varies according to the aluminum content in all  $\alpha$ -phases. The iron begins to precipitate when the temperature drops below about 1000°C, whilst the alloys with greater amounts of aluminum, do not precipitate the iron until the temperature has fallen, in some cases, as low as 850°C. The iron appears as finely precipitates evenly distributed throughout the structure with no apparent tendency for concentration at grain boundaries (Mrowka, 2010).



**Fig: 2.2.Cu-Al-Fe phase diagram.(Source: Copper Development Association.(2000). Equilibrium Diagrams.CDA Publication No.94, p. 19.)**

## 2.6.2 Copper-Aluminum – Nickel – Iron



*Fig: 2.3. Cu-Al- Ni-Fe phase diagram. (Source: Copper Development Association. (2000). Equilibrium Diagrams. CDA Publication No.94, pp. 21.)*

Alloys containing 9-12% aluminum with additions up to 6% each of iron and nickel represent a most important group of commercial aluminum bronzes. The common alloys, which normally contain 3-6% each of these two elements, have been fully investigated in view of their excellent combination of mechanical properties and corrosion resistance.

A vertical section throughout the Cu-Al-Ni-Fe quaternary diagram at 5% Fe and 5% Ni is shown in figure 2.3. It can be seen that the system remains essentially the same as the binary system with the introduction of an additional phase,  $k$ . The  $k$  phase is related to the iron-rich phase in the Cu-Al-Fe system (Nikanorov, 2005).

$K$  absorbs aluminum from the matrix and hence extends the apparent range of the  $\alpha$ -field. Thus, under equilibrium conditions,  $\beta$  is not retained below 600°C unless the aluminum exceeds 11%, as compared with 9.45 in the binary system. The  $k$  precipitate in the  $\alpha$ -matrix has a pronounced effect on the properties and considerably increases the mechanical strength. At the same time, the reduction in ductility is not as marked as would occur if  $\beta$  had been formed in the binary alloy to give an equivalent strength. This is the most outstanding advantage which alloys of this type have over other aluminum bronzes. The absence of  $\beta$  also avoids any danger from eutectoid formation or whatever the previous history of the material with respect to heat treatment or rate of cooling, unless the aluminum content in excess of about 10.5% (Mokhtari et al, 2012).

Nickel is the most frequent alloying element in aluminum bronze. Its addition has a strong influence in the stabilization of  $\beta$  phase. The  $\alpha/\beta$  structure is retained at low cooling rates even with 2% nickel. The addition of nickel to an alloy containing iron has a beneficial effect in modifying the stable structure.

The  $\gamma_2$  phase formation is suppressed and the  $\alpha$  phase solid solution range is extended towards higher aluminum contents. The combined effect of iron and nickel produces a kappa phase which has the same structure as the  $\beta$  aluminum bronze



(Muroga, 2005). The size and disposition of kappa can be controlled from fairly massive to fairly dispersed forms. Hence, in alloys containing less than 11% Al, the decomposition of  $\beta$  produces  $(\alpha+k)$ , when nickel and iron are also present (Norman, 2009). By regulating the speed of cooling, the transformation of  $\beta$ ,  $\alpha+k$  can then be arranged to obtain two very important effects:

- (1) Hardening by precipitation of (k in  $\alpha$ ) and
- (2) Simple martensitic straining to obtain relatively softer phase  $\beta$  decomposes to  $\alpha$ .

In making casting having nickel and iron, it is important to see that differential structure is avoided, as indeed should be the case for all complex castings.

Manganese is added to the complex alloy because it has deoxidizing effect in the melted metal. It improves the corrosion resistance of the aluminum bronze as it stabilizes the  $\beta$  phase and reduces the risk of decomposition of the eutectoid. The main drawback, however is that aluminum bronze with a low manganese addition is susceptible to corrosion. When the addition exceeds 11% a fully stable  $\alpha/\beta$  structure is obtained resulting to improved corrosion resistance of the production (Moradlou et al, 2011).

Silicon increases ductility, improves tensile strength but significantly reduce hardness in composition of 1-40% weight.

Hideous aluminum bronzes are tough and suitable for cold and also hot forming. Heterogeneous alloys are stronger, harder, but they have lower cold forming properties compared to the homogenous alloys. They are suitable for hot forming and have good cast properties. Aluminum bronzes are distinguished by good strength, even at elevated temperatures, and also very good corrosion resistance and wear resistance aluminum bronzes are used in the chemical and food industry for stressed components production. These alloys are used in mechanical engineering for much stressed gear wheels and worm wheels, armatures working at

elevated temperature etc. The aluminum bronze is divided into two groups, cast and wrought aluminum bronzes (Skocovsky. 2000).

Aluminum bronze with Al content from 4.5 to 11% is used for forming elementary or complex components while with Al content from 7.5 to 12% only complex components can be cast (Sadayappan et al, 1999).

## **2.7 Effects of Alloying Elements**

In addition to aluminum, which ranges from 5% to 14%, the alloying elements most commonly used in aluminum bronze are nickel, manganese, silicon and tin. The mechanical properties of aluminum bronze depend primarily on aluminum content; however, varying proportions of these secondary additions leads to sub-classification of the family into four types as follows:

- The low alloy. Single-phase (face-centered cubic) alpha alloys containing less than 8% aluminum. These alloys have good ductility when both hot and cold and are well suited for cold working into tubes, sheets, strips and wires. Alloys of this type containing 3% iron are single phase and also contain aluminum above 9%.
- The more highly alloyed, two phase (duplex) alloys containing from 8% to 11% aluminum and usually with additions of iron and nickel, for higher strength. As aluminum content is increased to between 8 to 10%, the alloys are progressively strengthened by appearance of the harder body-centered cubic beta phase, which additionally makes the bronzes more suitable for hot working and casting. Even greater strength and hardness is developed in alloys containing more than 10% Al. such alloys are favored for specialized applications requiring superior wear resistance.

The other alloying elements mentioned (Mn, Sn, and Si) also modify the structure and lower the strength and corrosion resistance: Iron improve tensile strength and acts as grain refiner, nickel improves yield (proof) stress and corrosion resistance

and has a beneficial stabilizing effect on the metallurgical structure while manganese also performs a stabilizing function.

- The copper-aluminum silicon alloys or silicon aluminum bronze. These are mainly alpha-phase alloys and therefore have good strength and ductility. Alloys having silicon contents ranging up to about 2% and aluminum to about 6% are known as aluminum-silicon bronzes; these are stronger than unmodified single phase aluminum bronzes and can be cast and hot-worked more readily. Like other aluminum bronzes, they have a low magnetic permeability and excellent resistance to shock loading. Silicon also improves machinability. The alloy is available in wrought and cast forms.
- The Copper–Manganese–Aluminum or Manganese–Aluminum bronzes. These alloys have good castability and were in fact, developed primarily for the manufacture of propellers. Manganese at about 13% is a major alloying addition in a series of manganese –aluminum bronzes in which aluminum levels range between 8 and 9%, although not so strong as other aluminum bronzes, they also have good resistance to impingement and cavitation and can be heat treated to give low magnetic permeability. They have excellent weldability (Skocovsky, 2006).

## **2.8 Review of previous studies on copper-aluminum alloy.**

Micro alloying is a technique used to strengthen and harden metals. In this technique, the atoms of the alloying elements (impurity atoms) go into either substitutional or interstitial solid-solution, and distort the lattice structure of the solvent and offer resistance to dislocation movement. This resistance is greater with interstitial element (Mustafa, 2009). Superior alloy with improved mechanical and corrosion properties can be obtained by addition of alloying elements in micro quantity (Abdul et al, 2013). Micro alloying technology was originally developed

for micro alloyed steels. Although the amount of micro alloying elements is usually less than 10%, they lead to improved combinations of strength and ductility, weldability, toughness, and corrosion resistance (Nikanorov et al 2003). The role played by the alloying elements varies. This can be seen from the following micro alloying elements which modify the form, quantity and the distribution pattern of the eutectic  $\alpha$ -solid solution +CuAl<sub>2</sub> phase, distributed in the form of individual isolated inclusions between the dendrite cell and grain body (Nnuka, 1991). The increase in hardness and tensile strength is due to the interaction of the stress field around the particles with the stress field of a moving dislocation and also due to physical obstruction by the hard particles to the moving dislocation (Nnuka, 2004). The extent to which strengthening is produced depends upon the amount of second phase particles, the characteristics and properties of the second phase, and the particle size, shape and distribution pattern (Nnuka, 1991). Micro alloying is basically to improve the mechanical properties such as strength, hardness, rigidity, corrosion resistance and machinability, and also sometimes to improve the fluidity and other casting properties (Haggins, 2004). Mechanical properties of marine propellers casting made of copper alloys are formed by the selection of chemical composition of the alloy and the use of casting technology ensuring efficient refinement as well as deoxidation and the removal of non-metallic inclusions, gases and slag from molten metal. The evaluation of the mechanical properties is made on the basis of sections taken from a test ingot cast separately or cast together with propeller screw. So established properties give only an approximate knowledge about the actual properties of the propeller casting that has different cooling conditions in sections thicker than test ingot, but they are reliable and accepted by the Marine Classification Societies (CDA, 2000). Maintaining the chemical composition of copper alloy in the range recommended by the standards does not guarantee the

required mechanical properties. Even small differences in the chemical composition of individual heats may significantly affect the mechanical properties while maintaining the same casting conditions. Copper alloys Cu3 category according to the Polish Register of Shipping (2008), have usually five components, so it is important to know the interactions of the components. Information about the effects of individual components (Al, Fe, Ni, Mn) on the mechanical properties of aluminum bronzes are available in the literature however there is no sufficient information on joint action of components (Piaseczny, & Rogowski, 2006). For this reason, statistical studies were undertaken to develop appropriate regression equations describing the mechanical properties of castings as a function of chemical composition of a copper alloy. Changes in chemical composition of the alloy are generally associated with specific changes in the microstructure of castings. Therefore, qualitative and quantitative study of the microstructure were undertaken in order to explain the reason for the changes in mechanical properties of obtained castings. The microstructure of aluminum bronze Cu-Al-Fe-Ni consists of a solid solution,  $\alpha+\gamma_2$  eutectoid and precipitations of iron rich k-phase. Based on the review of the literature it is assumed that the mechanical properties of Cu3 category screw propellers castings largely depend on the location, size and shape of the precipitates of intermetallic k-phase. k phase, which chemical composition consists of copper, aluminum, iron and nickel can occur in several forms in the alloy microstructure (Łabanowski & Olkowski, 2014). The research has shown that the shape, size and distribution of k-phase precipitates in cast bronze BA1055 microstructure significantly affect the mechanical properties. The strength and hardness of the alloy increase with increase in the percentage of copper in solution ( $\alpha$ -solid solution of copper in aluminium matrix), increase in the number and distribution of the intermetallic compound and decrease in grain size. Modern engineering materials must be able to carry high load and simultaneously possess

high reliability. The materials of choice have high strength, improved hardness, high-toughness, and low density (Rodney, 2009). Micro-structural variation in the high strength aluminum alloys exists over a range of scales. At the atomic and nanoscopic scale, the microstructure is related to the mechanical properties of the alloy and involves defect structures, hardening precipitates and dispersed particles (Rakhit, 2013). The basic structure consists of cored dendrites of aluminum solid solution, with a variety of constituents at the grain boundaries or interdendritic spaces forming a brittle, more or less continuous network of eutectics (Smith, 1993). Intermetallic particles such as constituent and impurity particles exist at larger scales with minimum sizes generally between 0.5 to 1.0  $\mu\text{m}$ . Some types of these particles can achieve local thermodynamic equilibrium during ingot formation (Guofa et al, 2011). From the perspective of obtaining the desired mechanical properties at the nanometer scale, characterization has focused on the evolution of the alloy microstructure. Corrosion initiation however, is much more closely related to the large constituent particles whose compositions are based on major alloying elements (Haydar et al, 2014). One useful method for modifying microstructure is addition of micro alloying elements, which introduce secondary phase particles, interact with dislocations, and also serve as heterogeneous nucleation sites in the metal matrix. Industrial variants of these alloys have considerably more complex compositions than the simple, “model” alloys. Compositions are frequently chosen so as to modify the structure through the formation of multiple precipitate phases (Sami et al, 2007). Multiple phase precipitation and the presence of additional elements in solution pose severe difficulties in clarifying the effect of any given element on  $\theta'$  precipitation (William et al; 2006). Recent work proposed a different interpretation of the origin of hardening in the alloy. This is thought to be related to a subtle redistribution of the solute through a preferred interaction between intermetallic compound and

dislocations. The diffusion of atoms to the dislocation loops and helices lead to heterogeneous precipitation of the secondary phase at these defects (Schmidt & Schmidt, 1997). One method to achieve required mechanical properties is the addition of alloying elements that dissolve in solid solution at elevated temperatures and precipitate out at lower temperature (Kear et al; 2007).

Careful design of alloys can further improve the mechanical properties of age hardenable alloys. One approach is the addition of trace amounts of elements, referred to as micro alloying elements, which enhance the nucleation rate by stimulating heterogeneous nucleation of the hardening phases (Cenoz, 2010). Donatus et al (2012) successfully studied the feasibility of producing a dual-phase aluminum bronze alloy and the use of selected treatments to manipulate the mechanical properties of the produced alloy, as a potential replacement for conventional structural materials, particularly steels. Sand casting technique was used and was found to be effective based on its advantages of low cost, ease of use and flexibility. Recently, it was recognized that trace additions of silicon is suitable for stimulating the precipitation of  $\theta'$  phase, resulting in enhanced peak hardness. Another approach for improving mechanical properties is the introduction of high density of dislocations by plastic deformation (ISO 428, 2000). Again it has been proposed that the relative ability of an element to modify the alloy structure can be determined from the atomic number of the element or from the position in the periodic table (Nnuka, 1991). Dispersoids can pin grain growth, limiting grain size, thus making a small contribution to increased strength. It is shown that the nature and kinetics of the precipitation process depend on the solute–solute interactions that produce solute clusters. The solute clusters precede the formation of GP zones or precipitation and have a defining role on the nature and kinetics of the subsequent precipitation processes (Lawrence & Vimod, 2006). Moreover, interactions between solute clustering and dislocations can have a significant

hardening effect, the origins of which seem to be distinctly different from the conventional notion of precipitation hardening (Gronostajski, 2001). Dispersion strengthening is a means of strengthening alloys where in small particles of usually less than 0.1  $\mu\text{m}$  of a hard, inert phase are uniformly dispersed within a load-bearing matrix phase (Norman, 2009). Subsequent evolution of the microstructure involves the replacement of the GP zones with more stable phases (Issac, 2010). Recently, it has been realized that the nano-scale clusters that generally form in the initial stage of phase decomposition are extremely important in controlling precipitate microstructures and the resultant alloy properties (Labanowski and Olkowski, 2014). Characterization of mechanical and micro structural sensitive properties of Zn-Al alloys was studied by Rodney (2009). In this study, the effect of copper content on the microstructural changes and mechanical behavior of Zn-Al alloy was investigated. Mechanical properties such as ultimate tensile strength, percentage elongation, impact strength, hardness and also micro structural features of Zn-Al containing 0-3%wt copper and 0.30wt% magnesium were systematically studied. The alloys were processed by liquid metallurgy route. X-ray diffraction technique was employed to identify the phases formed in the alloys and the elemental quantification was performed by means of energy dispersive X-ray spectroscopy analyzer. The microstructure of the alloys was examined by scanning electron microscope. It was observed that tensile strength and yield strength of the alloys increased continuously with increasing copper content. This was attributed to the formation of intermetallic compounds  $\text{CuZn}_5$ ,  $\text{CuZn}_2$ ,  $\text{CuAl}_2\text{O}_4$  and  $\text{Cu}_9\text{Al}_4$ . The impact strength decreased after 2wt% copper, ductility increased with increasing copper content. Micro hardness of the experimental alloys varied as a function of the copper content. The effects of copper as an alloying element are to raise the ultimate tensile strength and endurance limit and to improve the casting characteristics and machinability, but its resistance to



corrosion suffers (Pisarek, 2007). The formation and the distribution of various precipitates from supersaturated solid solution have a significant meaning in strengthening many engineering alloys (Pisarek, 2008a). The strength of the precipitation hardening alloy depends on the distribution, size and shape of the precipitated intermetallic phases (Nnuka, 1991). Regarding the type of the precipitates the corresponding hardness, tensile strength and ultimate tensile strength of the alloy is expected. The  $\text{CuAl}_2$  phase may be present at the boundaries of  $\alpha$ -Al and/or in the Al matrix. The  $\text{CuAl}_2$  phase serves as a reinforcing phase and improves both the strength and the wear resistance of the alloy (Gronostajski, 2001). The primary species of precipitation strengthening are secondary phase particles. These particles impede the movement of dislocations throughout the lattice (Pisarek, 2001). Physically, the strengthening effect can be attributed both to size and modulus effects, and to interfacial or surface energy (Nnuka, 1991). Precipitate particles also serve by locally changing the stiffness of a material. Dislocations are repulsed by regions of higher stiffness. Conversely, if the precipitates cause the material to be locally more compliant, then the dislocation will be attracted to that region (Nnuka, 1991). As the size of the secondary phase particles increases, the particles impede dislocation movement and it becomes increasingly difficult for the particles to cut through the material (Pisarek, 2008b). Due to the composition, aluminum bronze can be divided into two basic groups: Elementary (binary) alloys; i.e. Cu-Al alloys without any other alloying elements, Complex (multicomponent) alloys; besides the Al. These alloys contain other alloying elements like Fe, Ni, Mg, etc. whose content does not exceed 6% (Schmidi & Schmidt 1997). The role of alloying elements like nickel, iron and manganese which tend to stabilize the  $\beta$ -phase and effectively permit slower cooling rate, is very important. When iron and nickel are present at a normal level, they modify the structure of aluminum-bronze and instead of the normal  $\alpha + \beta$

structure with small but tolerable amounts of  $\beta$ , only the  $\beta$  phase is formed. The complex alloys which are notable for their high strength, corrosion and erosion resistance can be cast easily without the influence of the eutectoid structure (Chee& Mohamed, 2000). Commercial binary alloys of aluminum bronze usually contain about 8% Al but the best combination of properties can be obtained in the range of 9 to 11% Al. In the range of binary aluminum bronze, advantage can be taken of characteristic eutectoid transformations in which phase changes occur. This is very important in respect to engineering alloys where suitable heat treatment can confer desired properties in castings. The  $\alpha$  - aluminum bronze having an F.C.C structure is suitable for application where high corrosion resistance is important. Iron is frequently added to aluminum –bronzes. 1% iron improves the mechanical properties mainly due to grain refinement. Addition of up to 5.5% of iron is permitted but above 1.2%, it appreciably improves strength characteristics like tensile strength and hardness but with lowered ductility and also mildly stabilizes  $\beta$  phase. When iron is present in trace quantity, it is not termed as impurity but a useful addition. Beyond 1% it is present as a finely dispersed phase in the structure and has no adverse influence on corrosion resistance. Nickel is the most frequent alloying element in aluminum bronze because it has a strong influence on the stabilization of  $\beta$  phase. The  $\alpha/\beta$  structure is retained at low cooling rates even with 2% nickel. The addition of nickel to an alloy containing iron has a beneficial effect in modifying the stable structure. The  $\gamma_2$  phase formation is suppressed and the  $\alpha$  solid solution range is extended towards higher aluminum contents. The combined effect of iron and nickel produces a kappa phase which has the same structure as the  $\beta$  aluminum bronze (Labanowski& Olkowski, 2009). The size and disposition of kappa can be controlled from fairly massive to fairly dispersed forms. Hence, in alloys containing less than 11% aluminum, the decomposition of  $\beta$  produces  $(\alpha+k)$ , when nickel and iron are also present. By

regulating the speed of cooling, the transformation of  $\beta$ ,  $\alpha+k$  can then be arranged to obtain two very important effects: hardening by precipitation of ( $k$  in  $\alpha$ ) and simple martensitic straining to obtain relatively softer phase as  $\beta$  decomposes to  $\alpha$ . In making casting with bronzes containing nickel and iron, it is important to see that differential structure is avoided, as indeed should be the case for all complex castings. Manganese is added to the complex alloy because it has deoxidizing effect in the melted metal. It improves the corrosion resistance of the aluminum bronze as it stabilizes the  $\beta$  phase and reduces the risk of decomposition of the eutectoid. The stabilization of  $\beta$  phase can be achieved with addition of low level of manganese while addition of up to 4% is sufficient to retain  $\alpha/\beta$  structure. The main drawback, however is that aluminum bronze with a low manganese addition is susceptible to corrosion. When the addition exceeds 11% a fully stable  $\alpha/\beta$  structure is obtained resulting to improved corrosion resistance of the product (Labanowski & Olkowski, 2009). Silicon increases ductility, improves tensile strength but significantly reduces hardness in composition of 1-40% weight. Hideous aluminum bronzes are tough and suitable for cold and also hot forming. Heterogeneous alloys are stronger, harder, but they have lower cold forming properties compared to the homogenous alloys. They are suitable for hot forming and have good cast properties. Aluminum bronzes are distinguished by good strength, even at elevated temperatures, and also very good corrosion resistance and wear resistance. These alloys are used in mechanical engineering for much stressed gear wheels and worm wheels, armatures working at elevated temperature etc. Aluminum bronze with Al content from 4.5 to 11% is used for forming elementary or complex components while with Al content from 7.5 to 12% only complex components can be cast.  $CuAl_{15}$  bronzes are used for cold forming. It is supplied in the form of sheets, strips, bars wire and pipes. (Labanowski & Olkowski, 2014) In the soft state this alloy can reach the tensile strength of 380MPa, ductility

of 40% and hardness of 70 to 110HB. It is used in the boat building, chemical, food and paper making industries. Complex aluminum bronzes are normally used for hot forming.  $\text{CuAl}_{19}\text{Mn}_2$  is used for armatures (below  $250^\circ\text{C}$ ) production.  $\text{CuAl}_{19}\text{Fe}_3$  is used for bearings shells and valve seats production, etc.  $\text{CuAl}_{10}\text{Fe}_3\text{Mn}_2$  alloy has higher hardness and strength and is suitable for shells and bearings production and is replacing leaded bronzes at up to temperature of  $500^\circ\text{C}$ . For temperatures up till  $600^\circ\text{C}$ , the  $\text{CuAl}_{10}\text{Fe}_4\text{Ni}_{14}$  where Ni replaces Mn, is used. In sea water corrosion environment, this bronze attains better properties as compared to chrome- nickel corrosion steels. It is resistant against cavitation corrosion and stress corrosion.  $\text{CuAl}_{10}\text{FeN}_{14}$  is used for castings, water turbines and pumps construction, valve seats, exhaust valves and other components working at elevated temperatures and also in the chemical industry. Besides  $\text{CuAl}_{19}\text{Ni}_5\text{Fe}_1\text{Mn}_1$  the nickel alloy consists also of a higher content of manganese. It is suitable for cars worm wheels, compression rings, friction bearings for high pressure etc (Skocovsky, 2006). The influence of 3% iron on the copper-aluminum system, Iron addition of this order only slightly modifies the binary diagram and particles of iron are precipitated throughout both the  $\alpha$  and  $\beta$  phases. The solubility of iron at high temperature varies according to the aluminum content in all  $\alpha$ -phases. The iron begins to precipitate when the temperature drops below about  $1000^\circ\text{C}$ , whilst the alloys with greater amounts of aluminum, do not precipitate the iron until the temperature has fallen, in some cases, as low as  $850^\circ\text{C}$ . The iron appears as fine precipitates, evenly distributed throughout the structure with no apparent tendency for concentration at grain boundaries. The microstructure is typical of the general distribution of constituents produced by hot-working the duplex aluminum bronzes. The  $\alpha$  is surrounded by a matrix of dark etching  $\beta$ , both constituents having been elongated in the direction of working. Twins are present and could be revealed by heavy etching. The iron- rich phase is present as tiny particles

distributed throughout the microstructure, but not visible at the magnification of this micrograph(Labanowski& Olkowski, 2011).Alloys containing 9-12% aluminum with additions of up to 6% each of iron and nickel represent a most important group of commercial aluminum bronzes. The common alloys, which normally contain 3-6% each of these two elements, have been fully investigated in view of their excellent combination of mechanical properties and corrosion resistance.A vertical section throughout the Cu-Al-Ni-Fe quaternary diagram at 5% Fe and 5% Ni It can be seen that the system remains essentially the same as the binary system with the introduction of an additional phase, k. The k phase is related to the iron- rich phase in the Cu-Al-Fe system.K absorbs aluminum from the matrix and hence extends the apparent range of the  $\alpha$ -field. Thus, under equilibrium conditions,  $\beta$  is not retained below 600°C unless the aluminum exceeds 11%, as compared with 9.45 in the binary system. The k precipitate in the  $\alpha$ - matrix has a pronounced effect on the properties and considerably increases the mechanical strength. At the same time, the reduction in ductility is not as marked as would occur if  $\beta$  had been formed in the binary alloy to give an equivalent strength. This is the most outstanding advantage which alloys of this type have over other aluminum bronzes. The absence of  $\beta$  also removes any danger from eutectoid formation or whatever the previous history of the material with respect to heat treatment or rate of cooling, unless the aluminum content is in excess of about 10.5%.This is generally carried out at about 975°C. At this temperature the alloy has a  $\beta$ + k matrix containing areas that become elongated in the direction of working. The mechanical properties of aluminum bronze depend primarily on aluminum content. Alloys with up to about 8% aluminum have a ductile single phase structure and are the most suitable for cold working into tube, sheet, strip and wire (Mrowka, 2010). As the aluminum content is increased to between 8% and 10% the alloys are progressively strengthened by a second harder phase which

makes them more suitable for hot working and casting. Above 10%, even greater strength and hardness is developed for specialized wear resistant applications. Some of the aluminum bronze alloys are of comparable strength to low carbon steels and stronger than most stainless steels. They retain a substantial proportion of their strength at elevated temperature and gain strength slightly at lower temperatures, while retaining ductility (CDA, 2000). Nickel-aluminum bronze {NAB} is a group of aluminum bronzes, which contains 9-12wt% aluminum with additions of iron and nickel up to 4 wt % (Wharton et al, 2005). It has an excellent resistance to stress corrosion and corrosion fatigue. The freedom from oxide flaking combined with corrosion resistance, together with good creep and fatigue properties at elevated temperatures, make aluminum bronzes ideal for high temperature service. It is known aluminum bronzes further alloyed with iron and nickel form alpha and beta phases and homogenous distribution of FeNiAl<sub>9</sub> and Al + NiAl<sub>3</sub> in the alpha and beta phase enhance the formation of harder and more resistant structure by tightening the matrix (Kaplan & Yildiz 2003). Iron also improves the tensile strength because it acts as a grain refiner while nickel improves proof stress and corrosion resistance and has a beneficial stability effect on the metallurgical structure. Manganese also performs a stability function. Two other alloy types complete the range of commercial alloys: silicon up to about 2 % with aluminum up to about 6% form a range of alloys known as aluminum silicon bronzes. These bronzes have a higher strength than normal single – phase aluminum bronze but are cast and hot – worked more readily, have similarly low magnetic permeability and excellent resistance to shock loading. Silicon also improves machinability. The alloys are available in wrought and cast forms. Manganese (about 13%) is the major addition in a series of manganese aluminum bronzes with aluminum levels of 8 – 9%. Their foundry properties are better than for the aluminum bronzes and they have good resistance to

impingement and cavitations as well as being heat treatable. They have excellent welding properties (CDA, 2000). The microstructure of aluminum bronze with less than 11% aluminum consists of alpha solid solution and the iron and nickel rich kappa phase. The kappa phase absorbs aluminum from the alpha solid solution preventing the formation of the beta phase unless the aluminum content is above 11%. The kappa phase increases the mechanical strength of the aluminum bronzes without decrease in ductility. The decrease in ductility of the aluminum bronzes occurs only when the beta phase is formed. The beta phase is harder and more brittle than the alpha phase. Beta phase is formed if the material is quenched or fast cooled, which then transforms it into a martensite structure. Tempering the martensite results in a structure of alpha with kappa precipitates. The tempered structure is very desirable and has high strength and hardness. The slowly cooled, as cast structures consist of alpha and kappa phases. Kappa is present in the lamellar form and finely divided in all the alpha areas. Iron and nickel also suppress the formation of the gamma double prime phase which has deleterious effects on the properties of aluminum bronzes. Gronostajski (2001) investigated the effect of phase transformation on the limit of strain in aluminum bronze. The plastic formability of aluminum bronzes is determined by the structures which depend on the chemical composition, the temperature and strain rate. The tested bronzes have very good plastic properties at elevated temperatures. In such deformation condition in which  $\beta$  phase predominates in the structure in the range of  $\beta$  phase eutectoid transformation temperature, anomalous changes in the limit of strain occur. The aim of the research was to find out how the phase transformation affected the formability of aluminum bronzes at different temperatures and at different strain rates and to determine the regions of good strength. Result showed that at low temperatures below 700K, the formability of aluminum bronze decreases with increasing aluminum content and only  $\text{CuAl}_8\text{Fe}_3$  bronze has quite

enough good limit to strains. In this temperature range the limit of strain increases slightly with decreasing deformation rate. At higher temperature, dynamic reconciliation begins and CuAl<sub>10</sub>Fe<sub>3</sub>Mn<sub>2</sub> aluminum bronze containing mainly  $\beta$  phase become super plastic. The smaller the fraction of  $\beta$  phase in the aluminum bronze, the higher the temperature at which the bronze becomes super plastic. Magnesium was introduced into the melt in different proportions from 1-4 wt%. After the alloying process, the specimens were sectioned, ground, polished and etched before viewing under an optical metallographic microscope. Mechanical tests were carried out on the specimens to determine hardness, tensile strength, yield strength and ductility of each specimen. It was concluded that the addition of magnesium to aluminum bronze increased both hardness and yield strength of aluminum bronze and reduced ductility (Łabanowski & Olkowski, 2012). Pisarek (2008a) successfully studied aluminum bronze containing vanadium and having improved wear resistance. The alloy had the composition of Al 13-18%, Fe-2-6%, V 0.3-2.5% and Cu as balance. In addition to the above elements, small amount of impurities were presents in the alloy up to amount of about 0.50 wt % without affecting the basic properties of the alloy. The alloy was cast either statistically and centrifugally cast and had greatly improved wear resistance over the conventional aluminum-iron- copper alloy due to the addition of vanadium. The wear resistance of the alloy was determined on a rolling slip friction device (Ampler wear test machine). Moradlou et al (2011) studied the effect of magnesium and nickel on tribological properties of cast aluminum-bronzes. After casting, the samples were heat-treated, quenched and aged. Wear test was conducted by pin-on-disk apparatus and wear mechanisms and microstructure of the specimens, were studied by scanning electron microscopy. It was shown that addition of magnesium and nickel reduced the size of  $\alpha$  and  $\beta$ . phases. Increasing the amount of magnesium and nickel to certain percentage, improved the



mechanical and tribological properties of the alloys. Increased alloying elements decreased the wear mechanism. Sami (2007) successfully investigated the effect of impurity elements on the mechanical properties of aluminum bronze C95800. At present, there is no consensus on the maximum allowable limits for impurity elements such as Pb, Zn, Sn, Cr, Be, Bi, and Se in aluminum bronze castings. The impurities could adversely affect mechanical properties and promote cracking during welding and heat treatment. The study was undertaken to evaluate the effect of such impurity elements, on the mechanical properties, heat-treatment and weld ability of the most popular aluminum bronze C95800. To date, mechanical properties of single and two elements additions have been completed. UTS and YS values were in excess of the minimum specified in ASTM B/48. However, the % elongation was reduced to below or just above the specified minimum in a few cases. Labanowski & Olkowski (2011) investigated the effect of chemical composition on the mechanical properties of BA1055 bronze. Properties of over hundred melts were analyzed. Metallographic investigations were performed on the samples taken from five sand cast bars. It was shown from the research that small changes in chemical composition can significantly alter the mechanical properties of BA1055 bronzes. The effect of intermetallic *k*-phase, i.e. its chemical composition, shape and dimension of precipitates, seems to be the most important. Regression calculations showed that mechanical properties of BA1055 bronze casting can be predicted with specific probability on the chemical composition of the alloy. Higher amount of eutectoid  $\alpha+\gamma_2$  in bronze microstructure enhances tensile strength of the alloy. The strengthening effect of *k* -phase depends not on the amount of this phase in the microstructure, but on its morphology (dimension and shape of precipitates). Haydar et al (2014) studied the effect of graphite on mechanical and machining properties of aluminum bronzes. In the research, a base aluminum bronze alloy with chemical composition Cu-10%Al was produced based

on powder metallurgy technique with a determined suitable compacting pressure of 400MPa, sintering for 1 hour in 920°C in a vacuum furnace ( $10^{-4}$ ), and then quenching from 950°C in cold water and tempered at 450°C for 2 hours. Graphite particles of 0.05, 0.1, 0.3, 0.6, 1 and 3 weight percentages were added as reinforcing elements to the alloy. The influence of the graphite particles on the structure physical, mechanical and machining properties of the base alloy (aluminum bronze) were investigated. These include microstructure, hardness, compressive strength and the roughness of the machined surface. Tests analysis are conducted using scanning electron microscope (SEM), X-ray fluorescence (XRF), X-ray diffraction (XRD), micro hardness test, compressive strength test as well as machining test. The result of the investigations showed that 0.3 wt% of graphite particles had the greatest effect on the properties of the studied aluminum bronze increased the hardness by 7.93%, and the compressive strength by 11.62%. The result of the machining experiments shows that percentage of graphite particles reduced the surface roughness by (32.38% to 22.66%) when turned with the same machining conditions. Prasad (2004) studied the sliding wear behavior of aluminum bronze under varying material compositions, microstructure and test conditions. The aluminum bronze composition 9.9Al, 1.2Fe, balance of Cu and leaded-tin bronze were investigated. The alloy melts were solidified in cast iron moulds in form of long cylindrical tubes. The microstructure of the aluminum bronze revealed primary  $\alpha$ -phase dendrites surrounded by copper-aluminum compound and iron particles. The soft  $\alpha$ -phase, a solid solution of aluminum in copper provides ductility and compatibility to the alloy system. The copper-aluminum alloy and iron particles were hard and imparted strength. The alloy did not contain any solid lubrication and crack sensitivity phase like lead in the leaded-tin bronzes. The leaded-tin bronze attained high density but less hardness of 70-79HV, inferior strength and lower elongation as compared to aluminum bronze of

hardness 162 HV. Kaplan & Yildiz (2003) investigated on the mechanical properties and microstructures of an aluminum bronze subjected to some physical treatments. In particular, the solidification structure, the effects of solution treatment, tempering heat treatment and mold types on the microstructure of the aluminum bronze produced in two different molds were examined. The result showed that the heat treatments have some interesting effect on the mechanical properties, microstructures and phase transformation temperatures of the samples. It was observed that  $\alpha + \beta_i$  and  $\alpha + \beta_{ii}$  phase transformation were formed depending on both the die casting and the heat treatments, but in contrast  $\alpha + \beta$  phase were formed sand-casting specimen. Sami et al (2007) investigated the improvement of casting condition for some aluminum bronze alloys. They used two types of aluminum bronze alloys in order to determine the proper methods of melting and casting in two different conditions; with treating materials as (Albral 2, Lagos 50 and deoxidizing tube (E3) and without determining the effects of these conditions on mechanical properties of alloys. The alloys were (a) Aluminum bronze alloys (AB1) and (b) Nickel-aluminum bronze alloys (AB2). These alloys were produced with different melting processes and cast method. The first one was made by preparing the charge materials to be melted and then, to the cast process without using any types of additions and treatment materials. The second one was made with casting conditional control, proper techniques of casting were employed and protective layers were used to minimize the oxidation and other casting defects. The molten metals from both processes were poured into two types of moulds; sand and metal moulds, both types were in dimensions ( $\Theta 100 \times 250$ ) mm. The final products of each type of alloys in each type of conditions were used to perform many types of inspections; chemical analysis, visual test, structure examinations, hardness test and tensile test. The results of all processes and inspections showed that the properties of alloys which were treated and casted in metal moulds were

better than that casted in sand moulds. These alloy castings were free from shrinkage cavities, inclusions and porosities due to using suitable sequence in alloy contents melting, no overheat, reducing the melting time, selecting non-turbulence casting method and suitable selection of pouring temperatures. The mechanical properties (hardness and ultimate tensile strength) for treated nickel-aluminum bronze alloys (T-AB2) were found to be better than that for other alloys. Pisarek (2007) investigated on the crystallization of the aluminum bronzes with additions of Si, Cr, Mo and W. Additions of Cr, W, Mo and Si were introduced to create in the microstructure of aluminum bronze complex silicides of iron about high mechanical and physical properties to the bronze BA1044. The process of formation of the microstructure of bronze with use of the method of the thermal and derivative analysis (TDA) was analyzed. The examinations under the microscope and X-ray microanalysis of the surface distribution of elements were conducted. The result showed that in aluminum bronze BA1044 after addition of Si, Cr, Mo and/or W the phase KFe, KNi crystallized as complex silicides of iron. Elements such as: Fe and Si dissolved first in silicides in the smaller stage in the matrix of the bronze, Mn and Ni they dissolved in (matrix and silicides, Cr dissolved in the larger stage in silicides than in the matrix. W and Mo dissolved in silicides however they crystallized as nanocrystals in the metal matrix and create composite with it. Lee et al (2002) studied on the microstructure and mechanical properties of as-cast multi-aluminum bronze. The microstructure of as-cast multi-aluminum bronze was observed and analyzed by XRD, SEM and EDAX; also the mechanical properties were tested. The relations between the microstructure and mechanical properties were discussed. The results showed that the microstructures of as-cast multi-aluminum bronze were  $\alpha + \beta' + \gamma_2 + k$ , and the tensile strength, compressive strength impact strength and hardness were 563MPa, 1258MPa, 0.34J/cm<sup>2</sup>, and 38HRC respectively in room temperature. They also

suggested that because of the coarse structures and ill-distribution of the structures, subsequent heat treatment should be carried out to improve the microstructure and mechanical properties in order to meet the requirement of the usability in die materials. Abdul et al (2013) studied on the effect of microstructure of nickel-aluminum bronze alloy (NAB) on the corrosion behavior in artificial seawater using linear polarization, impedance and electrochemical noise tests. The alloy was heat treated in different heating cycles including quenching, normalizing and annealing. Microstructure of the specimen was characterized before and after heat treatment by optical microscopy and scanning electron microscopy. Results showed that the value of pearlite phase in the normalized alloy was much more than other specimens, leading to higher corrosion resistance. Polarization test showed that starting point of passivation in polarization of the normalized alloy was lower than other specimens. The dissolution of Mn and Fe rich phases increased the Mn and Fe content of solid solution, and this enhanced the passivation power of the surface of the alloy. The effect of the alloying elements were seen by a lower corrosion potential and an inflexion at around 280mV (SCE) in the polarization curve, indicating the preferential dissolution of some elements beyond that potential. The polarization curve showed that the anodic polarization behavior of the alloy in the solution was essentially controlled by the intermetallic phases, mainly containing Cu. Two types of corrosion, pitting and selective corrosion, were observed in the specimens after being exposed to artificial seawater (Abdul et al 2013). Despite some of the desirable characteristics most aluminum bronzes exhibit abysmally deficient responses in certain critical applications necessitating mechanical properties enhancement. Hence, the microstructure and mechanical properties of cast aluminum bronze reinforced with iron granules (millscale) were investigated in this paper. Cast samples of the composite made from metal mould contain

millscale in varied amount from 2-10 wt %. Standard specimens were prepared from these homogenized samples for tensile, charpy impact and micro hardness tests while the composite microstructures were studied using an optical microscope (Abdul et al, 2013). Results show that optimum improved mechanical properties were achieved at 4 wt % millscale addition with ultimate tensile strength (UTS) of 643.8MPa which represented 10.1% improvement over conventional aluminum-bronze. The composite also demonstrated impact resilience of 83.9J and micro-hardness value of 88.7HRB. Millscale presence in the aluminum bronze system induced a stable reinforcing kappa phase by nucleation mechanism which resulted to enhancement of mechanical properties. However, the composite properties were impaired on millscale addition above 4 wt% due to grain clustering (Anup, 2014). Oh-Ishi & McNelley (2004) studied the effect of iron on structure and mechanical properties of aluminum bronze. The mechanical properties of aluminum bronze apart from aluminum depend on the extent to which other alloying elements modify the structure. In this regard, iron has been found to be both effective and efficient grain refiner in aluminum bronze systems. The presence of iron in the system enables the inducement of a hard reinforcing phase,  $\text{CuAl}_{10}\text{Fe}_3$ , in proportion to the amount of iron and other alloying agents. According to this structure has proven to be responsible for the significant improvement in tensile strength while other desirable properties are not compromised. In particular, the precipitation of different stable  $\alpha$ , and  $\beta$  phases with intermetallic precipitates of  $\text{Al}_3\text{Fe}$ ,  $\text{Al}_5\text{Fe}_2$  and  $\text{Al}_{13}\text{Fe}_4$  (depending on both the quantity of Fe in the system and other processing conditions) impact significantly the alloy mechanical characteristics. Iron (Fe) granules can be cheaply obtained in commercial quantity from its generic oxide for the purpose of alloying same with aluminum bronze. Granulated iron oxide, commonly called millscale is usually formed on the surface of hot rolled profiles such as plates, sheets, bars, etc. Millscale formation invariably

represents a significant level of yield loss to millers as it often reflects in huge differences between input stock and final output tonnages. The accumulation of millscale on the shop floor over time usually create handling and disposal challenges. Consequently, researchers have proposed various efficient methods and possible areas of its application (Skocovsky, 2006). For example, in the construction industry, the mixing of millscale in varying proportions has demonstrated increase in soil permeability, strength characteristics and decrease plasticity. Another veritable area in which millscale has found application is in cement mortars (Lawrence& Vimod, 2006). The study reported impressive results on several mortar mixes of concrete made from millscale aggregates in terms of their compressive and flexural strengths including the drying shrinkage. The foregoing indicates high potentials for millscale usage in different engineering materials for enhanced performance. This is also capable of increasing the quantity recyclable thereby reducing drastically the environmental challenges pose by its accumulation on the shop floor. This has been demonstrated in the recycling of aluminum dwarf by direct incorporation in aluminum melts (Piaseczny& Rogowski, 2006). The current study investigates the quantity of iron particles weight percent addition in aluminum bronze that confers improved mechanical properties that makes the material suitable for applications requiring high strength combined with low wear rate. The increase of aluminum content, in two-phase (duplex) Cu-Al alloys, from 8% up to 10% leads to a progressive strengthening, due to the appearance of harder, body centered cubic beta-phase, which additionally makes the bronzes more suitable for hot working and casting. Ni alloyed, heat treated aluminum bronzes are ones with the highest strength among the nickel-bearing aluminum bronzes. They exhibit excellent yield and compressive strength, high hardness and adequate elongation. They are a good load-bearing material, suitable for heavy-duty machine details and such exposed of

high impact. The additional increase of the strength is achieved through a heat treatment. The material exhibits excellent corrosion and heat resistance, good machinability and weldability. It is used for bushings and bearings of heavy duty, gears, and wear parts. It find application in the marine as pump parts, machine tool parts, aircraft parts, as well for military applications. The most often is used in the aircraft landing gear as bearing components (Mokhtari, et al. 2012). Moradlou, et al (2011) investigated the microstructures and mechanical properties of the reinforced cast aluminum bronze by modified nano-SiC powders. The results show that the structures and micrograph of the samples are obviously refined, and the  $\beta$  phase was obviously reduced, while the strength and toughness are significantly increased by 14% and 15% simultaneously. Kaplan & Yildiz (2003) studied the effect of production methods on the microstructure and mechanical properties of aluminum bronze. In the study, the solidification structure treatment, tempering, and mold types of the aluminum bronze produced were examined. According to the results of the experiment, the metallographic structure of the aluminum bronze was heterogeneous in pre-heated die casting specimen but homogenous in the sand cast ones. After applying tempering treatment, the structures of the material become considerably homogenous, and the hardness and tensile strengths increased significantly. The main reason for the increase of the mechanical properties was the formation of  $\alpha + \beta$  phases and the homogeneously distribution of compounds such as  $\text{FeNiAl}_9$  and  $\text{Al}+\text{NiAl}_3$  in the phases as a result of the heat treatments. In other words new phase formation caused the formation of a harder and more resistant structure by tightening the main matrix. The subject of this paper was the evaluation of the effect of a small change in composition of aluminum bronzes on the microstructure, as well the evaluation the effect of alloying with Ni, modifying by Molybdenum and heat treatment on the hardness. Based on only three compositions and four regimes of heat treatment we make the next primary



conclusions: Nickel alter and improve uniformity of the cast structure, raises hardness, and act as microstructure stabilizer. Nickel-aluminum bronzes are complex alloys in which small variations in composition can result in the development of markedly different microstructures, which can, in turn to result in wide variations in properties (Guofa,et al 2011).

## **2.9 Summary of Literature**

According to ISO 428 (2000) specifications, most categories of aluminum bronze contain 4-10 wt% aluminum in addition to other alloying elements such as iron, nickel, manganese and silicon in varying proportions. William, (2010)modified copper alloys containing 9-12% aluminum with addition of up to 6% each of iron and nickel.Daniel and Alan(2002)studied aluminum bronze containing 3-6% each of iron and nickel in view of their excellent combination of mechanical properties and corrosion resistance.Pisarek (2007) investigated on the crystallization of the aluminum bronzes with additions of Si, Cr, Mo and W. Additions of Cr, W, Mo and Si were introduced to create in the microstructure of aluminum bronze complex silicides of iron about high mechanical and physical proprieties to the aluminium bronze. Oh-Ishi & McNelley (2004) studied the effect of iron on structure and mechanical properties of aluminum bronze.Haydar et al (2014) studied the effect of graphite on mechanical and machining properties of aluminum bronzes. Graphite particles of 0.05, 0.1, 0.3 0.6, 1 and 3 weight percentages were added as reinforcing elements to the alloy. The influence of the graphite particles on the structure physical, mechanical and machining properties of the base alloy (aluminum bronze) were investigated.Cenoz (2010) researched on the microstructures and mechanical properties of the reinforced cast aluminum bronze by modified nano-SiC powders.Standard specimens were prepared from the homogenized samples for tensile, charpy impact and micro hardness tests while the

composite microstructures were studied using an optical microscope (Abdul et al, 2013). Abdul et al (2013) studied on the effect of microstructure of nickel-aluminum bronze alloy (NAB) on the corrosion behavior in artificial seawater using linear polarization, impedance and electrochemical noise tests. Lee et al (2002) studied on the microstructure and mechanical properties of as-cast multi-aluminum bronze. The microstructure of as-cast multi-aluminum bronze was observed and analyzed by XRD, SEM and EDAX; also the mechanical properties were tested. Sami et al (2007) studied the improvement of casting condition for some aluminum bronze alloys. They used two types of aluminum bronze alloys in order to determine the proper methods of melting and casting in two different conditions. Kaplan & Yildiz (2003) studied on the mechanical properties and microstructures of an aluminum bronze subjected to some physical treatments. Labanowski & Olkowski (2011) investigated the effect of chemical composition on the mechanical properties of BA1055 bronze. Properties of over hundred melts were analyzed. Metallographic investigations were performed on the samples taken from five sand cast bars. Moradlou et al (2011) studied the effect of magnesium and nickel on tribological properties of cast aluminum-bronzes.

Pisarek (2008b) successfully studied aluminum bronze containing vanadium and having improved wear resistance. The alloy had the composition of Al 13-18%, Fe-2-6%, V 0.3-2.5% and Cu as balance. Gronostajski (2001) investigated the effect of phase transformation on the limit of strain in aluminum bronze. Schmid and Schmidt, (1997) investigated copper alloys contain other alloying elements like Fe, Ni, Mg, etc. whose content does not exceed 6% composition. Donatus et al (2012) successfully studied the feasibility of producing dual-phase aluminum bronze and the use of selected treatments to manipulate the mechanical properties of the produced alloy. Sekunowo et al (2013) studied the microstructure and

mechanical properties of cast aluminium bronze reinforced with iron granules (millscale). Cast samples of the composite made from metal mould contain millscale in varied amount from 2-10 wt.%. The samples were homogenised at 11000C for 10 minutes in order to relieve the as-cast structures. Millscale presence in the aluminium bronze system induced a stable reinforcing kappa phase by nucleation mechanism which resulted to enhancement of mechanical properties. However, the composite properties were impaired on millscale addition above 4 wt.% due to grain clustering. Nwaeju et al(2015) investigated the effect of niobium on the structure and mechanical properties of aluminium bronze. The study shows that tensile strength, yield strength, impact strength and ductility increased by 10% respectively. Microstructural analysis revealed the primary  $\alpha$ -phase,  $\beta$ -phase (intermetallic phases) and fine stable reinforcing kappa phase and these phases resulted to the enhanced mechanical properties. Adeyemi et al (2013) researched on the effect of addition of magnesium on the microstructure and mechanical properties of Aluminum Bronze. Magnesium was introduced into the cast in different proportions from 1-4 wt% also a cast with 0wt% of magnesium. At the end of the experiments, it was concluded that the addition of magnesium to aluminum bronze increases both hardness and yield strength of aluminum bronze and reduces its ductility.

Therefore given the ever increasing demands of copper and its alloys in engineering designs and construction especially in the areas that require high values of mechanical properties like propeller of sea-going vessel, it is important to take care of coarse intermetallic compound present in the alloy, the instability of beta phase and the formation of gamma phase which affects the alloy properties and then to develop veritable alloys that can stand a test of time with relative high

strength which in turn will reduce the causes of failure in engineering designs and constructions, as a potential replacement for conventional structural materials.

So this study intends to enhance the structural sensitive properties of copper-10% aluminium alloy using carbide forming elements such as titanium, zirconium, manganese, vanadium, nickel, chromium, molybdenum and tungsten. The microstructure of alloys would be observed and analyzed with SEM *and* EDX, also the mechanical and physical properties would be tested and design expert used to model the process parameter.

## **CHAPTER THREE**

### **MATERIALS AND METHOD**

#### **3.1 Materials and Equipment**

The following materials and equipment were used to carry out this project: Aluminum ingot (99.9% pure), copper ingot(99.9% pure) were the major elements while zirconium powder, titanium powder, manganese powder, vanadium powder, chromium powder, molybdenum powder, nickel powder and tungsten powder are the carbide forming elements used for modification in the experiment. The equipment used are bailout crucible furnace, hack saw and iron table, weighing balance, steel crucible pot, atomic absorption spectroscopy (Model: L3007A) was used for chemical analysis, The microstructural examination was conducted using optical metallurgical microscope (Model: L2003A), and scanning electron microscope (LEO-430i) equipped with energy dispersive spectroscope (LINK-ISIS-300).Digital hydraulic universal tensile testing machine (Satec series, Instron 600DX) was used for ultimate tensile strength and yield strength. Impact testing machine (Model no; UI820) was used for impact strength and portable dynamic hardness testing machine (Model: DHT-6) was used for hardness test. Emery papers of different grades, air-gun drying machine, Keller's reagents, lathe machine, forceps, grinding machine, and vice.Design expert 10.0.6.software was utilized to designthe experiment.

#### **3:2 Materials Sourcing and Preparation**

The major raw materials used for the project were pure copper and pure aluminum. About 150 kg of copper and 30kg of aluminum were sourced from Cutis cable Plc Nnewi. Titanium, zirconium, manganese, vanadium, nickel, chromium, molybdenum and tungsten were sourced from Kermel Chemical Reagent Co., Ltd. Hebei, Tianjin, China. The first step of the experiment was chemical analysis on these

elements and then charge analysis and quantification, in order to achieve the composition of Copper-10%wtAluminium. This was done by weight percentage calculation. The first element was copper (99.9% pure), which was melted in a bailout furnace. Then the corresponding third elements of composition 0.5 to 10% at interval of 0.5% were added and stirred and cast. Subsequently, the specimens were machined and tested for the required mechanical properties.

**Table 3.1:** chemical composition for control sample (wt%)

<b>Cu</b>	<b>Al</b>	<b>Fe</b>	<b>Ca</b>	<b>K</b>	<b>S</b>	<b>Cl</b>	<b>O</b>	<b>C</b>
89.69	9.67	0.08	0.04	0.03	0.07	<0.01	<0.01	0.4

### 3:3 Charge Calculations

Total Charge = 30kg or 30000g

Materials used were 90% copper and 10% aluminum

$$\frac{\% \text{ of metal}}{100} \times \frac{\text{Total Charge}}{1} = \text{Quantity of metal} \quad (3.1)$$

For quantity of copper (Cu):

$$\frac{90}{100} \times \frac{30000}{1} = 27000 \text{ g}$$

$$\text{Oxidation loss of copper (Cu)} = \frac{1}{100} \times \frac{27000}{1} = 270 \text{ g}$$

Total mass of copper charged:  $(27000 + 270) \text{ g} = 27270 \text{ g}$

For quantity of aluminum (Al)

$$\frac{10}{100} \times \frac{30000}{1} = 3000g.$$

$$\text{Oxidation loss of aluminum} = \frac{2}{100} \times \frac{3000}{1} = 60g$$

$$\text{Total mass of aluminum} (3000 + 60)g = 3060g$$

$$\text{Total mass of Cu \& Al charged} = 27270 + 3060 = 30330g$$

The additives (titanium (Ti), zirconium (Zr), manganese (Mn), vanadium (V), nickel (Ni), chromium (Cr), molybdenum (Mo) and tungsten (W) were added in various composition from 0.5% to 10%. The weights of the base alloy and one hundred and thirty treated alloys were measured out with electronic weighing balance before melting and casting with metallic mould.

**Table 3.2:** Compositions of base alloy and treated alloys.

S/No	Alloy Composition	S/No	Alloy Composition	S/No	Alloy Composition
1	Cu-10% Al	42	Cu-10% Al+0.5% Ni	83	Cu-10% Al+6.0% Ti
2	Cu-10% Al+0.5% Ti	43	Cu-10% Al+1.0% Ni	85	Cu-10% Al+6.5% Ti
3	Cu-10% Al+1.0% Ti	44	Cu-10% Al+1.5% Ni	86	Cu-10% Al+7.0% Ti
4	Cu-10% Al+1.5% Ti	45	Cu-10% Al+2.0% Ni	87	Cu-10% Al+7.5% Ti
5	Cu-10% Al+2.0% Ti	46	Cu-10% Al+2.5% Ni	88	Cu-10% Al+8.0% Ti
6	Cu-10% Al+2.5% Ti	47	Cu-10% Al+3.0% Ni	89	Cu-10% Al+8.5% Ti
7	Cu-10% Al+3.0% Ti	48	Cu-10% Al+3.5% Ni	90	Cu-10% Al+9.0% Ti
8	Cu-10% Al+3.5% Ti	49	Cu-10% Al+4.0% Ni	91	Cu-10% Al+10% Ti
9	Cu-10% Al+4.0% Ti	50	Cu-10% Al+4.5% Ni	92	Cu-10% Al+5.5% Zr
10	Cu-10% Al+ 4.5% Ti	51	Cu-10% Al+5.0% Ni	93	Cu-10% Al+6.0% Zr
11	Cu-10% Al+5.0% Ti	52	Cu-10% Al+0.5% Cr	94	Cu-10% Al+6.5% Zr
12	Cu-10% Al +0.5% Zr	53	Cu-10% Al+1.0% Cr	95	Cu-10% Al+7.0% Zr
13	Cu-10% Al+1.0% Zr	54	Cu-10% Al+1.5% Cr	96	Cu-10% Al+7.5% Zr
14	Cu-10% Al+1.5% Zr	55	Cu-10% Al+2.0% Cr	97	Cu-10% Al+8.0% Zr
15	Cu-10% Al +2.0% Zr	56	Cu-10% Al+2.5% Cr	98	Cu-10% Al+8.5% Zr
16	Cu-10% Al+2.5% Zr	57	Cu-10% Al+ 3.0% Cr	99	Cu-10% Al+9.0% Zr
17	Cu-10% Al+3.0% Zr	58	Cu-10% Al+3.5% Cr	100	Cu-10% Al+9.5% Zr
18	Cu-10% Al+3.5% Zr	59	Cu-10% Al+4.0% Cr	101	Cu-10% Al+10% Zr
19	Cu-10% Al+4.0% Zr	60	Cu-10% Al+4.5% Cr	102	Cu-10% Al+5.5% W
20	Cu-10% Al+4.5% Zr	61	Cu-10% Al+5.0% Cr	103	Cu-10% Al+6.0% W
21	Cu-10% Al+5.0% Zr	62	Cu-10% Al+0.5% Mo	104	Cu-10% Al+6.5% W
22	Cu-10% Al+0.5% Mn	63	Cu-10% Al+1.0% Mo	105	Cu-10% Al+7.0% W
23	Cu-10% Al+1.0% Mn	64	Cu-10% Al+1.5% Mo	106	Cu- 10% Al+7.5% W
24	Cu-10% Al+1.5% Mn	65	Cu-10% Al+2.0% Mo	107	Cu-10% Al+8.0% W
25	Cu-10% Al+2.0% Mn	66	Cu-10% Al+2.5% Mo	108	Cu-10% Al+8.5% W
26	Cu-10% Al+ 2.5% Mn	67	Cu-10% Al+3.0% Mo	109	Cu-10% Al+9.0% W
27	Cu-10% Al+3.0% Mn	68	Cu-10% Al+3.5% Mo	110	Cu-10% Al+9.5% W
28	Cu-10% Al+3.5% Mn	69	Cu-10% Al+4.0% Mo	111	Cu-10% Al+10% W
29	Cu-10% Al+4.0% Mn	70	Cu-10% Al+4.5% Mo	112	Cu-10% Al+5.5% Cr
30	Cu-10% Al+4.5% Mn	71	Cu-10% Al+5.0% Mo	113	Cu-10% Al+6.0% Cr
31	Cu-10% Al+5.0% Mn	72	Cu-10% Al+0.5% W	114	Cu-10% Al+6.5% Cr
32	Cu-10% Al+0.5% V	73	Cu-10% Al+1.0% W	115	Cu-10% Al+7.0% Cr
33	Cu-10% Al+1.0% V	74	Cu-10% Al+1.5% W	116	Cu-10% Al+7.5% Cr
34	Cu-10% Al+1.5% V	75	Cu-10% Al+2.0% W	117	Cu-10% Al+8.0% Cr
35	Cu-10% Al+2.0% V	76	Cu-10% Al+2.5% W	118	Cu-10% Al+8.5% Cr
36	Cu-10% Al+2.5% V	77	Cu-10% Al+3.0% W	119	Cu-10% Al+9.0% Cr
37	Cu-10% Al+3.0% V	78	Cu-10% Al+3.5% W	120	Cu-10% Al+9.5% Cr
38	Cu-10% Al+3.5% V	79	Cu-10% Al+4.0% W	128	Cu-10% Al+10% Cr
39	Cu-10% Al+4.0% V	80	Cu-10% Al+4.5% W	129	Cu-10% Al+9.0% Mo
40	Cu-10% Al+4.5% V	81	Cu-10% Al+5.0% W	130	Cu-10% Al+9.5% Mo
41	Cu-10% Al+5.0% V	82	Cu-10% Al+5.5% Ti	131	Cu-10% Al+10% Mo



### **3.4 Sand Preparation and Moulding**

Sand mould was prepared and used for the casting of the specimens. One barrow of sand was mixed in a sand mixing machine with ten litres of water, meanwhile impurities such as metals, hard lumps; stones etc. were removed using sieves of sizes 500 $\mu$ m and 400 $\mu$ m to obtain fine grain sizes. The sand was mixed well to ensure uniform distribution of the ingredients. The foundry floor was cleared of dirt and the floor board was put in place. Some moulding sands were sprinkled on the floorboard surface and then the patterns were introduced. Sand was introduced and rammed; the ingate runner and risers, plumbago (parting materials), rammers were used to prepare the mould. The patterns were removed and the cavities created repaired. The pattern removal was done slowly to prevent mould damage. After the pattern was removed and mould repaired, Ash was then sprinkled to the cavities to enhance easy flow of the molten metal inside.

### **3:5 Furnace Preparations**

A charcoal fired crucible furnace was used in melting and alloying of the metals. It has a maximum temperature of 2000°C. The inner part of the furnace was built with high refracting bricks. The furnace has a crucible steel pot of high melting temperature with a capacity of 35kg inserted into the furnace in which the charge metals were heated. For each charge, the crucible pot was removed, thoroughly cleaned to avoid other materials inclusions. The oil from the drum was allowed to flow through a leading hole to the furnace, with air blast from the blower continually meeting the burner thereby energizing the already lit fire. The roof of the furnace was then closed.

### **3.6 Melting and Casting**

For the base or master alloy, copper and aluminum were measured out and charged into the furnace. Considering the melting points of the metals, copper was charged in first and was heated for about 6mins then aluminum was added and heated for another 6mins at temperature of 1200°C. The charge was held for about 2-3minutes to super heat and then removed with the use of a pair of tongs and hand gloves. As melting progressed the melt was stirred manually from time to time in order to ensure a homogenous mixture and to facilitate dissolution of the alloying elements.

### **3.7 Machining**

The developed alloys samples were machined to the required dimension according to the British Standard (BS); BSEN ISO 6892-1:2016 for tensile, BSEN ISO 6505-4:2004 for hardness, BSEN ISO 148-1:2016 for impact strength, using a lathe machine at Delta State Polytechnic, Ogwashi-Uku. The tensile test samples were machined to 120mm in length and 10mm in diameter with a gauge diameter and length of 8mm and 50mm respectively. The samples for impact strength test were machined to 55mm x 10mm x 10mm in size with a 2mm deep notch ( $\Delta 45^\circ$ ) inscribed at the Centre of the sample while the hardness samples were machined to 20mm in length and 16mm in diameter. The machined samples were stored for structural, mechanical and physical properties investigations

### **3.8 Mechanical Tests**

The objectives of mechanical testing were to; provide data for use in the design of engineering structures; determine whether a particular specimen conforms to the properties assumed in its design and determine the response of materials to forces and loads. Digital hydraulic universal tensile testing machine (Satec series, Instron 600DX) was used for ultimate tensile strength and yield strength. Impact testing

machine (Model no; UI820) was used for impact strength and portable dynamic hardness testing machine (Model: DHT-6) was used for hardness test. These tests were carried out at Cutix Cable Plc Nnewi and Delta State Polytechnic Ogwashiuku.

### 3.8.1 Impact Test

The impact strength (BSEN ISO 148-1:2016) of the developed alloys was carried out with the Impact testing machine (Model no; UI820). The purpose of the impact testing was to determine the behavior of the specimens when subjected to high-rate loading, usually in bending, tensions or torsion. The specimens were placed horizontally as a single supported beam between the anvils 400mm apart. The striking hammer was used to strike the specimen on the face opposite to the notch. The energy required to break away the specimen was calculated and recorded using the formula:

$$Energy = WR(COS\beta - COS\alpha) \quad (3.2)$$

Where,

**W** = weight of the pendulum

**$\alpha$**  = angle through which the pendulum falls.

**$\beta$**  = angle through which the pendulum rises

**R**= distance between the centre of gravity of the pendulum and the axis of rotation.

### 3.8.2 Hardness Test:

The hardness test was conducted according British Standard for hardness (BSEN ISO 6505-4:2004) using portable dynamic hardness testing machine (Model: DHT-6). The brinell tester which consists of a hand operated vertical hydraulic press

designed to force a ball indenter into the test specimen was used. The specimen was placed on the anvil; the hand wheel was rotated so that the specimen along with the anvil moved up and contacted the steel ball of 10mm diameter. A load of 100N was applied hydraulically (by oil pressure) and the ball pressed into the sample. The diameter of the indentation made in the specimen by the pressed ball was measured by the means of a micrometer microscope containing an ocular scale, usually graduated in tenths of a millimeter, permitting estimation to the nearest 0.05mm. The indentation diameter was measured at three places at right angle to each other, and the average of the three readings was taken

$$BHN = \frac{F}{A} \quad (3.3)$$

$$A = \frac{\Pi D}{2} (D - \sqrt{D^2 - d^2}) \quad (3.4)$$

Where

BHN=Brinell hardness number

A=Surface area of Indentation

L = applied force/load, kg

D = diameter of steel ball, mm

d = diameter of the Indentation, mm

### 3.8.3 Tensile Test

Tensile test experiment is one of the widely used mechanical tests. It was performed according to British Standard for tensile strength(BSEN ISO 6892-1:2016)using a digital hydraulic universal tensile testing machine, Satec series,

Instron 600DX. The basic idea of tensile test is to place a sample of a material between two fixtures called “grips” which clamp the material. The samples were machined to 120mm in length and 10mm in diameter with a gauge diameter and length of 8mm and 50mm respectively. The machine was incorporated with a computer system which shows different properties of the specimen such as: percentage elongation, stress strain, yield point and stress/ strain graph when subjected to tensile pull. The specimen was fixed between the lower and upper jaw of the machine after this was completed, the machine was controlled to pull the specimen apart, putting the specimen under tension which caused the specimen to break at a breaking force. During the pull of the specimen, the tensile properties were measured. The tensile test is a destructive characterization technique. The various tensile properties were calculated as follows:

$$UTS = \frac{P \max}{A_o} \quad (3.5)$$

$$Y = \frac{Load}{A_o} \quad (3.6)$$

$$Elongation = \frac{L_f - L_o}{L_o} \quad (3.7)$$

$$Strain = \frac{Extension}{L_o} \quad (3.8)$$

$$E = \frac{P L_o}{A_o L_f - L_o} \quad (3.9)$$

Y is yield strength

P is load at any point up to the elastic limit.

L<sub>o</sub> is the gauge length

$A_0$  is original area

$\Delta L$  is the elongation or change in  $L_0$  at any Load P.

### 3.9 Physical Properties

#### 3.9.1 Electrical Resistivity

Electrical resistivity (also known as resistivity, specific electrical resistance, or volume resistivity) is an intrinsic property that quantifies how strongly a given material opposes the flow of electric current. A low resistivity indicates a material that readily allows the flow of electric current. Resistivity is commonly represented by the Greek letter  $\rho$  (rho). The SI unit of electrical resistivity is the ohm-metre ( $\Omega \cdot m$ ). A multimeter device was used to measure the resistance of the rod. The standard test method used for determining the electrical resistivity and conductivity of the samples is ASTM B193-87.

$$\rho = R \frac{A}{L} \quad (3.10)$$

Where

$R$ = is the electrical resistance of a uniform specimen of the material

$L$ = is the length of the piece of material

$A$ = is the cross-sectional area of the specimen

#### 3.9.2 Electrical Conductivity

Electrical conductivity is the reciprocal of electrical resistivity, and measures a material's ability to conduct an electric current. It is commonly represented by the Greek letter  $\sigma$  (sigma), but  $\kappa$  (kappa) (especially in electrical engineering) or  $\gamma$  (gamma) are also occasionally used. Its SI unit is siemens per metre (S/m).

$$\sigma = \frac{1}{\rho} \quad (3.11)$$

$\sigma$  (sigma) = resistivity

$\rho$  (rho) = Electrical conductivity

### 3.10 Structural Analysis

The microstructural analysis was done at National Metallurgical Training Institute (NMTI), Onitsha and Sheda Science and Technology, Abuja. The following procedures were observed before the specimens were viewed under the microscope;

- ❖ **Grinding:** this was carried out to ensure that the specimen is flat. This was achieved using fairly coarse file. After grinding, the specimens were washed to remove chips after which fine grinding was carried out using grades of emery papers in the order of 220, 500, 800, 1200 and 2200 grits respectively.
- ❖ **Polishing:** the well-grounded specimens were taken to a polishing machine to remove the fine scratches and obtain a mirror like finish. After polishing, the specimens were rinsed with water and dried.
- ❖ **Etching:** the purpose of etching is to make visible the structural characteristics of the alloy. The etchant used was iron (II) chloride solution.
- ❖ **Metallurgical microscopes:** after etching, the specimens were viewed under the microscope, to examine the structure of the alloys.

### 3.11 Experimental Design and Optimization Parameters

Response Surface Methodology (RSM) was used to investigate the influence of carbide forming elements on the structural sensitive properties of copper-

10% aluminium alloy. Table 4.2 represents the factors and levels of performance characteristics.

**Table 3.3: Represents the Design expert (Levels, Responses)**

Std	Run	Factor 1 A:% Zr	Response 1 Yield Strength MPa	Response 2 UTS MPa	Response 3 Hardness BHN	Response 4 Elongation %	Response 5 Impact Strength J	Response 6 Resistivity Mm	Response 7 Conductivity S/m
13	1	0.5							
6	2	1.5							
11	3	2.5							
9	4	3.5							
3	5	4.5							
1	6	5							
12	7	6							
8	8	6.5							
10	9	7.5							
7	10	8.5							
4	11	9							
5	12	9.5							
2	13	10							

### **Analysis of Variance (ANOVA)**

Analysis of variance is a statistical technique for testing whether the means of three or more populations are all equal. It has been utilized to obtain the best combination of materials that will give the optimum performance measures.

This technique separates the total variation displayed by a set of observation, as measured by the sum of square of the deviation from the mean into components associated with defined sources of variation used as criteria of classification for the observations.



**CHAPTER FOUR**  
**RESULTS AND DISCUSSION**

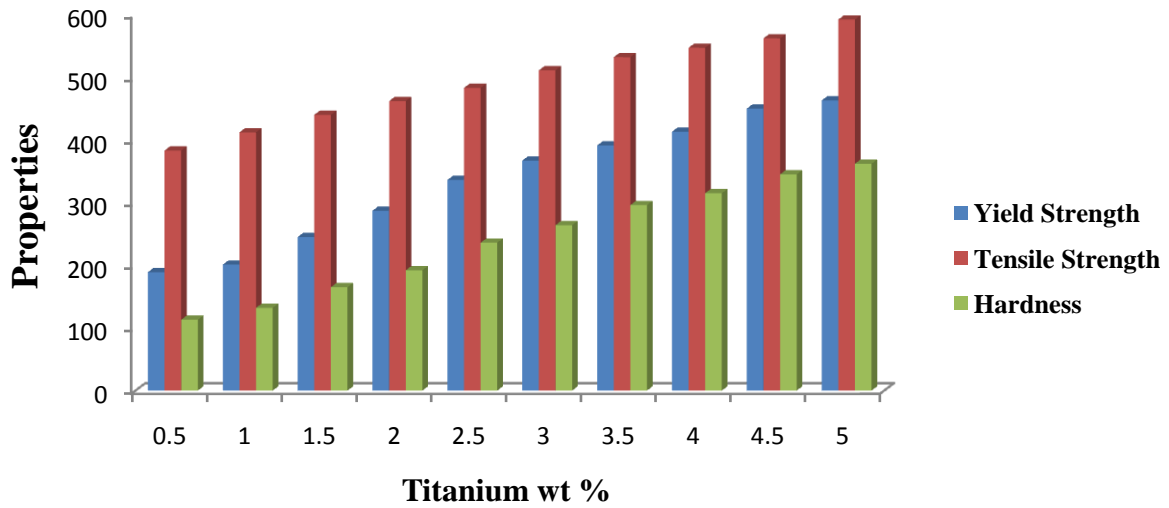
**Table 4.1: Mechanical and Physical Properties of Cu-10%Al Alloy and modified samples.**

Alloy	Yield Strength (MPa)	Ultimate Tensile Strength (MPa)	Hardness (BHN)	Elongation %	Impact Strength (Joules)	Resistivity $\rho$ ( $\Omega \cdot m$ ). $\times 10^{-8}$	Conductivity $\sigma$ S/m $\times 10^7$
Cu-10%Al	167	331	104	36.04	42.34	5.28	9.35
Cu-10%Al+0.5Ti	189	383	113	25.61	38.94	6.06	8.84
Cu-10%Al+1.0Ti	201	412	132	24.39	37.57	6.82	8.30
Cu-10%Al+1.5Ti	245	440	165	23.12	35.23	7.21	7.35
Cu-10%Al+2.0Ti	287	462	192	22.31	34.13	7.84	6.56
Cu-10%Al+2.5Ti	336	483	236	21.41	32.63	8.43	5.10
Cu-10%Al+3.0Ti	367	511	264	20.56	31.03	8.97	4.57
Cu-10%Al+3.5Ti	391	532	296	18.14	29.67	9.37	4.35
Cu-10%Al+4.0Ti	413	547	315	17.36	28.32	9.87	4.14
Cu-10%Al+4.5Ti	450	562	345	16.48	27.05	10.45	3.93
Cu-10%Al+5.0Ti	463	592	362	15.83	26.87	12.32	3.46
Cu-10%Al+0.5Zr	207	369	118	25.21	38.57	6.29	8.61
Cu-10%Al+1.0Zr	213	383	131	24.83	37.69	6.83	8.10
Cu-10%Al+1.5Zr	254	408	173	23.46	36.45	7.18	7.31
Cu-10%Al+2.0Zr	289	435	197	23.06	34.43	7.92	6.75
Cu-10%Al+2.5Zr	324	472	228	22.76	32.42	8.56	5.21
Cu-10%Al+3.0Zr	363	491	275	21.45	29.31	9.17	4.43
Cu-10%Al+3.5Zr	389	514	309	20.24	28.87	9.87	4.03
Cu-10%Al+4.0Zr	413	536	326	18.76	27.82	10.69	3.81
Cu-10%Al+4.5Zr	435	565	347	16.55	27.16	12.13	3.65
Cu-10%Al+5.0Zr	467	580	362	15.78	26.66	13.79	3.21
Cu-10%Al+0.5Mn	192	381	111	25.68	36.75	6.06	8.86
Cu-10%Al+1.0Mn	238	405	137	24.41	34.14	6.82	8.15
Cu-10%Al+1.5Mn	276	437	161	23.64	33.56	7.31	7.48
Cu-10%Al+2.0Mn	313	463	199	22.48	32.16	7.84	6.71
Cu-10%Al+2.5Mn	358	493	237	21.58	30.56	8.38	5.13
Cu-10%Al+3.0Mn	397	507	263	20.36	28.76	8.86	4.93
Cu-10%Al+3.5Mn	425	522	284	18.34	27.67	9.16	4.19
Cu-10%Al+4.0Mn	452	549	296	17.66	26.81	9.24	4.04
Cu-10%Al+4.5Mn	483	562	310	16.47	26.41	9.46	3.83
Cu-10%Al+5.0Mn	497	586	341	15.63	26.25	9.93	3.66
Cu-10%Al+0.5V	203	378	109	25.46	37.57	6.02	8.67
Cu-10%Al+1.0V	222	391	128	24.63	35.63	6.76	8.32
Cu-10%Al+1.5V	268	407	157	23.43	34.64	7.37	7.38
Cu-10%Al+2.0V	318	433	195	22.96	32.45	8.11	6.78
Cu-10%Al+2.5V	335	473	243	22.46	31.26	8.84	5.03
Cu-10%Al+3.0V	372	498	297	21.35	30.42	9.37	4.78
Cu-10%Al+3.5V	420	512	325	19.28	28.63	9.89	4.39

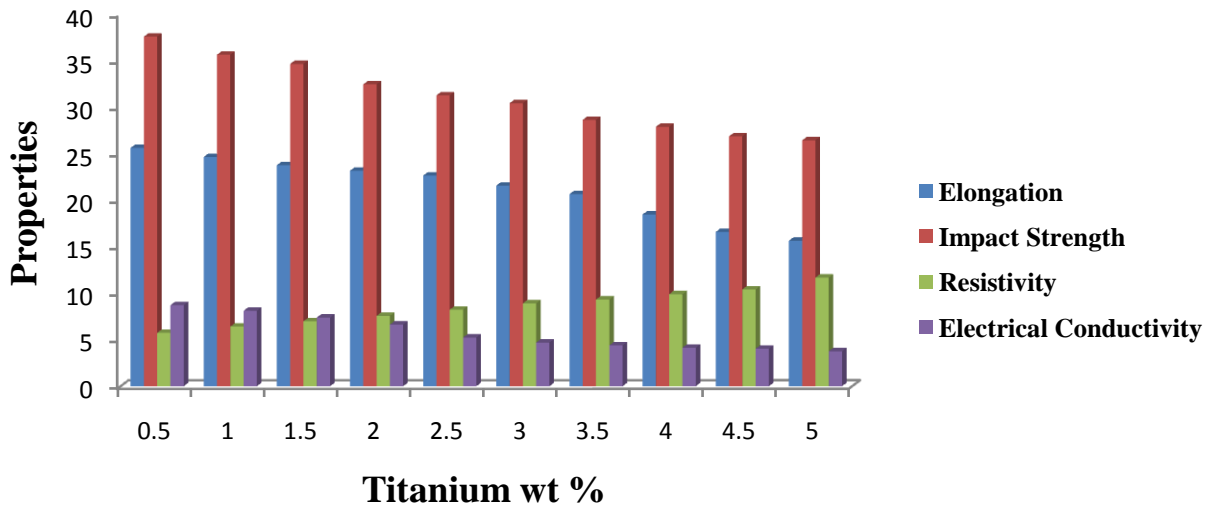
Alloy	Yield Strength (MPa)	Ultimate Tensile Strength (MPa)	Hardness (BHN)	Elongation %	Impact Strength (Joules)	Resistivity $\rho$ ( $\Omega \cdot m$ ). $\times 10^{-8}$	Conductivity $\sigma$ S/m $\times 10^7$
Cu-10%Al+4.0V	467	561	363	16.46	26.88	8.56	4.14
Cu-10%Al+4.5V	428	532	341	16.58	27.06	7.78	3.65
Cu-10%Al+5.0V	402	506	316	17.64	27.43	7.07	3.36
Cu-10%Al+0.5W	201	373	110	25.58	36.75	6.95	8.68
Cu-10%Al+1.0W	234	412	128	24.40	34.14	7.53	8.21
Cu-10%Al+1.5W	266	434	156	23.32	33.56	8.21	7.45
Cu-10%Al+2.0W	298	456	187	22.42	32.16	8.92	6.79
Cu-10%Al+2.5W	346	481	215	21.32	30.56	9.34	5.03
Cu-10%Al+3.0W	385	502	288	20.46	28.76	10.27	4.58
Cu-10%Al+3.5W	393	522	302	18.24	27.67	10.79	4.31
Cu-10%Al+4.0W	421	546	321	17.36	26.81	11.21	4.05
Cu-10%Al+4.5W	434	554	345	16.42	26.41	12.62	3.71
Cu-10%Al+5.0W	457	576	365	15.53	26.25	13.36	3.46
Cu-10%Al+0.5Cr	189	362	112	25.36	37.57	5.73	8.71
Cu-10%Al+1.0Cr	228	385	129	24.64	35.63	6.41	8.11
Cu-10%Al+1.5Cr	267	411	144	23.76	34.64	6.98	7.38
Cu-10%Al+2.0Cr	298	433	181	23.16	32.45	7.56	6.63
Cu-10%Al+2.5Cr	367	466	229	22.64	31.26	8.23	5.23
Cu-10%Al+3.0Cr	381	484	247	21.56	30.42	8.91	4.69
Cu-10%Al+3.5Cr	397	508	266	20.64	28.63	9.32	4.39
Cu-10%Al+4.0Cr	416	529	293	18.46	27.88	9.89	4.14
Cu-10%Al+4.5Cr	429	556	317	16.58	26.86	10.40	4.03
Cu-10%Al+5.0Cr	441	586	330	15.64	26.43	11.68	3.76
Cu-10%Al+0.5Mo	201	384	113	25.58	37.57	6.13	8.64
Cu-10%Al+1.0Mo	223	407	137	24.38	35.63	6.83	8.15
Cu-10%Al+1.5Mo	266	434	162	23.22	34.64	7.51	7.71
Cu-10%Al+2.0Mo	295	466	188	22.32	32.45	8.22	6.49
Cu-10%Al+2.5Mo	342	492	223	21.51	31.26	8.89	5.23
Cu-10%Al+3.0Mo	352	504	267	20.48	30.42	9.43	4.49
Cu-10%Al+3.5Mo	381	521	289	18.34	28.63	9.56	4.29
Cu-10%Al+4.0Mo	398	545	296	17.46	27.88	9.83	4.14
Cu-10%Al+4.5Mo	415	561	318	16.44	26.86	10.27	4.03
Cu-10%Al+5.0Mo	432	582	335	15.64	26.56	12.75	3.86
Cu-10%Al+0.5Ni	205	373	115	25.32	38.48	6.08	8.91
Cu-10%Al+1.0Ni	229	386	138	24.64	37.66	6.75	8.20
Cu-10%Al+1.5Ni	267	410	156	23.38	36.53	7.16	7.48
Cu-10%Al+2.0Ni	288	428	193	23.02	34.21	7.88	6.79
Cu-10%Al+2.5Ni	327	463	229	22.68	32.42	8.37	5.03
Cu-10%Al+3.0Ni	361	488	239	21.44	30.36	8.82	4.58
Cu-10%Al+3.5Ni	383	533	284	19.62	28.56	9.28	4.39
Cu-10%Al+4.0Ni	456	564	326	17.18	26.89	9.83	4.14
Cu-10%Al+4.5Ni	426	532	308	17.45	27.08	10.18	4.03
Cu-10%Al+5.0Ni	384	517	293	17.64	27.48	12.83	3.86

Alloy	Yield Strength (MPa)	Ultimate Tensile Strength (MPa)	Hardness (BHN)	Elongation %	Impact Strength (Joules)	Resistivity $\rho$ ( $\Omega \cdot m$ ). $\times 10^{-8}$	Conductivity $\sigma$ S/m $\times 10^7$
Cu-10%Al	167	331	104	36.04	42.34	5.28	9.35
Cu-10%Al+5.5Ti	489	609	369	15.61	26.64	12.45	3.24
Cu-10%Al+6.0Ti	501	612	378	15.39	26.07	12.82	3.10
Cu-10%Al+6.5Ti	558	694	410	15.12	25.23	13.21	3.02
Cu-10%Al+7.0Ti	497	582	362	16.81	26.83	12.84	3.56
Cu-10%Al+7.5Ti	466	573	346	17.21	27.83	11.43	3.90
Cu-10%Al+8.0Ti	447	561	334	18.56	28.33	11.07	4.37
Cu-10%Al+8.5Ti	435	532	326	19.14	29.67	10.87	4.85
Cu-10%Al+9.0Ti	423	532	315	20.36	30.92	10.37	5.14
Cu-10%Al+9.5Ti	412	528	310	21.78	31.05	09.85	5.63
Cu-10%Al+10Ti	405	520	302	22.83	32.87	09.42	5.86
Cu-10%Al+5.5Zr	479	596	363	15.41	26.57	13.48	3.11
Cu-10%Al+6.0Zr	487	603	376	15.25	26.09	13.83	3.01
Cu-10%Al+6.5Zr	504	618	385	14.76	25.45	14.08	2.81
Cu-10%Al+7.0Zr	529	625	390	14.56	25.13	14.42	2.75
Cu-10%Al+7.5Zr	544	656	394	14.36	24.82	14.86	2.21
Cu-10%Al+8.0Zr	553	632	399	14.15	24.31	15.17	2.03
Cu-10%Al+8.5Zr	549	514	376	16.24	26.87	14.87	2.43
Cu-10%Al+9.0Zr	423	506	365	17.76	28.82	13.69	2.86
Cu-10%Al+9.5Zr	415	485	350	18.55	29.16	12.13	4.35
Cu-10%Al+10.Zr	407	480	342	20.78	31.66	11.32	4.78
Cu-10%Al+5.5W	481	483	370	15.43	26.05	13.45	3.28
Cu-10%Al+6.0W	494	592	388	15.20	25.84	13.83	3.06
Cu-10%Al+6.5W	516	608	396	14.12	25.36	14.11	2.85
Cu-10%Al+7.0W	528	676	407	14.02	24.16	14.62	2.59
Cu-10%Al+7.5W	512	591	385	14.32	26.56	13.84	2.86
Cu-10%Al+8.0W	485	572	378	15.56	27.76	12.87	3.48
Cu-10%Al+8.5W	463	562	362	16.74	28.67	12.29	3.91
Cu-10%Al+9.0W	451	546	351	17.16	29.21	11.81	4.05
Cu-10%Al+9.5W	434	534	345	18.42	30.71	11.62	4.31
Cu-10%Al+10.W	427	536	336	20.53	31.25	11.26	4.86
Cu-10%Al+5.5Cr	459	592	338	15.36	26.27	11.93	3.54
Cu-10%Al+6.0Cr	478	605	349	15.14	26.03	12.41	3.11
Cu-10%Al+6.5Cr	497	611	354	15.06	25.64	12.98	2.88
Cu-10%Al+7.0Cr	518	653	381	14.86	25.45	13.36	2.63
Cu-10%Al+7.5Cr	507	586	379	16.44	26.86	12.23	2.93
Cu-10%Al+8.0Cr	488	574	364	17.76	27.42	11.91	3.49
Cu-10%Al+8.5Cr	467	568	356	18.84	28.33	11.22	3.89
Cu-10%Al+9.0Cr	456	549	343	20.46	29.88	10.89	4.14
Cu-10%Al+9.5Cr	449	526	337	21.68	30.46	10.30	4.63
Cu-10%Al+10Cr	441	516	330	16.84	31.93	09.68	4.86
Cu-10%Al+5.5Mo	451	594	343	15.48	26.47	13.13	3.44
Cu-10%Al+6.0Mo	483	607	357	15.38	26.13	13.83	3.23
Cu-10%Al+6.5Mo	496	614	362	15.22	25.74	14.21	3.05
Cu-10%Al+7.0Mo	525	626	371	15.02	25.35	14.92	2.69
Cu-10%Al+7.5Mo	538	642	383	14.81	24.76	15.39	2.23
Cu-10%Al+8.0Mo	518	584	367	16.48	26.42	14.43	2.49
Cu-10%Al+8.5Mo	496	561	359	17.84	27.83	13.56	2.89
Cu-10%Al+9.0Mo	478	555	346	18.46	28.38	12.83	3.14
Cu-10%Al+9.5Mo	465	551	338	19.84	29.86	12.27	3.83
Cu-10%Al+10.Mo	452	549	335	20.64	30.58	11.75	4.26

## 4.1 Mechanical and Physical Properties Analysis



**Figure 4:1: The effect of titanium composition on yield strength, UTS and hardness of Cu-10%Al alloy**



**Figure 4:2: The effect of titanium composition on properties of Cu-10%Al alloy.**

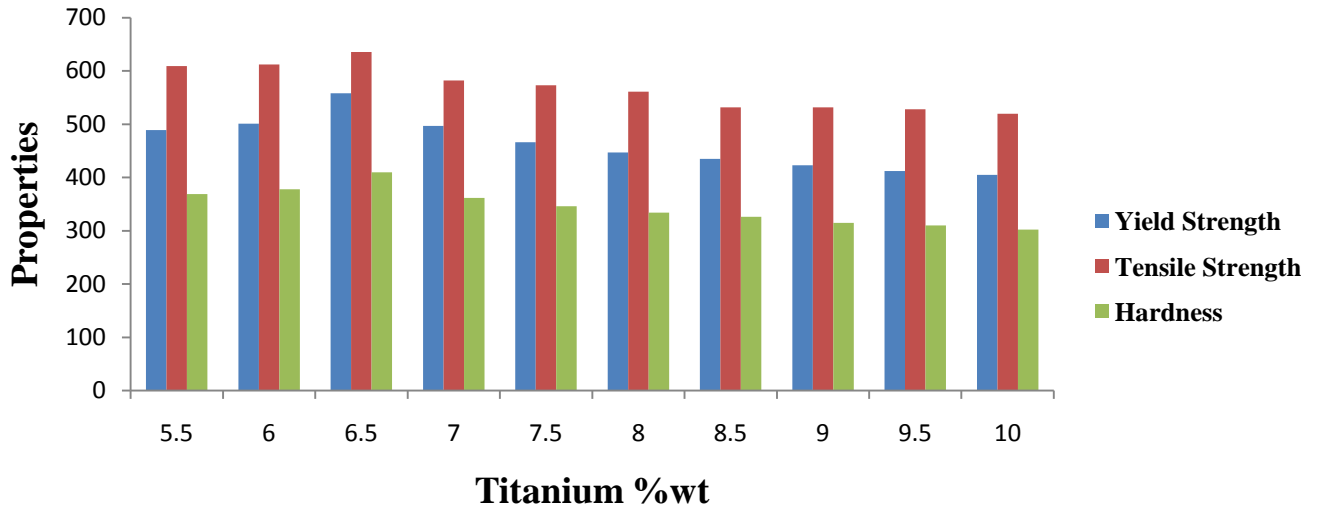


Figure 4:3: The effect of titanium composition on properties of Cu-10%Al alloy.

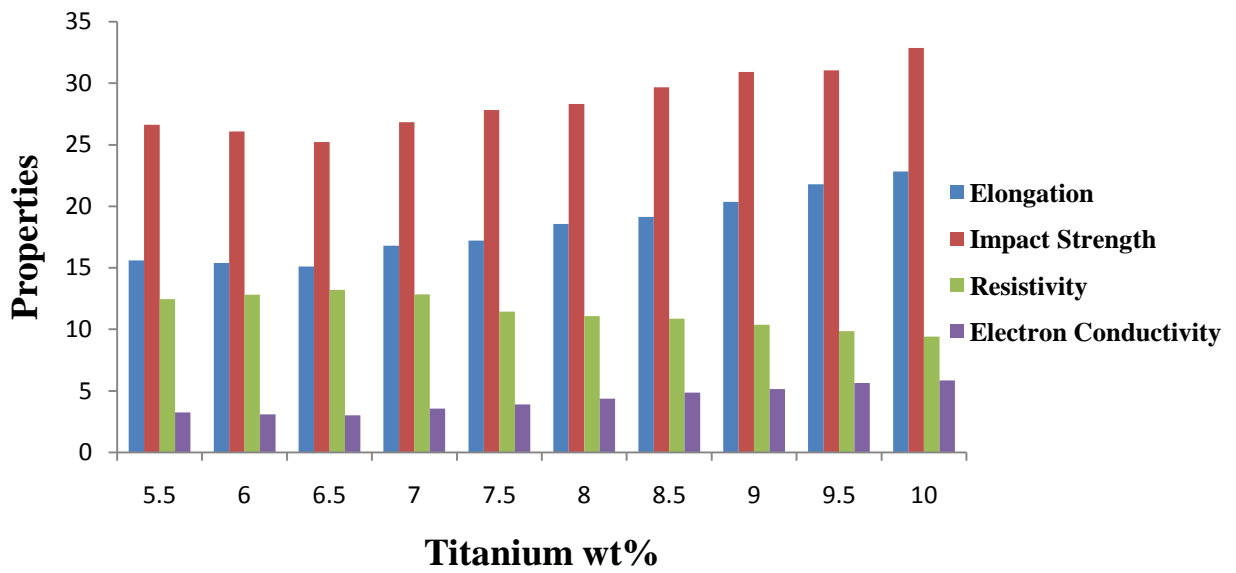


Figure 4:4: The effect of titanium composition on properties of Cu-10%Al alloy.

## **The effects of titanium composition on the mechanical properties of Cu-10%Al alloys**

Table 4.1 and Figures 4.1-4.4 show values of the mechanical properties for the modified alloy containing titanium. From the micrograph, it was observed that  $\alpha+k$  eutectoid phase precipitated from the  $\beta$  phase structure, finer agglomerates of the phases  $\alpha+k$  were precipitated out and this enhanced the mechanical properties of the modified specimens compared to the base specimen. Plates 10 and 11, revealed the finest precipitate of  $\alpha+k$  phases. This corresponded to the highest mechanical and physical properties (yield, tensile strength, hardness and resistivity), while impact strength, percentage elongation and electron conductivity had the lowest values as titanium content increased. In order to have a better combination of mechanical properties in terms of strength and ductility, the specimens were modified at different percentages. However, the peak values of tensile strength, yield strength and hardness are 636MPa, 558MPa, 410BHN at 6.5% composition with corresponding impact strength and %E values; 25.23J and 15.12% respectively. In the specimens that contains 7.0% titanium; it was observed that the coarse intermetallic  $\text{Cu}_9\text{Al}_4$  compound in the specimen resulted in the decrease in the values of the properties.

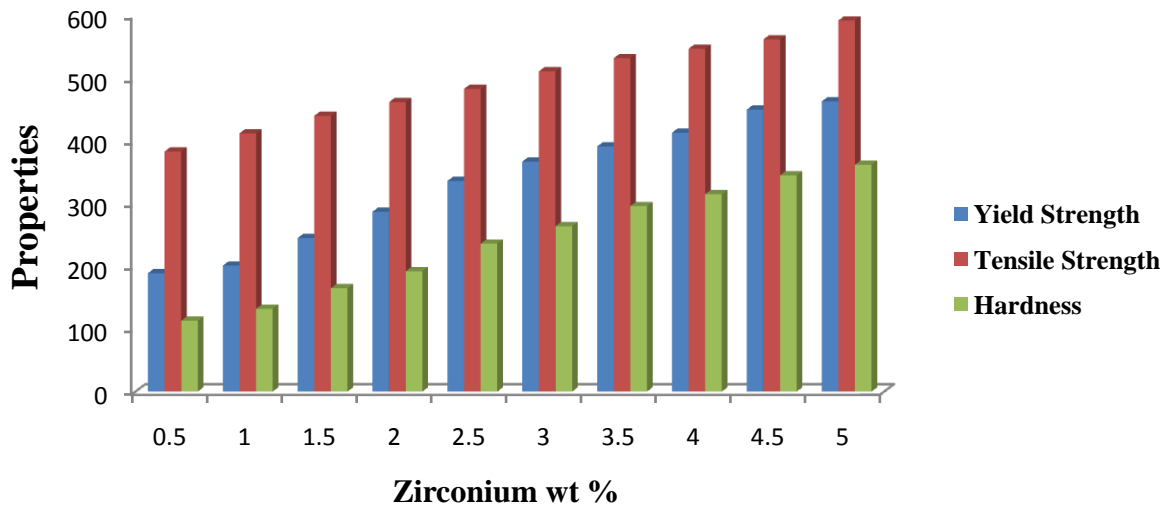


Figure 4:5: The effect of zirconium composition on yield strength, UTS and hardness of Cu-10%Al alloy

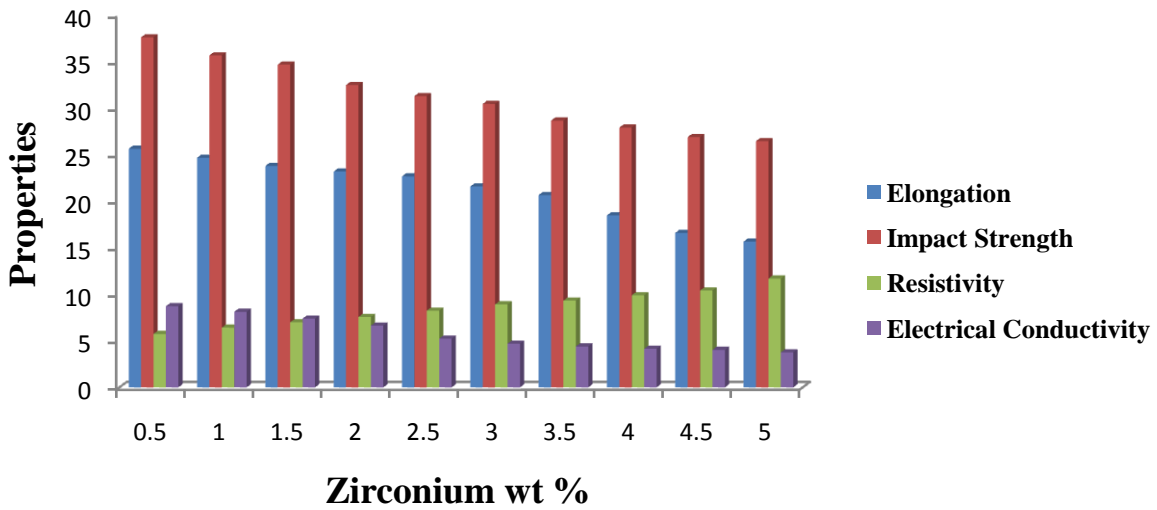


Figure 4:6: The effect of zirconium composition on properties of Cu-10%Al alloy.

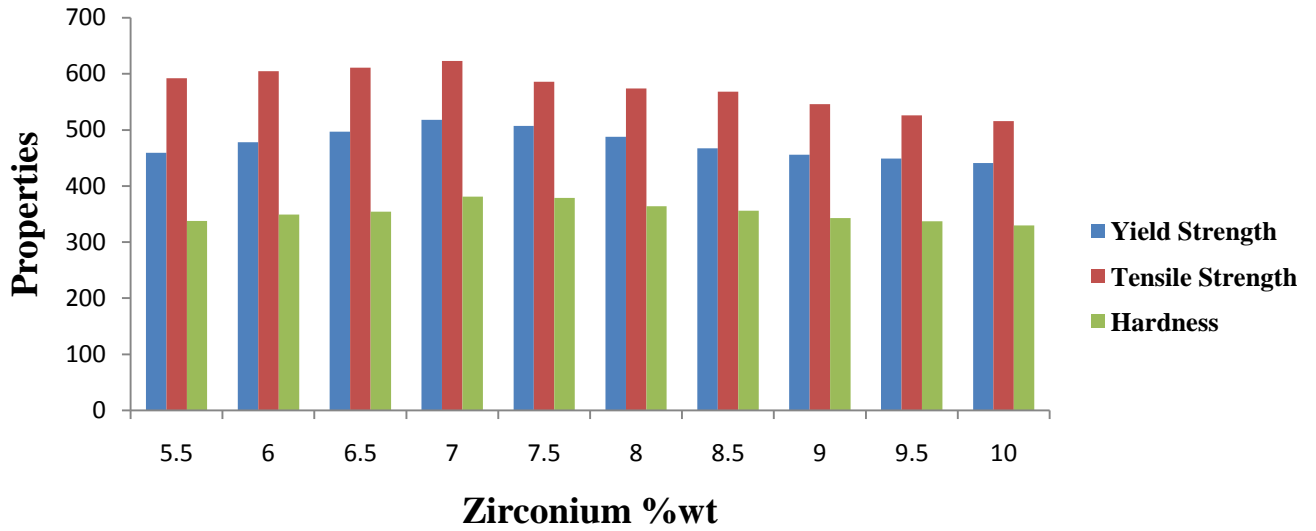


Figure 4:7: The effect of zirconium composition on properties of Cu-10%Al alloy.

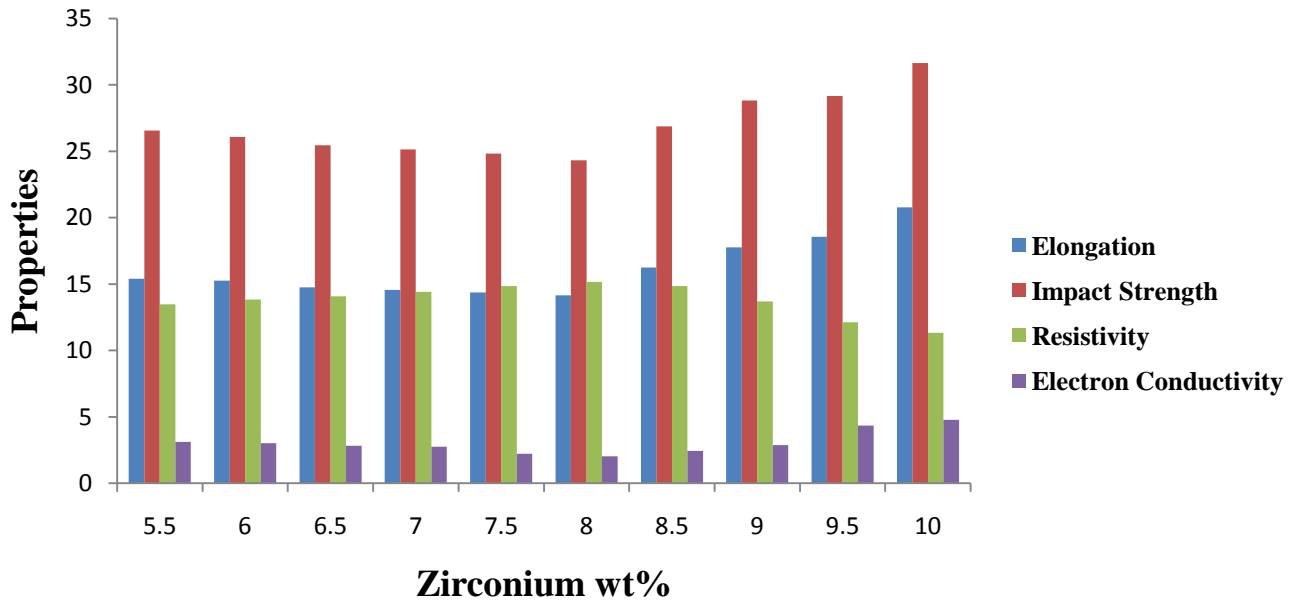
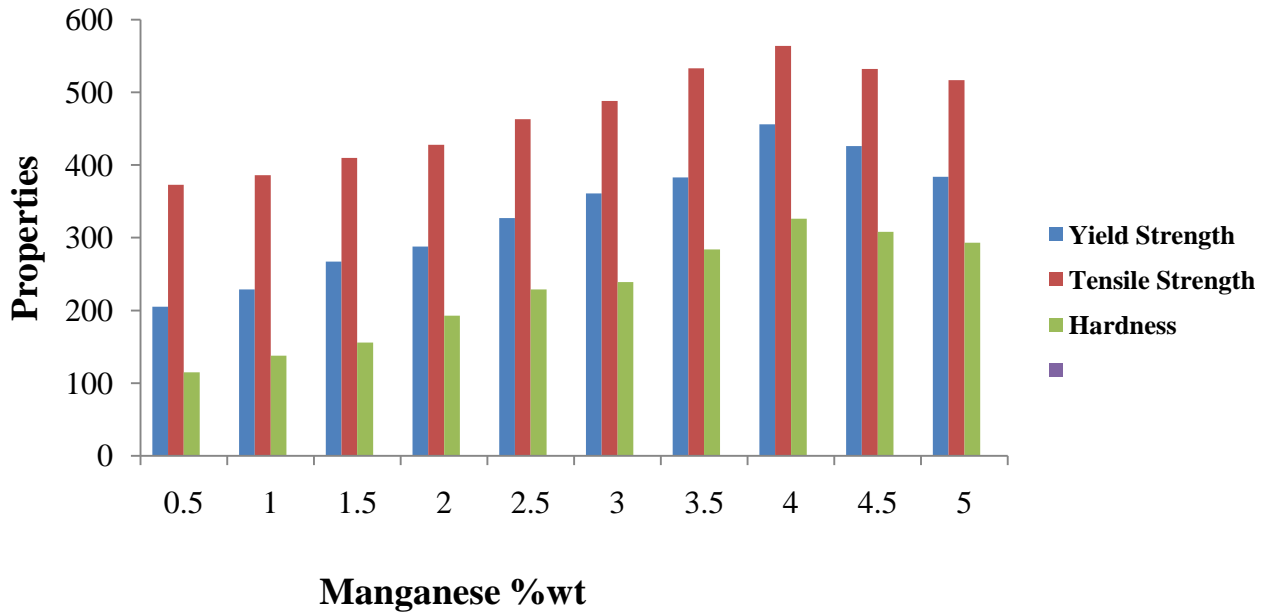


Figure 4:8: The effect of zirconium composition on properties of Cu-10%Al alloy.

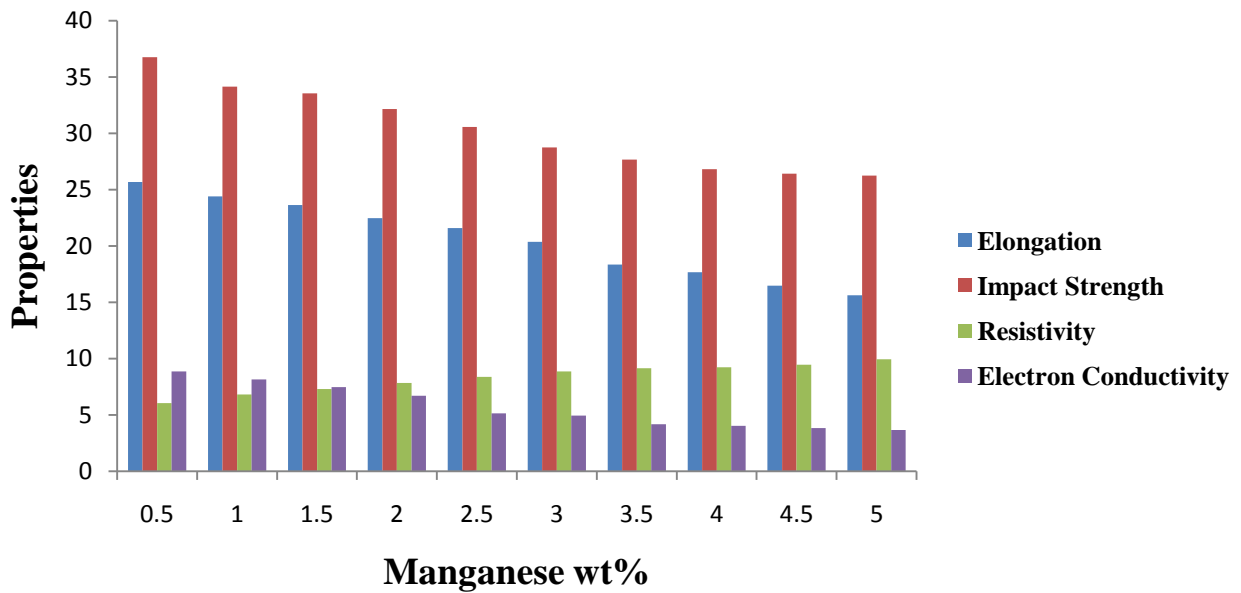


## **The effects of zirconium composition on the mechanical properties of Cu-10%Al alloys**

Table 4.1 and Figures 4.5-4.8 show the values of the mechanical properties for the modified alloy containing zirconium. It was observed that the yield, tensile, hardness and resistivity values of modified specimen increase above the base alloy as zirconium content continued to increase, while impact strength, percentage elongation and electron conductivity decreased. As the composition of the modifying element increased, it caused hindrance to the dislocation movement in the copper alloy lattice and hence increase in strength was noted as the composition increased. The inducement of varying composition of kappa ( $\kappa$ ) precipitate in the  $\alpha$ -matrix, their morphology and size significantly influence the alloy thereby enhancing a stronger metallic bond between the alloys. It was also observed that the yield, hardness, UTS and resistivity values decreased after the peak values at 8.0%-10% of zirconium composition. These decreases were as a result of the coalescence and coarsening of the finely dispersed precipitates of  $\alpha$  and  $\gamma_2$  phases.



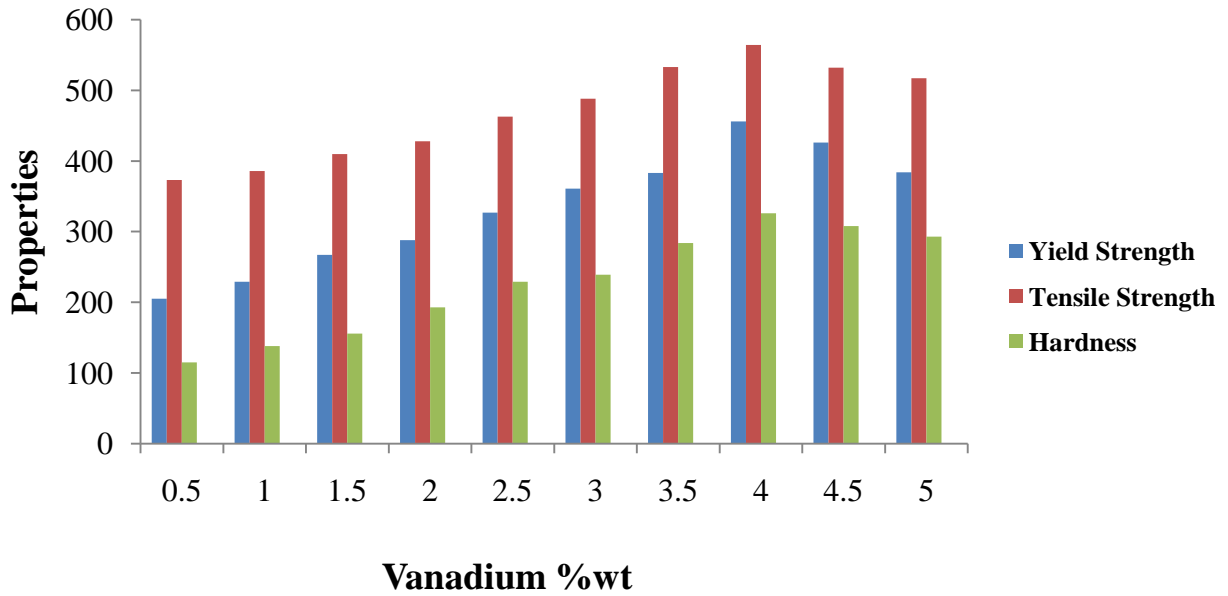
**Figure 4:9: The effect of manganese composition on yield strength, UTS and hardness of Cu-10%Al alloy**



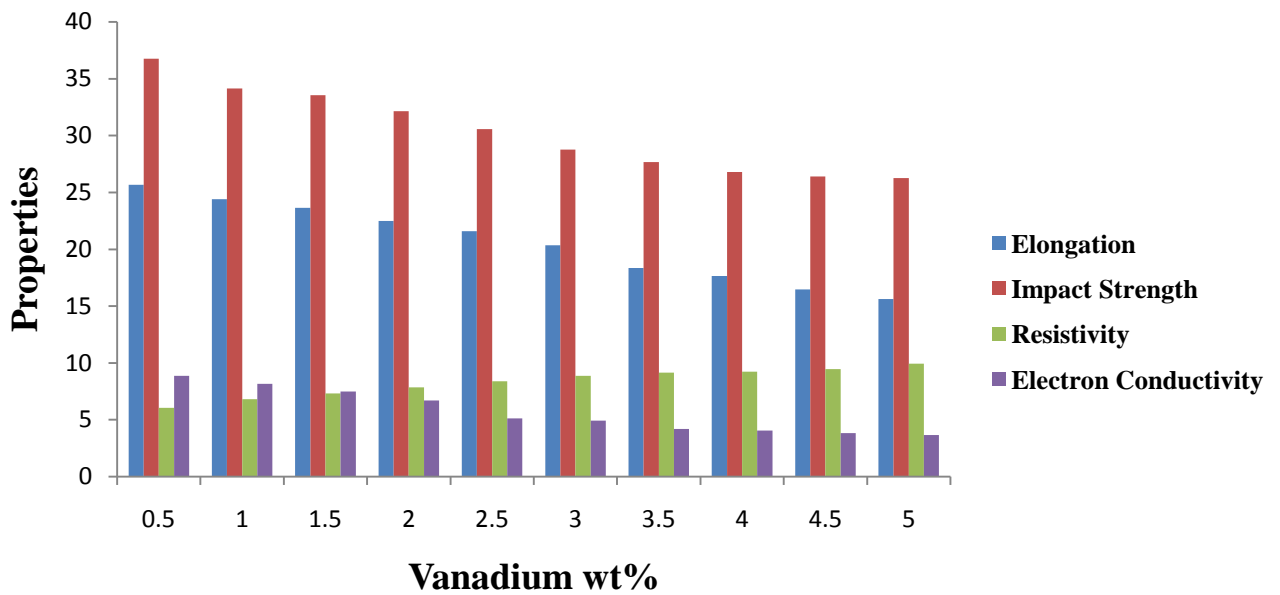
**Figure 4:10: The effect of manganese composition on properties of Cu-10%Al alloy**

## **The effects of manganese composition on the mechanical properties of Cu-10%Al alloys**

Table 4.1 and Figures 4.9-4.10 show the values of the mechanical properties for modified alloy containing manganese. It was observed that the yield strength, tensile strength, hardness and resistivity values of modified specimen increased and impact strength, percentage elongation and electron conductivity decreased as manganese content increased. This is as a result of body-centered cubic structure developed because of the formation of interstitial solid solution between the copper lattice and manganese atom. The structure retarded the breakdown of  $\alpha$  to  $\beta + \gamma_2$  phase boundaries. Manganese increased hardenability which contributes the formation of manganese sulphide (MnS) during casting. It was also observed that the yield, hardness, UTS and resistivity values decreased after the peak values at 4.0%-5% of zirconium composition. The decrease was as a result of the coalescence and coarsening of the finely dispersed precipitates.



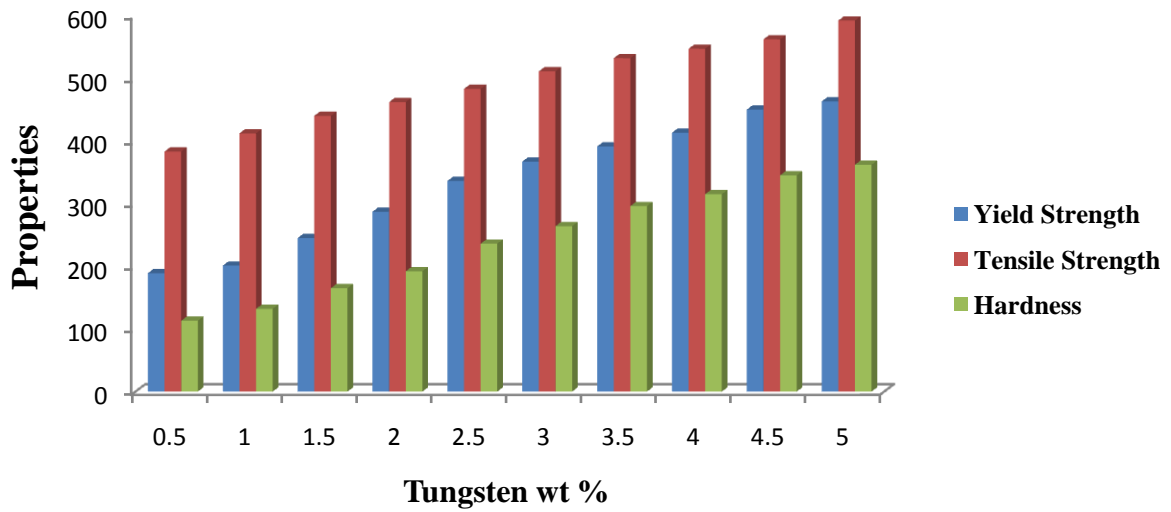
**Figure 4:11: The effect of vanadium composition on yield strength, UTS and hardness of Cu-10%Al alloy**



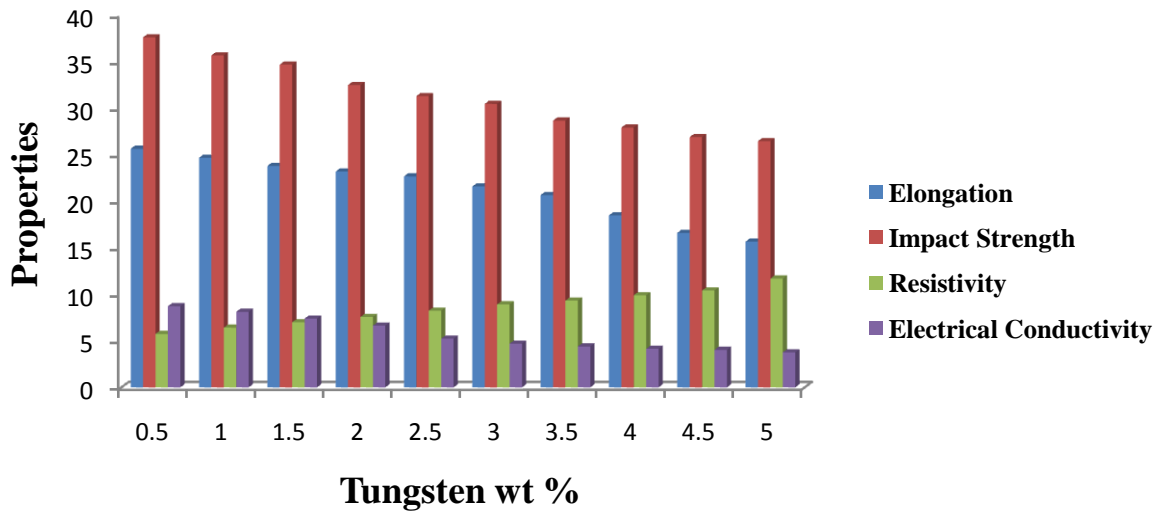
**Figure 4:12: The effect of vanadium composition on properties of Cu-10%Al alloy.**

### **The effects of vanadium composition on the properties of Cu-10%Al alloys**

Table 4.1 and Figures 4.10-4.12 show the values the mechanical and physical properties for modified alloy containing vanadium. The yield strength, tensile strength, hardness and resistivity values of modified specimen were observed to increase while impact strength, percentage elongation and electron conductivity decreased as vanadium content increased. These values were obviously higher than the value of the base alloy specimen which was indications that the finely dispersed precipitates of  $\alpha$  and  $\kappa$  phases formed during the modification process impeded dislocation movement during deformation and thereby strengthened the alloy. Secondly body centered cubic (BCC) structure of vanadium atom which occupied the substitutional site of copper lattice. However, the highest UTS values of 561MPa, yield strength value of 467, hardness value of 363BHN were obtained with the specimen at 4.0% composition of vanadium. It was also observed that the UTS, yield and hardness strength values decreased after the peak values at the modifying composition of 4.5% and 5.0% of vanadium respectively. This decrease is as a result of coalescence and coarsening of the finely dispersed precipitates of  $\alpha$  and  $\gamma_2$  phases.



**Figure 4:13: The effect of Tungstencomposition on yield strength, UTS and hardness of Cu-10%Al alloy**



**Figure 4:14: The effect of tungsten composition on properties of Cu-10%Al alloy.**

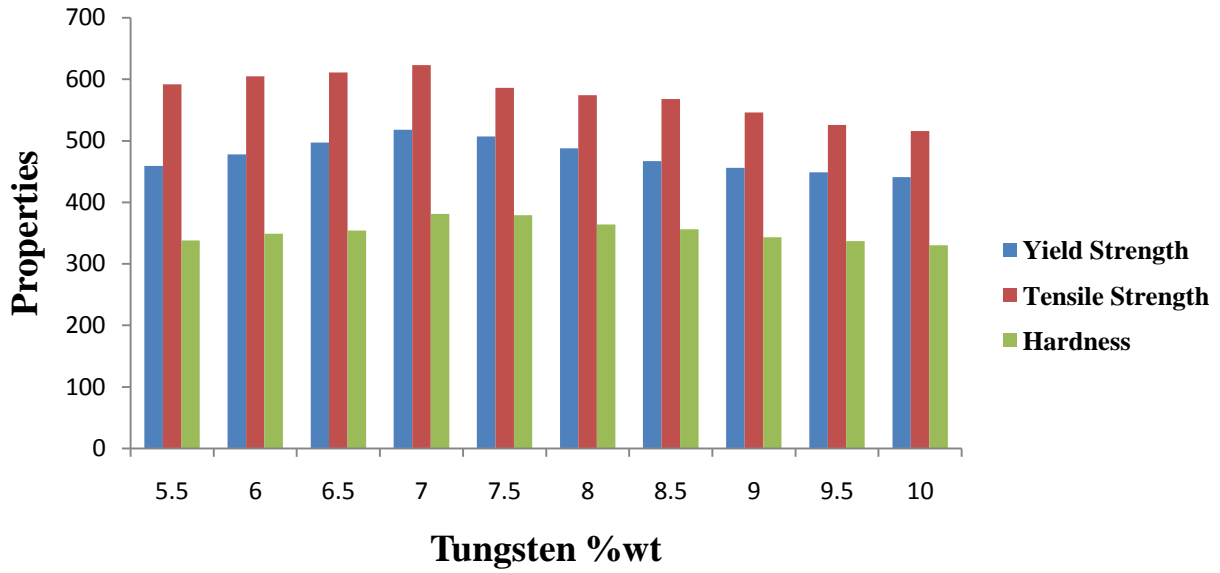


Figure 4:15: The effect of tungsten composition on properties of Cu-10%Al alloy.

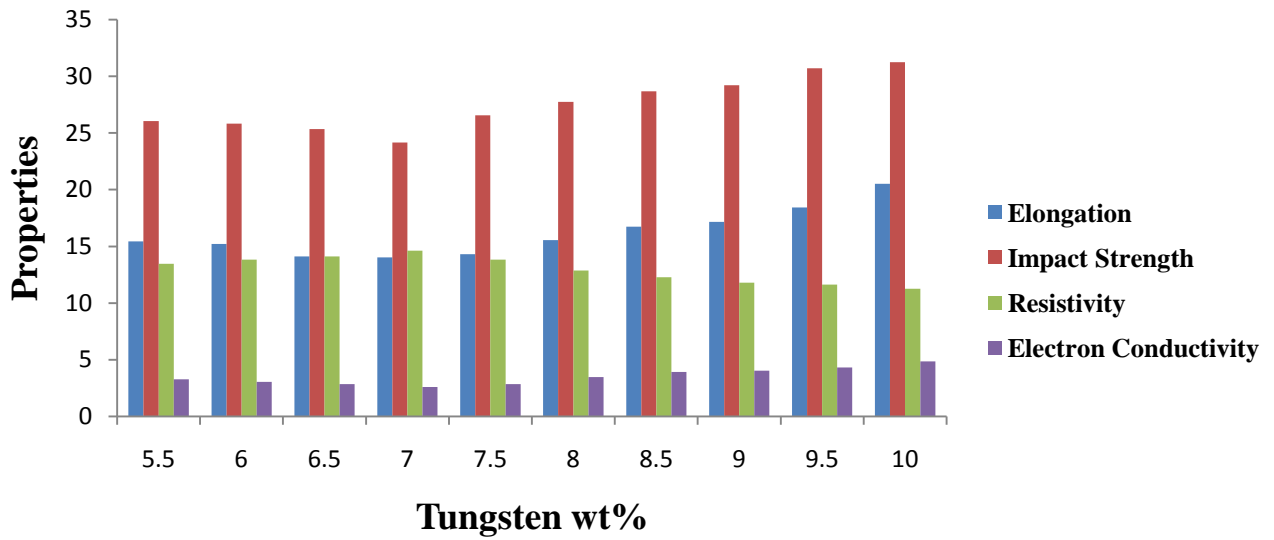
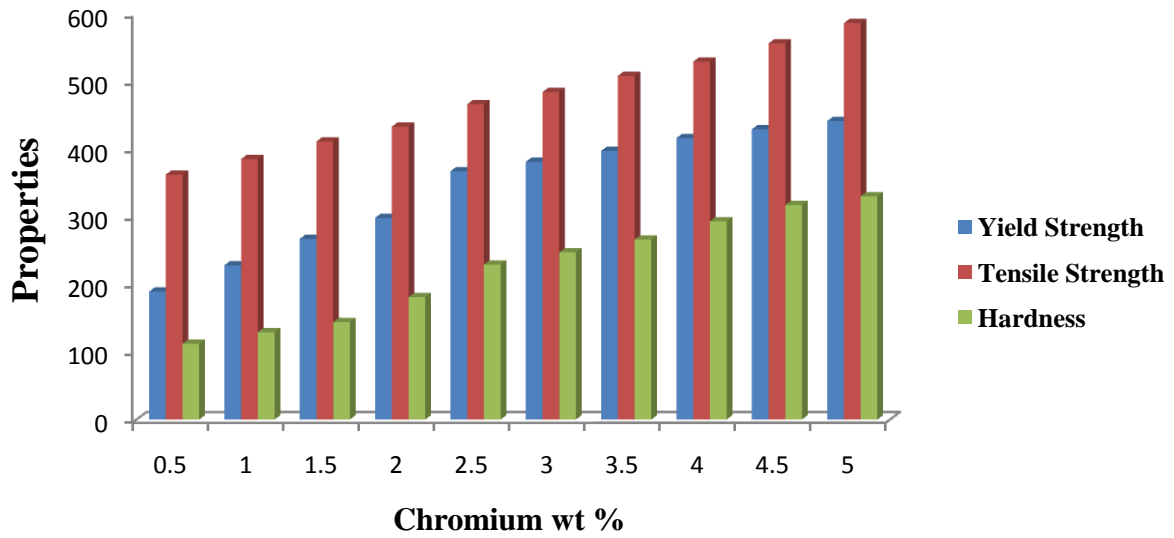


Figure 4:16: The effect of tungsten composition on properties of Cu-10%Al alloy.

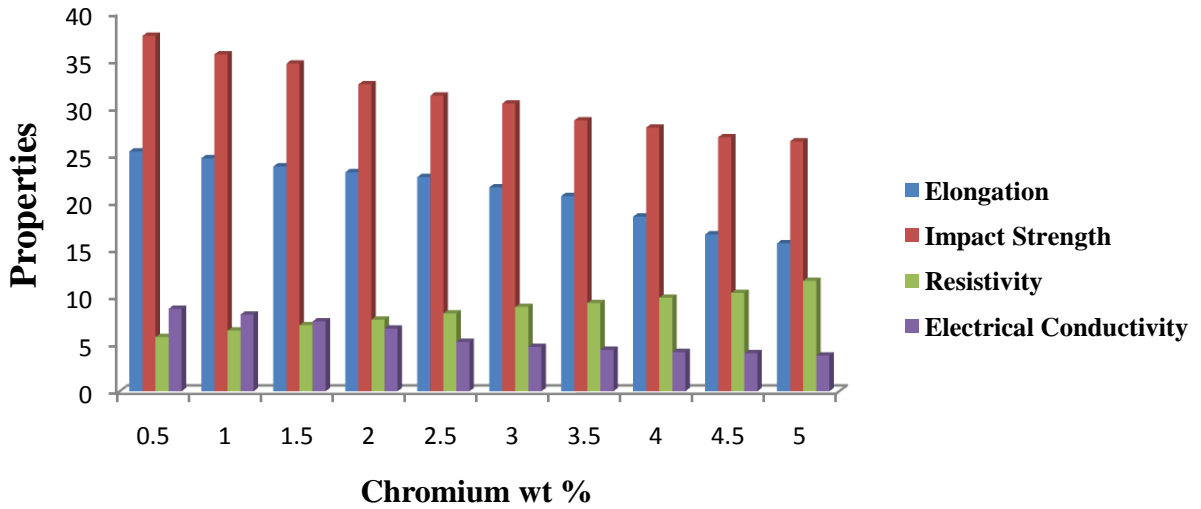
## **The effects of tungsten composition on the properties of Cu-10%Al alloys**

Table 4.1 and Figures 4.13-4.16 reveal the values of the mechanical and physical properties for modified alloy containing tungsten. It was observed that the yield strength, tensile strength, hardness and resistivity values of modified specimen increased while impact strength, percentage elongation and electron conductivity decreased with increase in tungsten content. The improvement of the properties was an indication that finely dispersed precipitates of  $\alpha$  and  $\kappa$  phases formed during the modification process, thereby impeding the movement of dislocation during solid solution strengthening. The  $\kappa$  precipitates, being a stable and coherent secondary phase in the matrix provided substantial level of impediment to dislocation motion which increased the composition to the amount of fine lamellar  $\kappa$  present. It was observed that the yield, hardness, UTS and resistivity values decreased after the peak values at 7.5%-10% of tungsten composition. These decreases were as a result of the coarsening of the finely dispersed precipitates.





**Figure 4:17: The effect of chromium composition on yield strength, UTS and hardness of Cu-10%Al alloy**



**Figure 4:18: The effect of chromium composition on properties of Cu-10%Al alloy.**

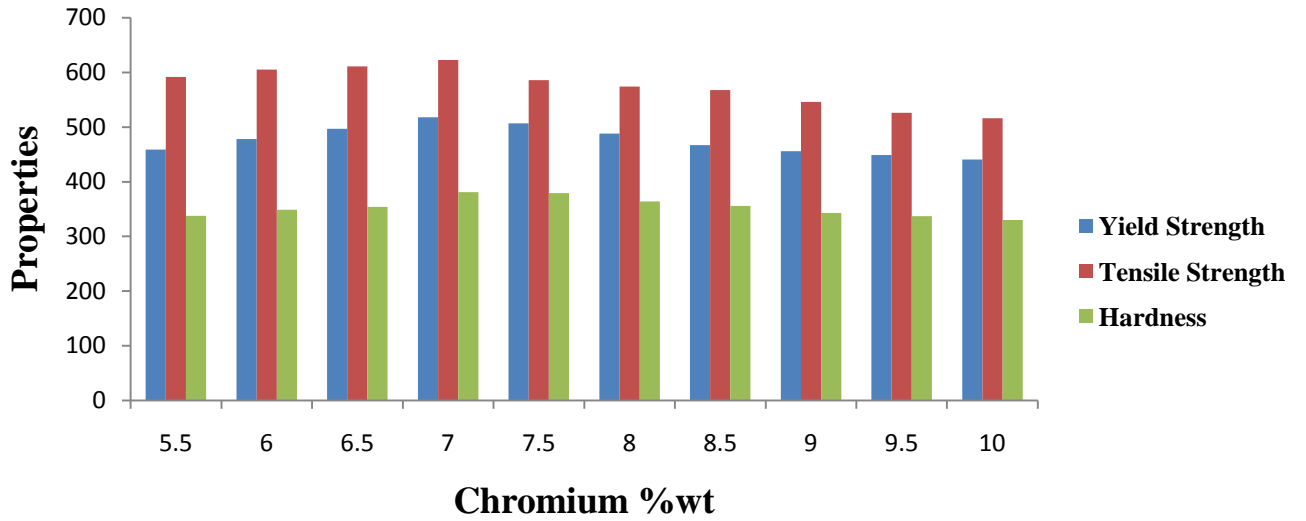


Figure 4:19: The effect of chromium composition on properties of Cu-10%Al alloy.

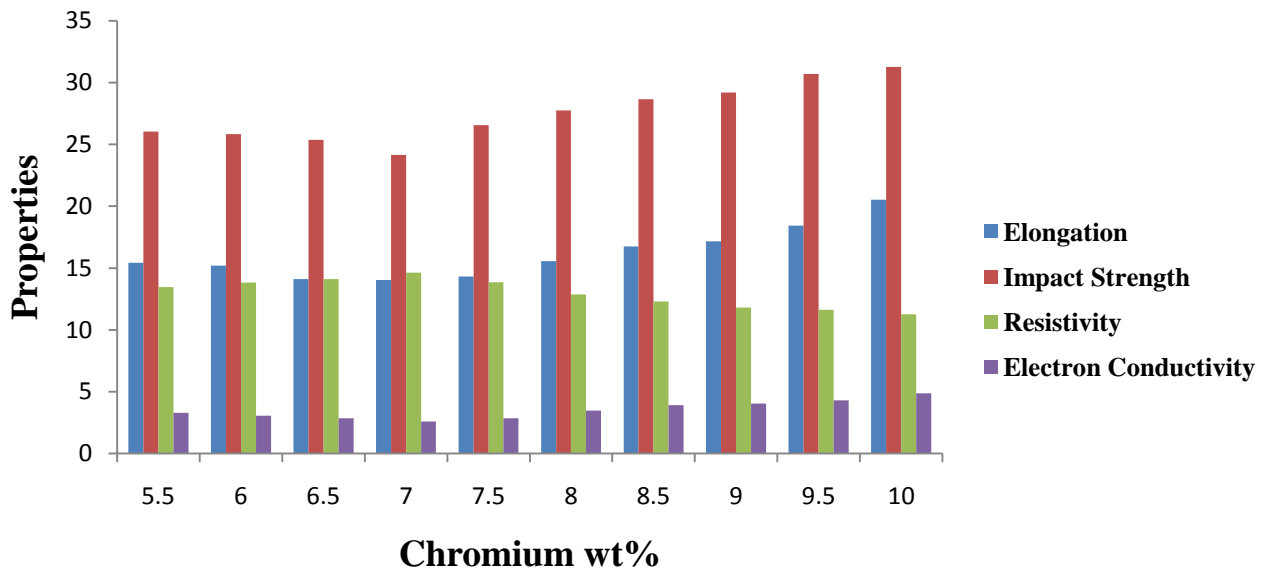
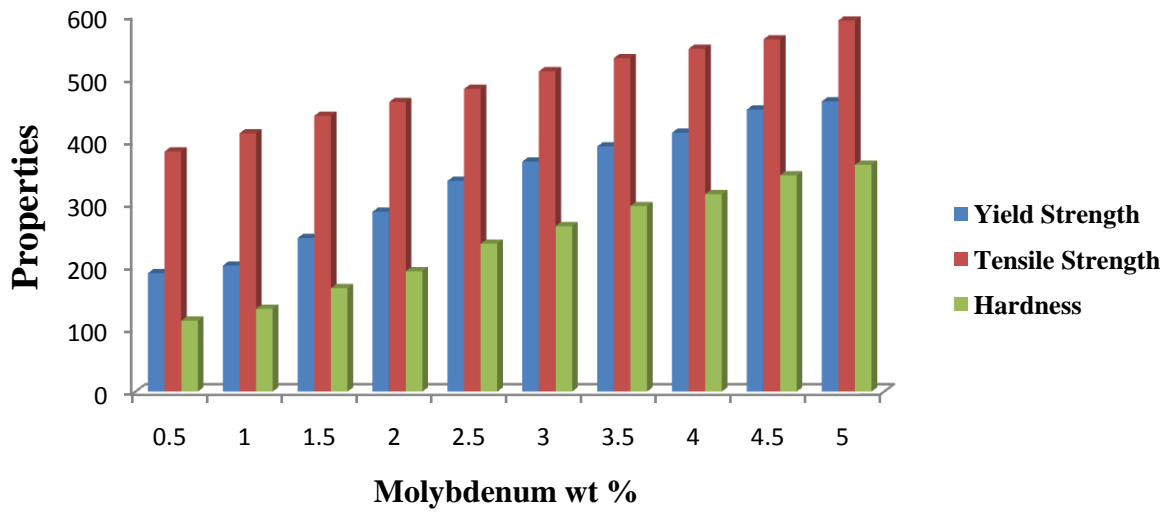


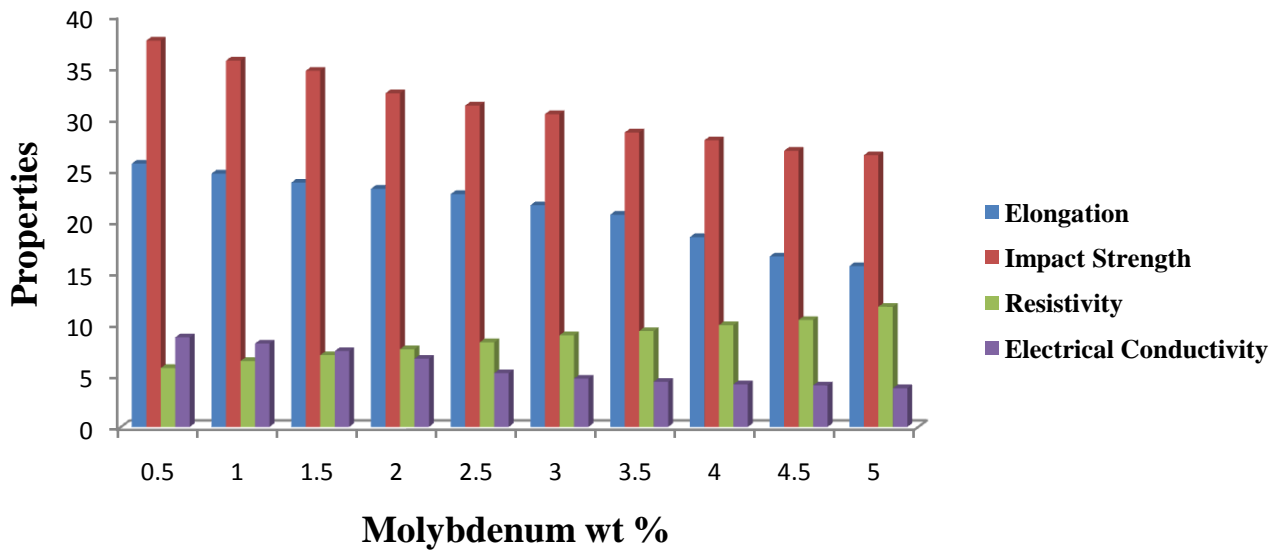
Figure 4:20: The effect of chromium composition on properties of Cu-10%Al alloy.

### **The effects of chromium composition on the properties of Cu-10%Al alloys**

Table 4.1 and Figures 4.17-4.20 show the values of the mechanical and physical properties for modified alloy containing chromium. The micrograph indicates that eutectoid phase precipitated from the martensitic  $\beta'$  phase structure, finer agglomerates of the phases  $\alpha+k$  were precipitated out and this brought about the improved mechanical properties of the modified specimens over the base specimen. Plates 57 and 58 showed the finest precipitate of  $\alpha+\gamma_2$  phases which corresponded to the highest mechanical and physical properties (yield, tensile strength, hardness and resistivity). But impact strength, percentage elongation and electron conductivity had their lowest values as chromium content increased. However, the highest UTS value of 623MPa, yield strength value of 518MPa, and hardness value of 381BHN were obtained with the specimen modified at 7.0% composition. It was also observed that the yield, hardness, UTS and resistivity values decreased after the peak values at 7.0%-10% of chromium composition. The decrease was a result of the coarsening of the finely dispersed precipitates.



**Figure 4:21: The effect of molybdenum composition on yield strength, UTS and hardness of Cu-10%Al alloy**



**Figure 4:22: The effect of molybdenum composition on properties of Cu-10%Al alloy.**

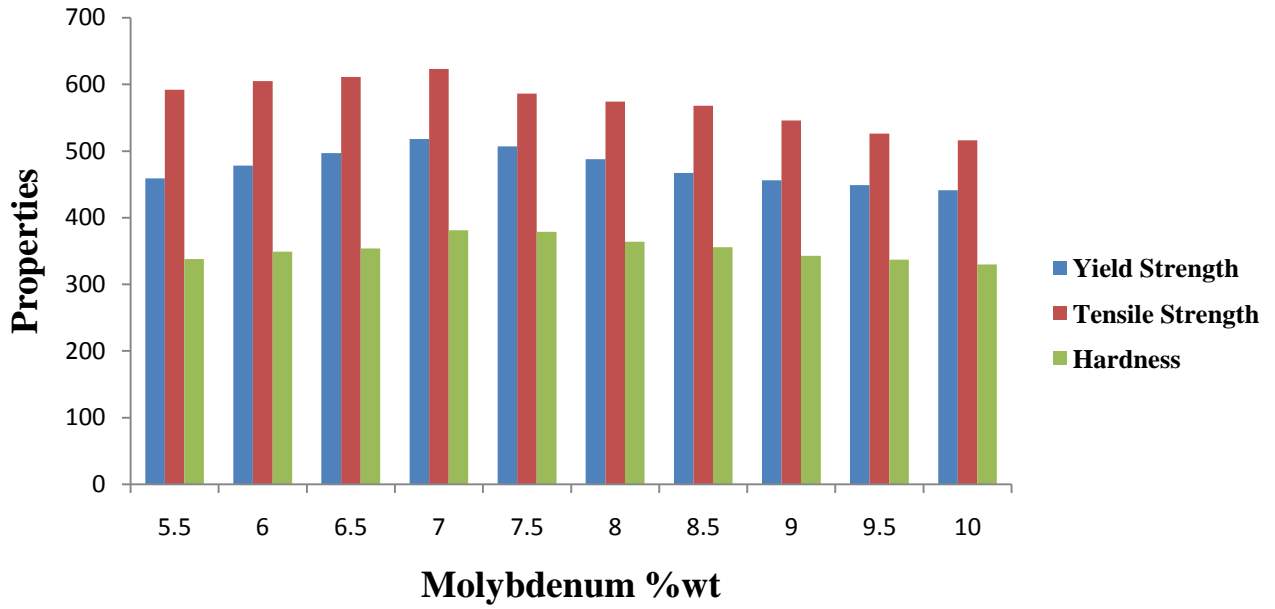


Figure 4:23: The effect of molybdenum composition on properties of Cu-10%Al alloy.

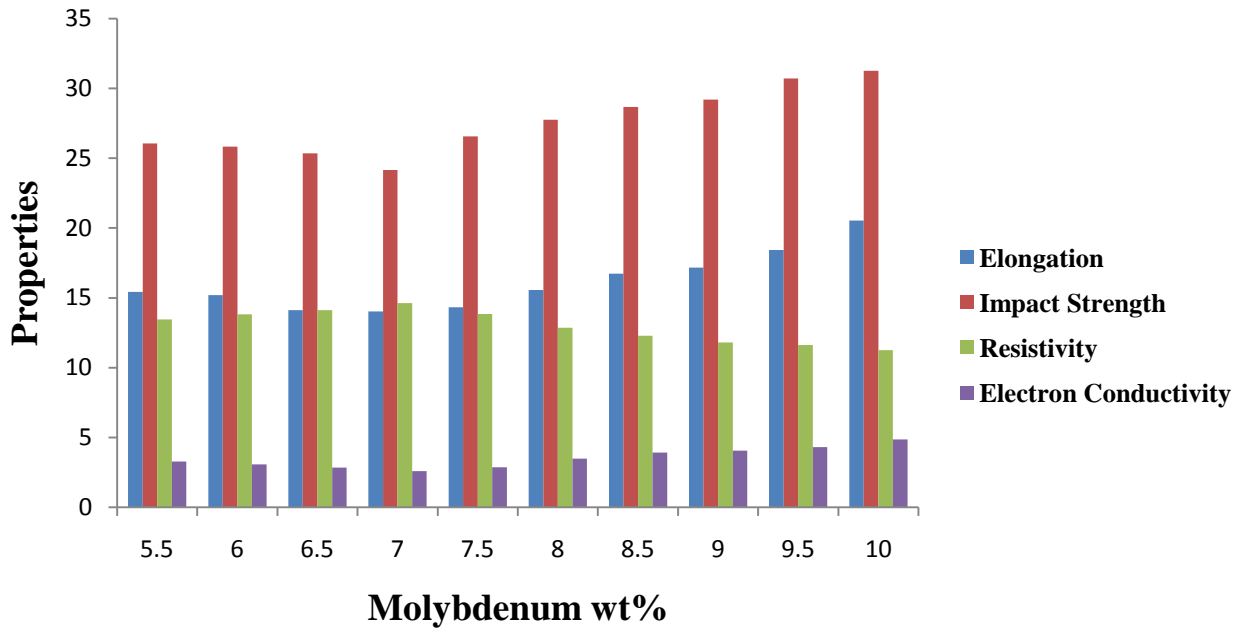
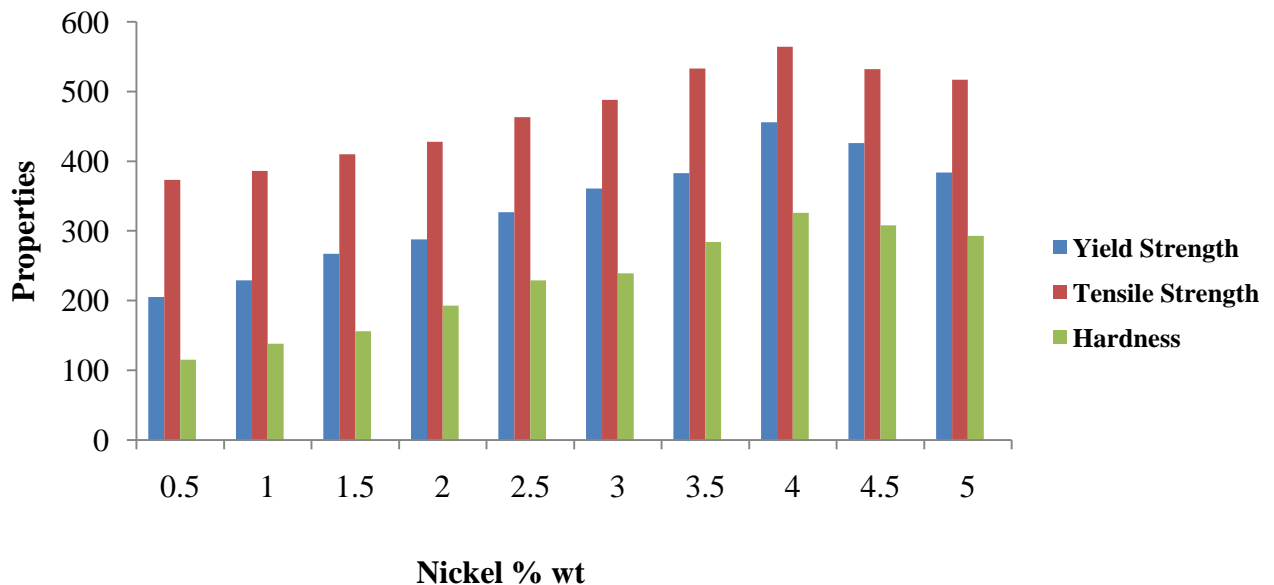


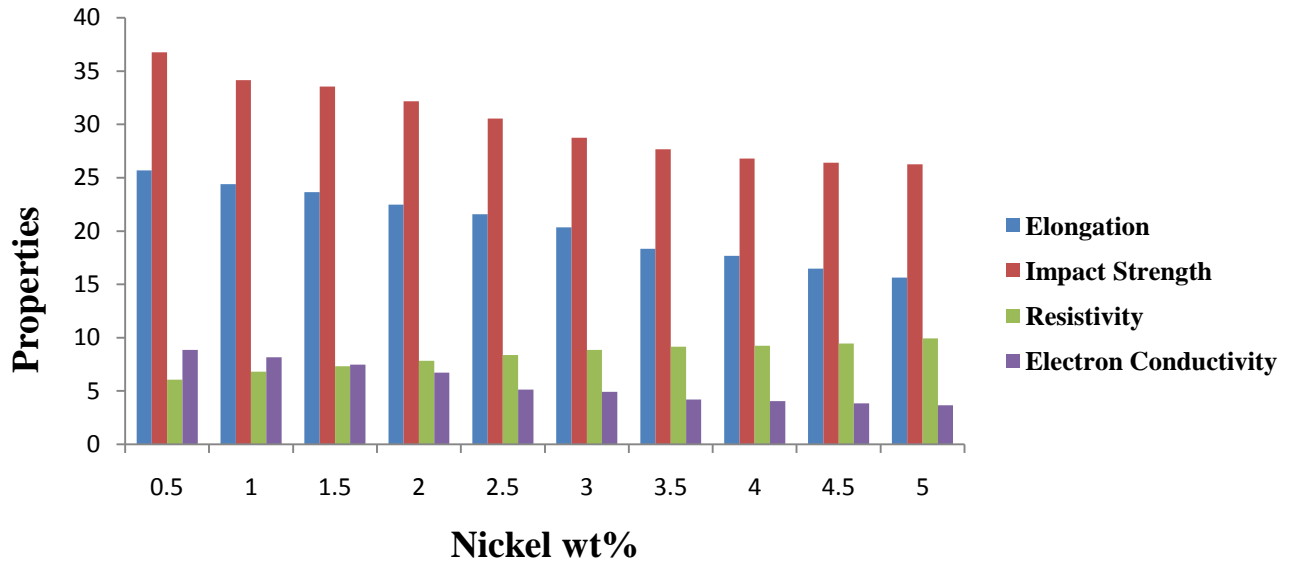
Figure 4:24: The effect of molybdenum composition on properties of Cu-10%Al alloy.

## The effects of molybdenum composition on the properties of Cu-10%Al alloys

The tensile strength, hardness, yield strength and resistivity of the Cu-10%Al alloys treated with molybdenum increased as the concentration of molybdenum increased but impact strength, percentage elongation and electron conductivity decreased. This is because molybdenum particles impeded on the dislocation movement in the copper alloy lattice. The kappa precipitates, being a stable and coherent secondary phase in the matrix provided substantial level of impediment to dislocation motion and increased the composition to the amount of fine lamellar kappa present. From the experimental results, these specimens had reduced yield, tensile strengths, hardness and resistivity values from the peak values, and from their micrographs, the reduction in the values was caused by, grain growth which led to coarsening of the  $\alpha+\gamma_2$  phase precipitates and thus softening the alloys.



**Figure 4:25: The effect of nickel composition on yield strength, UTS and hardness of Cu-10%Al alloy.**



**Figure 4:26: The effect of nickel composition on properties of Cu-10%Al alloy**

### **The effects of nickel composition on the properties of Cu-10%Al alloys**

Table 4.1 and Figures 25-26 depict the effect of nickel addition on the mechanical properties of Cu-10%Al alloys. The results of the experiment showed a mutual solid solubility between the copper matrix and nickel element, nickel acts as austenite stabilizer having face centered cubic (FCC) structure. The yield strength, tensile strength, hardness and resistivity values of nickel modified alloys were increased above the value of the base alloy but impact strength, percentage elongation and electron conductivity decreased. Further analysis of the results obtained from the modifying experiments shows that the mechanical properties were dependent on the modifying element composition. It was revealed on nickel addition; yield, tensile strength and hardness increased from 0.5-4.0wt% and decreased from 4.5-5.0wt%. This decrease is as a result of alloy solidifies with an alpha – beta structure from which the gamma phase begins to precipitate as coarse particles.

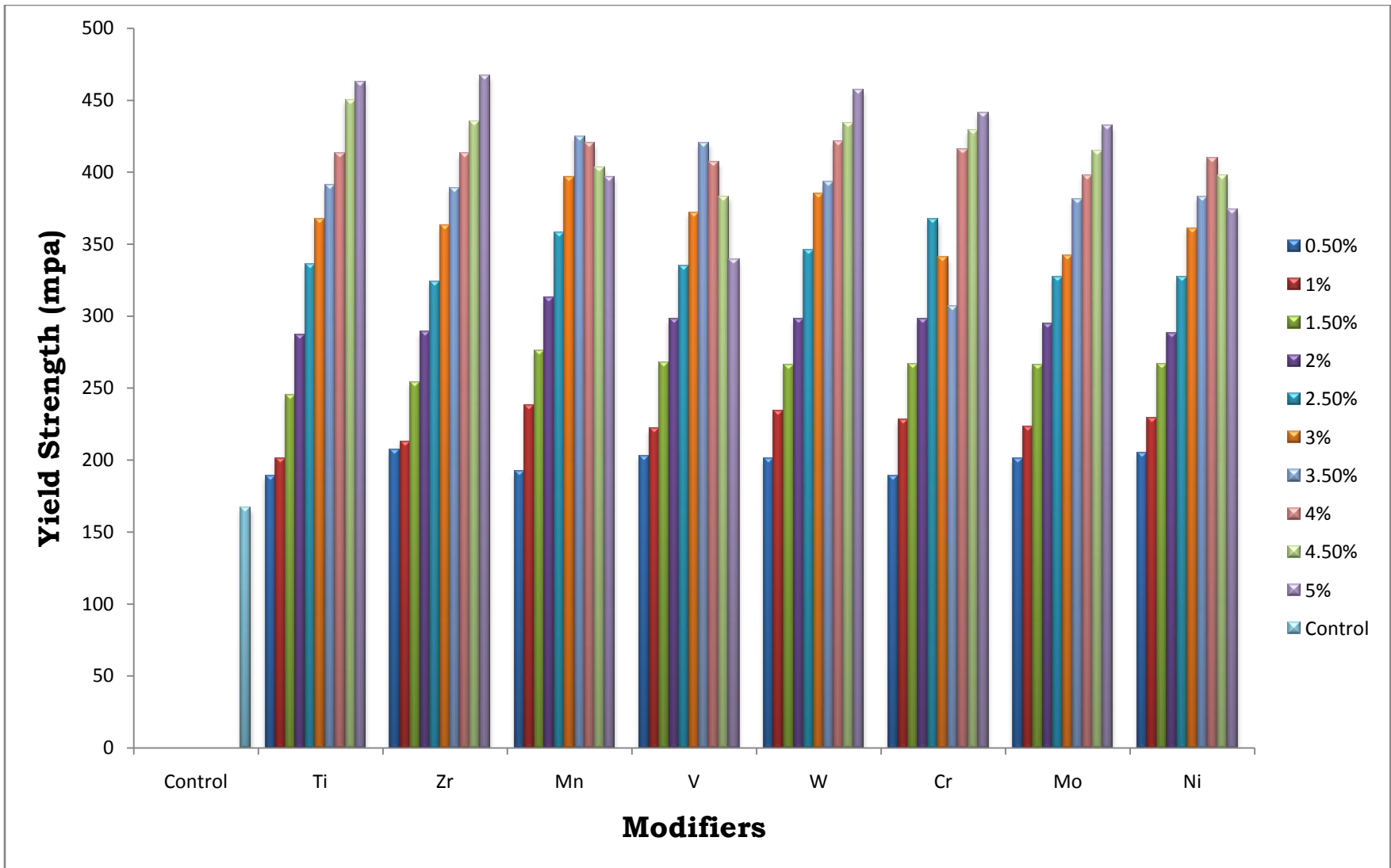
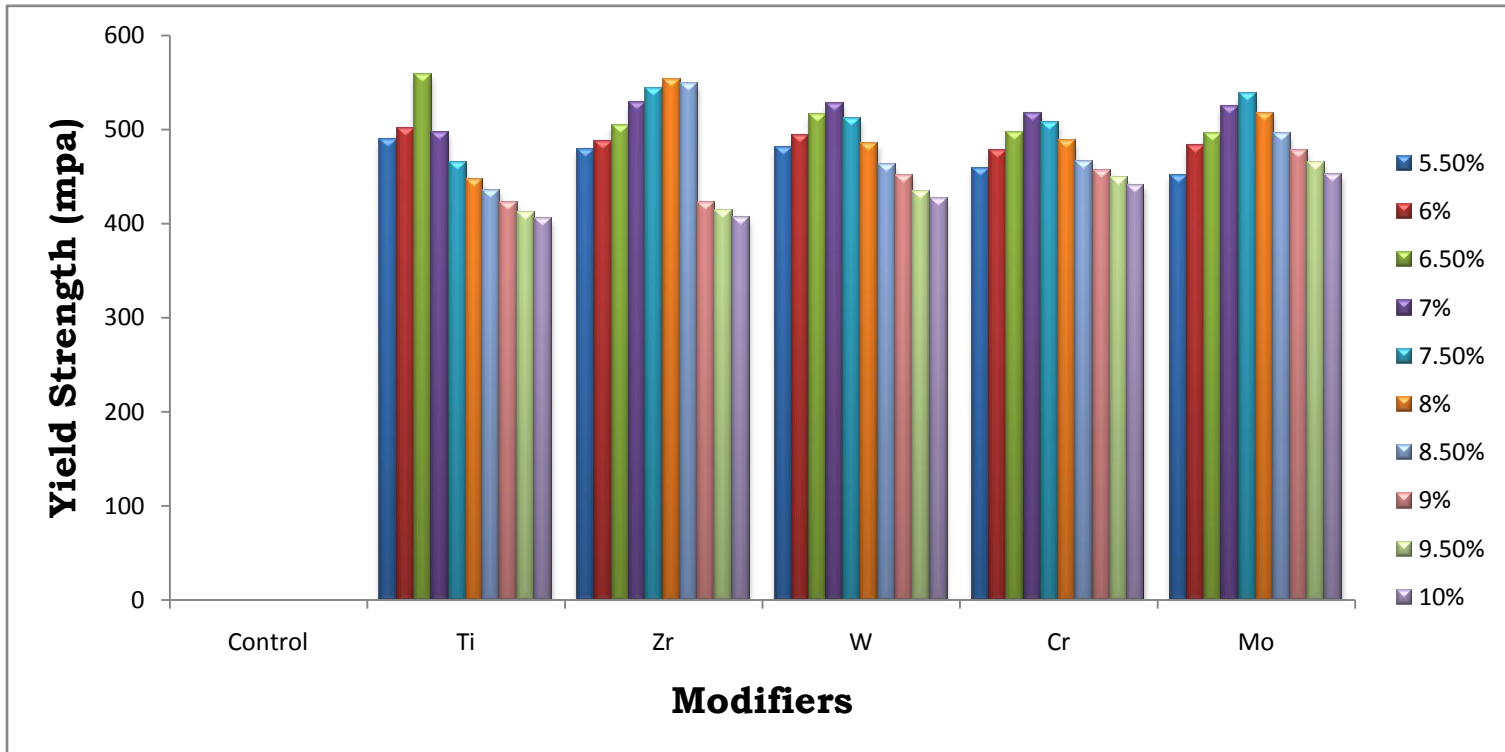


Figure 4.27: The effect of additives on Yield strength (MPa) of Cu-10%Al alloy





**Figure 4.28: The effect of additives on Yield strength (MPa) of Cu-10%Al alloy**

From Table 4.1, Figure 4.27 and Figure 4.28, it was also observed that the yield strength values increased with increased percentage composition of modifying elements. Base alloy had yield strength of 167MPa which was far less than the yield strength of all the modified specimens. The titanium modified specimen has the highest yield strength of 558MPa which was as a result of its martensitic structure that is very hard. After the peak values of yield strength, it was observed that yield strength decreased due to grain growth which led to coarsening of the  $\alpha+\gamma_2$  phase precipitates.

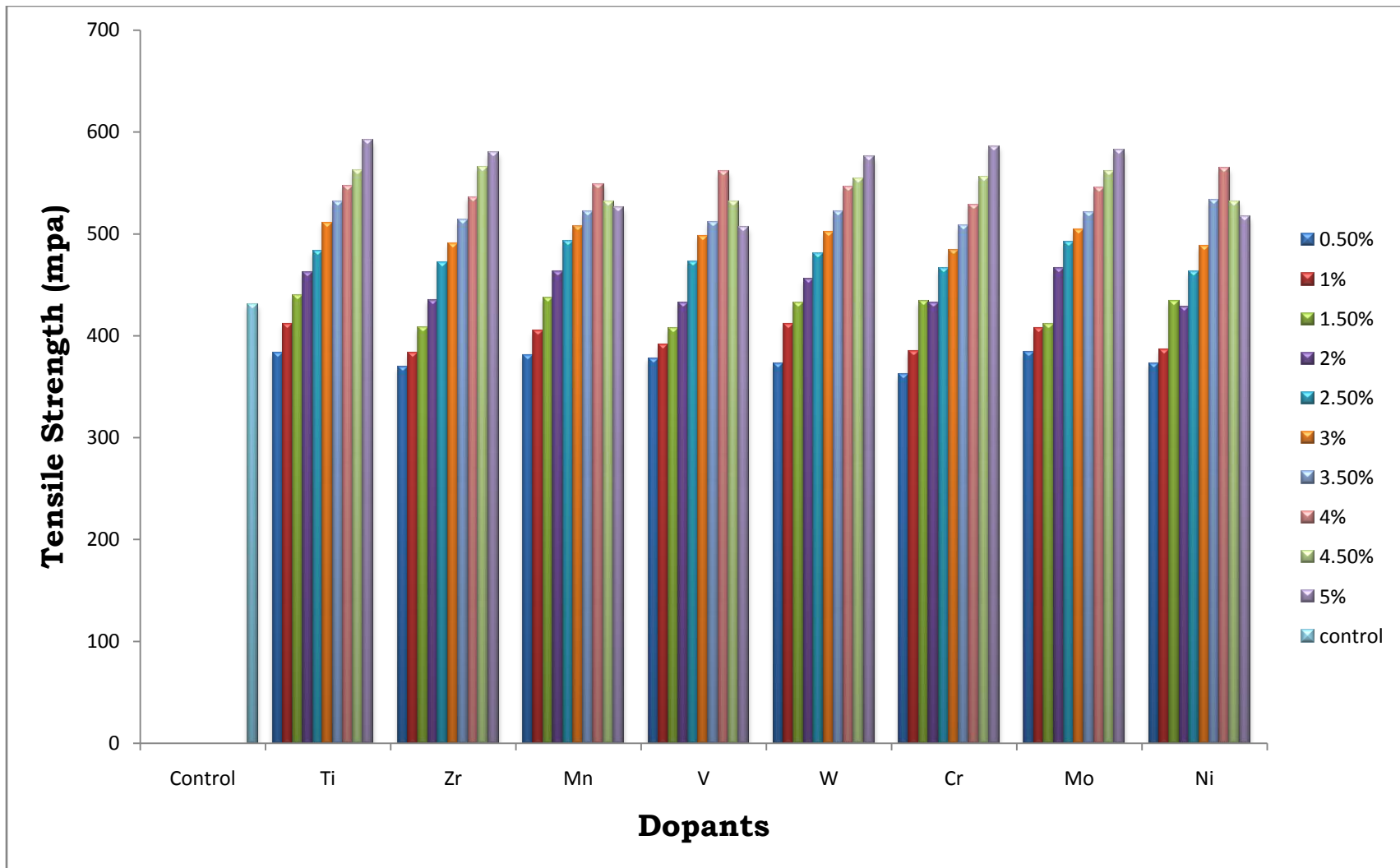
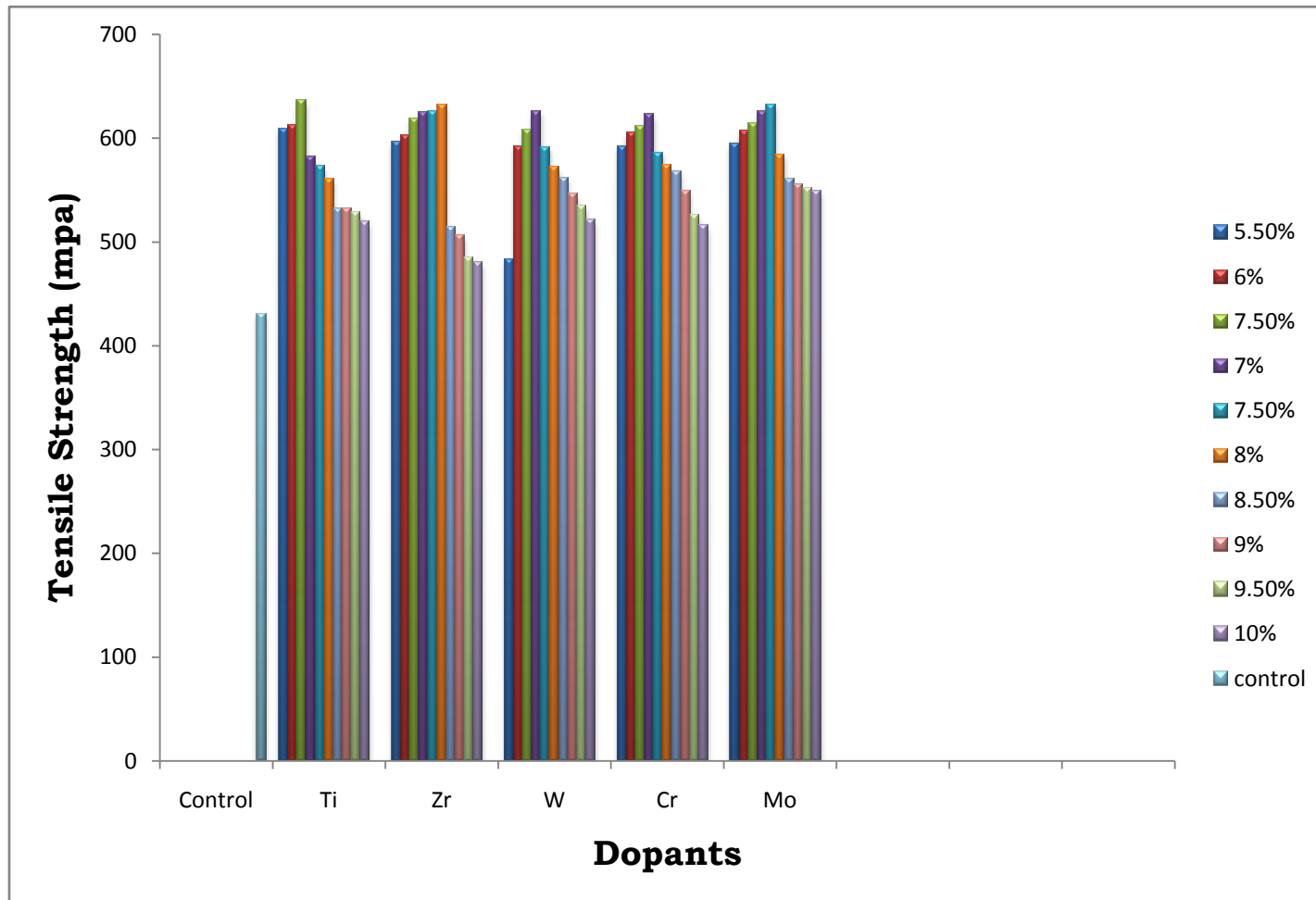


Figure 4.29: The effect of additives on Tensile strength (MPa) of Cu-10%Al alloy.



**Figure 4.30: The effect of additives on Tensile strength (MPa) of Cu-10%Al alloy.**

Table 4.1, Figure 4.29 and Figure 4.30 show the values of ultimate tensile strength for the modified specimens. It reveals that the ultimate tensile strength increased with increase in modifying elements. The highest ultimate tensile strength values of titanium, zirconium, tungsten, chromium, manganese and molybdenum were obtained at 6.5%, 8.0%, 7.0%, 7.0%, and 7.5% composition of these modifying elements while vanadium, manganese and nickel obtained theirs at 4%. These values were obviously higher than the value of the base alloy specimen which was 331MPa indicating that the finely dispersed precipitates of  $\alpha$  and  $\kappa$  phases formed during the modification treatment impeded dislocation movement during deformation and thereby strengthened the alloy. It was also observed that the UTS values decreased after their peak values at the different composition. This decrease could be as a result of casting defect or higher content of modifying elements which led to the coalescence and coarsening of the finely dispersed precipitates of  $\alpha$  and  $\gamma_2$  phases.

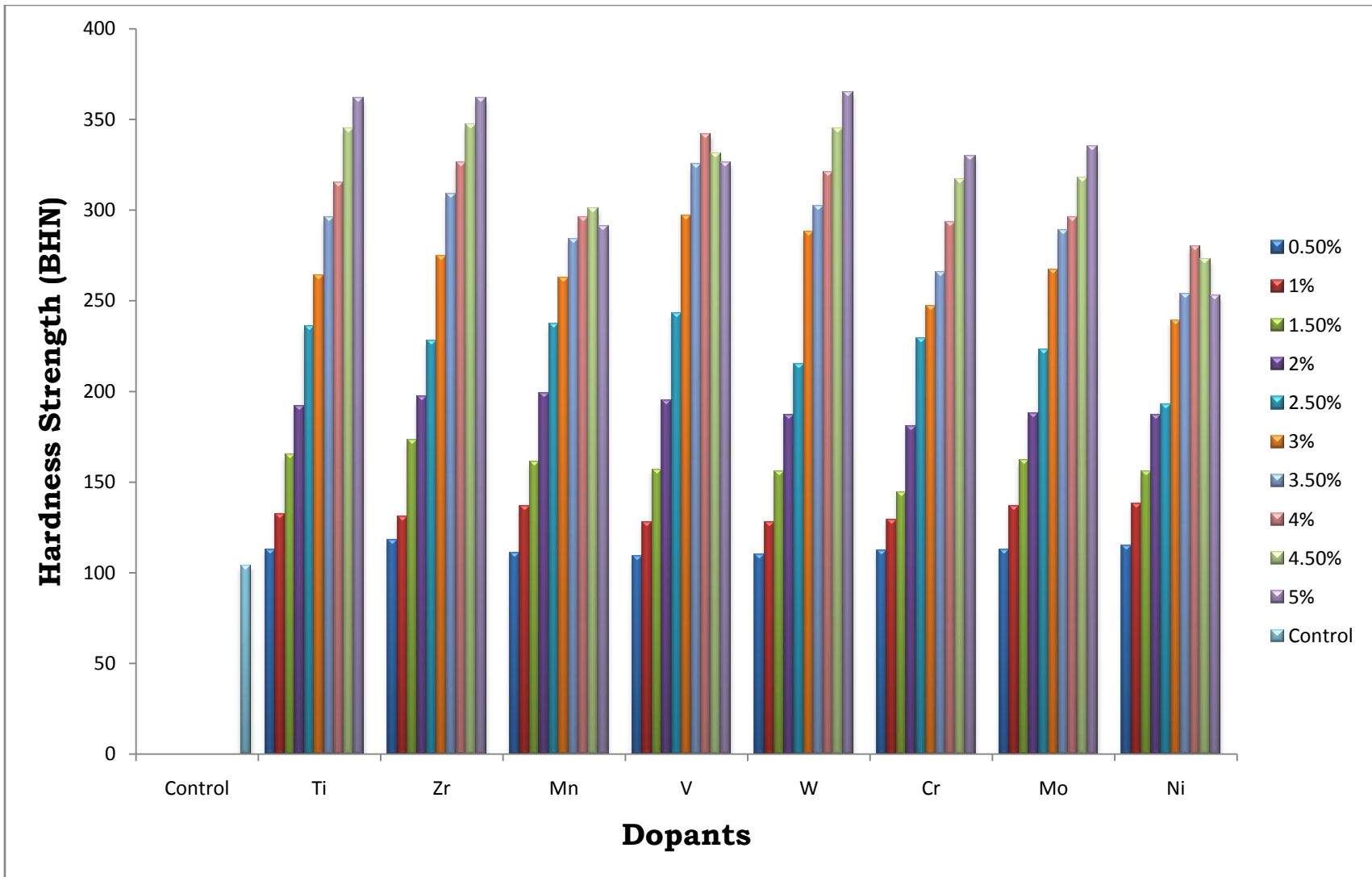


Figure 4.31: The effect of additives on Hardness strength (BHN) of Cu-10%Al alloy

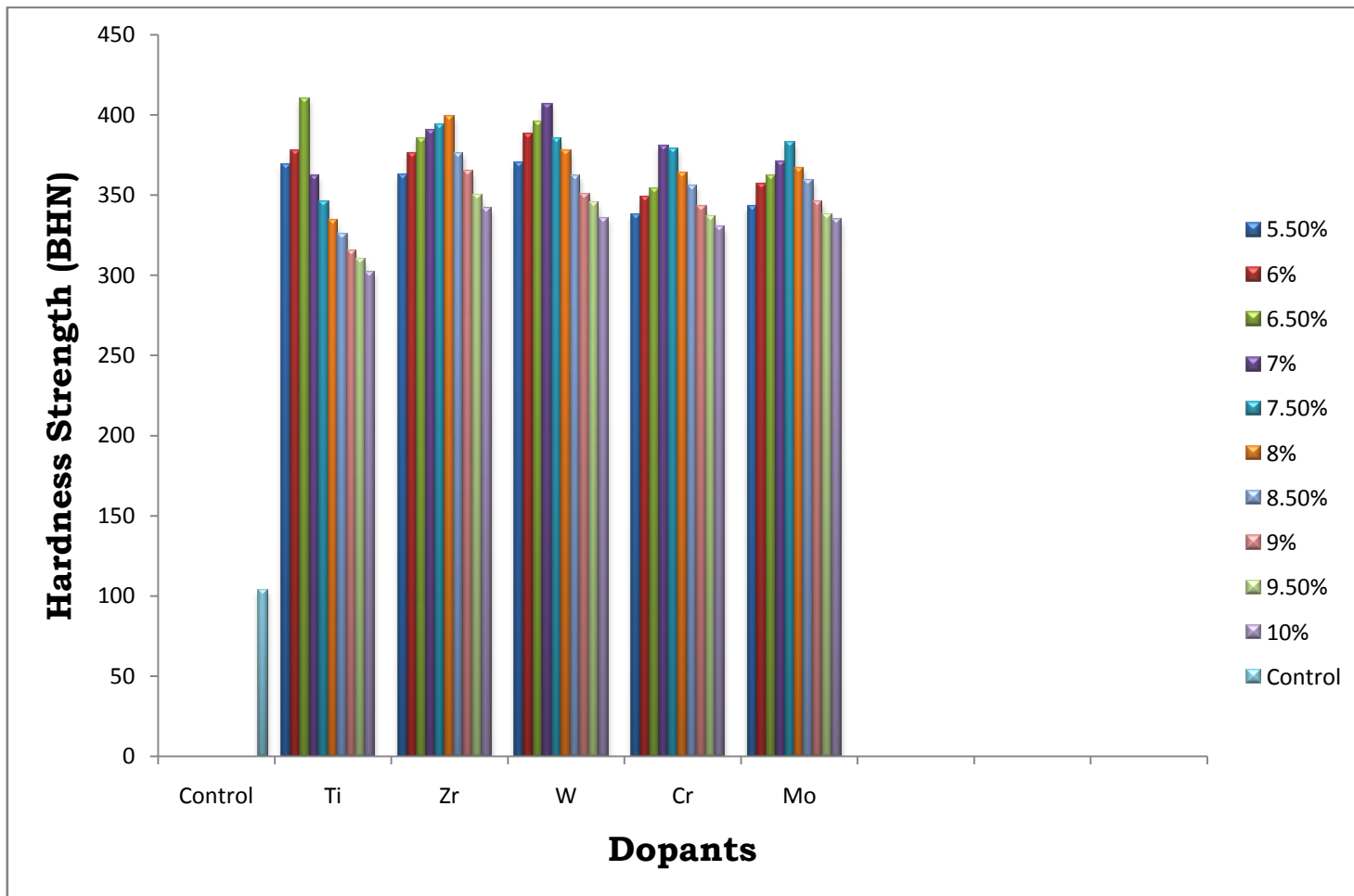
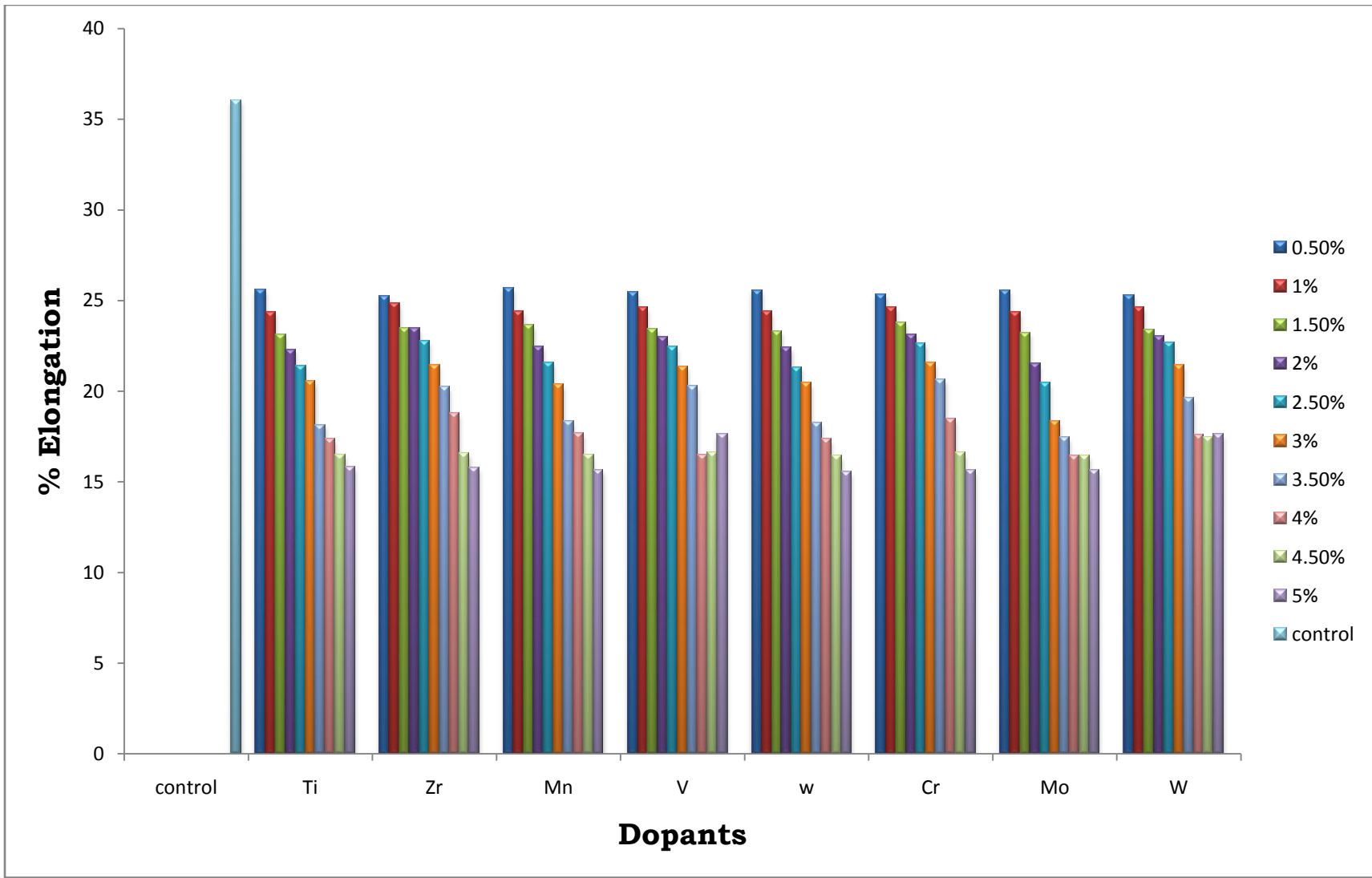


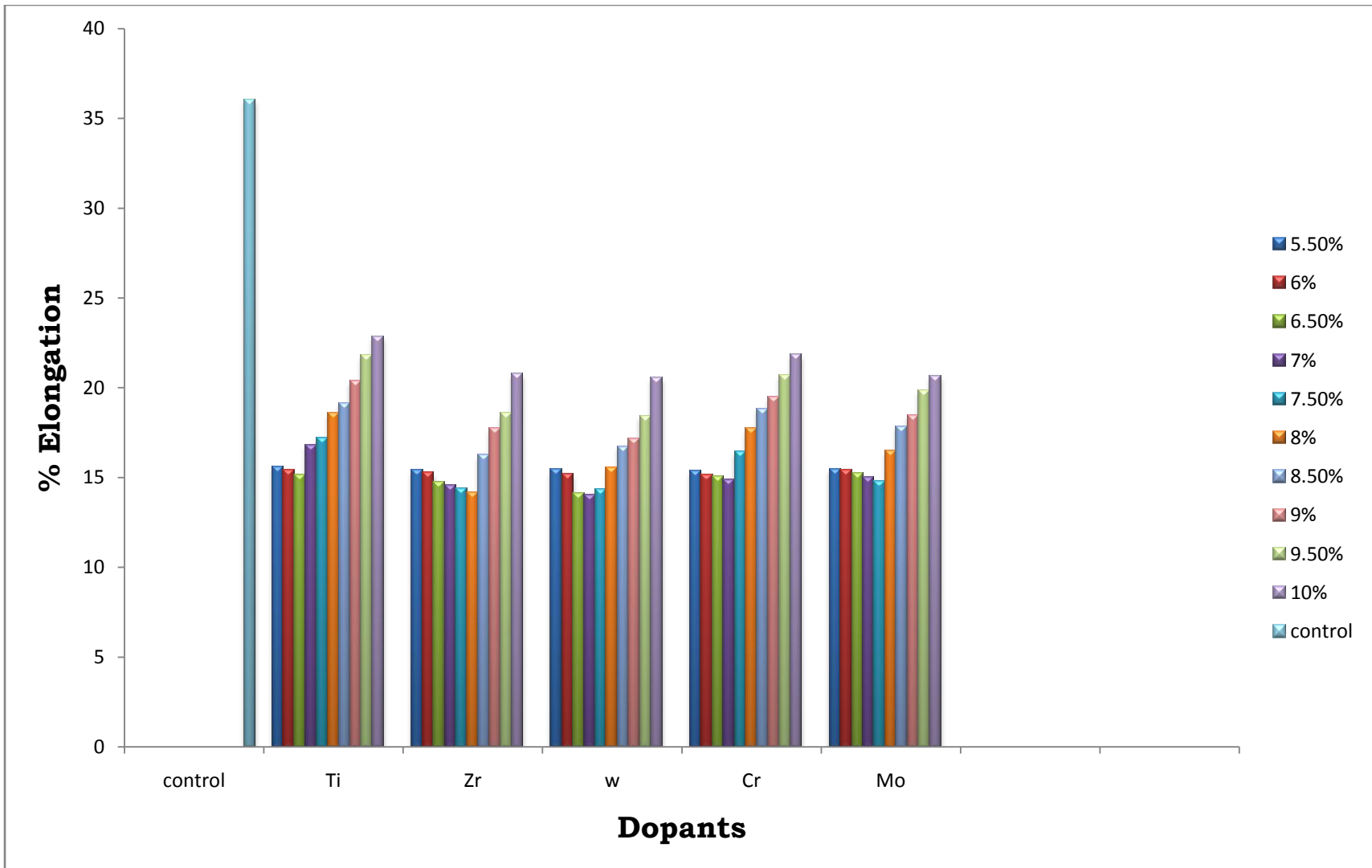
Figure 4.32: The effect of additives on Hardness strength (BHN) of Cu-10%Al alloy

The results of the hardness test values are shown in Table 4.1 and Fig 4.31 & 4.32, it was observed that the hardness values increased with increased percentage composition of modifying elements, thus confirming hardening of the aluminum bronze alloy by modification process. The hardness value of base alloy specimen was 104Hv, which is less than the hardness values of the modified specimens. The highest hardness value of 410BHN was obtained with the modified specimen with titanium as compared with the base and other modified alloy specimens. A sharp decrease in the hardness was observed at different composition of the modifying elements. This decrease is as a result of coarsening of the fine precipitates of  $\alpha$  and  $\gamma_2$  phases.



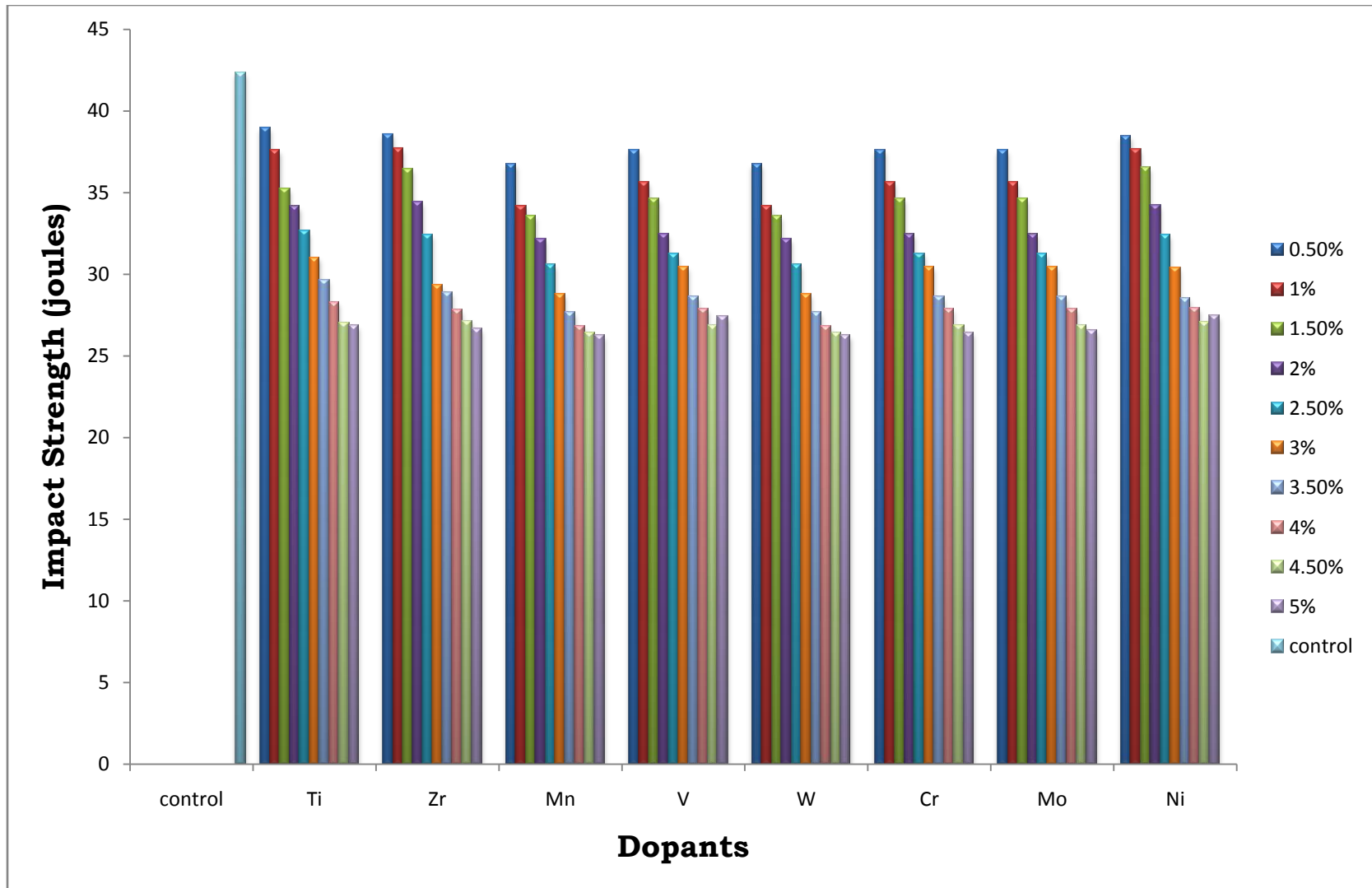


**Figure 4.33: The effect of additives on % Elongation of Cu-10%Al alloy**

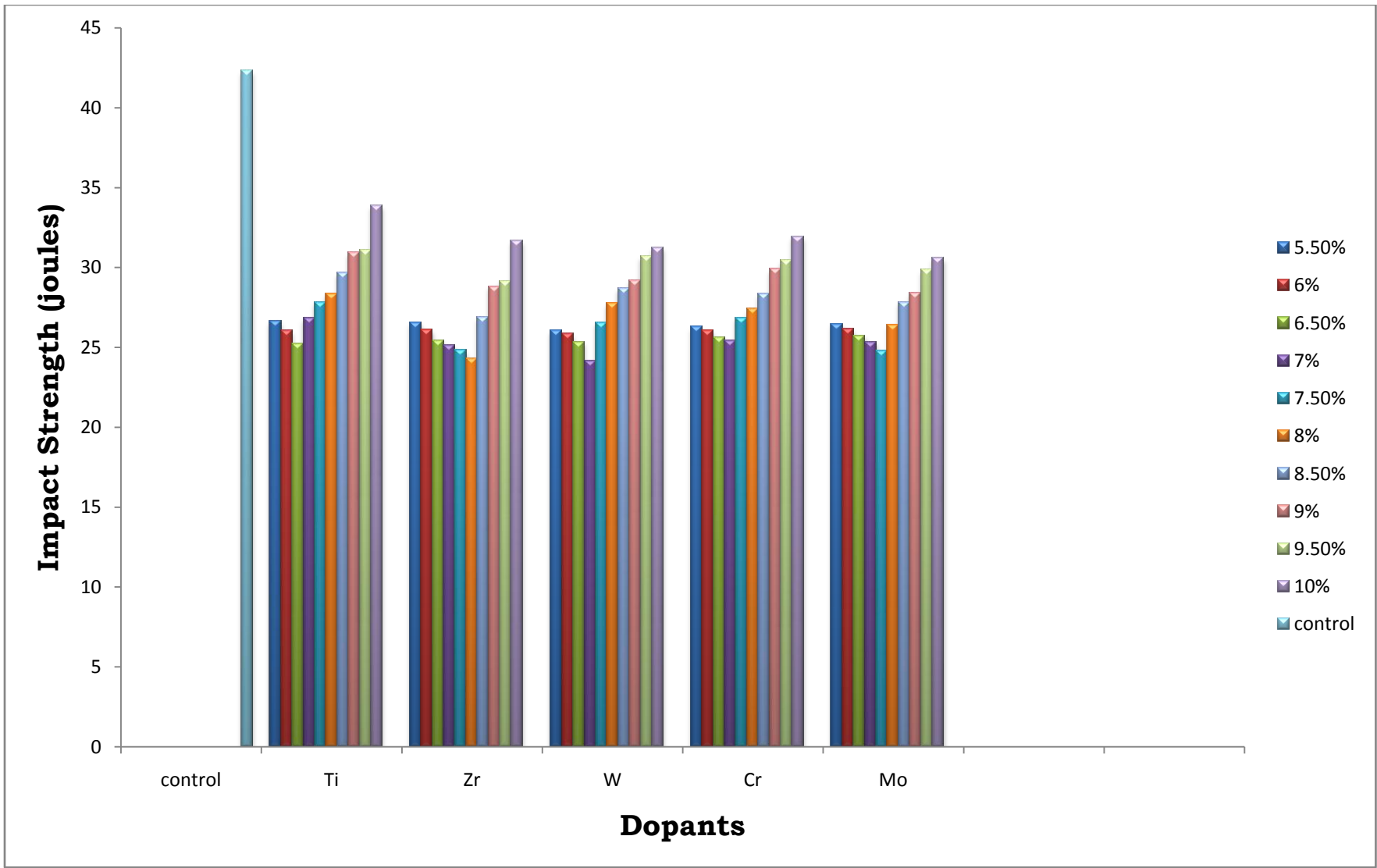


**Figure 4.34: The effect of additives on % Elongation of Cu-10%Al alloy**

Comparative analysis of table 4.1, Figure 4.33 and Figure 4.34 show that the percentage elongation of the modified specimens decreased with increase in composition of modifying elements when compared with the value of the base alloy. The base alloy has the highest percentage elongation of 36.04% which indicates that it is more ductile than modified specimens. This was as a result of the alloy becoming stronger and harder and thus less ductile. This could be as result of solid solubility of the elements in copper-aluminum alloys (intermetallic compound). Addition of those modifying elements to copper- aluminum alloys decreases the solubility of aluminum in copper matrix. This developed a structure that will substitute a compressive stress in the copper lattice which retarded the movement of dislocation and made the alloy to have a good combination of mechanical properties.



**Figure 4.35: The effect of additives on impact strength (J) of Cu-10%Al alloy**



**Figure 4.36: The effect of additives on impact strength (J) of Cu-10%Al alloy**

The results of the impact strength values are shown in Table 4.1 and Fig 4.35& 4.36. From the result, it was observed that the impact strength values of the modified specimens decreased with increase in composition of the modifying elements. The base alloy specimen has energy absorption of 42.34J, which is bigger compared to the values of the modified specimens. This was as a result of the alloy becoming stronger and harder and thus less ductile. The solid solubility of the elements in copper-aluminum alloys contributed also to the decrease of the impact strength values. The structural components such as intermetallic compound, the beta phase and the gamma phase in the copper-aluminum alloys were enhanced, thereby producing better combination of mechanical and physical properties.

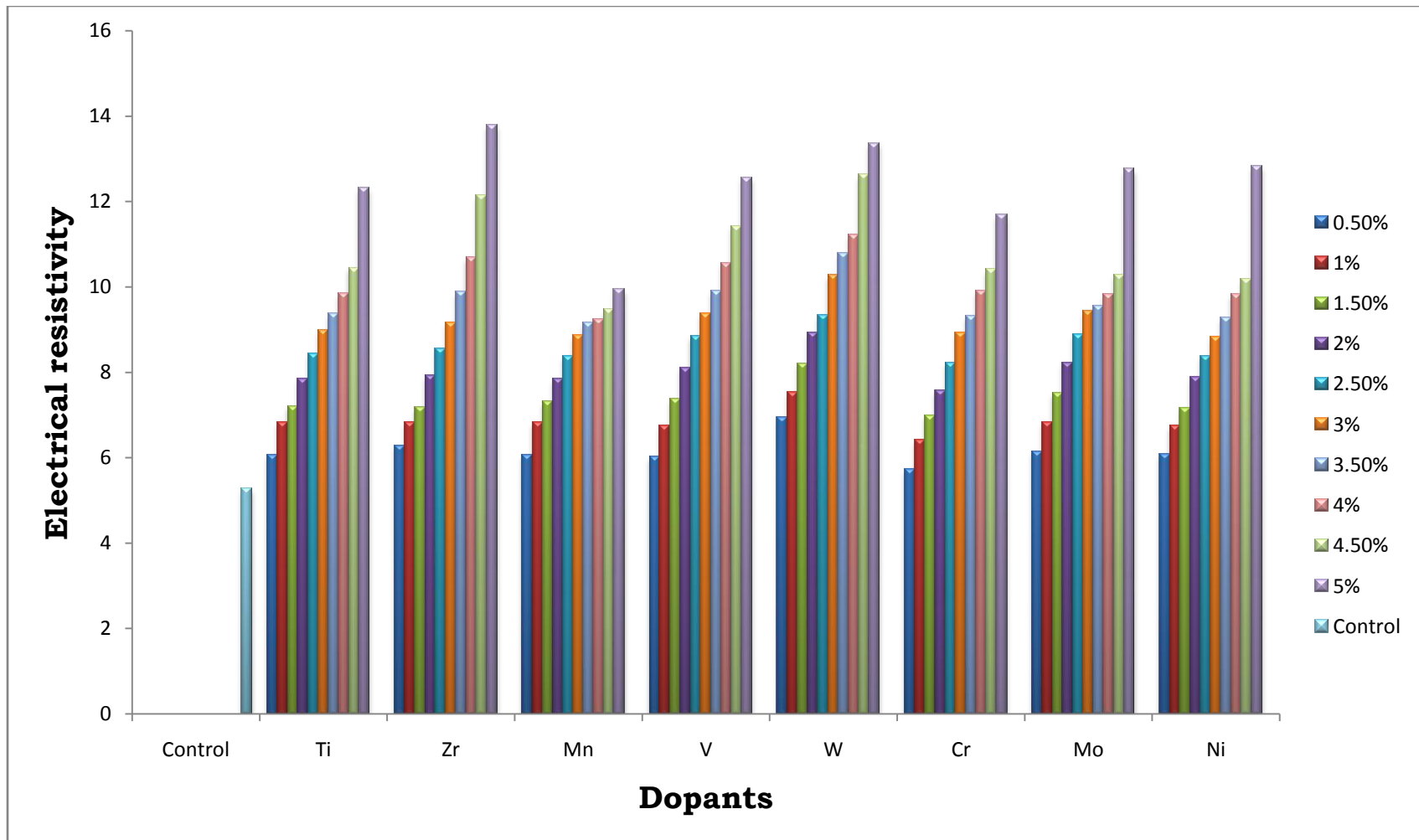


Figure 4.37: The effect of additives on electrical resistivity ( $\rho$ ) of Cu-10%Al alloy

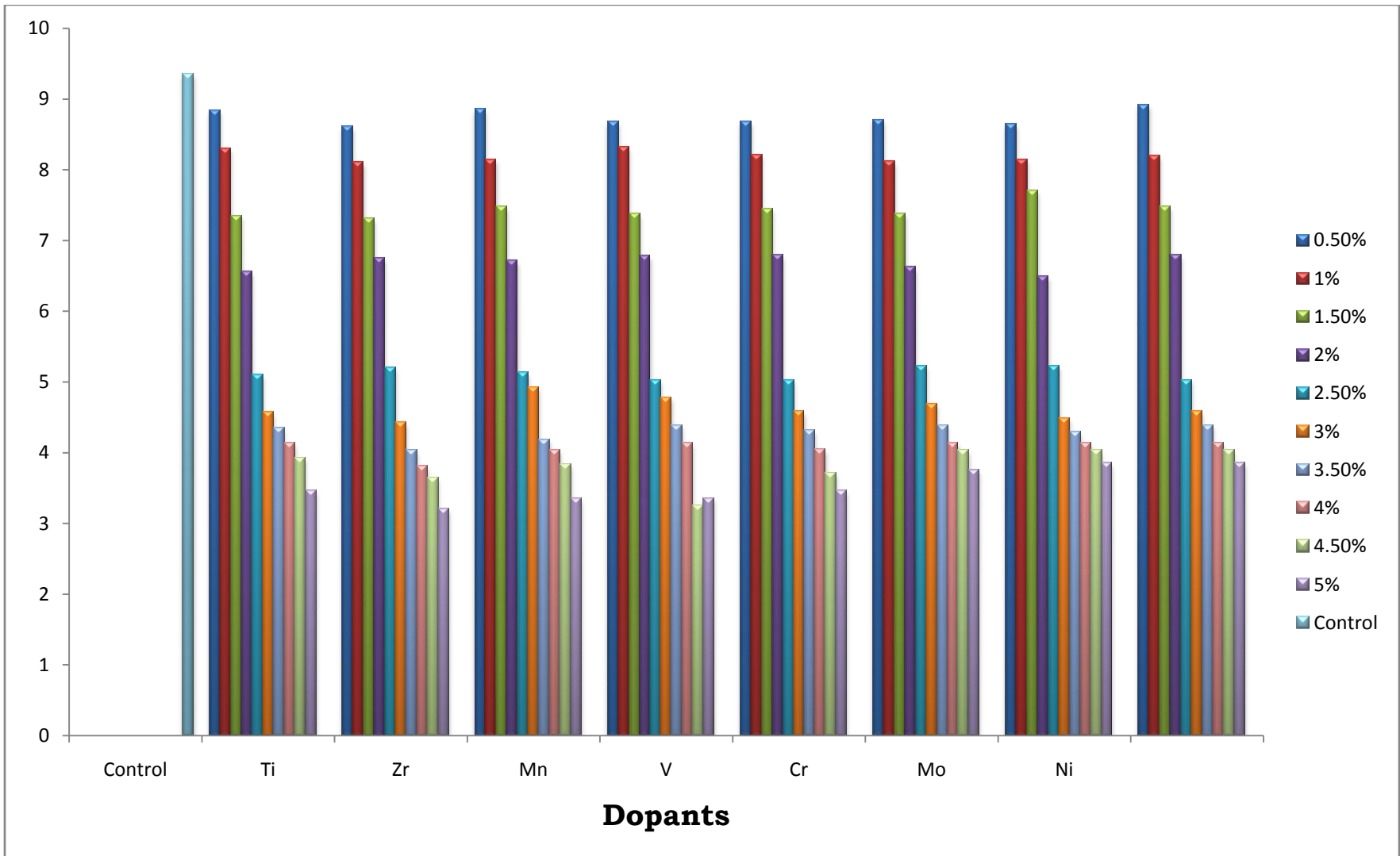


Figure 4.38: The effect of additives on electrical conductivity of Cu-10%Al alloy



### **4.3 Physical properties analysis**

#### **The effects of modifying elements on the physical properties of Cu-10%Al alloys**

Table 4.1 and Figures 4.37 & 4.38 show that resistivity was increased while electrical conductivity decreased in all the alloys cast thus; agreeing with the known principle that resistivity of a metal is increased by even a small amount of impurity. The increase in resistivity as a result of doping element of Cu-Al alloys is probably connected with lattice defects induced by the doping elements (Nwankwo, 2015) and because of the introduction of scattering centers by these dopants that reduced mean free path of the alloys (Nnuka, 2014). The changes in resistivity in all categories of the studied alloys were however very small. This means that the energy band structure was not much altered and the electronic characteristics of the solid Cu-Al alloys were preserved. It is evidenced however that the physical properties of electrical conductivity and resistivity of the alloys were sensitive to structural changes in the studied alloys and that these properties like the mechanical properties depend mainly on the percentage of the modifying elements.

### **4.4 Structural analysis**

The micrographs are shown in Plates 4.1 to 4.81. From Plate 4.1, it was observed that the microstructure consists of large coarse interconnected intermetallic  $\text{Cu}_9\text{Al}_4$  compounds and  $\alpha+\gamma_2$  phases. This alloy exhibited the lowest mechanical and physical properties because of its coarse microstructure.

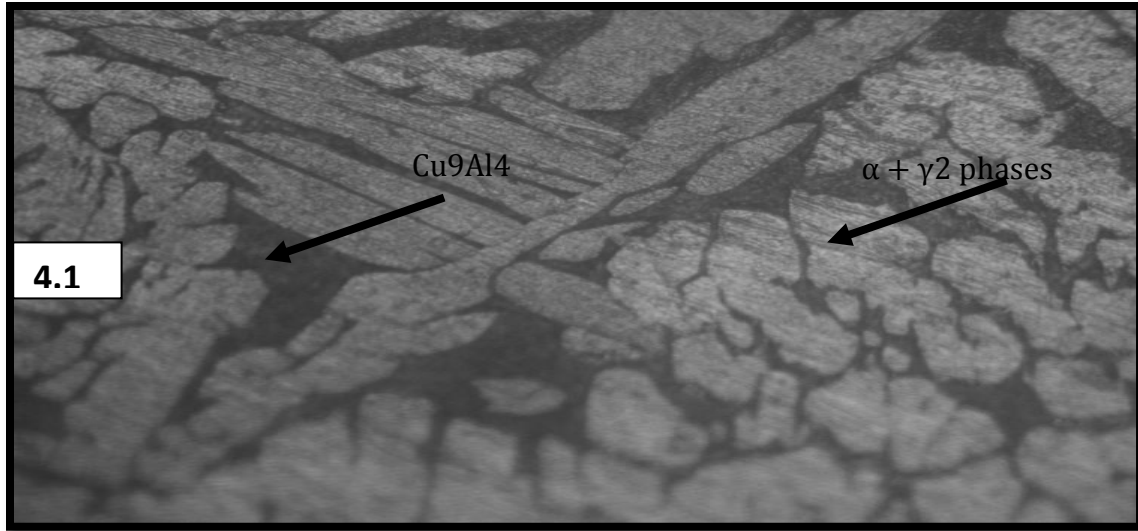


Plate 4.1: Micrograph of Cu-10wt%Al (base alloy)

(x400)

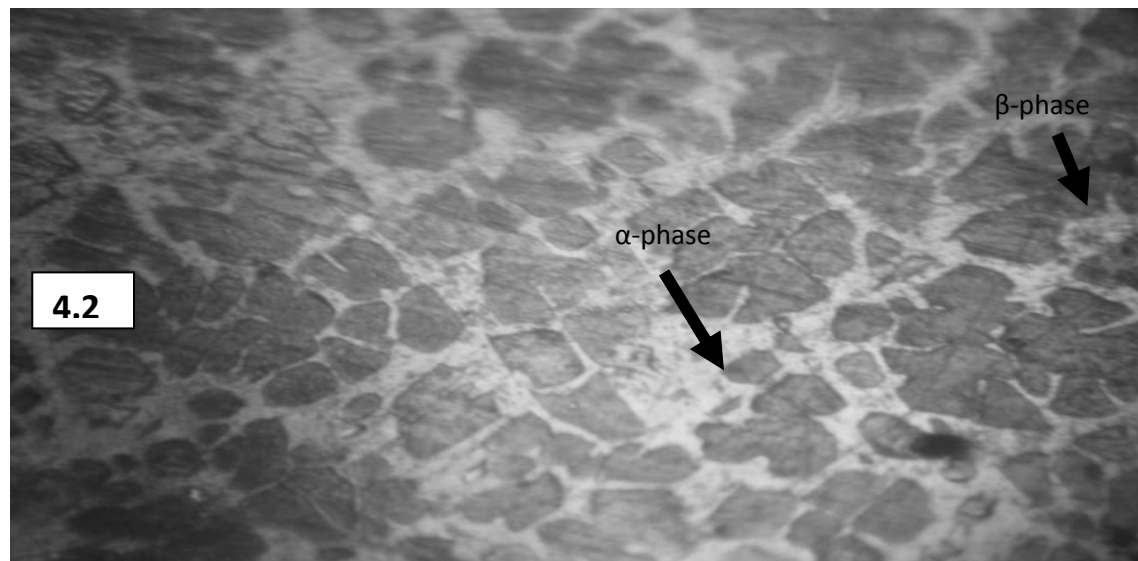


Plate 4.2: Micrograph of Cu-10wt%Al +0.5wt%Ti

(x400)

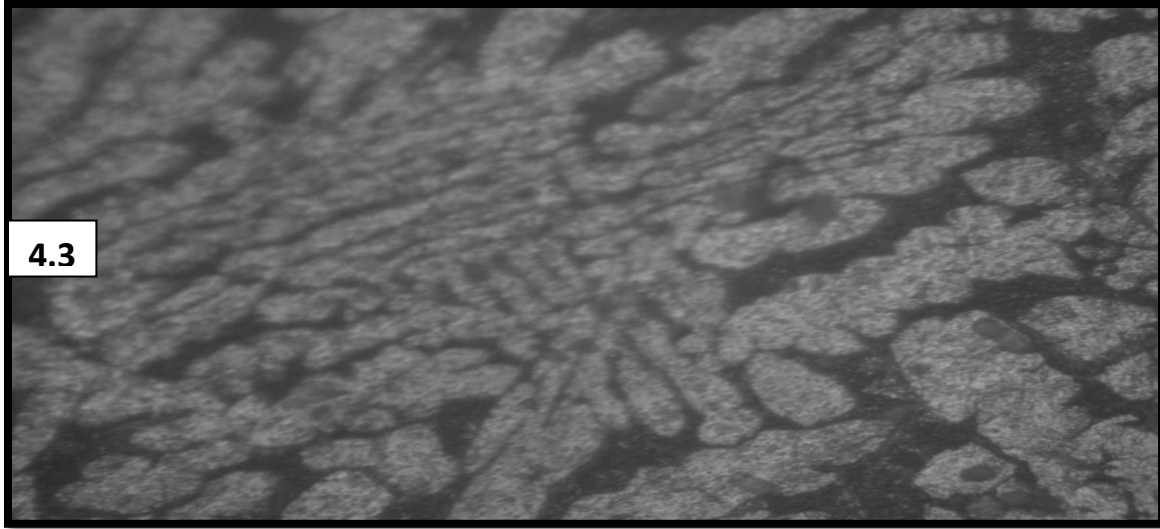


Plate 4.3: Micrograph of Cu-10%Al +2.0wt%Ti (x400)

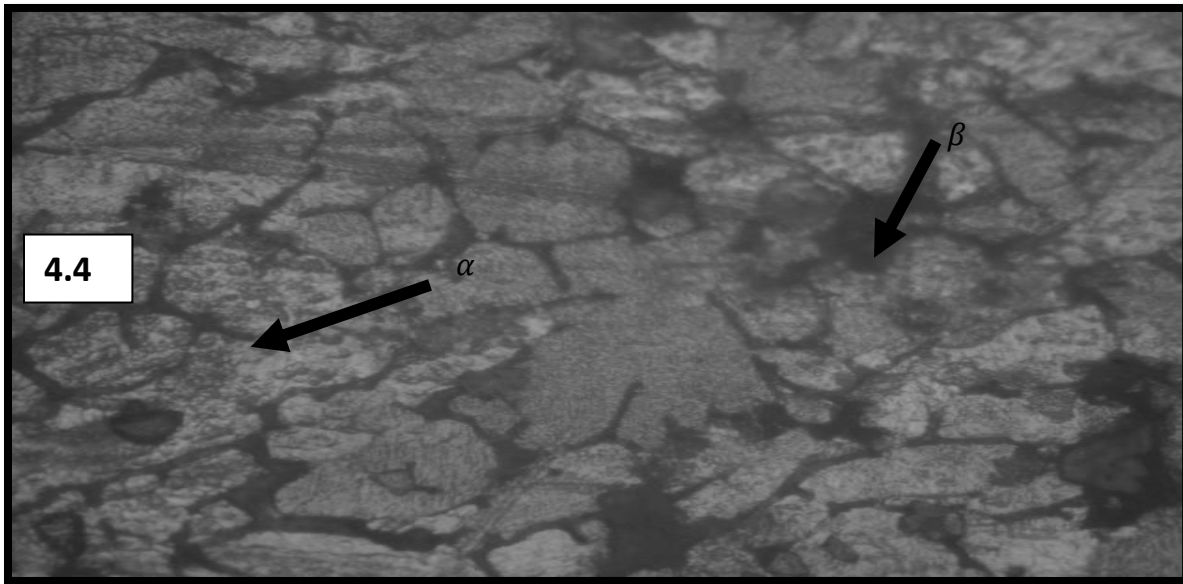
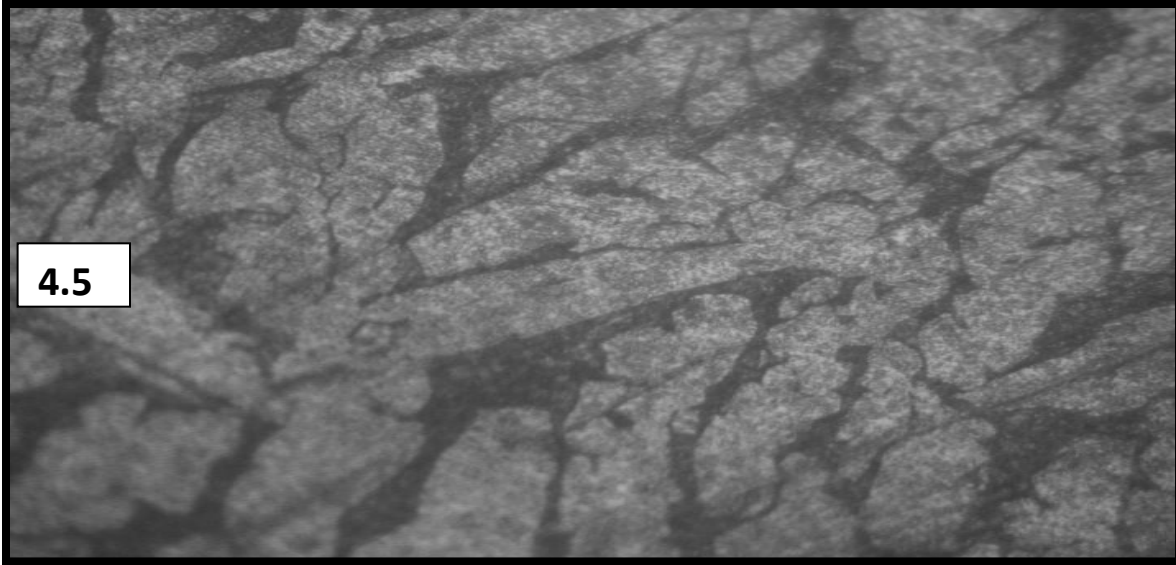
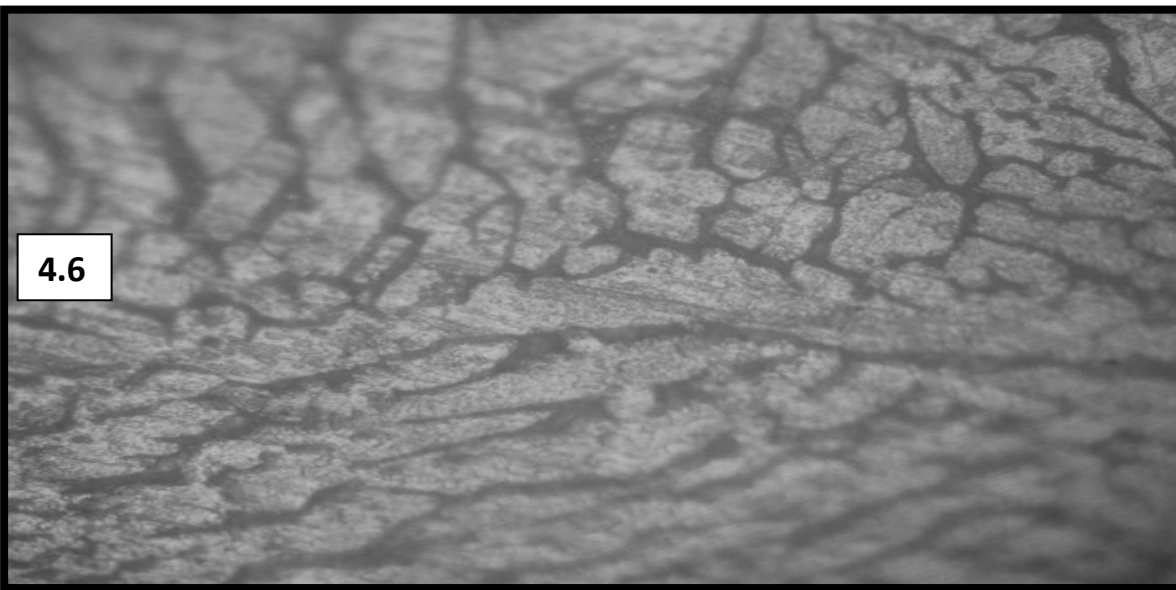


Plate 4.4: Micrograph of Cu-10%Al +4.5wt%Ti (x400)



**Plate 4.5: Micrograph of Cu-10%Al +5.0wt%Ti (x400)**



**Plate 4.6: Micrograph of Cu-10%Al +5.5wt%Ti (x400)**

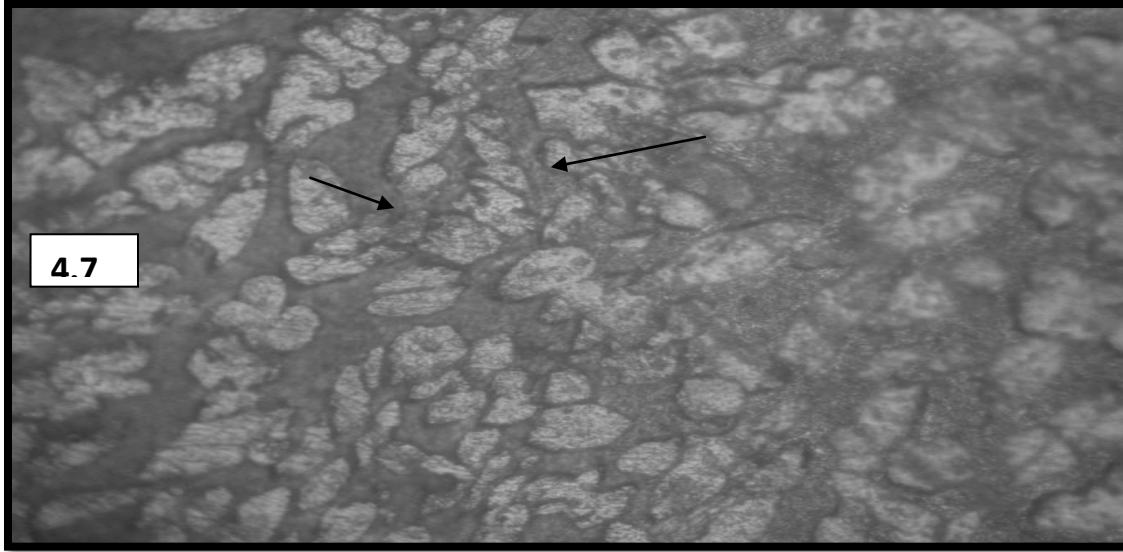


Plate 4.7: Micrograph of Cu-10%Al +7.5wt%Ti

(x400)

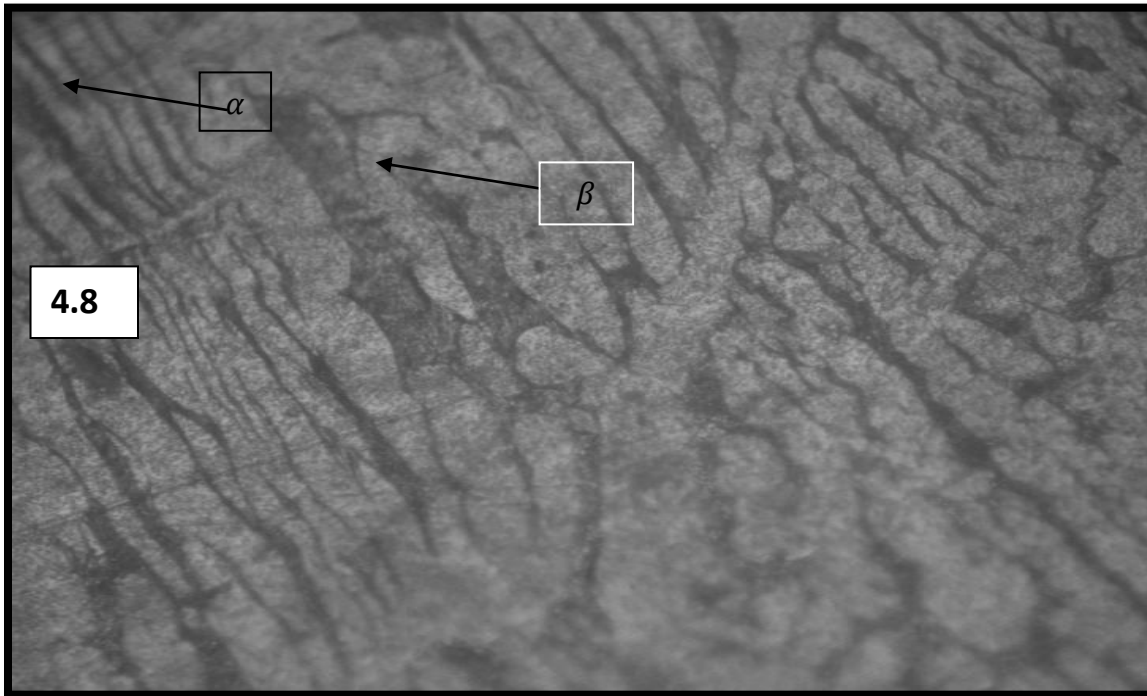
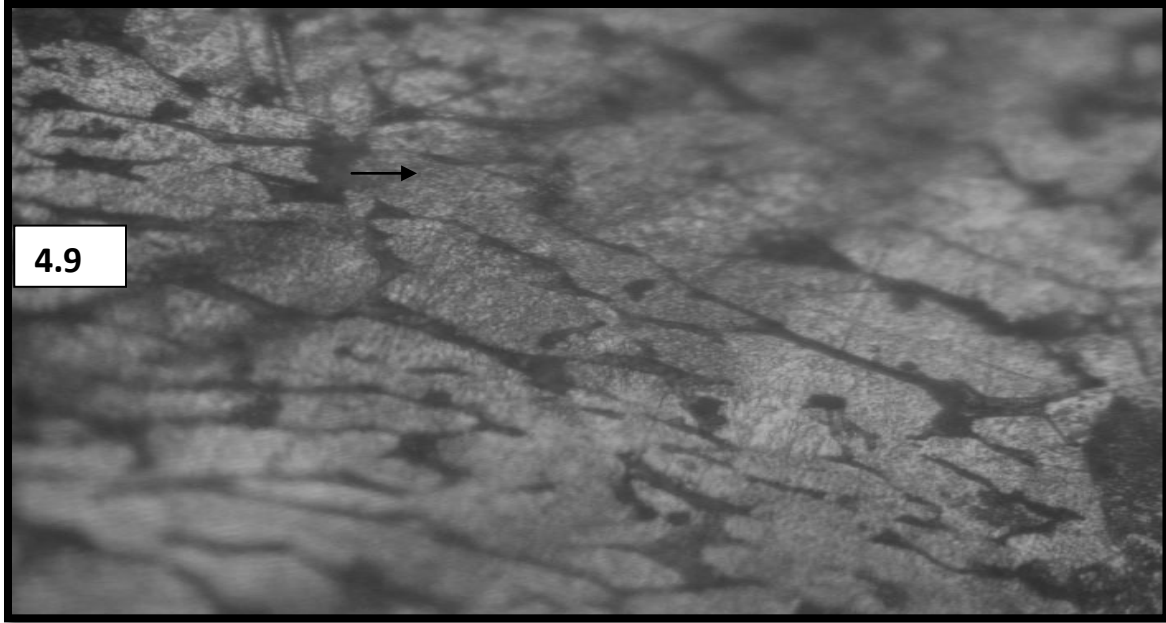
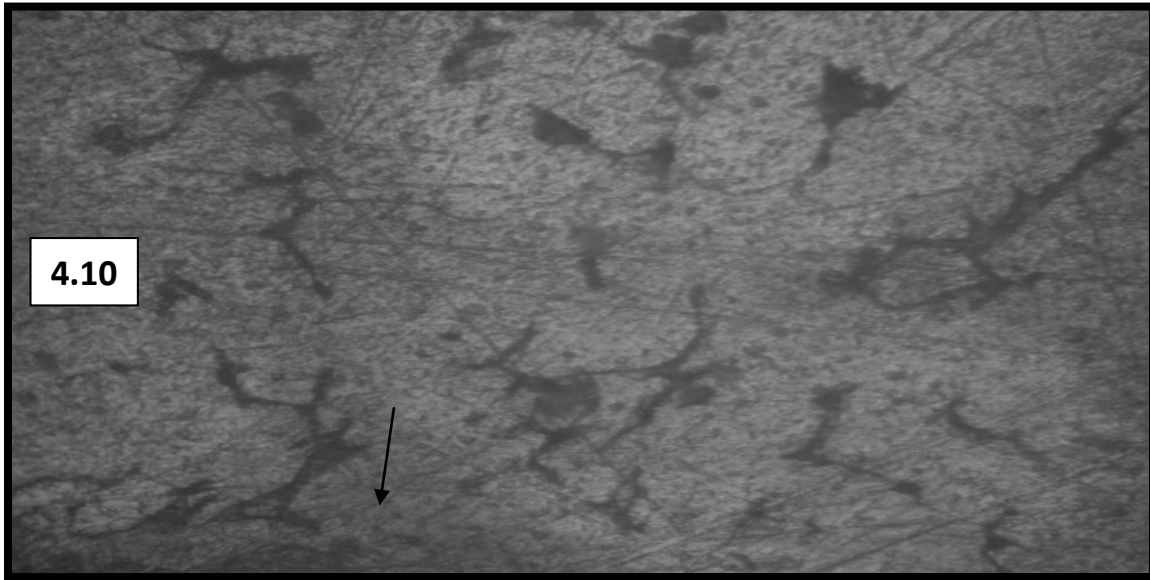


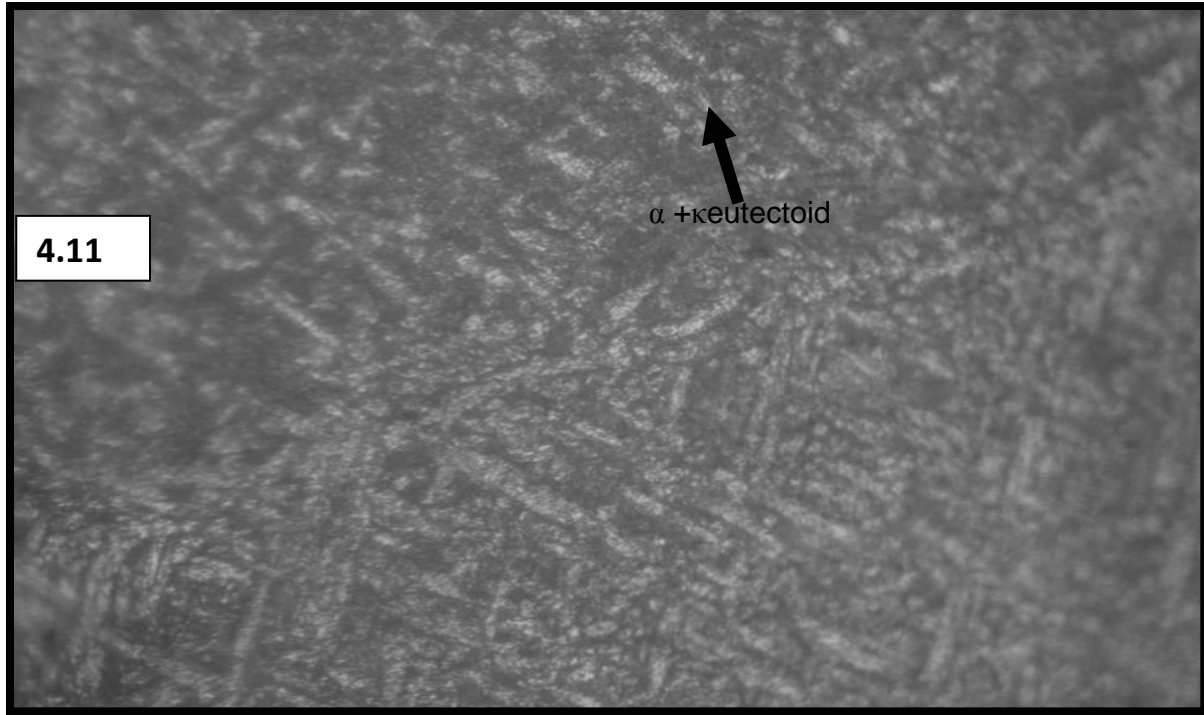
Plate 4.8: Micrograph of Cu-10%Al +10wt%Ti (x400)



**Plate 4.9: Micrograph of Cu-10%Al + 7.0wt%Ti (x400)**



**Plate 4.10: Micrograph of Cu-10%Al + 6.0wt%Ti. (x400)**



**Plate 4.11: Micrograph of Cu-10%Al+6.5wt%Ti.**

**(x400)**

Plate 4.2 to Plate 4.11 show the microstructure of copper-10% aluminium (Cu-10%Al) alloy treated with (0.5 to 10%) wt% titanium. From the micrographs, addition of titanium to Cu-10%Al alloy stabilizes the formation of  $\beta$ -phases which showed small grains of alpha ( $\alpha$ ) and few amount of kappa ( $\kappa$ ) phase grains in black lamellar form. The large size of  $\alpha$ -phase with white colour and small amount of  $\gamma_2$ -phase and  $\beta$ - phase were observed from the alloys as titanium increased in the microstructure. As the composition of titanium atom increased, it was also observed that  $\gamma_2$ -phase was suppressed which brought about improvement in the alloy properties.



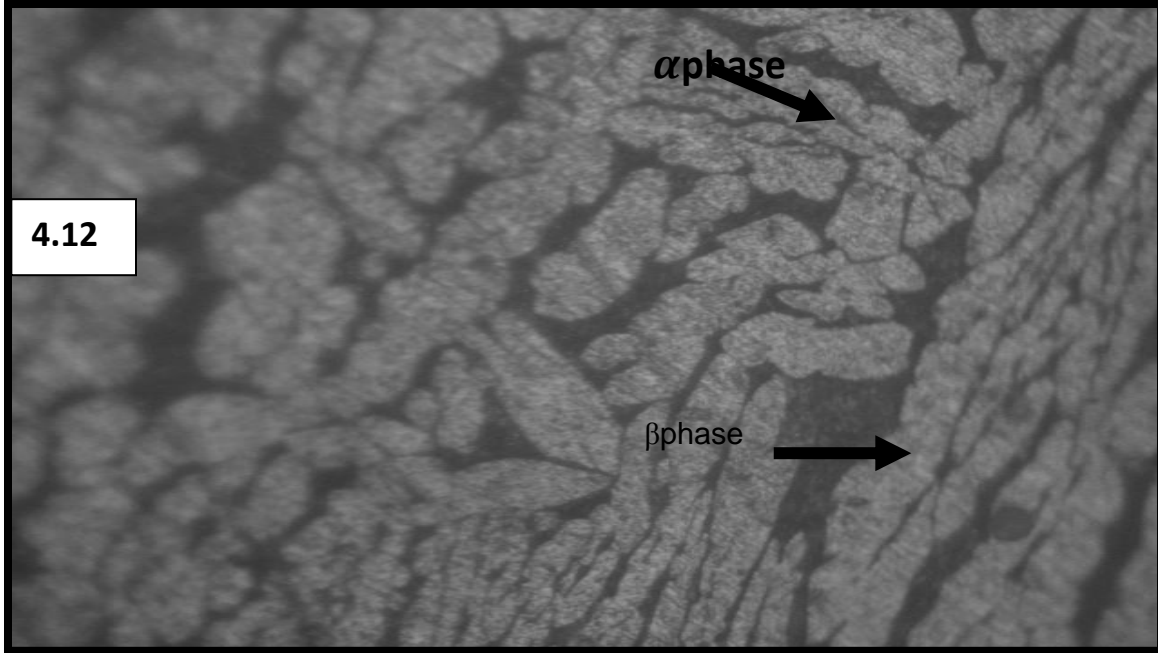


Plate 4.12: Micrograph of Cu-10%Al+0.5wt%Zr. (x400)

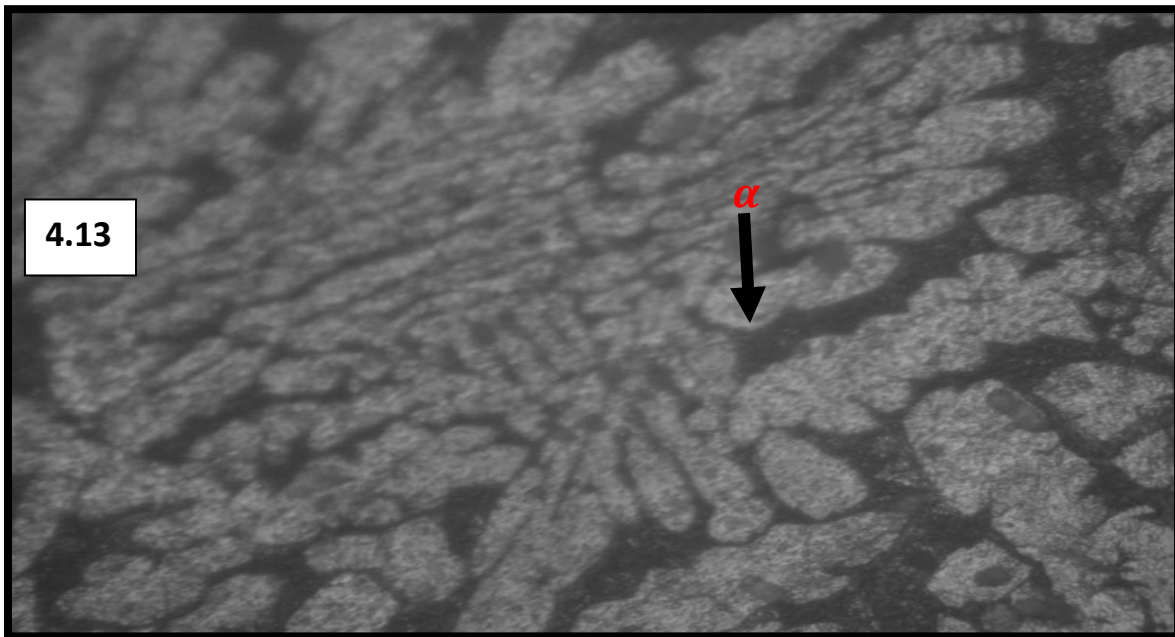
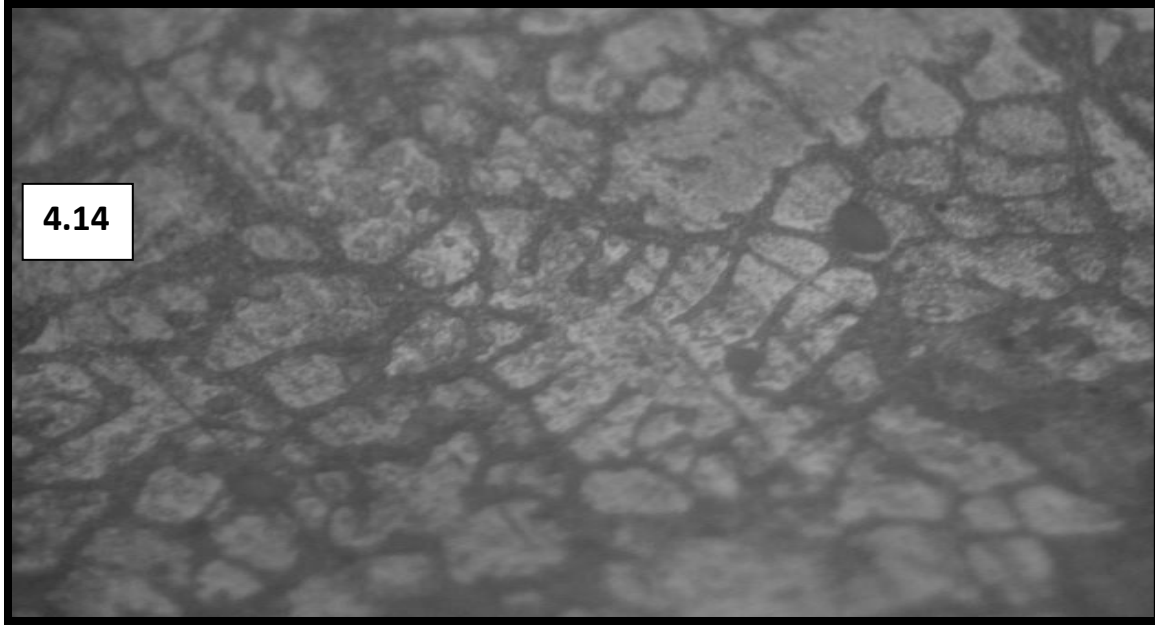
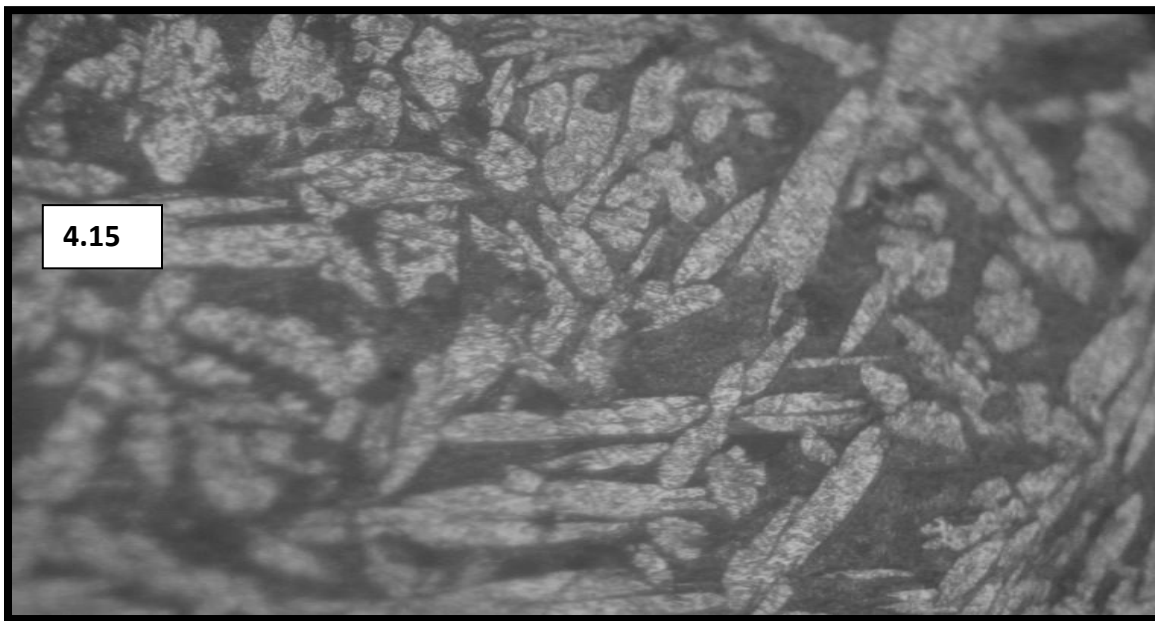


Plate 4.13: Micrograph of Cu-10%Al+3.0wt%Zr. (x400)





**Plate 4.14: Micrograph of Cu-10%Al+5.5wt%Zr. (x400)**



**Plate 4.15: Micrograph of Cu-10%Al+6.0wt%Zr (x400)**

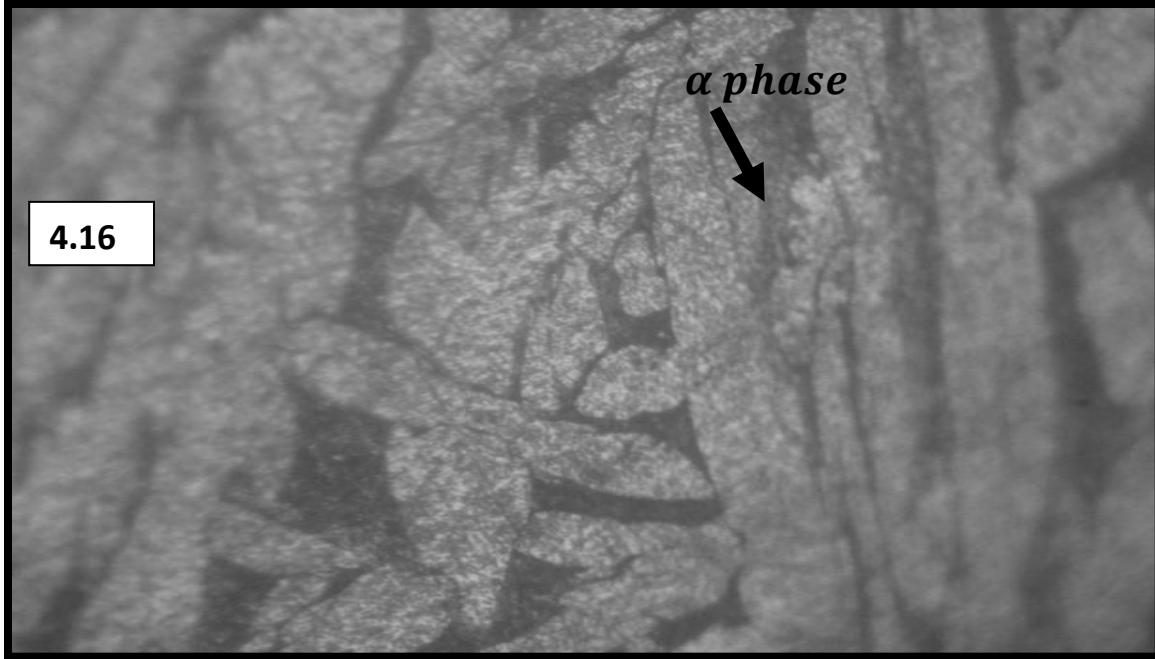


Plate 4.16: Micrograph of Cu-10%Al+7.0wt%Zr.

(X400)

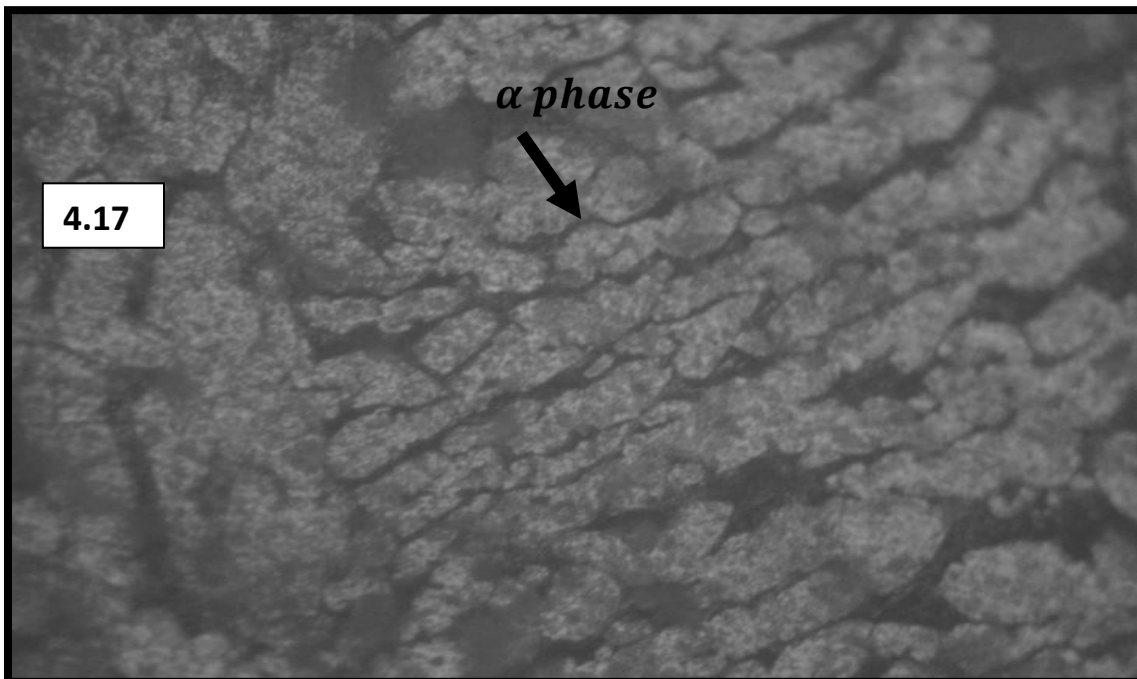


Plate 4.17: Micrograph of Cu-10%Al+6.5wt%Zr.

(x400)

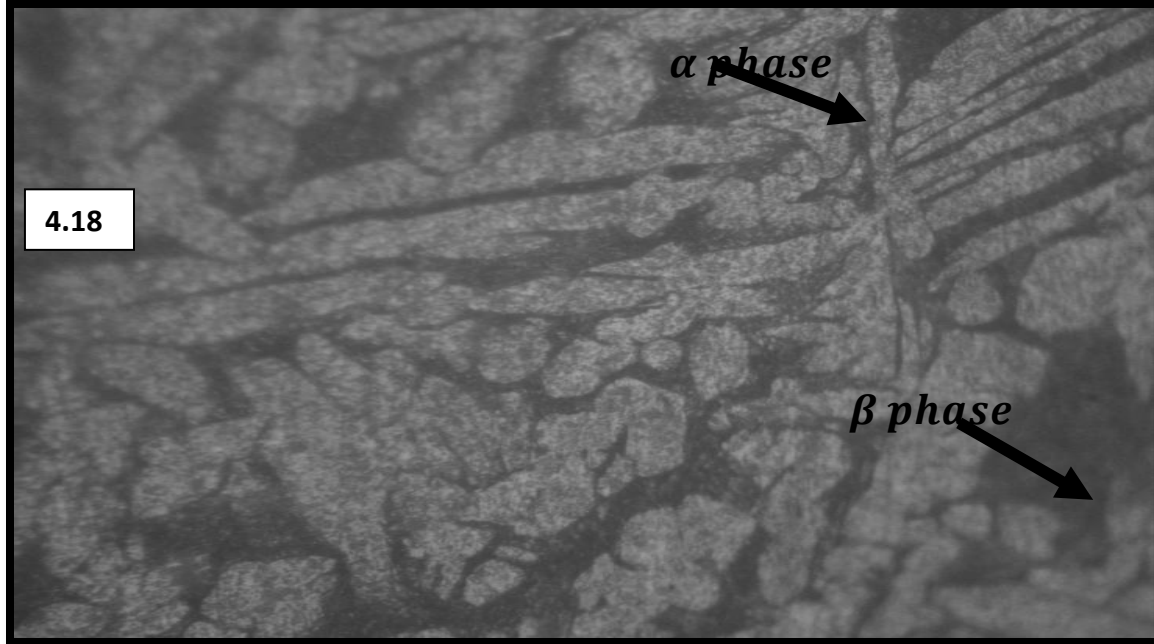


Plate 4.18: Micrograph of Cu-10%Al+9.5wt%Zr.

(x400)

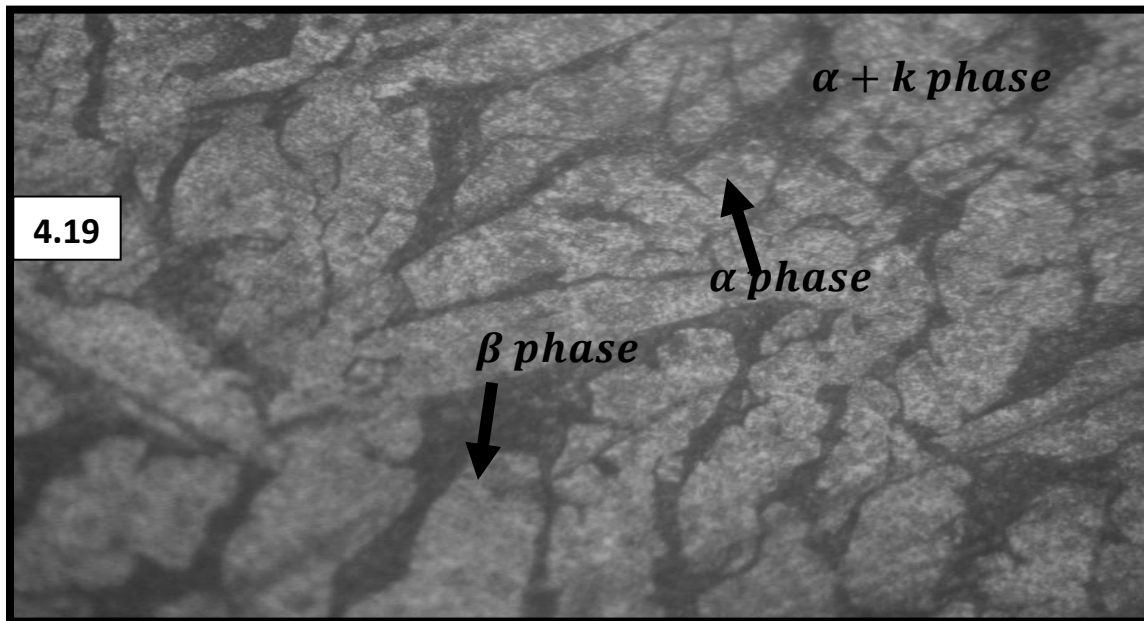


Plate 4.19: Micrograph of Cu-10%Al+10wt%Zr.

(x400)

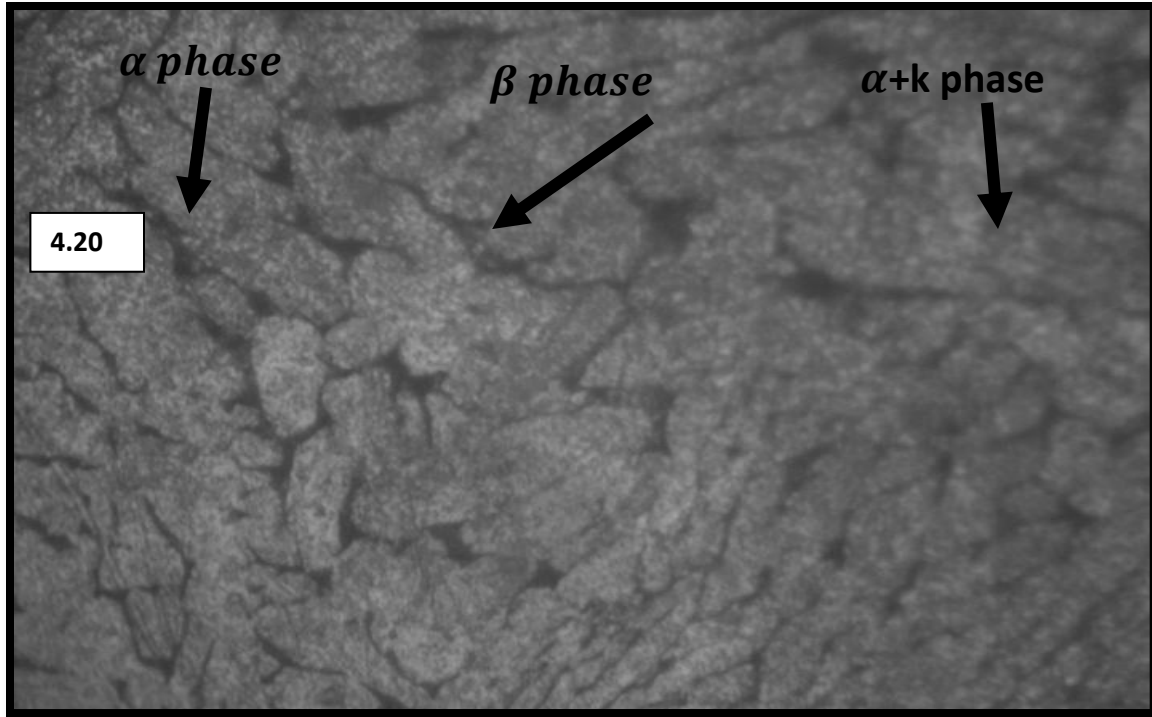


Plate 4.20: Micrograph of Cu-10%Al+7.5wt%Zr.

(x400)

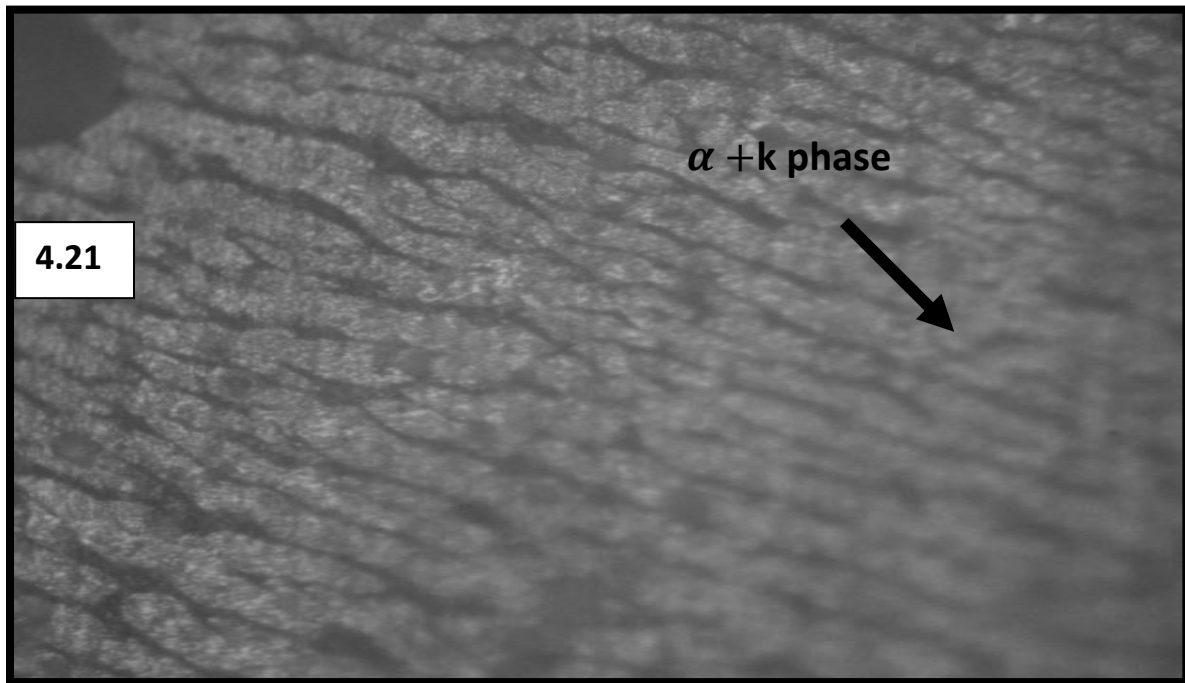
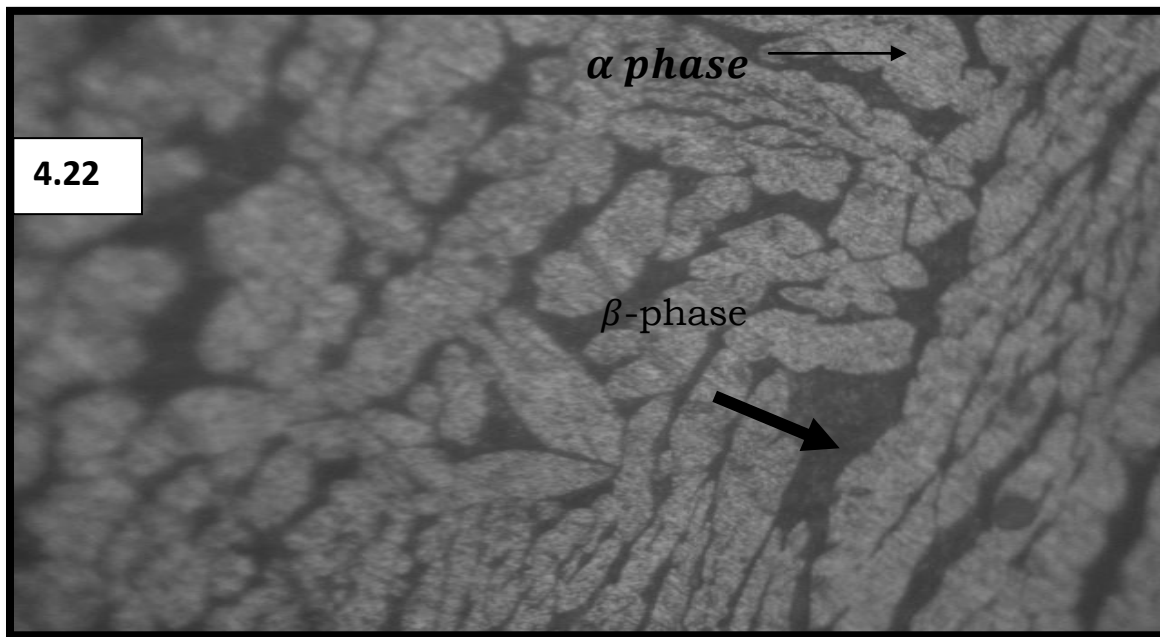


Plate 4.21: Micrograph of Cu-10%Al+8.0wt%Zr.

(x400)

Plate 4.12 to Plate 4.21 show the microstructure of Cu-10%Al alloy treated with (0.5 to 10) wt% zirconium. The  $\alpha$ -phase increased in size as the composition of zirconium increased. This leads to formation of fine lamellar form of kappa ( $\kappa$ ) precipitates present in the microstructures.  $\beta$ -phase decreased in size as the weight composition of zirconium atom increased thereby allowing little or no  $\gamma_2$  phase to precipitate. Presence of sparse distribution of kappa precipitates in the predominated  $\alpha$  matrix caused smaller grains to development which leads to improvement in the alloy properties.



**Plate 4.22: Micrograph of Cu-10%Al+0.5wt%Mn. (x400)**

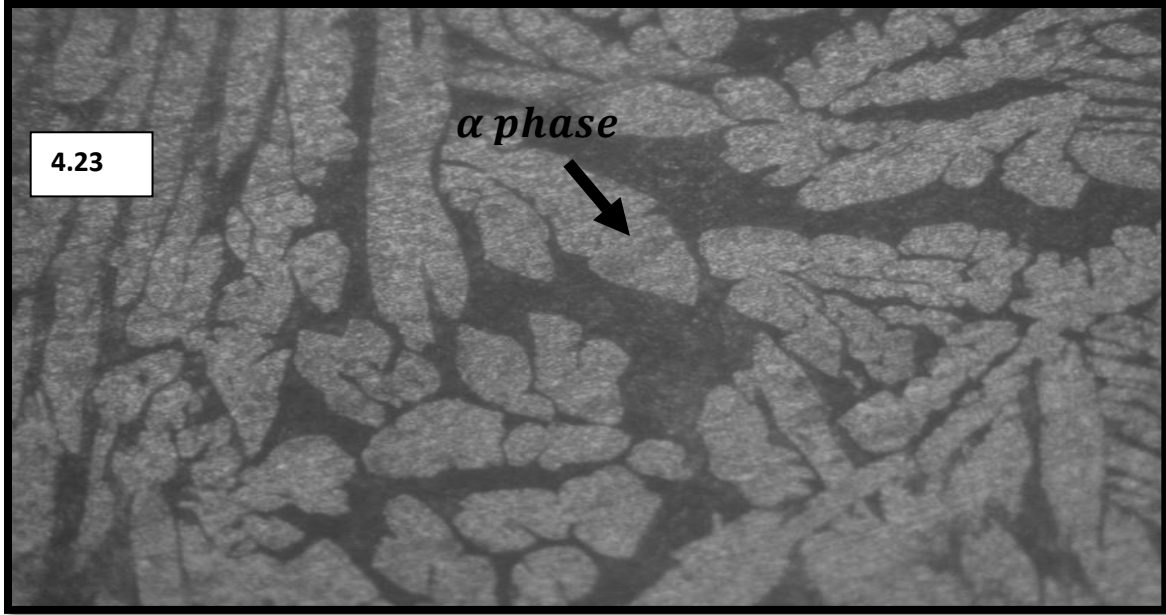


Plate 4.23: Micrograph of Cu-10%Al+1.0wt%Mn. (x400)

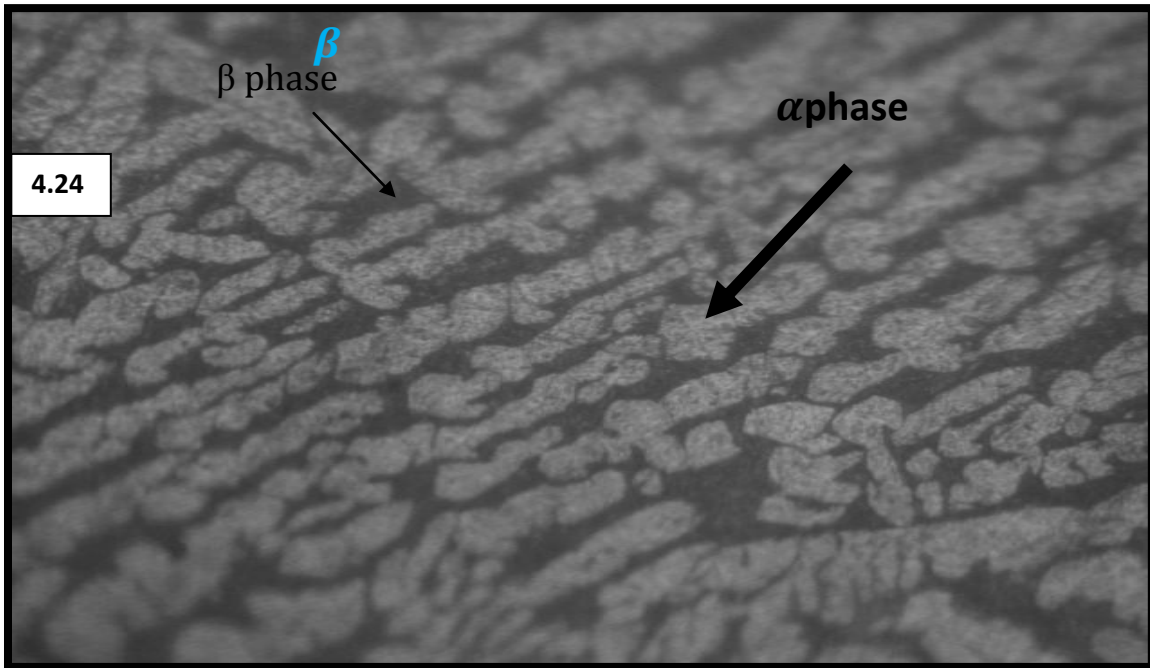


Plate 4.24: Micrograph of Cu-10%Al+1.5wt%Mn. (x400)

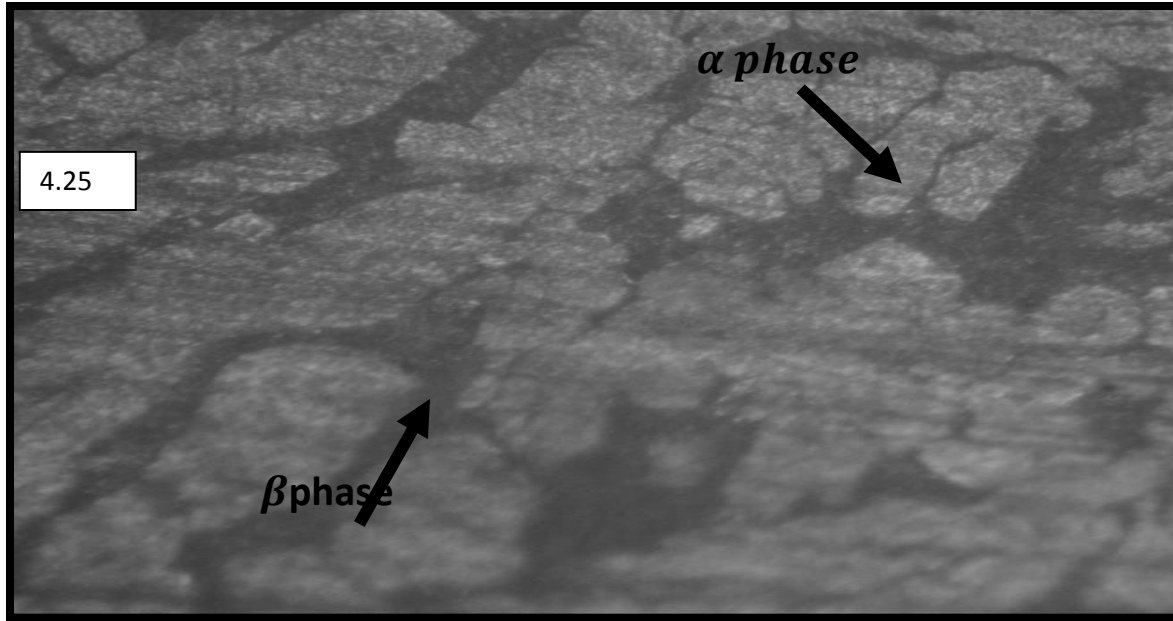


Plate 4.25: Micrograph of Cu-10%Al+2.0wt%Mn. (x400)

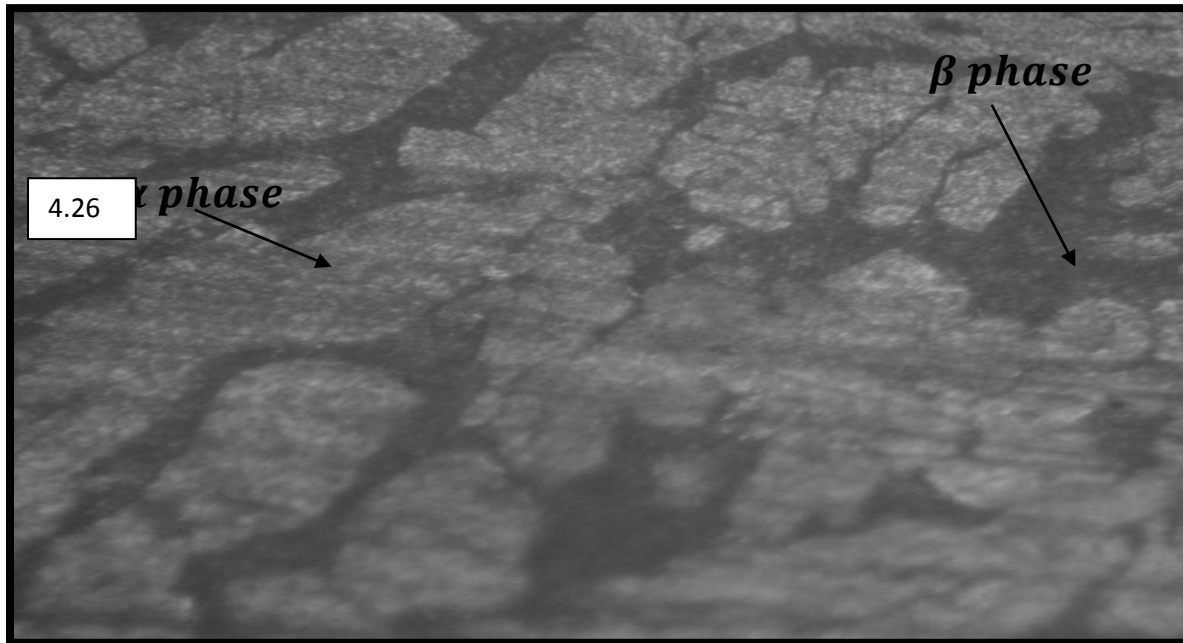


Plate 4.26: Micrograph of Cu-10%Al+2.5wt%Mn. (x400)



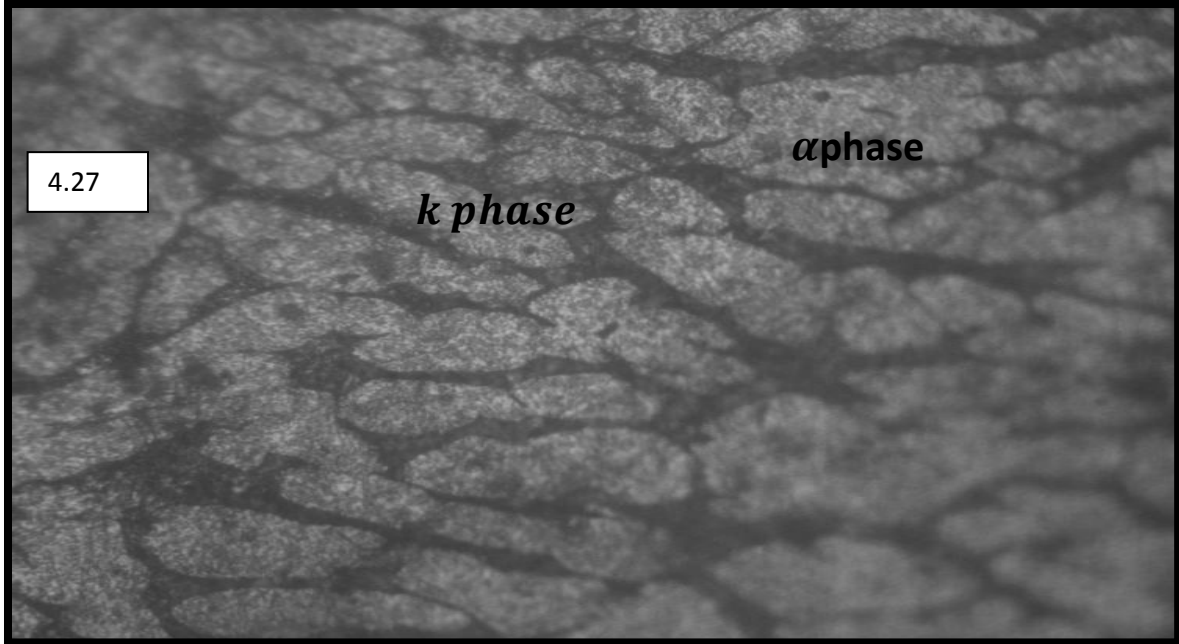


Plate 4.27: Micrograph of Cu-10%Al+3.0wt%Mn. (x400)

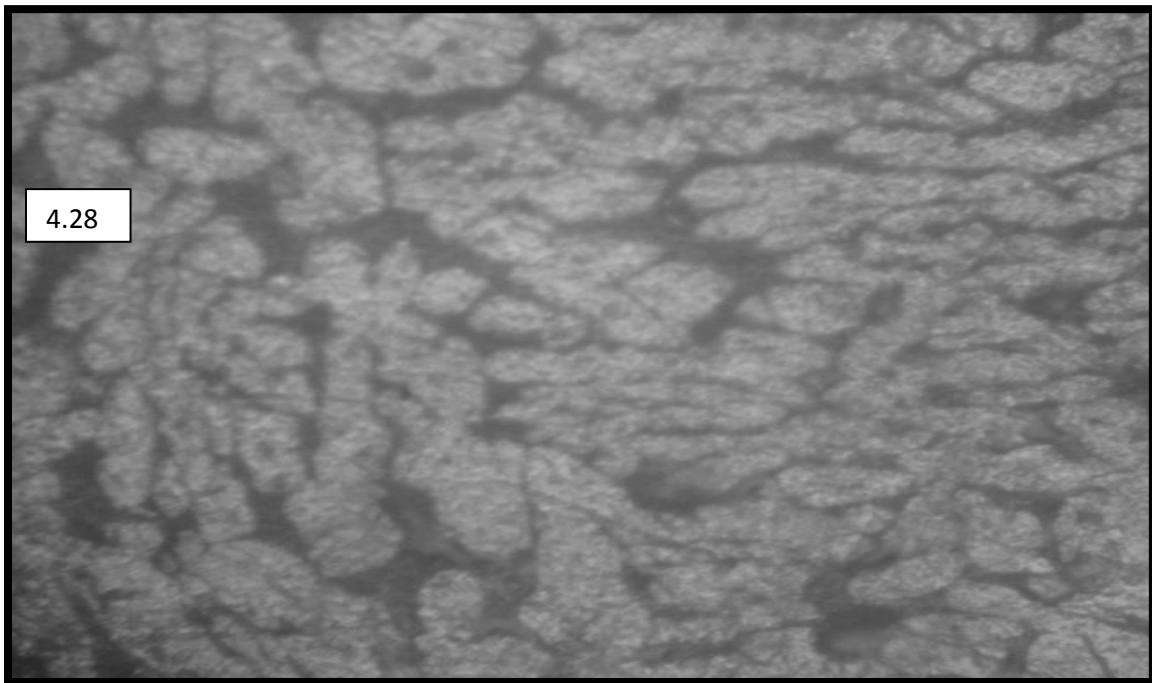
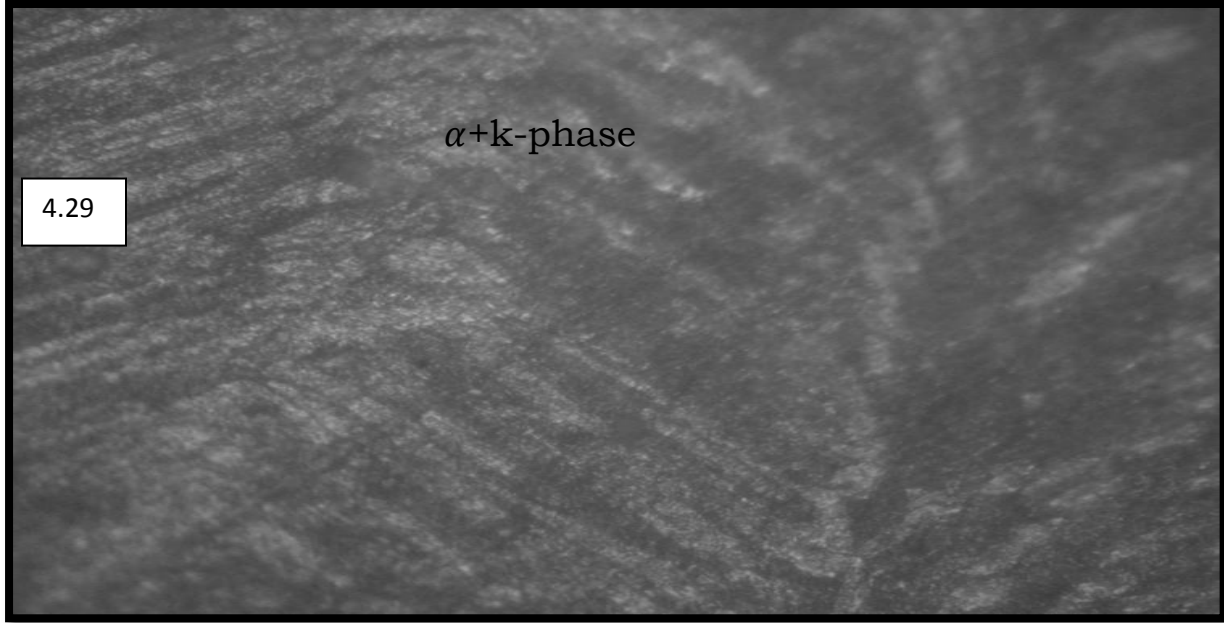
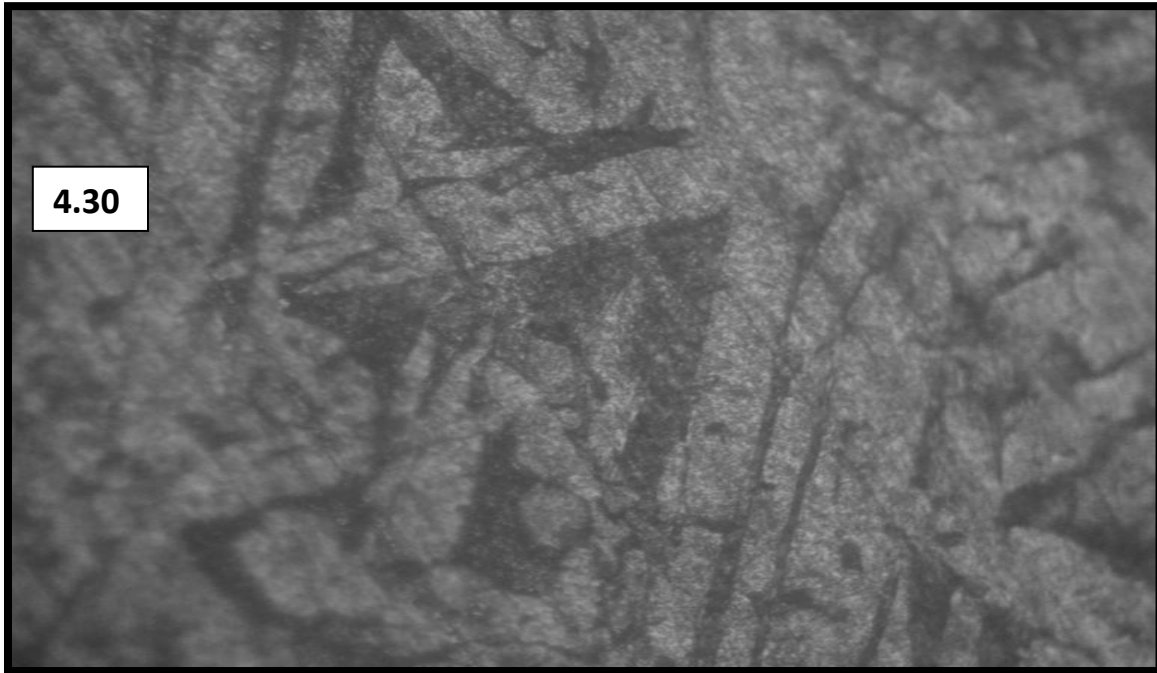


Plate 4.28: Micrograph of Cu-10%Al+3.5wt%Mn. (x400)

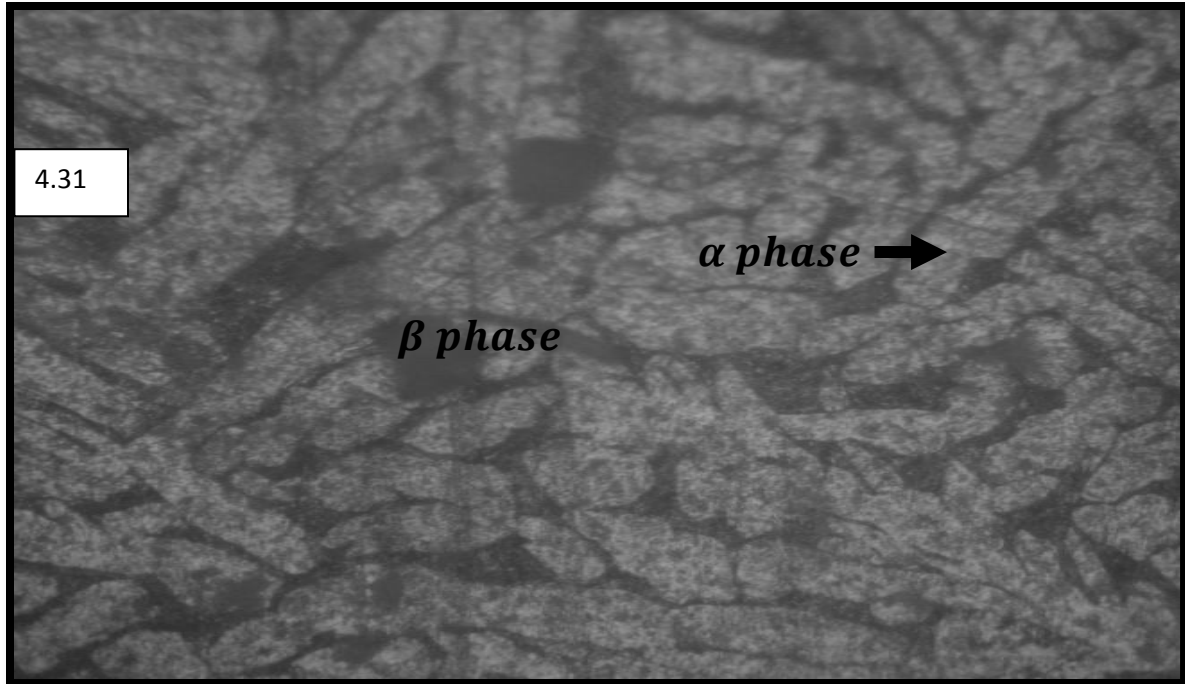




**Plate 4.29: Micrograph of Cu-10%Al+4.0wt%Mn (x400)**



**Plate 4.30: Micrograph of Cu-10%Al+4.5wt%Mn. (x400)**



**Plate 4.31: Micrograph of Cu-10%Al+5.0wt%Mn. (x400)**

Plate 4.22 to Plate 4.31 show the microstructure of Cu-10%Al alloy treated with (0.5 to 5.0) wt% manganese. Because of the solubility of manganese in copper, the addition of manganese to the alloy refines the grain structure and stabilizes the  $\beta$ -phases. It also suppressed the formation of  $\gamma_2$  phase. Manganese forms intermetallic phase with aluminum which produces the fine structure.

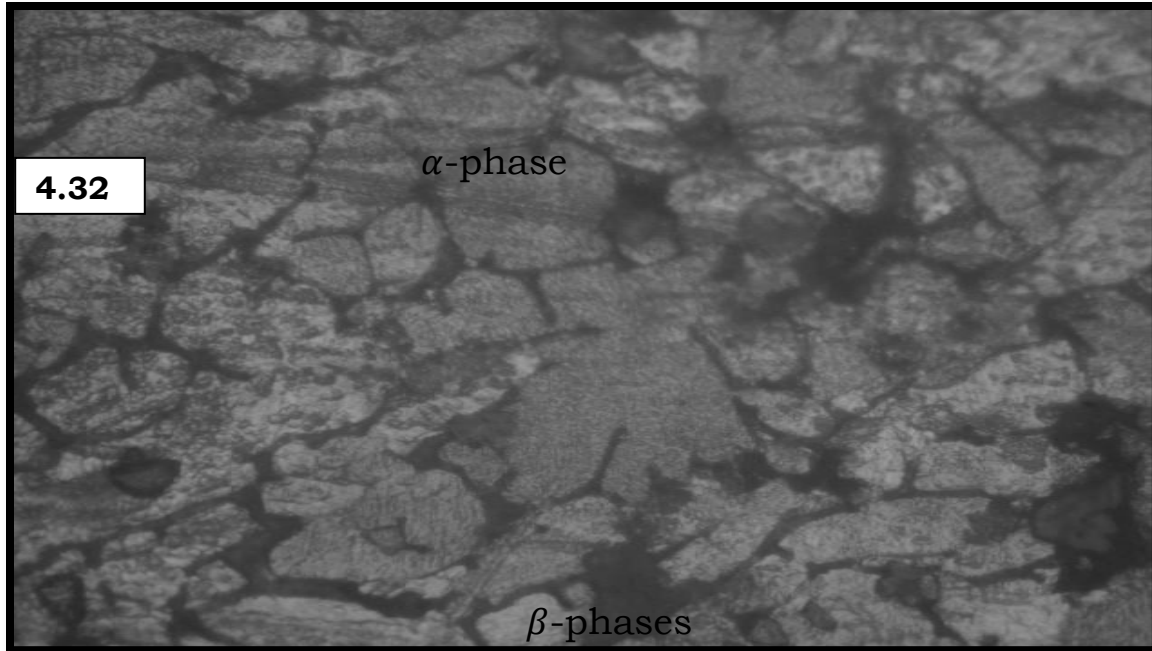


Plate 4.32: Micrograph of Cu-10%Al +0.5wt%V.(x400)

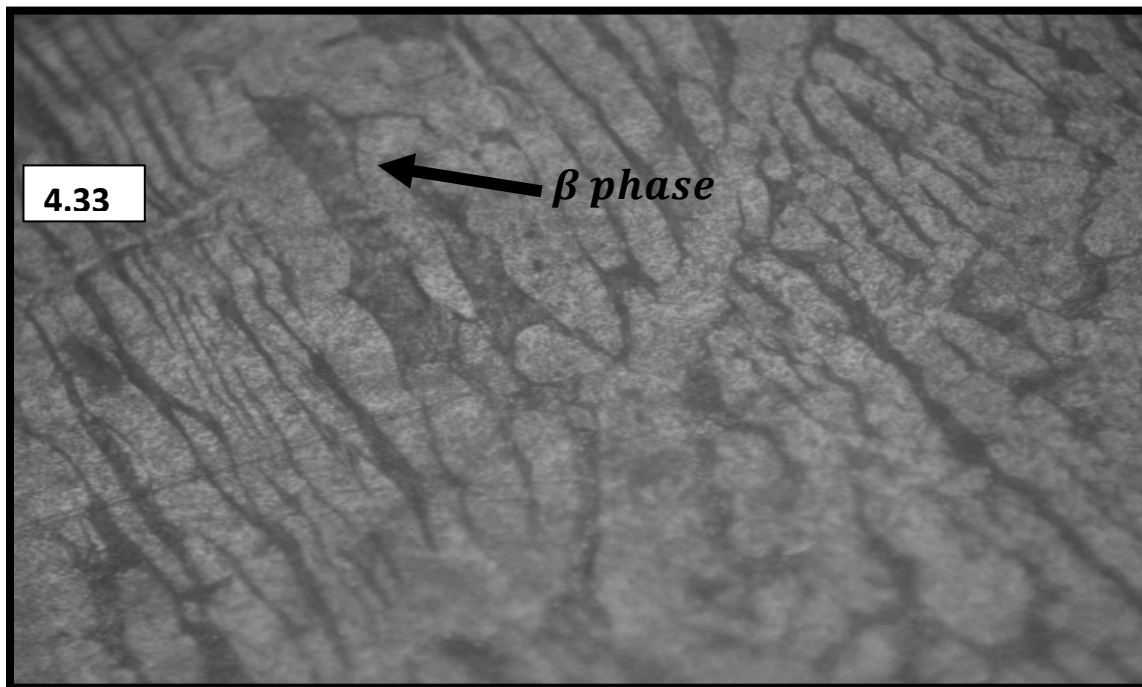
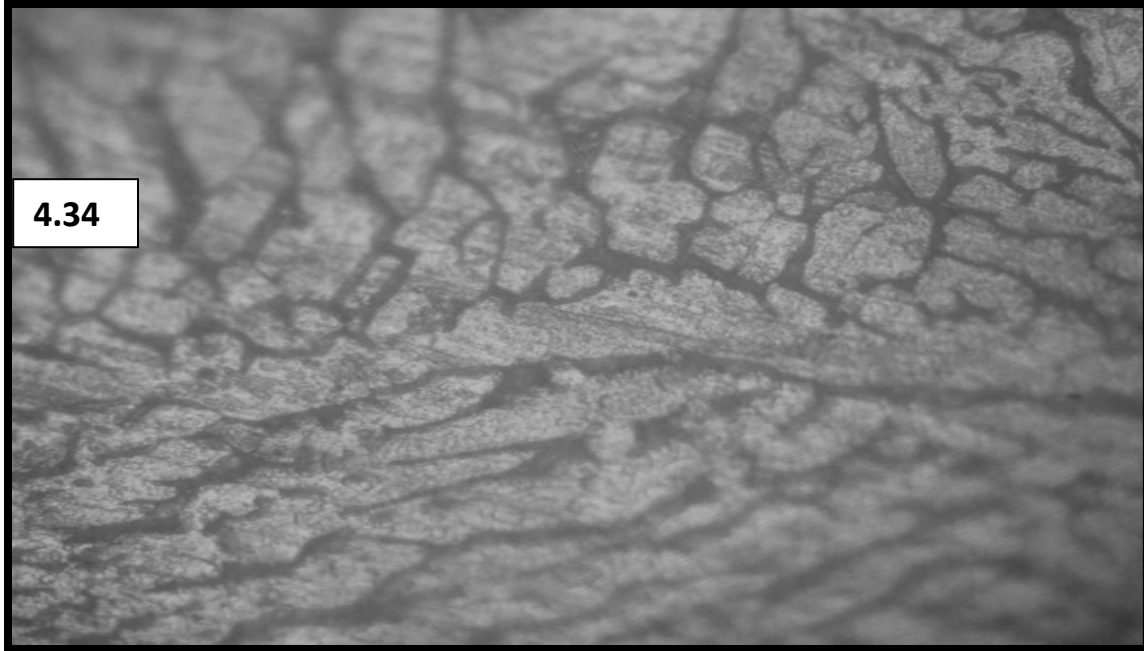
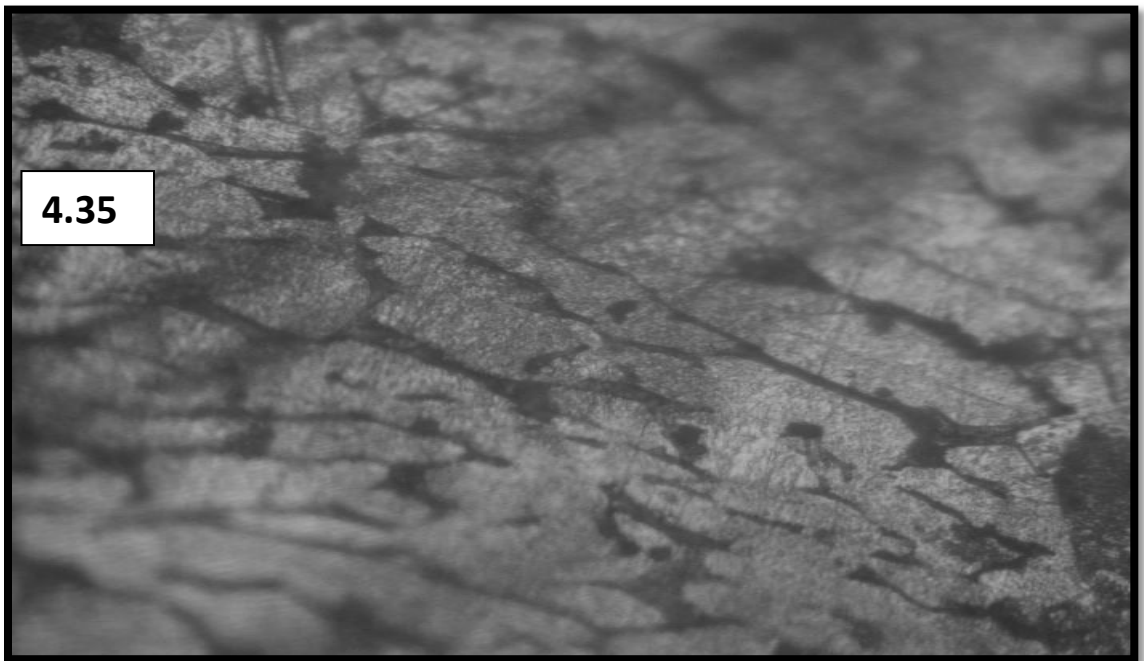


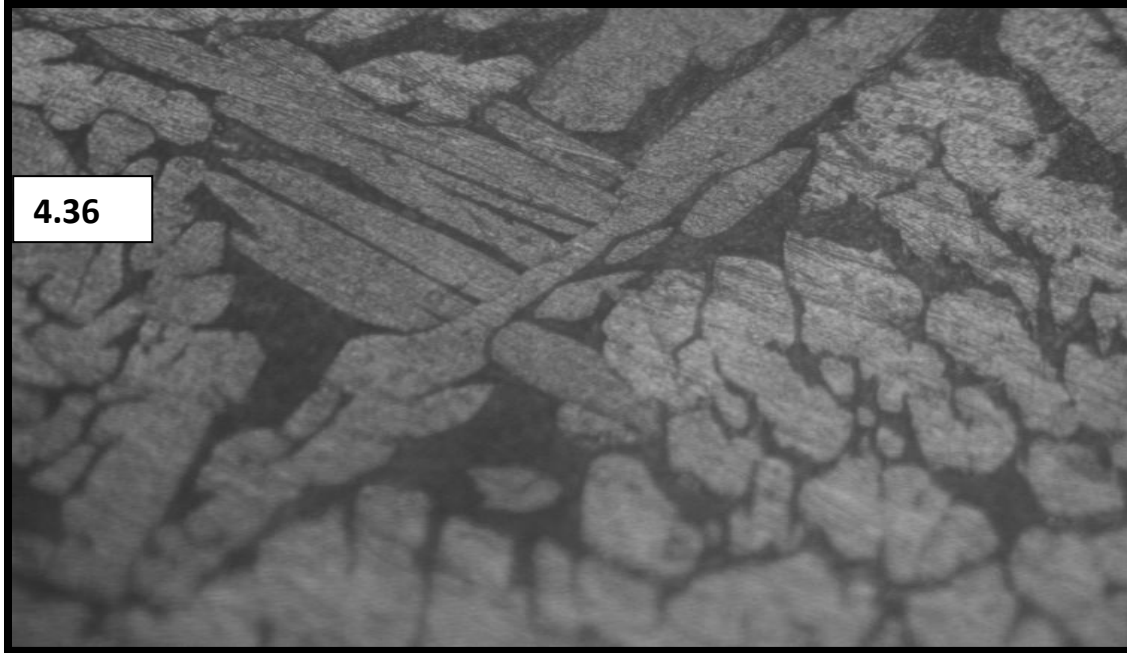
Plate 4.33: Micrograph of Cu-10%Al +1.0wt%V (x400)



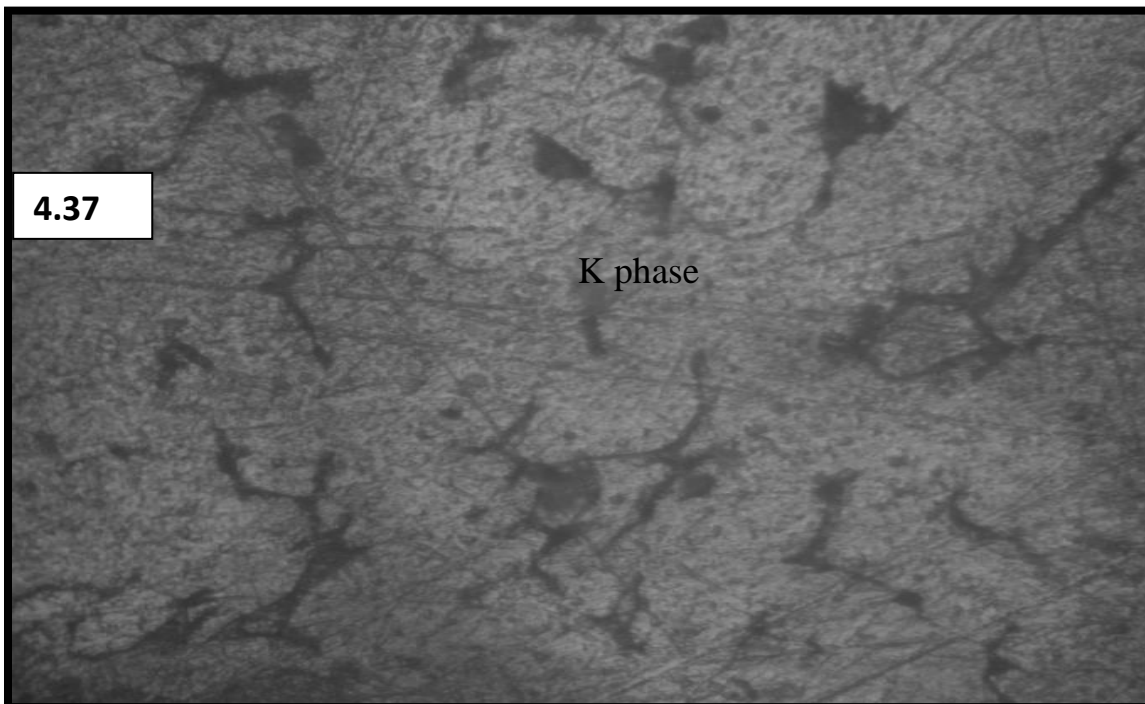
**Plate 4.34: Micrograph of Cu-10%Al +1.5wt%V (x400)**



**Plate 4.35: Micrograph of Cu-10%Al +2.0wt%V (x400)**



**Plate 4.36: Micrograph of Cu-10%Al +2.5wt%V (x400)**



**Plate 4.37: Micrograph of Cu-10%Al +3.0wt%V (x400)**

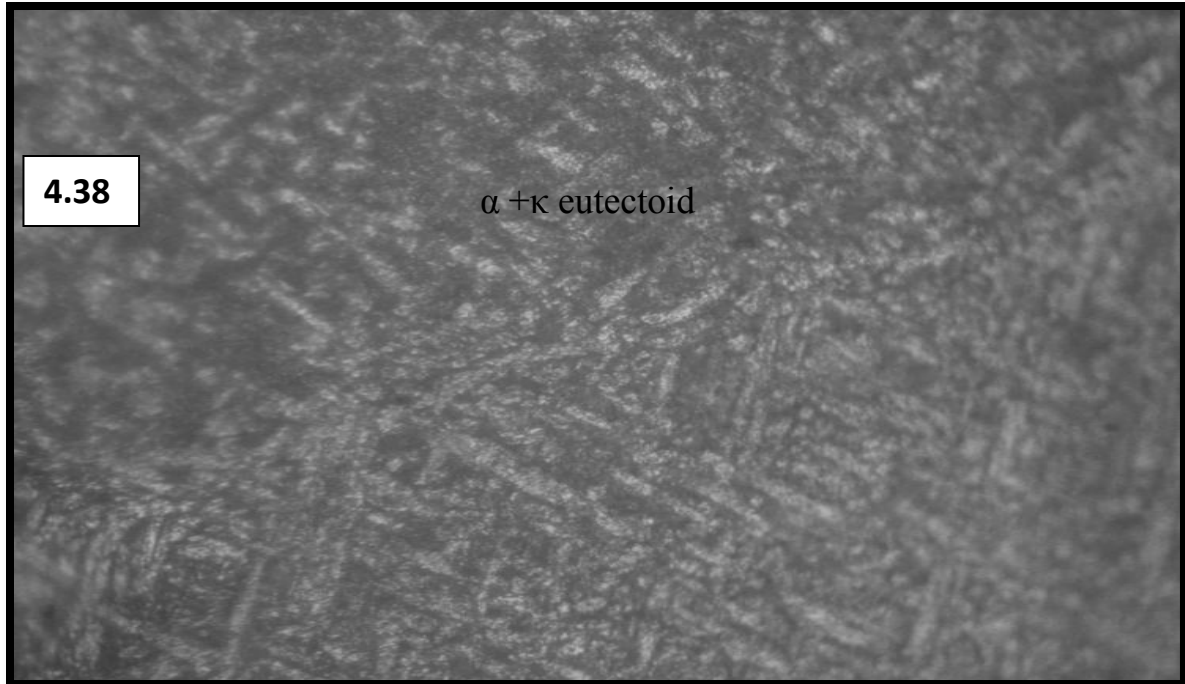


Plate 4.38: Micrograph of Cu-10%Al +3.5wt%V (x400)

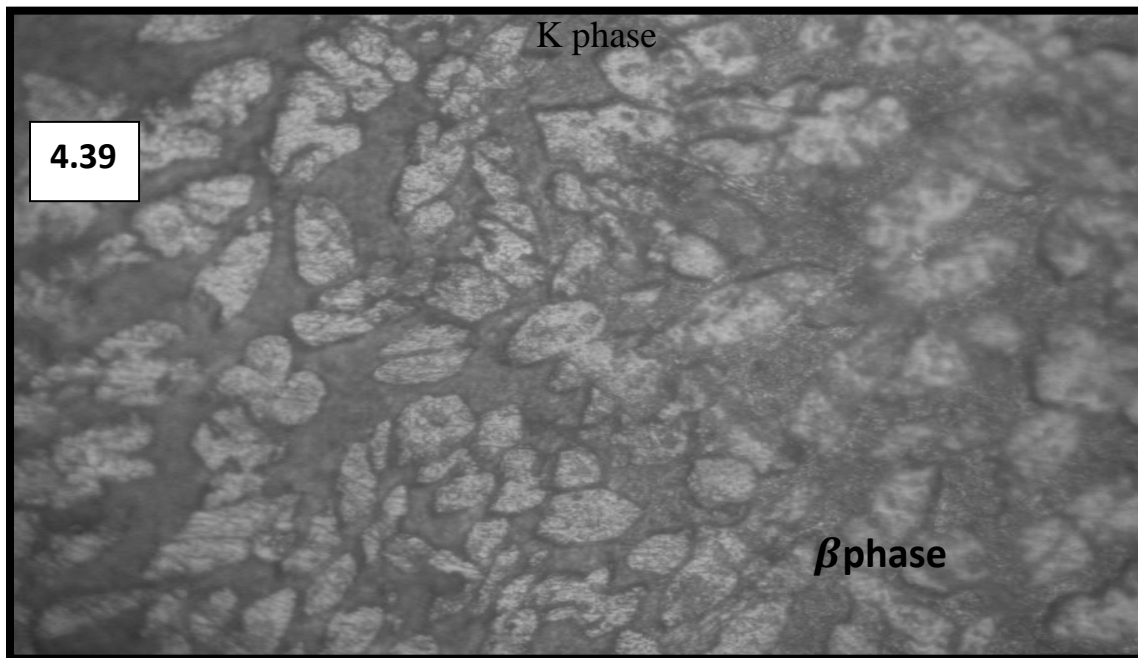


Plate 4.39: Micrograph of Cu-10%Al +4.0wt%V (x400)

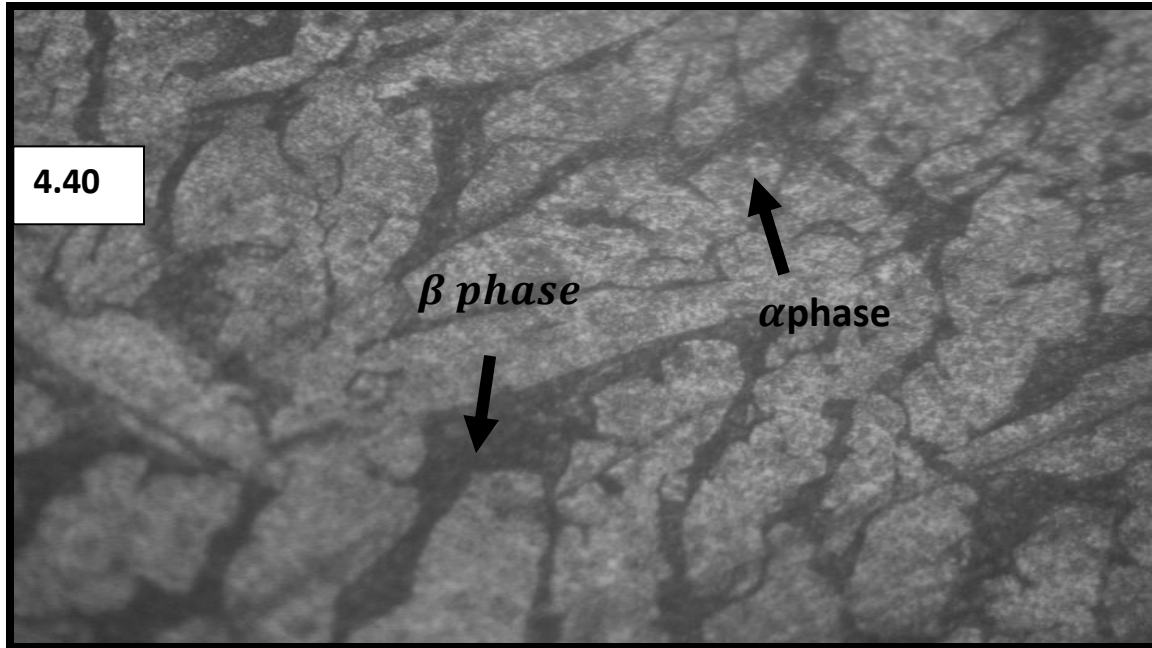


Plate 4.40: Micrograph of Cu-10%Al+4.5wt%V. (x400)

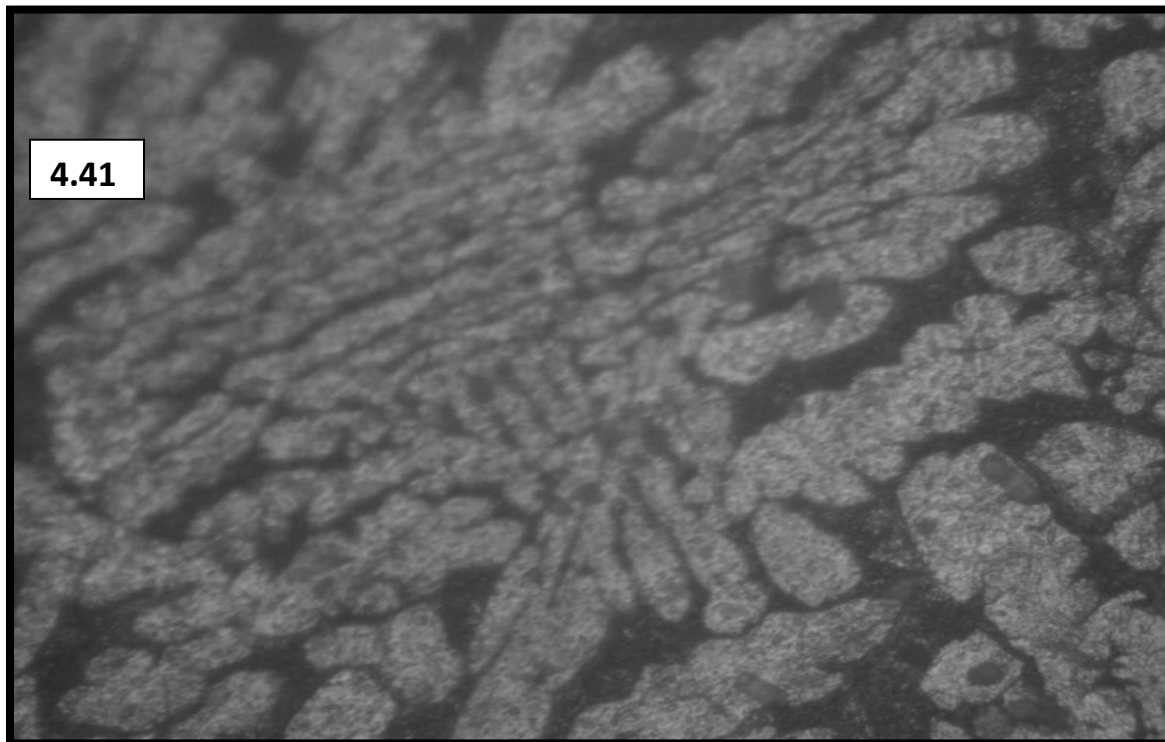
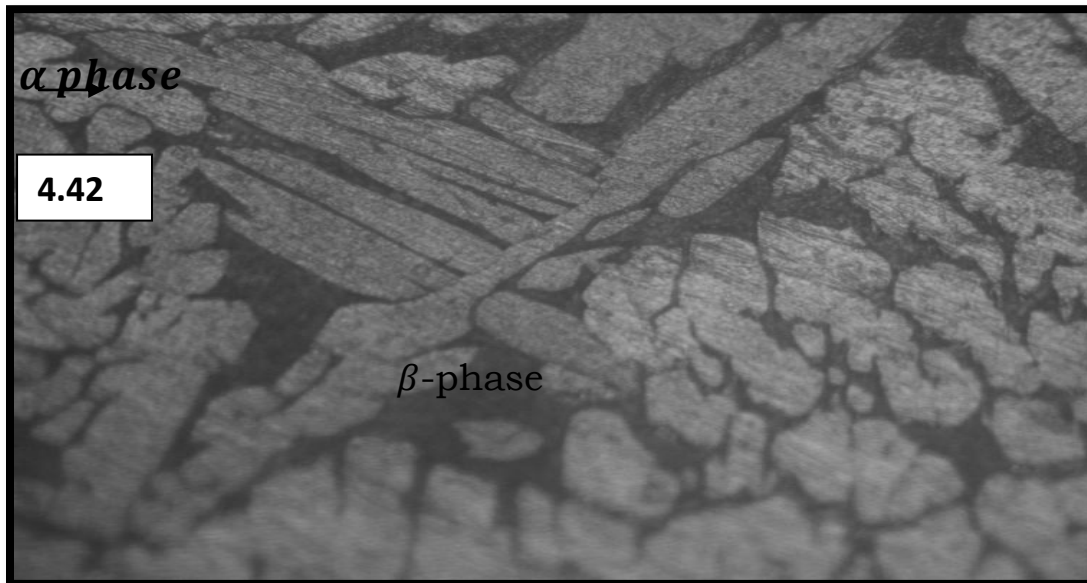


Plate 4.41: Micrograph of Cu-10%Al+5.0wt%V. (x400)



Plate 4.32 to Plate 4.41 represent the micrographs of Cu-10%Al alloy treated with (0.5 to 5.0) wt% vanadium. The micrographs show that as vanadium increased the quantity of  $\alpha$ -phase increase in copper matrix. The presence of more vanadium in the alloy matrix provided increased in nucleation sites for the transformation of  $\beta$ -phase. Vanadium reduced kinetics of kappa phase precipitates due to small grains of  $\alpha$ -phase, this smaller grains of  $\alpha$ -phase lead to formation of  $\alpha + \kappa$  eutectoid which brought about improvement in alloy properties.



**Plate 4.42: Micrograph of Cu-10%Al +0.5wt%W(x400)**



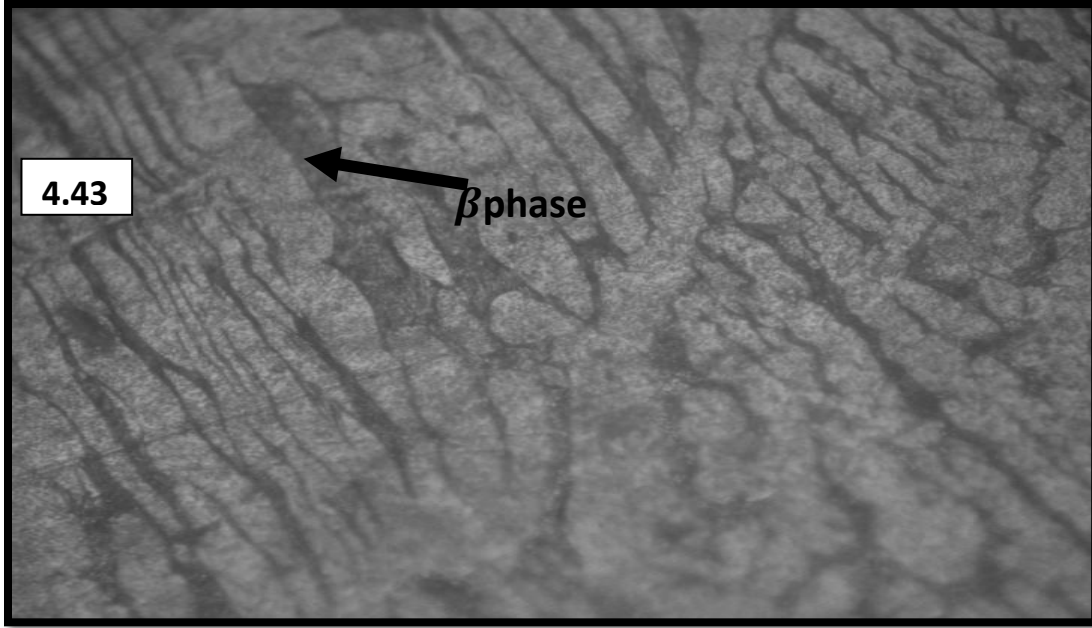


Plate 4.43: Micrograph of Cu-10%Al + 3.0wt%W(x400)

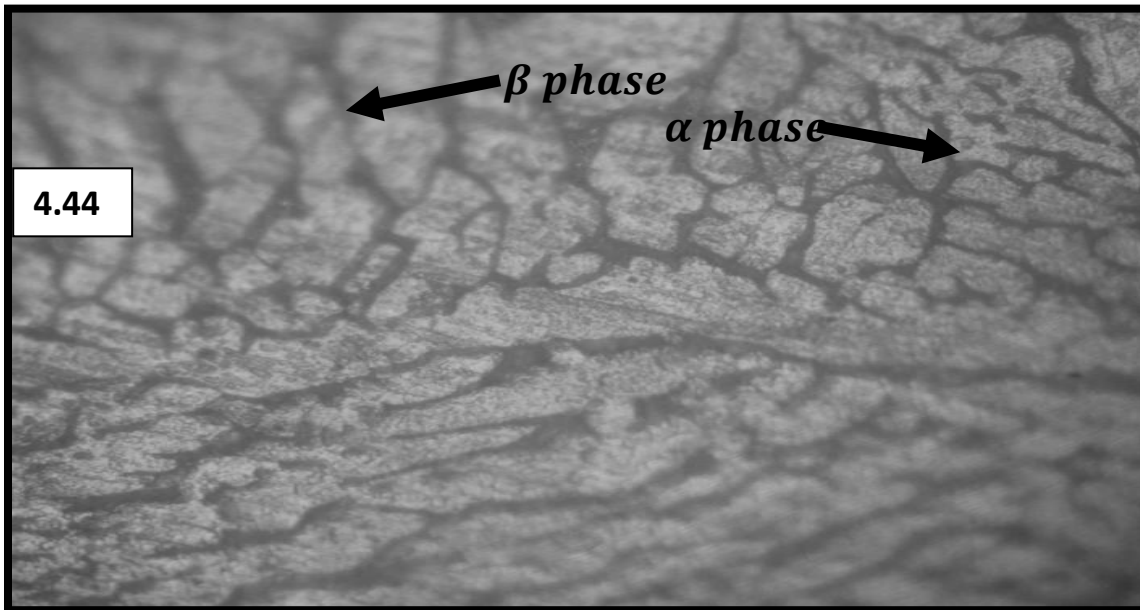


Plate 4.44: Micrograph of Cu-10%Al + 4.5wt%W(x400)

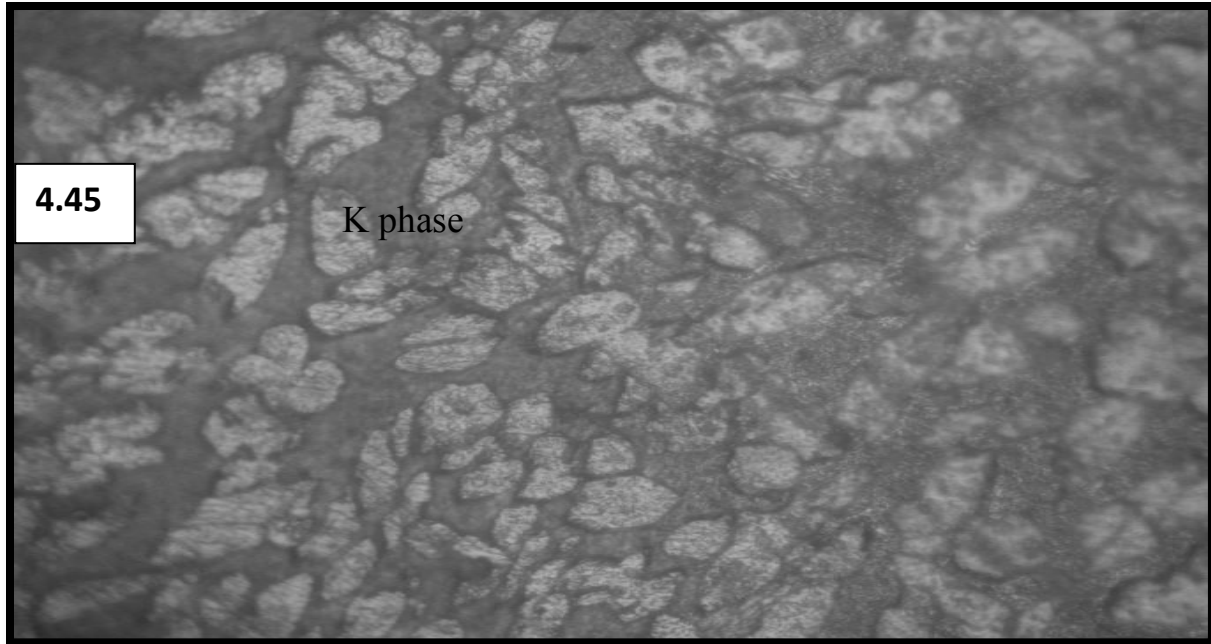


Plate 4.45: Micrograph of Cu-10%Al+5.0wt%W (x400)

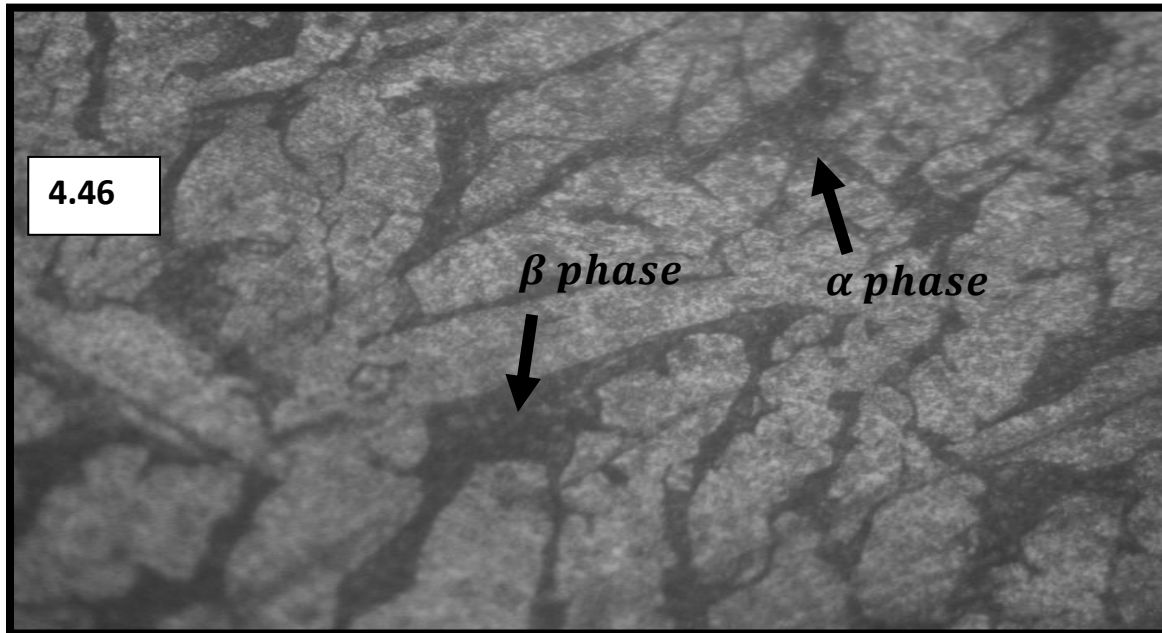


Plate 4.46: Micrograph of Cu-10%Al+6.0wt%W.(x400)

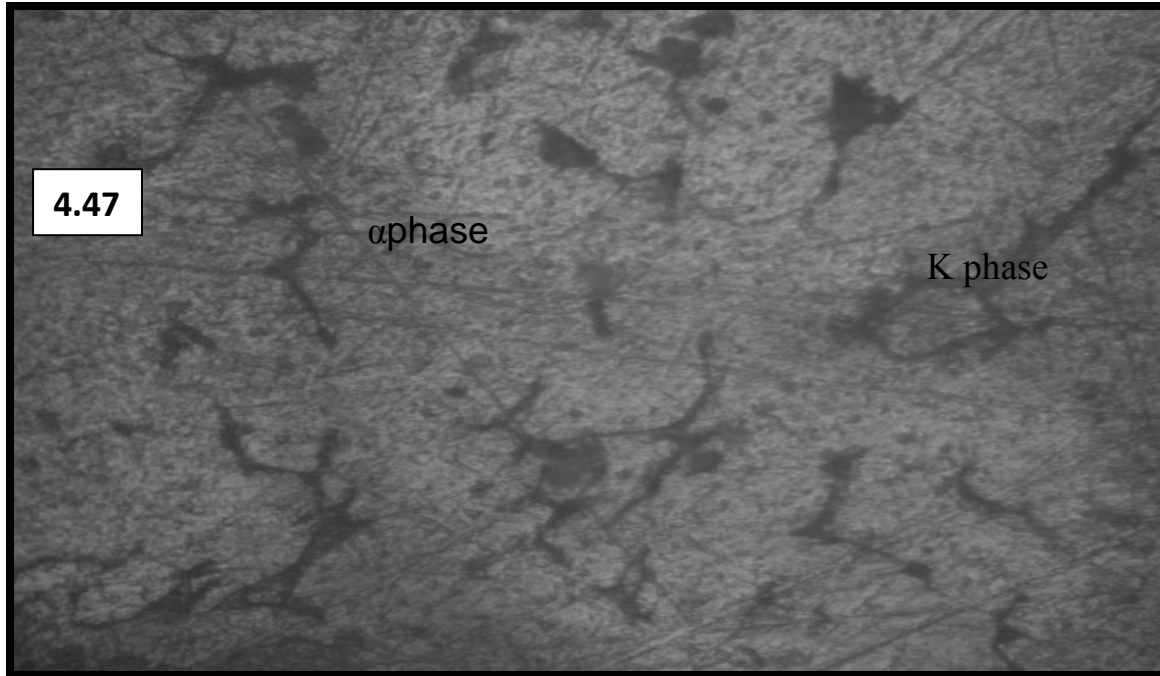


Plate 4.47: Micrograph of Cu-10%Al +6.5wt%W(x400)

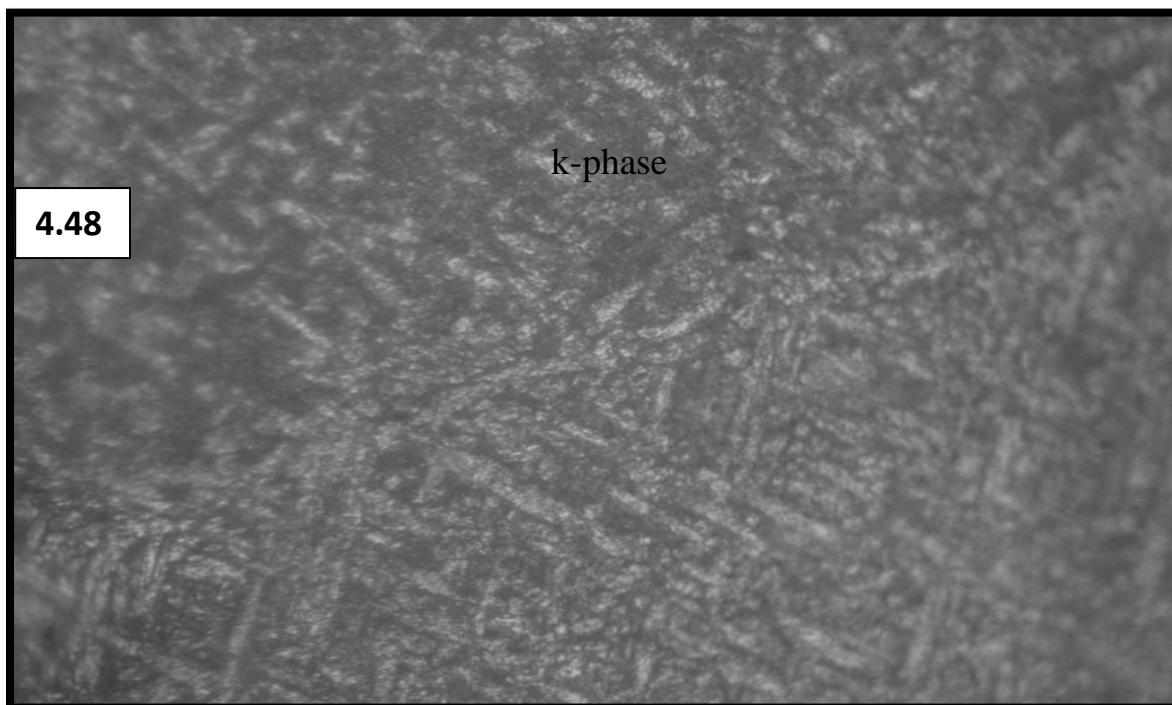
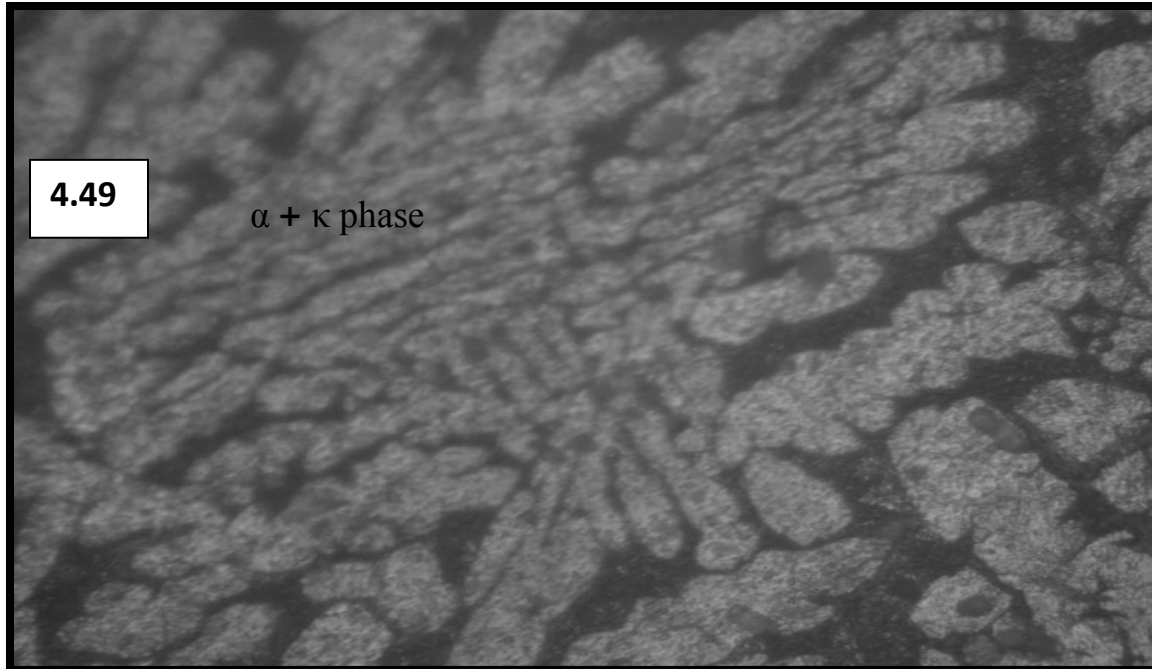
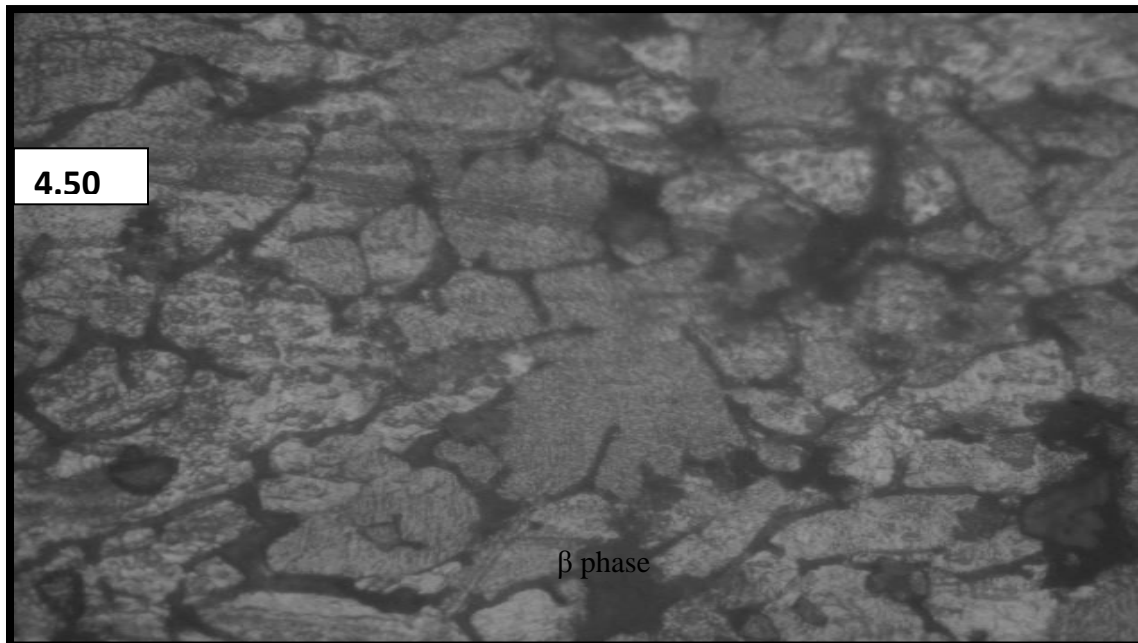


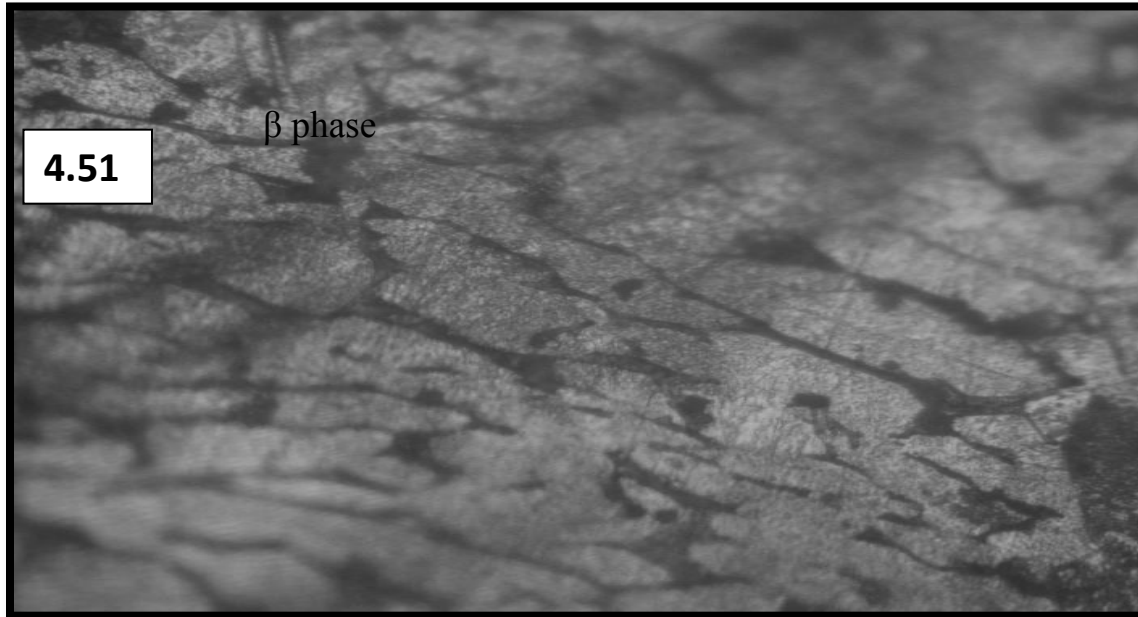
Plate 4.48: Micrograph of Cu-10%Al +7.0wt%W(x400)



**Plate 4.49: Micrograph of Cu-10%Al+8.0wt%W. (x400)**



**Plate 4.50: Micrograph of Cu-10%Al +9.5wt%W. (x400)**



**Plate 4.51: Micrograph of Cu-10%Al +4.0wt%W (x400)**

Plate 4.42 to Plate 4.51 show the microstructure of Cu-10%Al alloy treated with (0.5 to 10) wt.% tungsten. The overall grain size was reduced considerably with increase in tungsten content. Addition of tungsten to the alloy refines the grain structure and stabilizes the  $\beta$ -phases. It also suppressed the formation of  $\gamma_2$  phase. This could be attributed to the presence of sparse distribution of suspected  $\alpha$ -phase precipitates in a predominant  $\beta$  matrix which brought improvement in the properties.

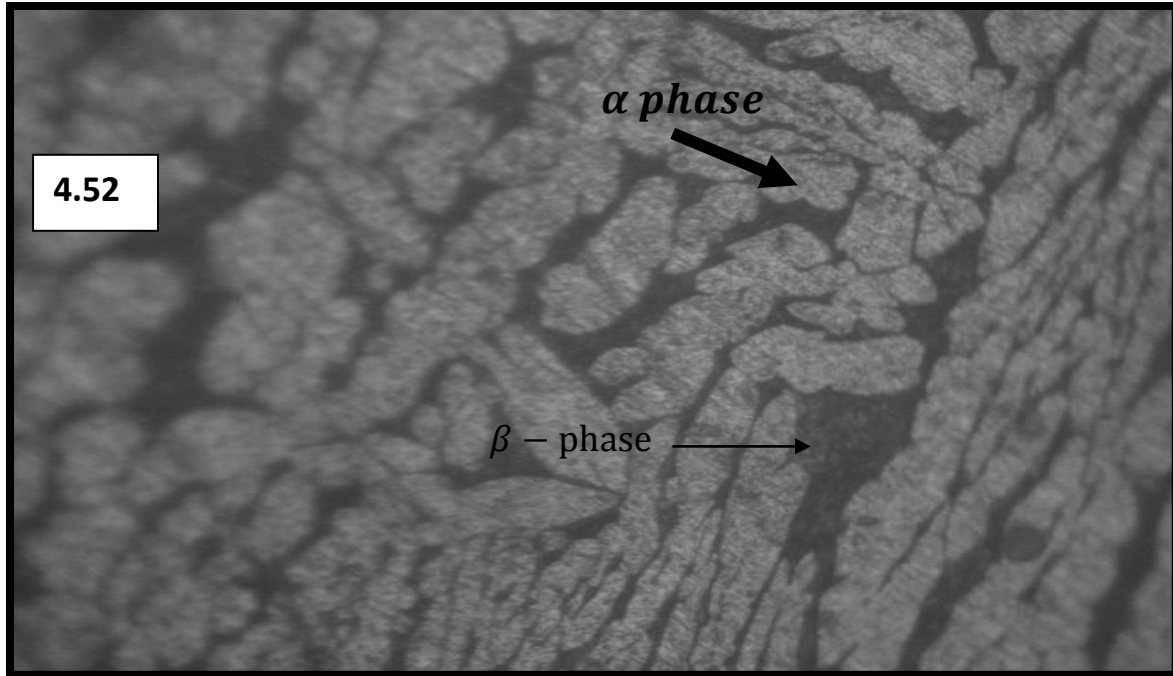


Plate 4.52: Micrograph of Cu-10%Al+0.5wt%Cr.

(x400)

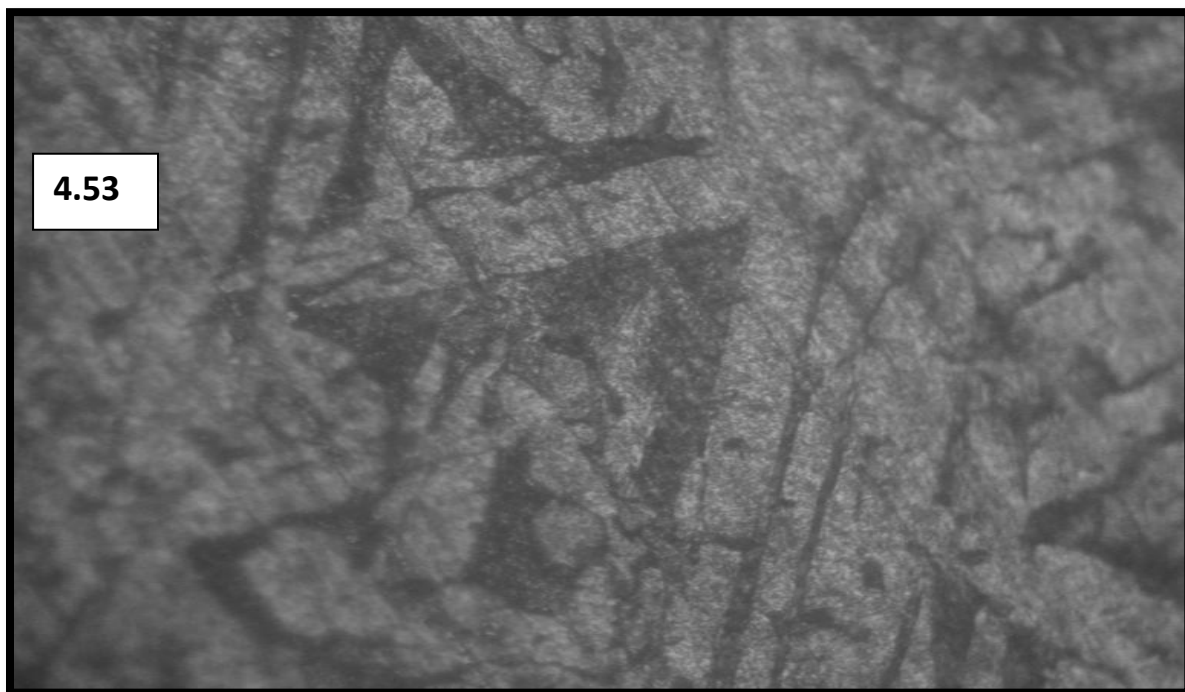


Plate 4.53: Micrograph of Cu-10%Al+1.0wt%Cr.

(x400)

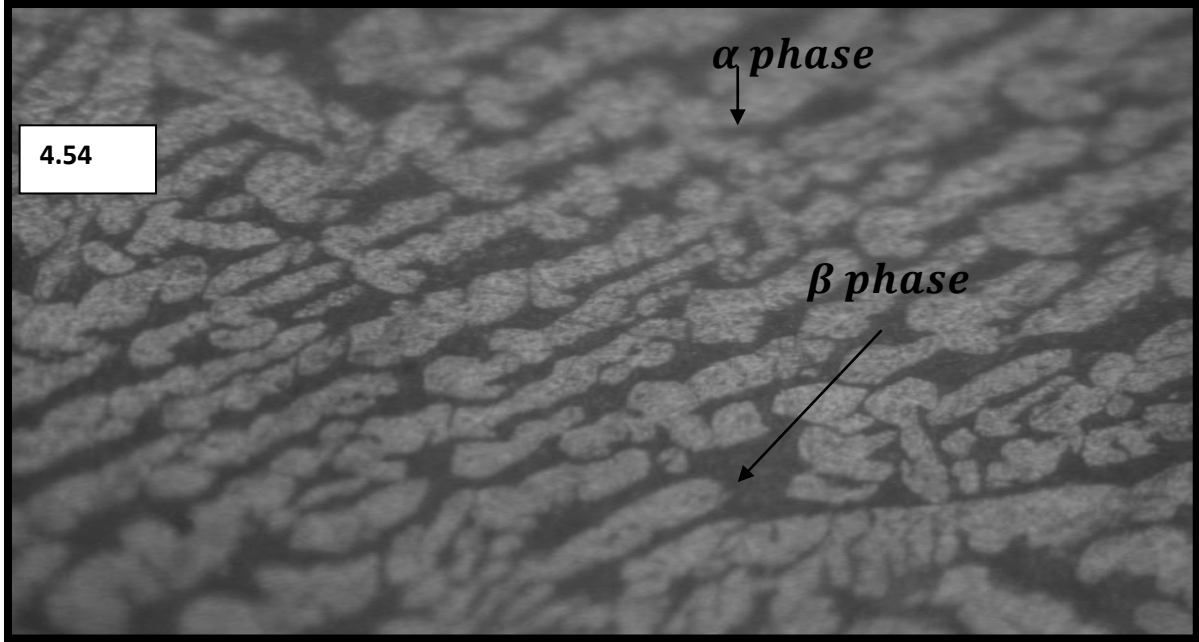


Plate 4.54: Micrograph of Cu-10%Al+1.5wt%Cr. (x400)

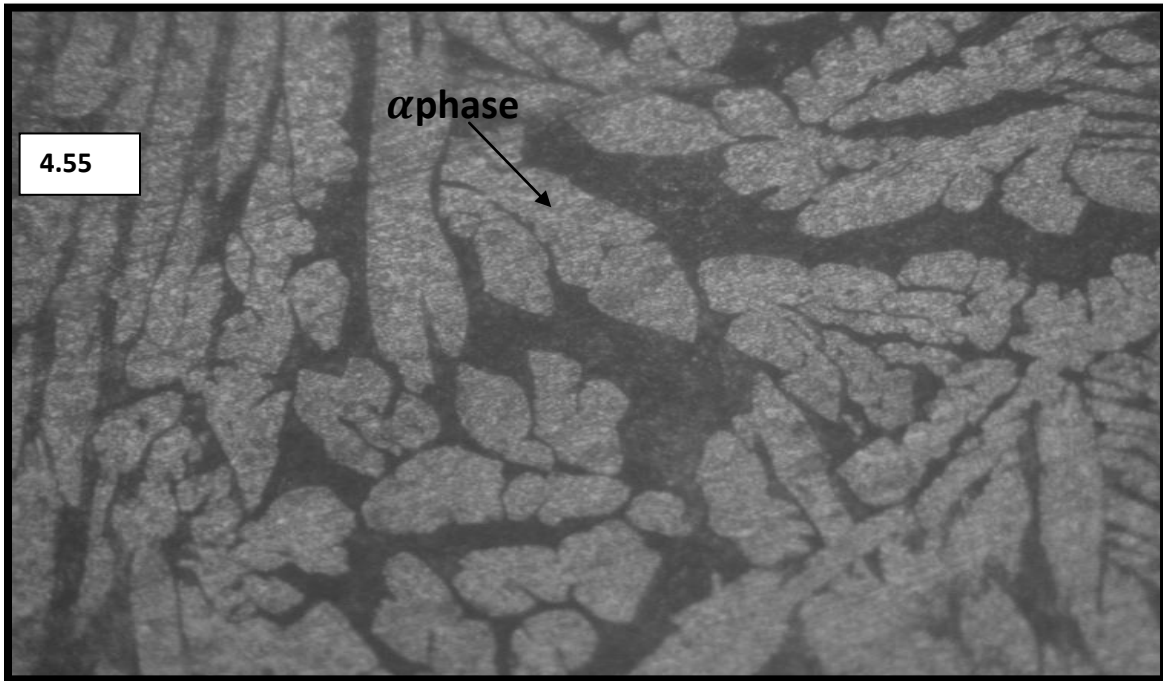


Plate 4.55: Micrograph of Cu-10%Al+2.0wt%Cr. (x400)



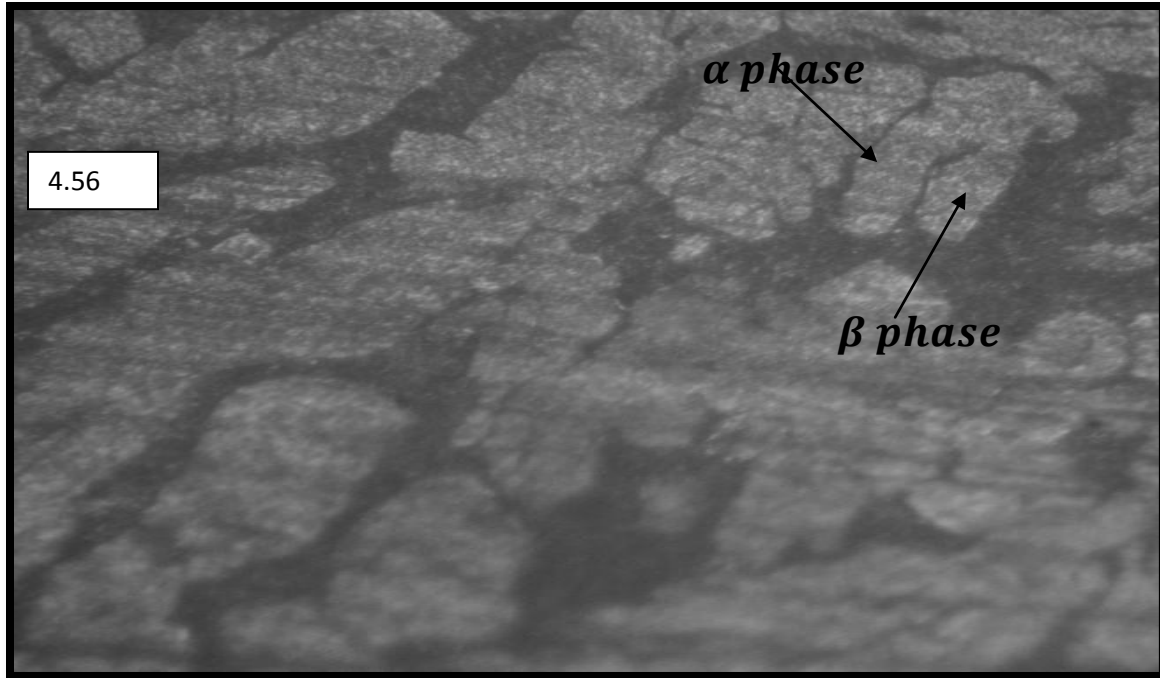


Plate 4.56: Micrograph of Cu-10%Al+2.5wt%Cr. (x400)

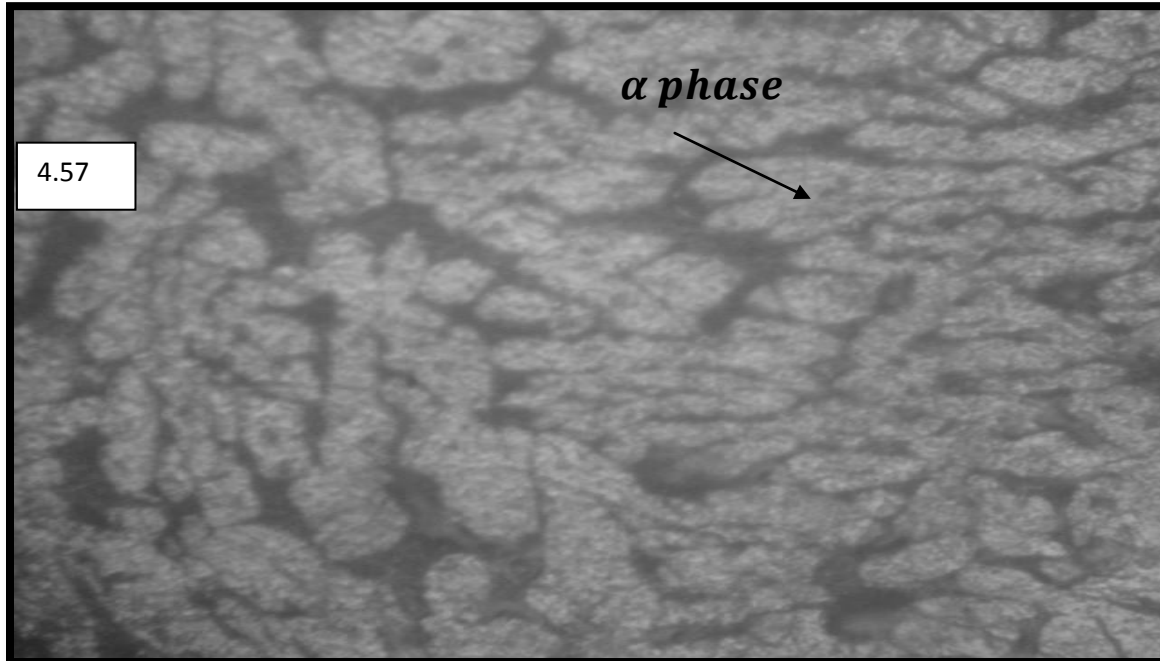


Plate 4.57: Micrograph of Cu-10%Al+3.0wt%Cr. (x400)



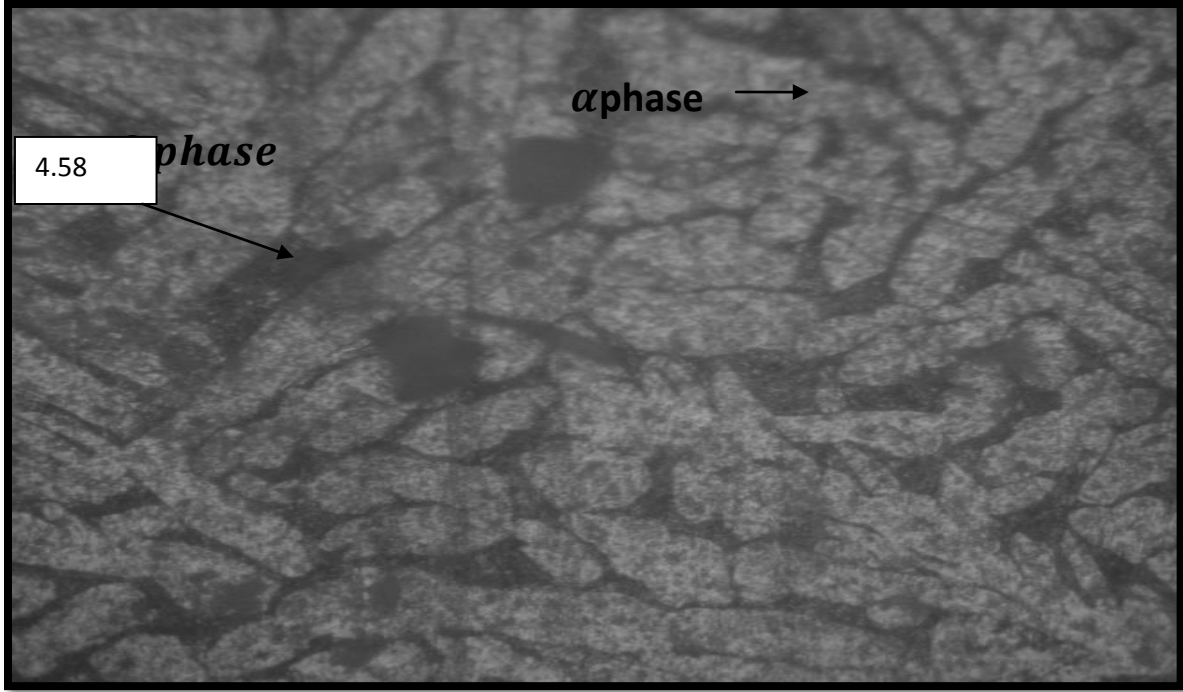


Plate 4.58: Micrograph of Cu-10%Al+3.5wt%Cr. (x400)

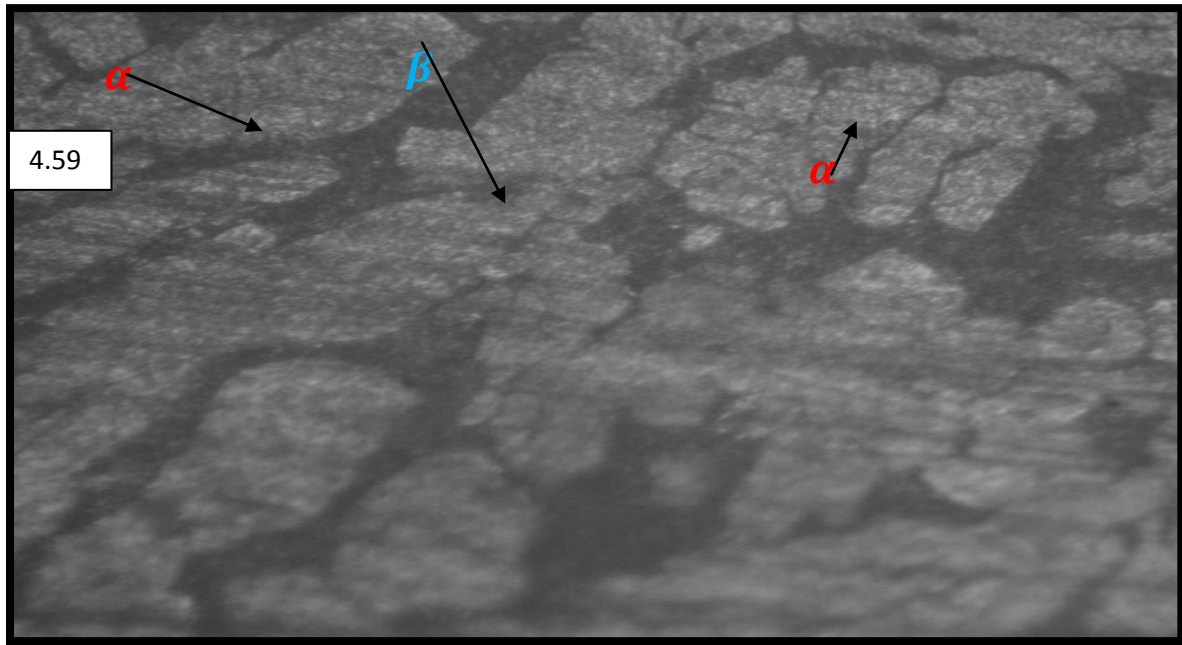
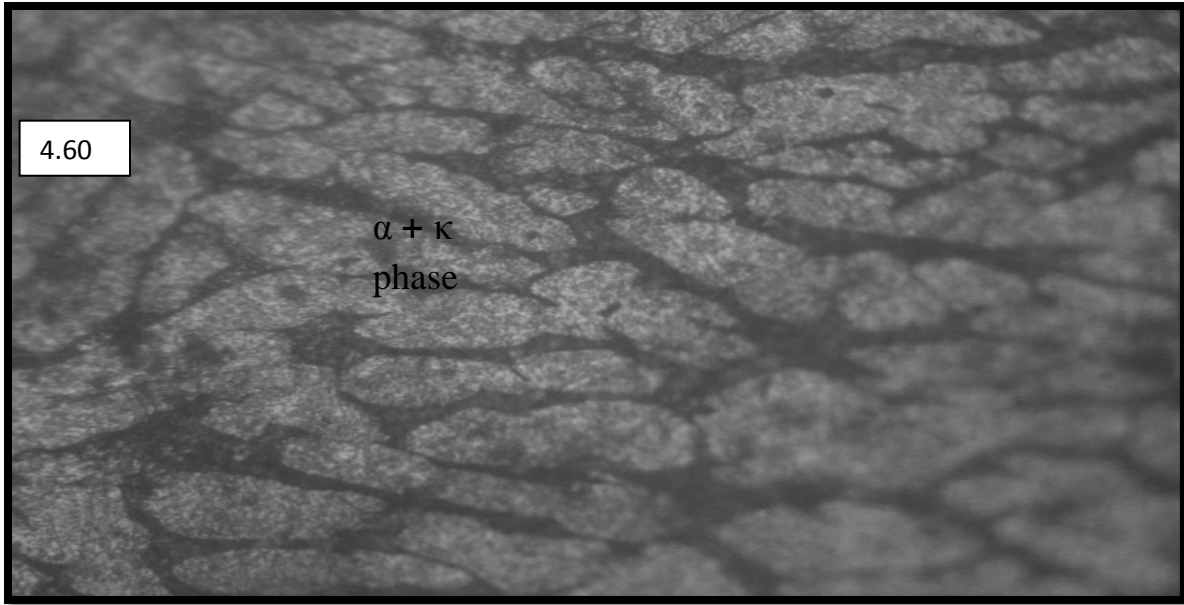
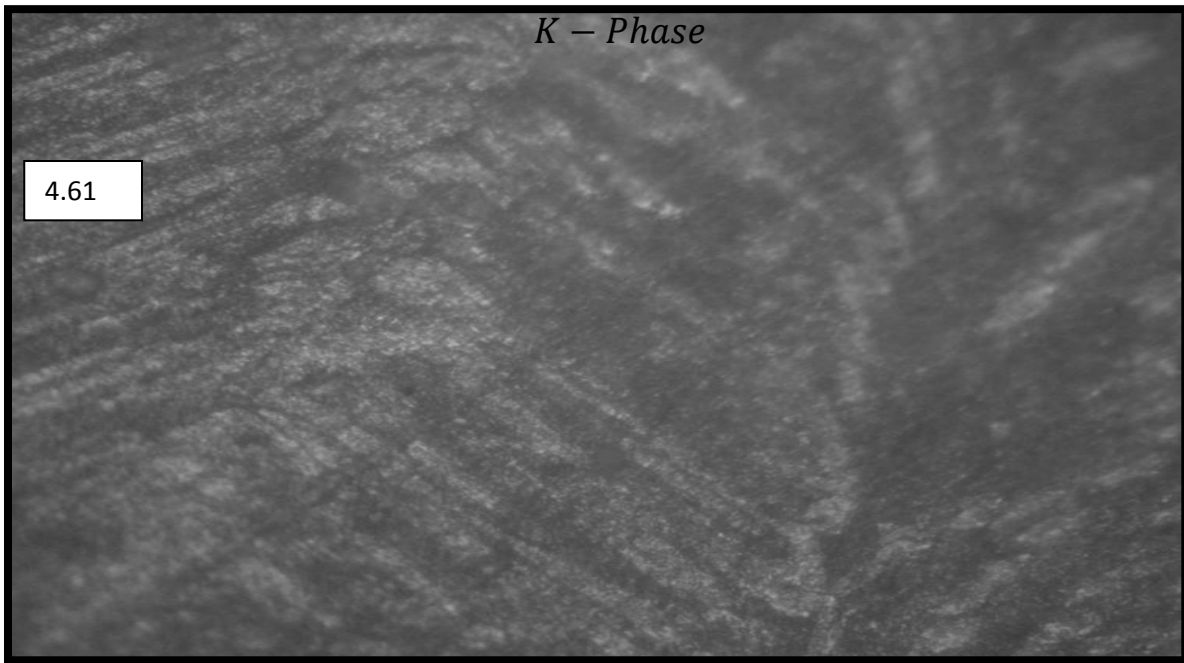


Plate 4.59: Micrograph of Cu-10%Al+5.0wt%Cr. (x400)



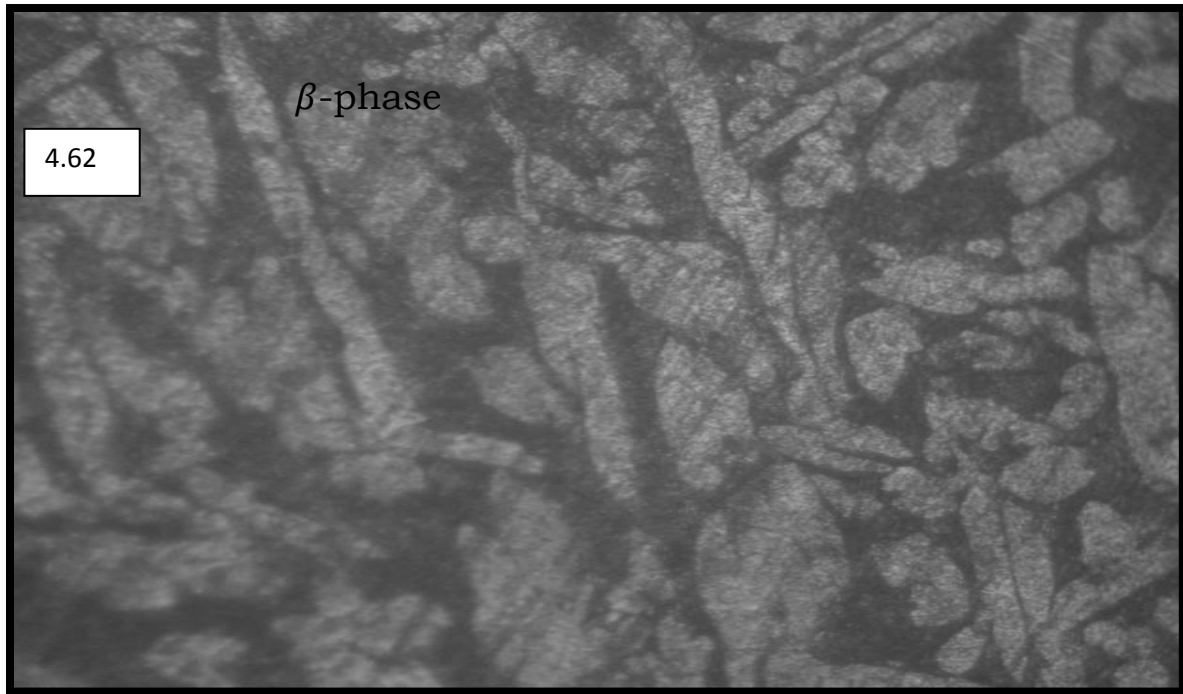
**Plate 4.60: Micrograph of Cu-10%Al+7.5wt%Cr. (x400)**



**Plate 4.61: Micrograph of Cu-10%Al+7.0wt%Cr. (x400)**

Plate 4.52 to Plate 4.61 show the microstructure of Cu-10%Al alloy treated with (0.5 to 10) wt% chromium. The micrographs show more dispersed precipitates of

$\alpha$ -phase in a more refined  $\beta$ - matrix with finer grain structure. The pearlite structure has been altered, with the lamellar structure transforming to give  $k$ -phases at the grain boundaries and with no undesirable  $\gamma_2$  phase, which has deleterious effect on mechanical properties of aluminum bronze.



**Plate 4.62: Micrograph of Cu-10%Al+0.5wt%Mo. (x400)**

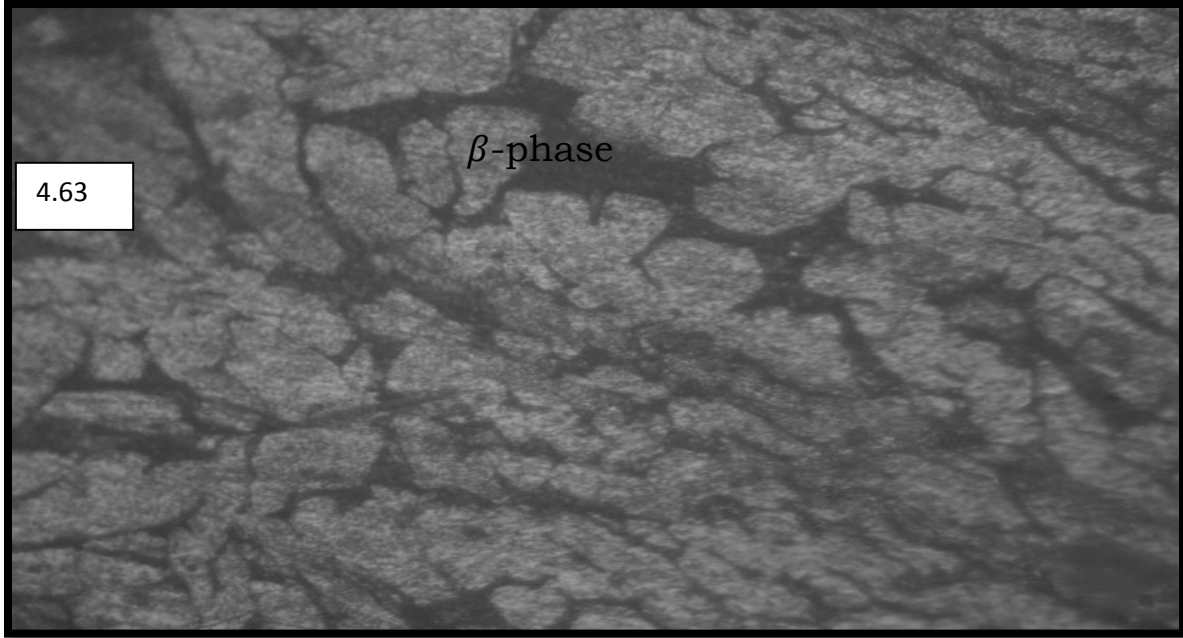


Plate 4.63: Micrograph of Cu-10%Al+1.0wt%Mo.

(x400)

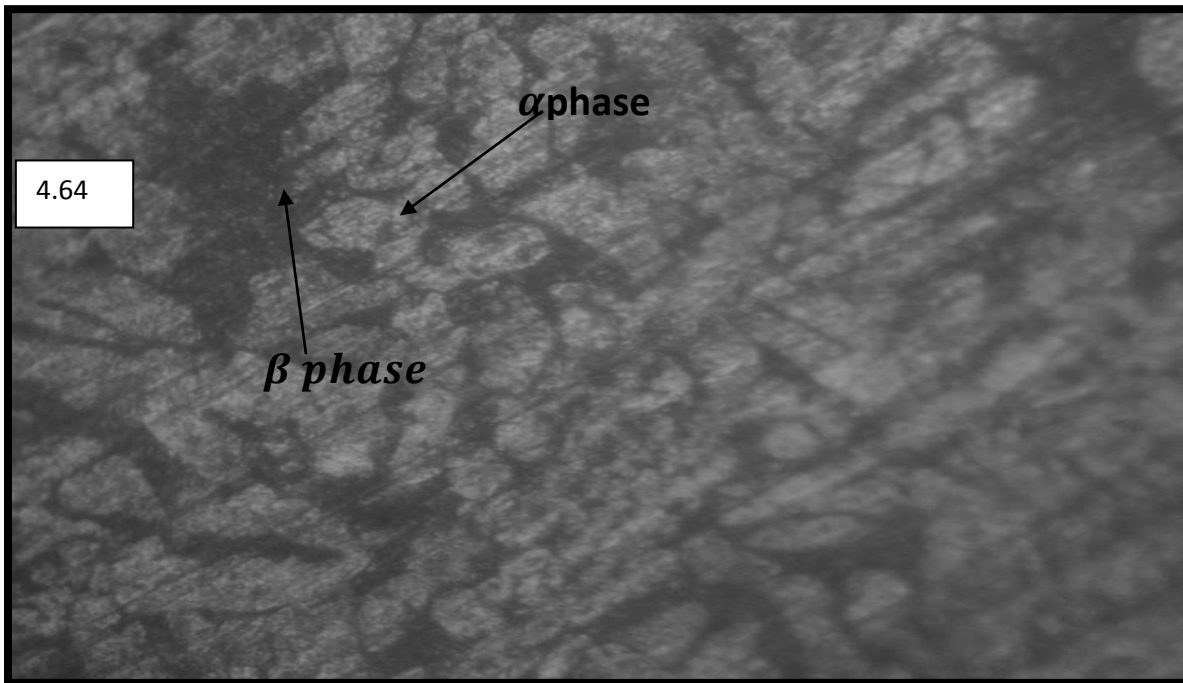


Plate 4.64: Micrograph of Cu-10%Al+1.5wt%Mo.

(x400)

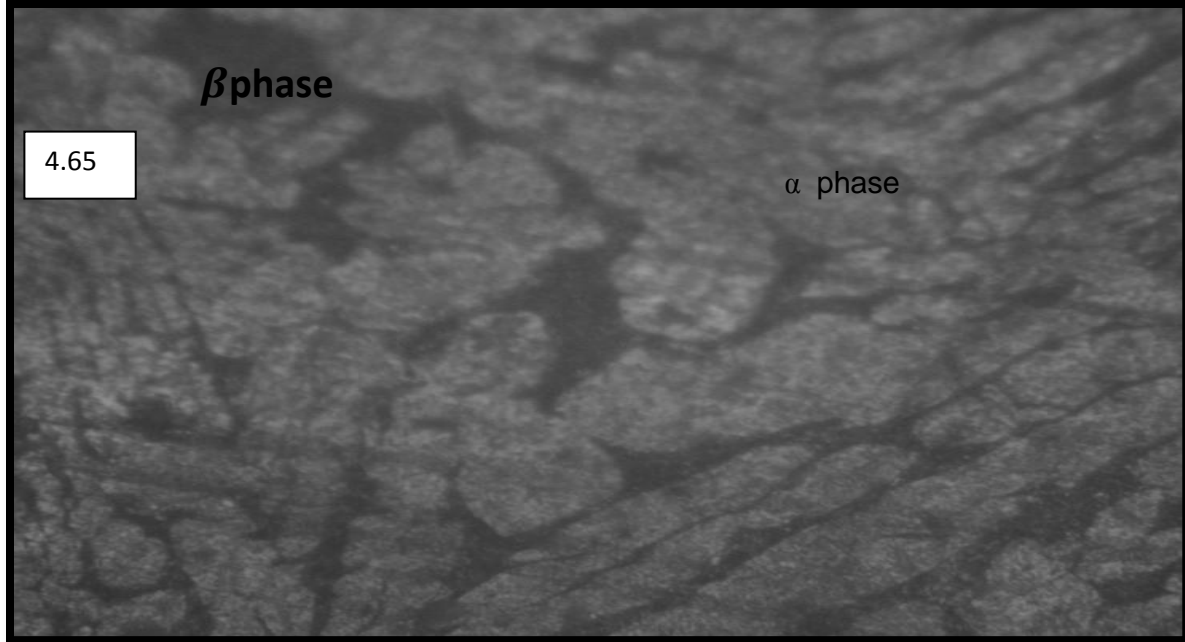


Plate 4.65: Micrograph of Cu-10%Al+2.0wt%Mo. (x400)

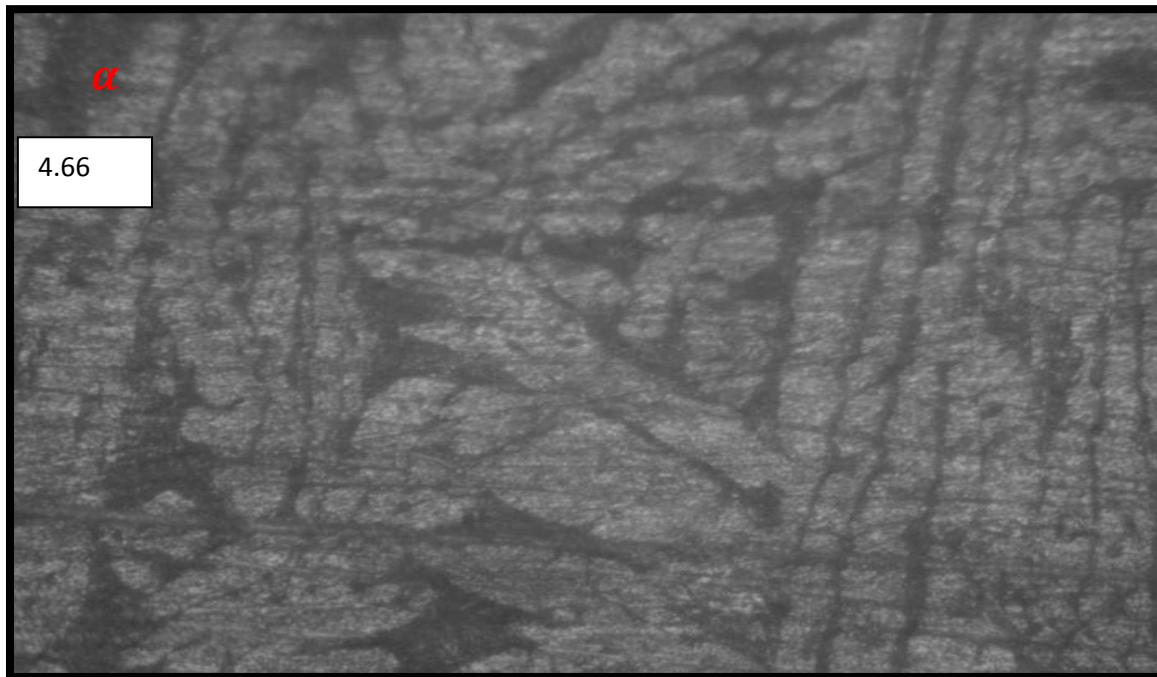
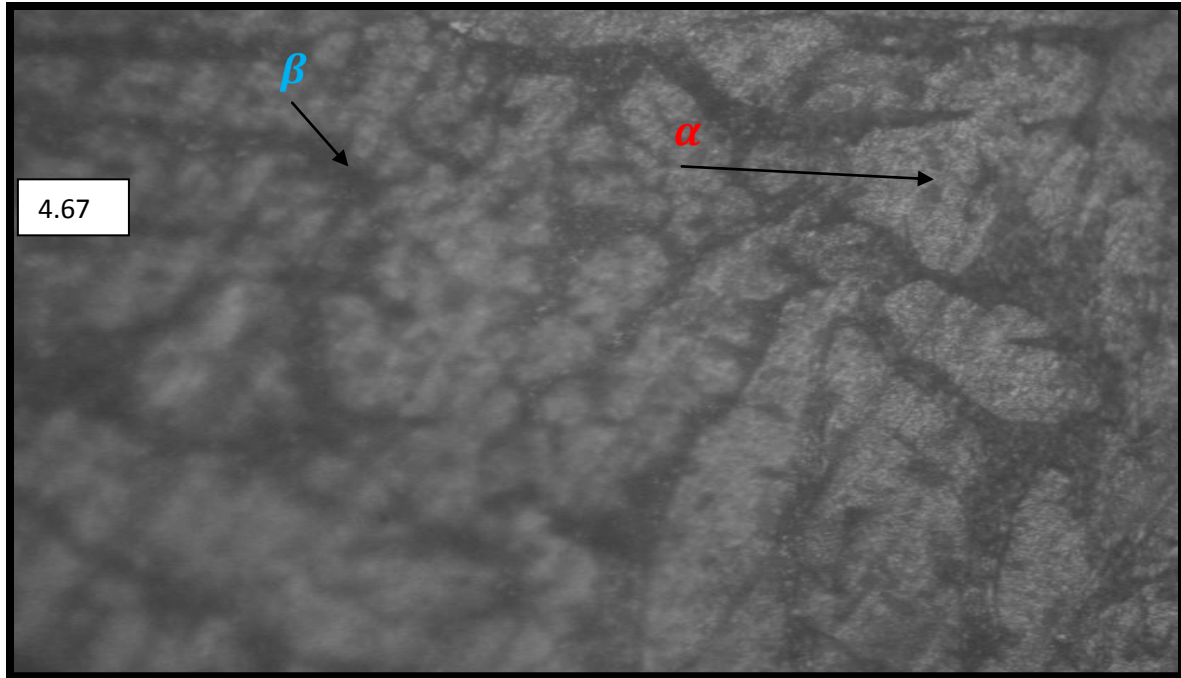
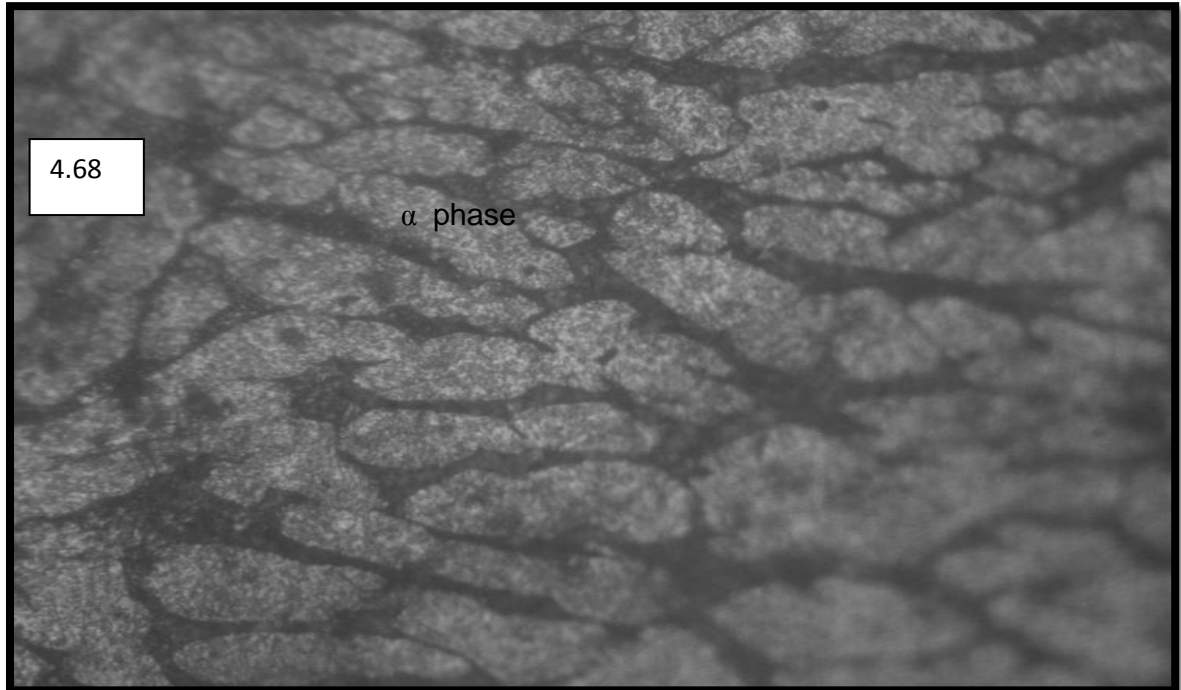


Plate 4.66: Micrograph of Cu-10%Al+2.5wt%Mo. (x400)





**Plate 4.67: Micrograph of Cu-10%Al+3.0wt%Mo. (x400)**



**Plate 4.68: Micrograph of Cu-10%Al+3.5%Mo. (x400)**

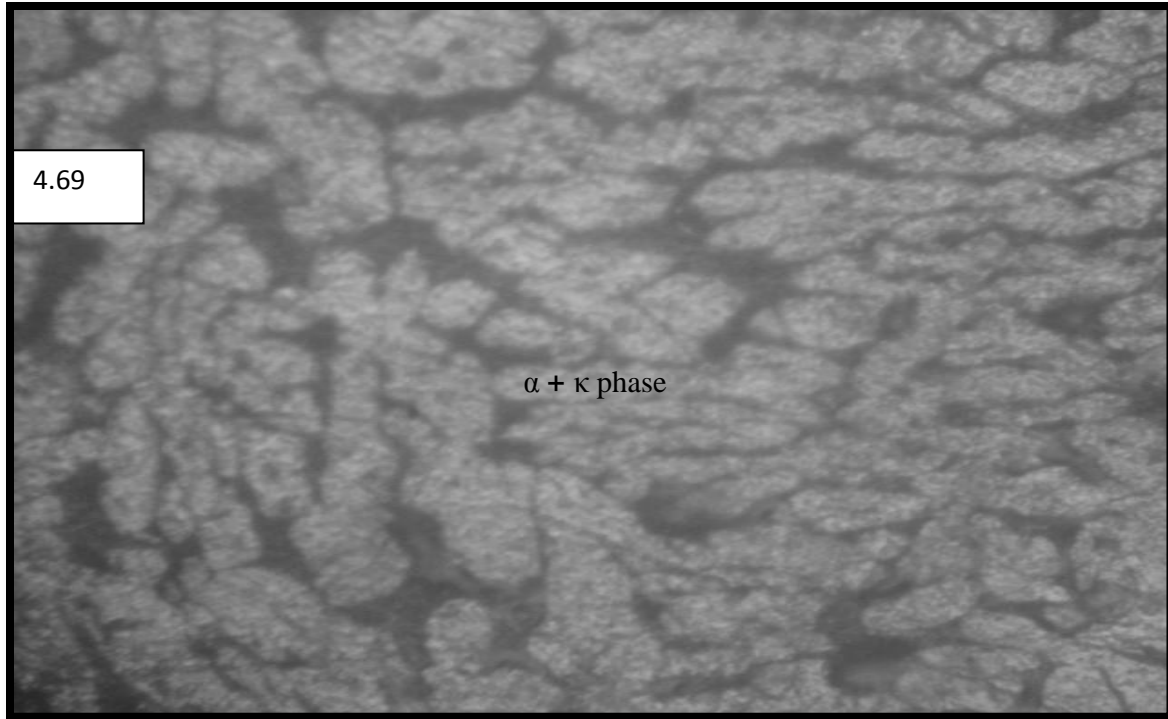


Plate 4.69: Micrograph of Cu-10%Al+5.5wt%Mo (x400)

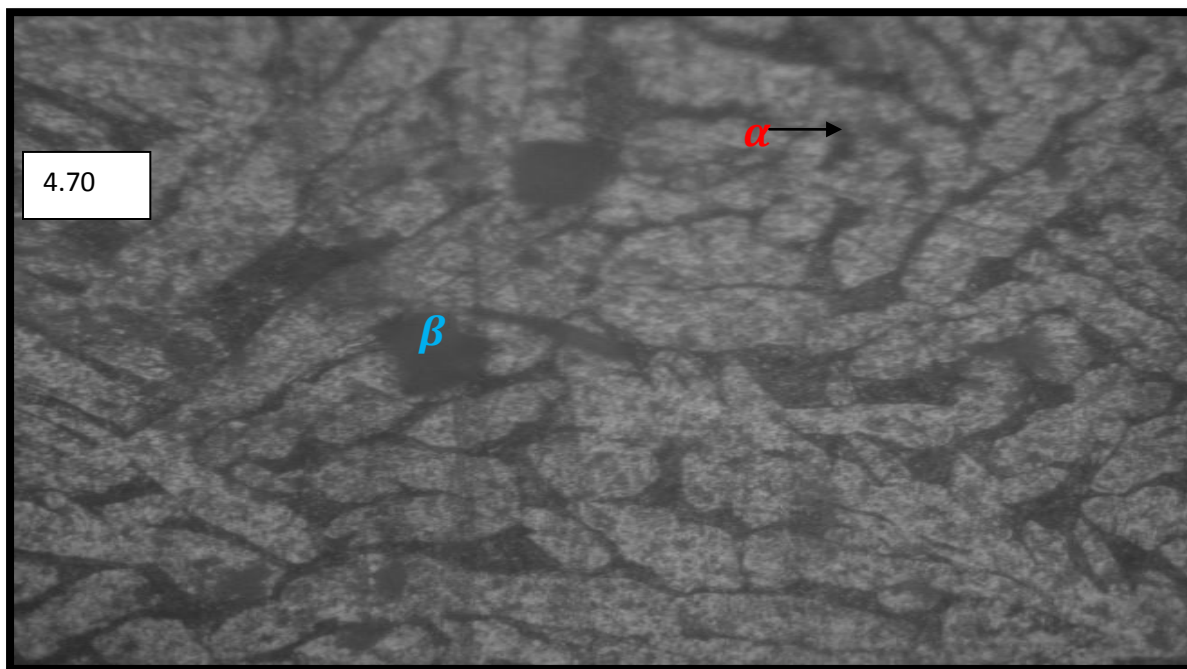
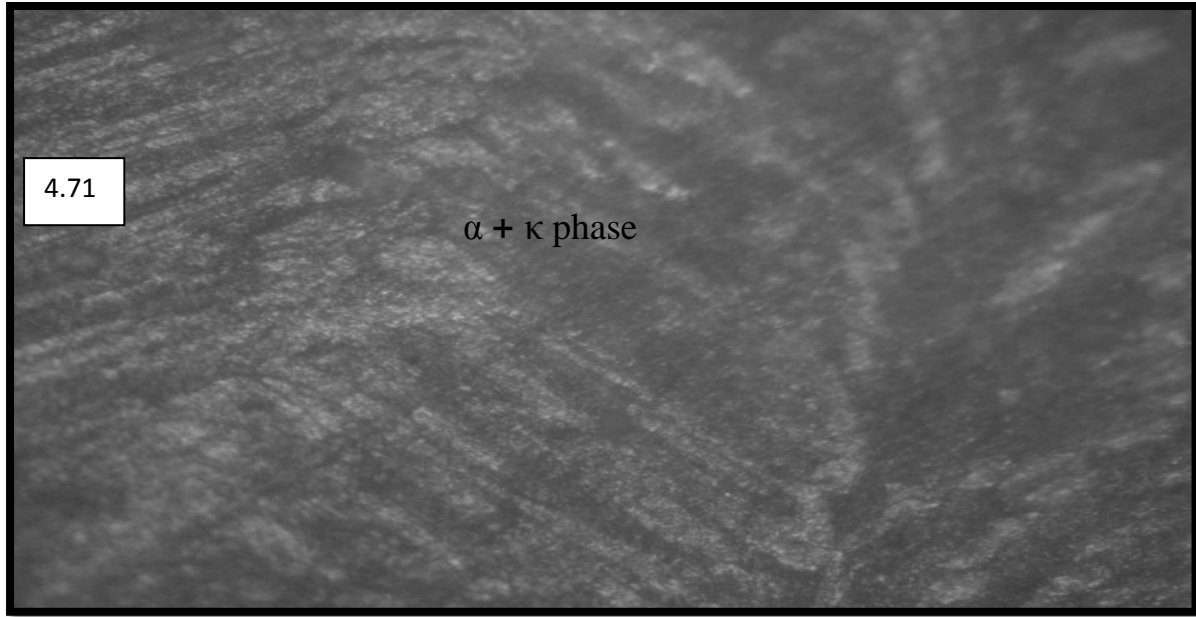


Plate 4.70: Micrograph of Cu-10%Al+7.0wt%Mo. (x400)



**Plate 4.71: Micrograph of Cu-10%Al+7.5wt%Mo. (x400)**

Plate 4.62 to Plate 4.71 show the microstructure of Cu-10%Al alloy treated with (0.5 to 10) wt% molybdenum. Molybdenum stabilized  $\beta$ -phase and hence increased toughness and strength. The microstructure showed that molybdenum increased the quantity of  $\alpha$ -phase in Cu-10%Al alloy system. Presence of sparse distribution of kappa precipitates in the predominated  $\alpha + \kappa$  caused smaller grains to increase in the microstructure which enhanced mechanical properties of the alloy.



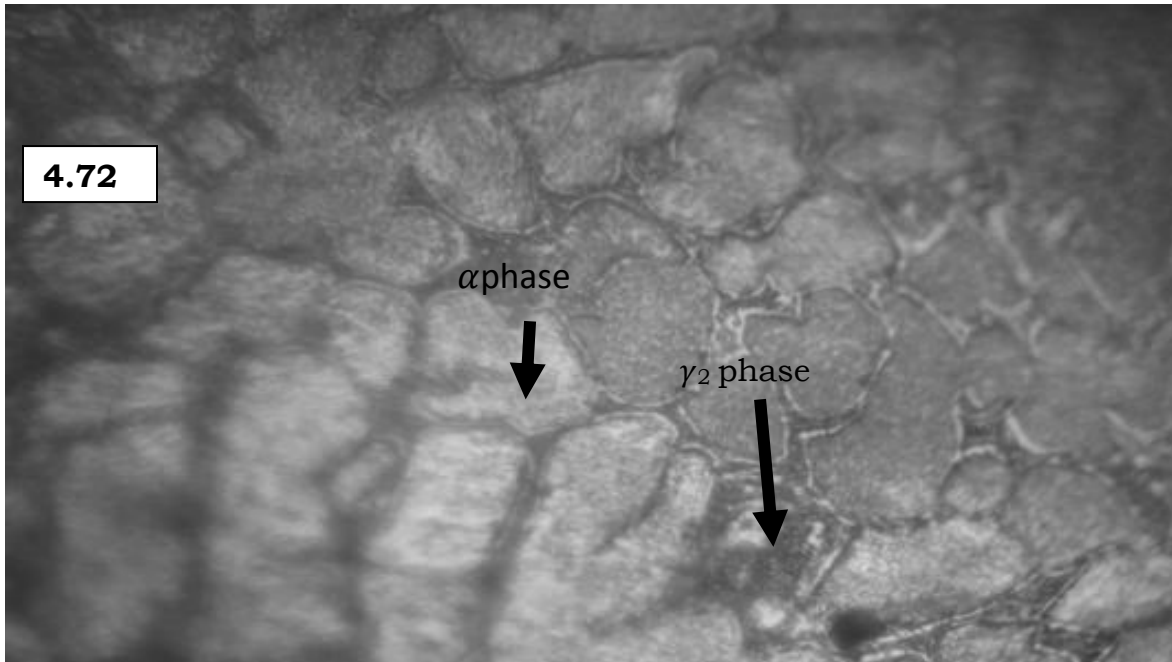


Plate 4.72: Micrograph of Cu-10%Al +0.5wt%Ni.

(x400)

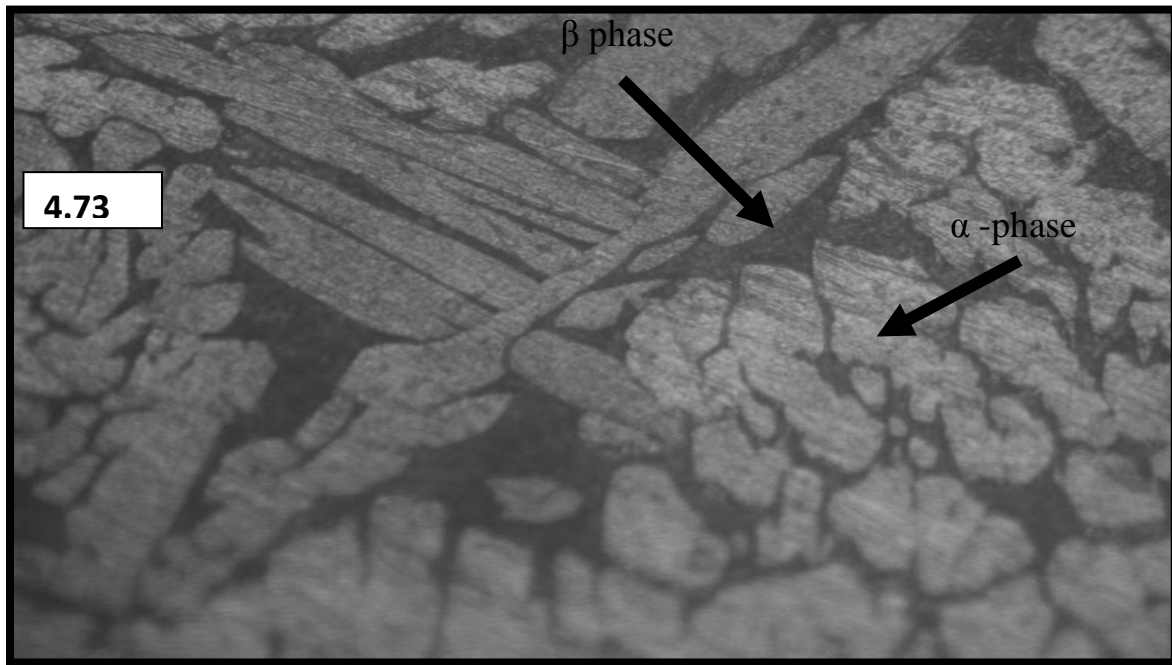
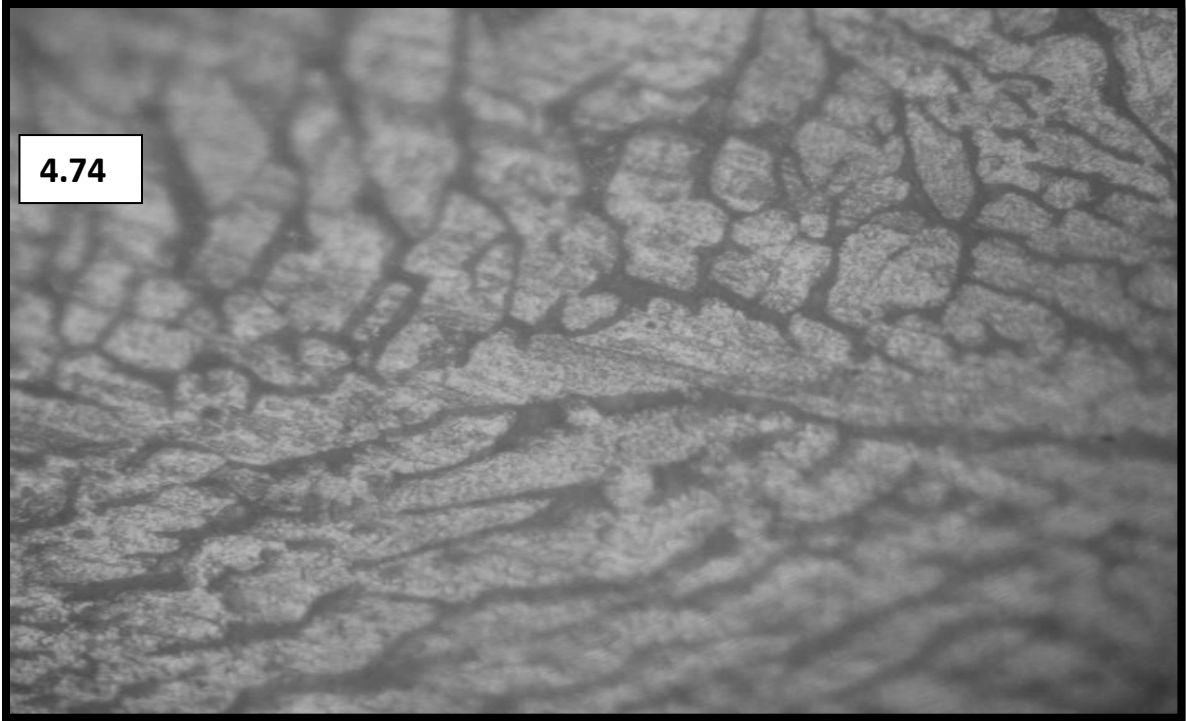
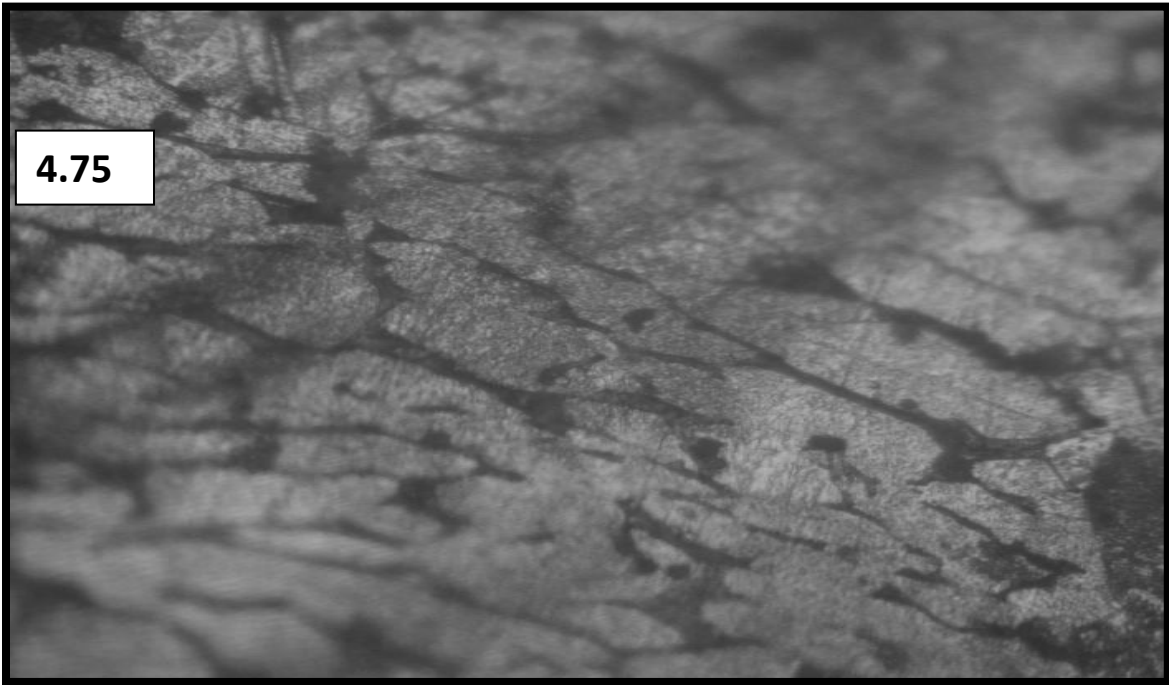


Plate 4.73: Micrograph of Cu-10%Al +1.0wt%Ni

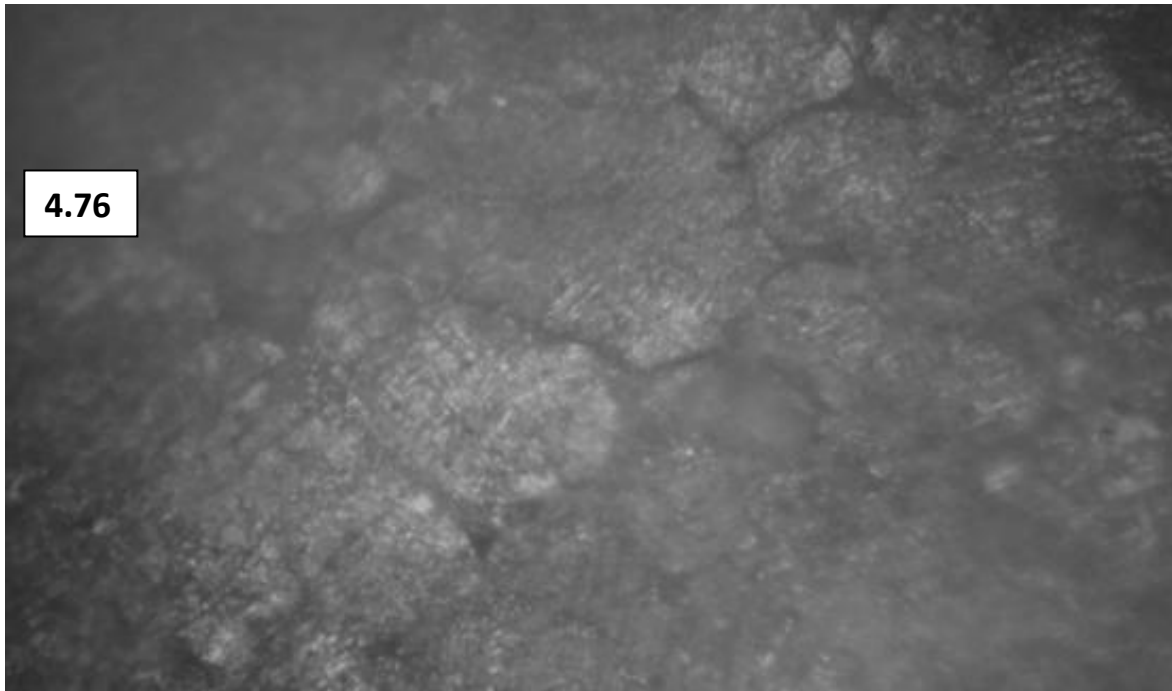
(x400)



**Plate 4.74: Micrograph of Cu-10%Al +1.5wt%Ni (x400)**



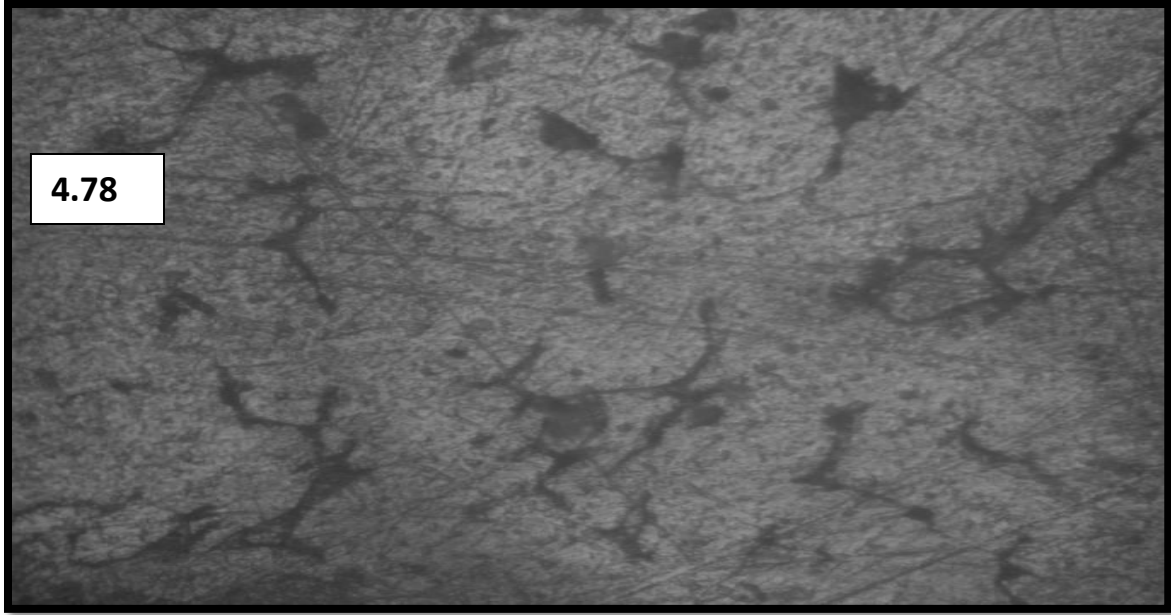
**Plate 4.75: Micrograph of Cu-10%Al +2.0wt%Ni (x400)**



**Plate 4.76: Micrograph of Cu-10%Al +2.5wt%Ni (x400)**

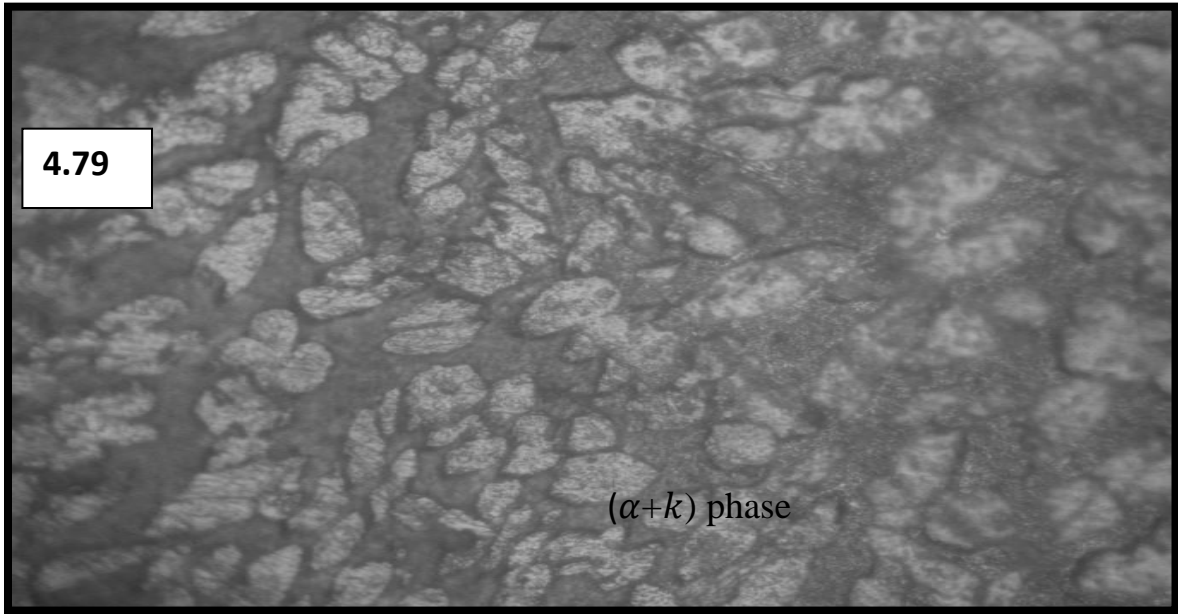


**Plate 4.77: Micrograph of Cu-10%Al +3.0%Ni (x400)**



4.78

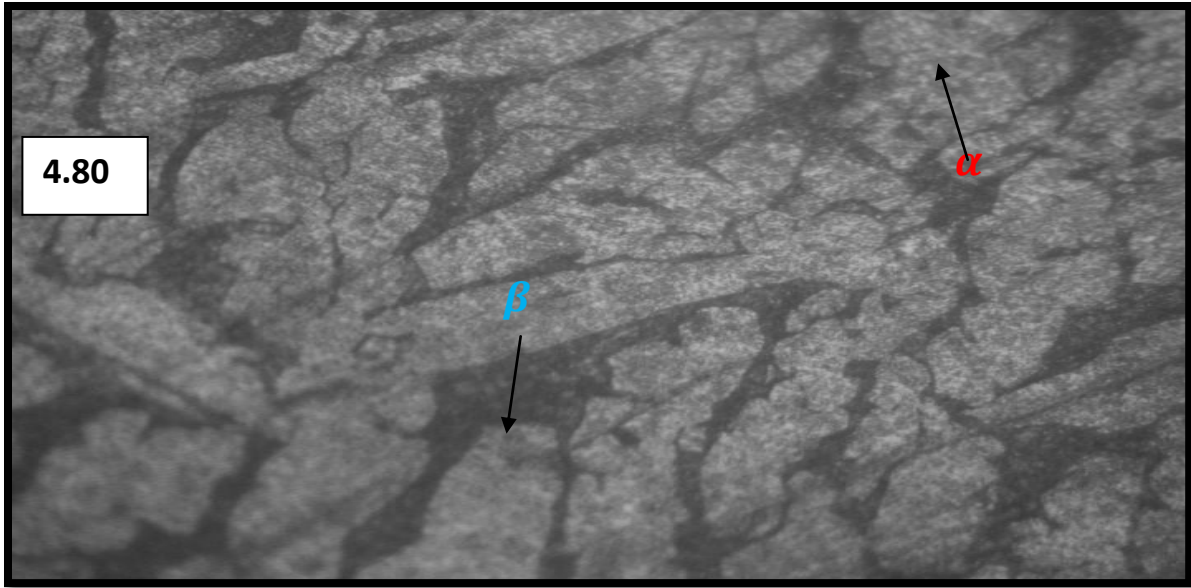
Plate 4.78: Micrograph of Cu-10%Al +3.5wt%Ni (x400)



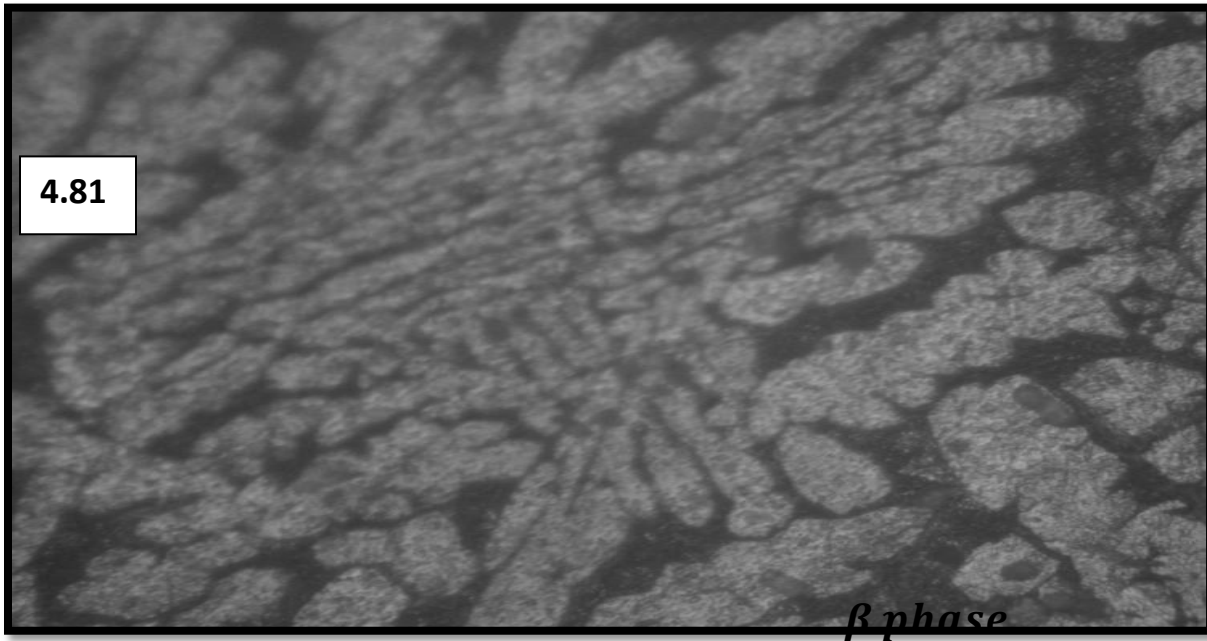
4.79

$(\alpha+k)$  phase

Plate 4.79: Micrograph of Cu-10%Al +4.0wt%Ni (x400)



**Plate 4.80: Micrograph of Cu-10%Al+4.5wt%Ni (x400)**



**Plate 4.81: Micrograph of Cu-10%Al+5.0wt%Ni (x400)**

Plate 4.72 to Plate 4.81 represent the micrographs of Cu-10%Al alloy treated with (0.5 to 5.0) wt% nickel. Addition of nickel has a strong influence on the stabilization of  $\beta$ -phases. Therefore nickel added improved the properties and



stabilized the effect of  $\beta$ -phase on the metallurgical structure. It also suppressed the formation of  $\gamma_2$  phase and modifies the characteristics of  $\beta$ -phases making it more susceptible to corrosion. It forms  $\text{Ni}_3\text{Al}$  intermetallic phase with Al which has precipitation hardening effect.

### 4.3 Structural Analysis with Scanning Electron Microscope (SEM) and Energy Dispersive Spectroscopy (EDS)

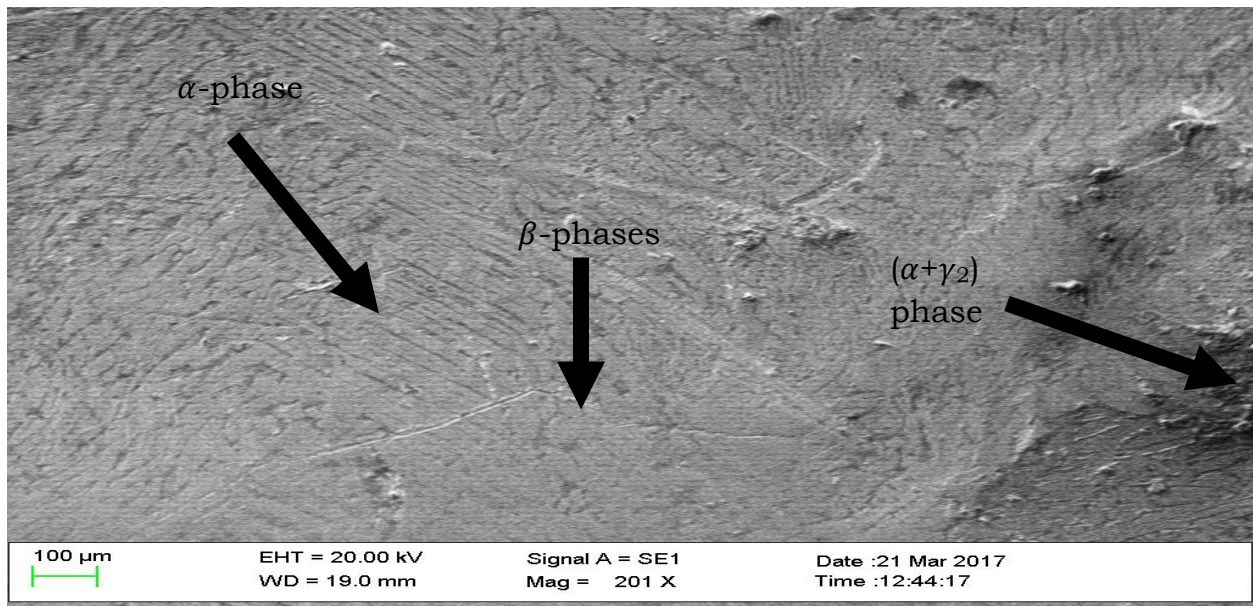


Plate4.82: Scanning Electron Microscope of Cu-Al alloy (base alloy)

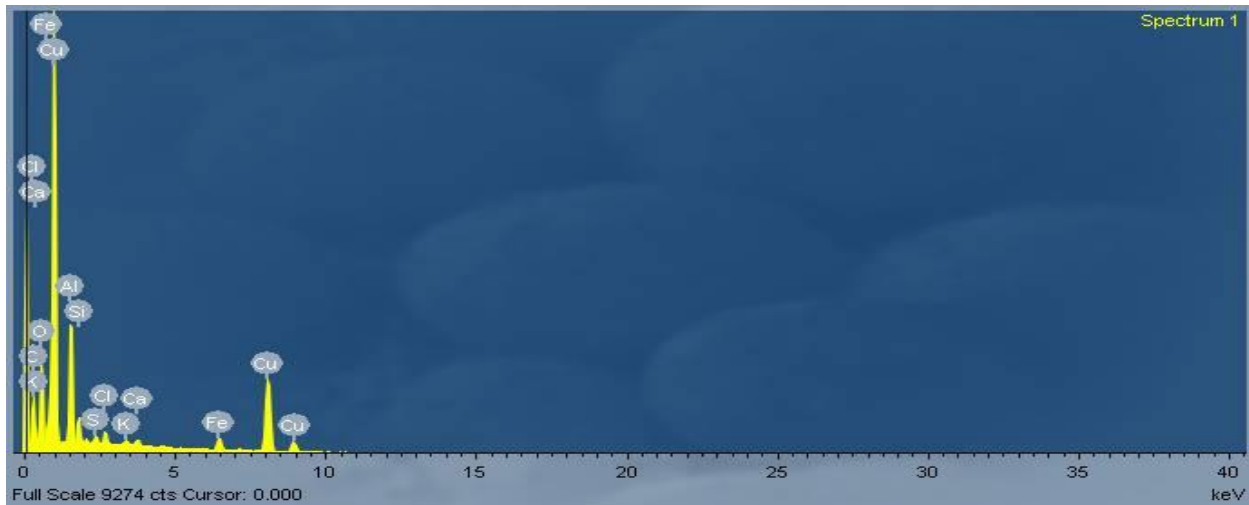


Figure 4.39: Energy dispersive spectrum (EDS) of Cu-10Al alloy

Plate 4.82 and Figure 4.39 represent the scanning electron microscope and energy dispersive spectroscopy on copper-aluminum alloy. SEM reveals the presence of  $\alpha$ -phase,  $\beta$ - phase and ( $\alpha+\gamma_2$ ) phase in white and dark spots in the alloy while EDX reveals the peak and presence of nine major elements such as Cu, Al, Fe, Ca, K, S, Cl, O and C.

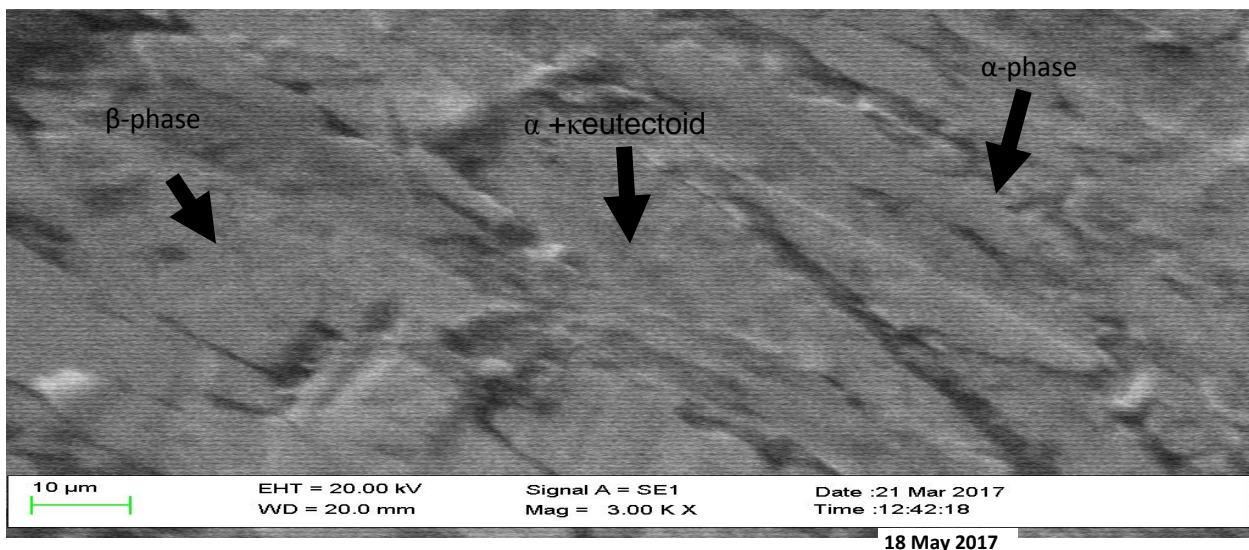


Plate 4.83: Scanning Electron Microscope of Cu-Al alloy with Chromium

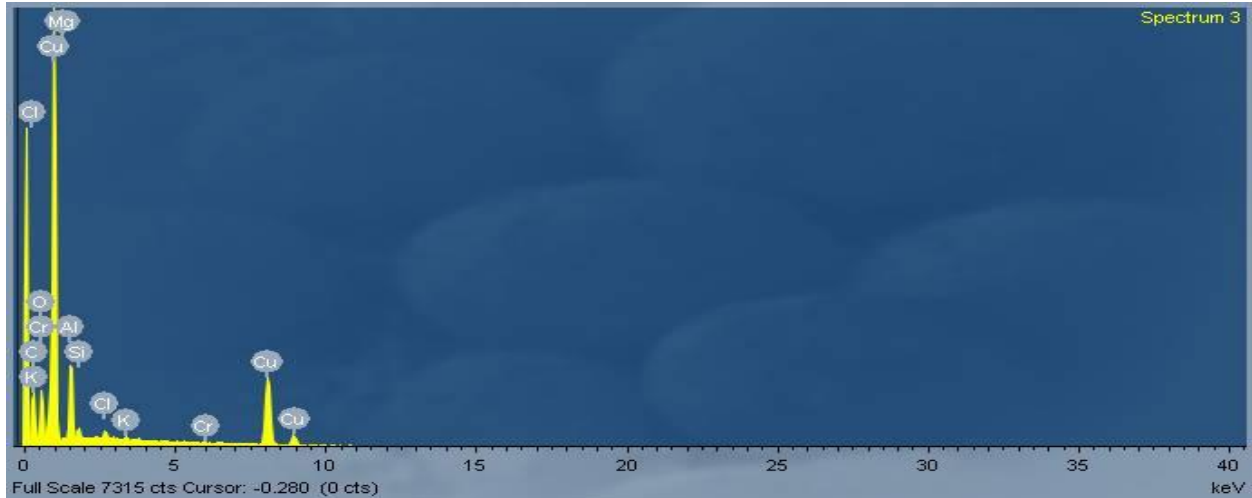


Figure 4.40: Energy dispersive spectrum (EDS) of Cu-10Al ally with Cr

Plate 4.83 and Figure 4.40 represent a detailed analysis of the scanning electron microscopy and energy dispersive spectroscopy of copper-aluminum modified with chromium. The results of the analysis revealed  $\beta$  transforms to  $\alpha + \kappa$  eutectoid and suppression of  $(\alpha + \gamma_2)$  phase, further secondary  $\kappa$  precipitated from the structure. As modification progresses, more of the  $\beta$  is transformed and, in doing so, the ductility decreased, as hardness, tensile and yield strength increase. It showed that addition of chromium refined and modified the intermetallic compound thereby cause the properties to improve.



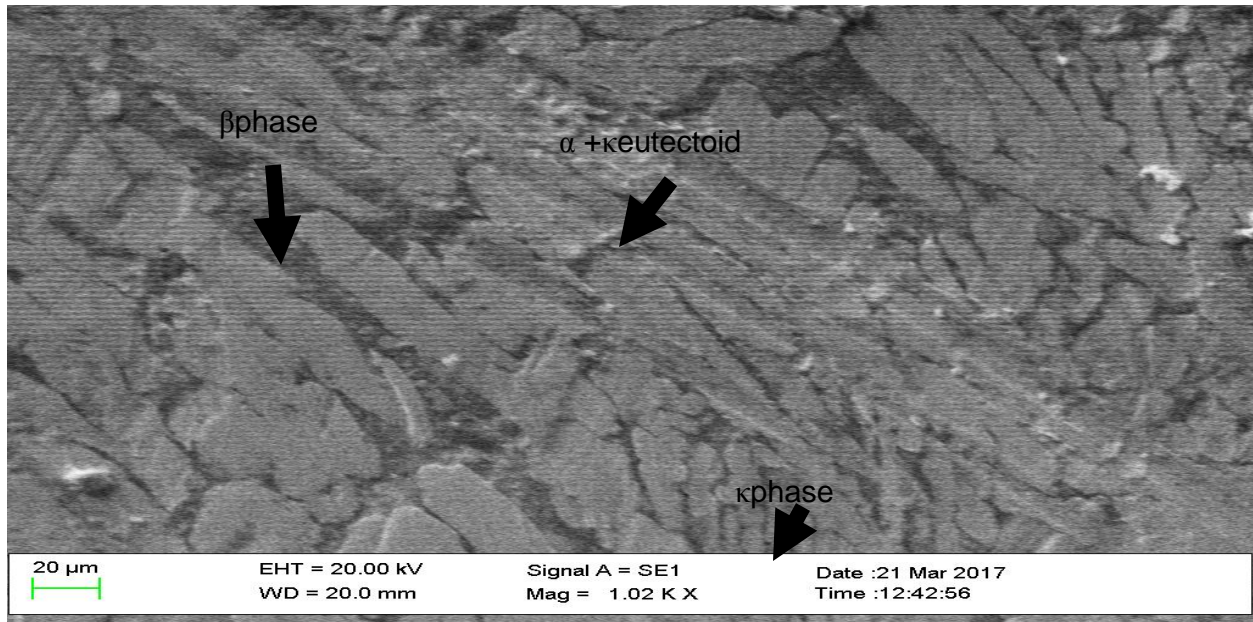


Plate 4.84: Scanning Electron Microscope of Cu-Al alloy with Mn

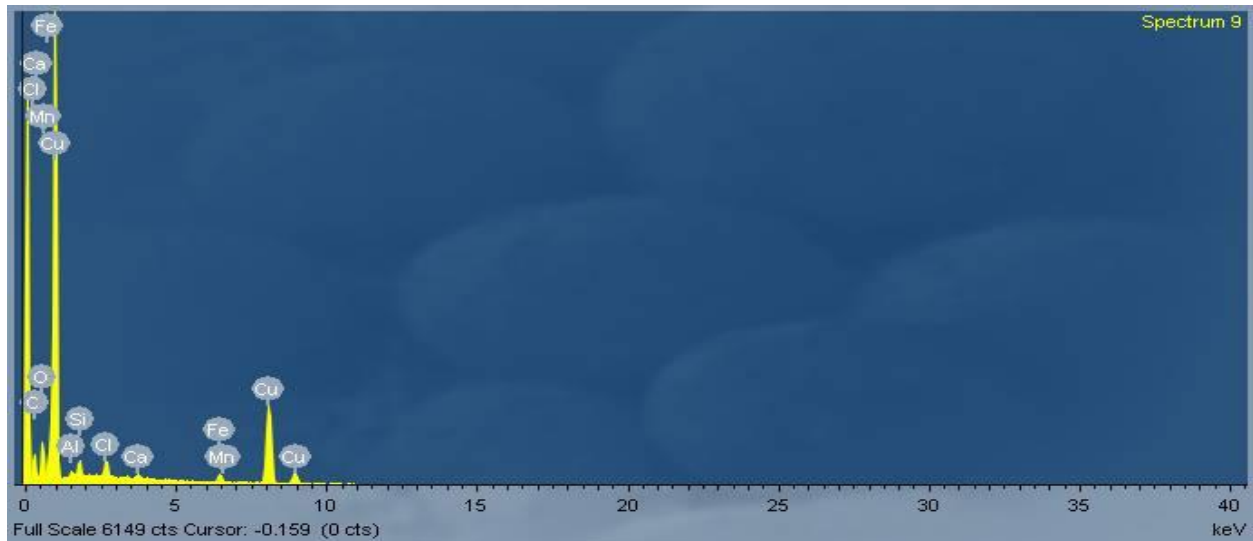


Figure 4.41: Energy dispersive spectrum (EDS) of Cu-10Al ally with Mn

The scanning electron microscopy and energy dispersive spectroscopy of copper-aluminum modified with manganese is shown in Plate 4.83 and Figure 4.41 respectively. The results of the analysis revealed the presence of two independent intermetallic phases such as  $\kappa$  phase and  $\alpha + \kappa$  eutectoid shown in white and dark

spots. An increase in the number of small k-phase precipitates increases the tensile strength while the presence of large globular precipitates improves hardness. It shows that addition of manganese refined and modified intermetallic compound.

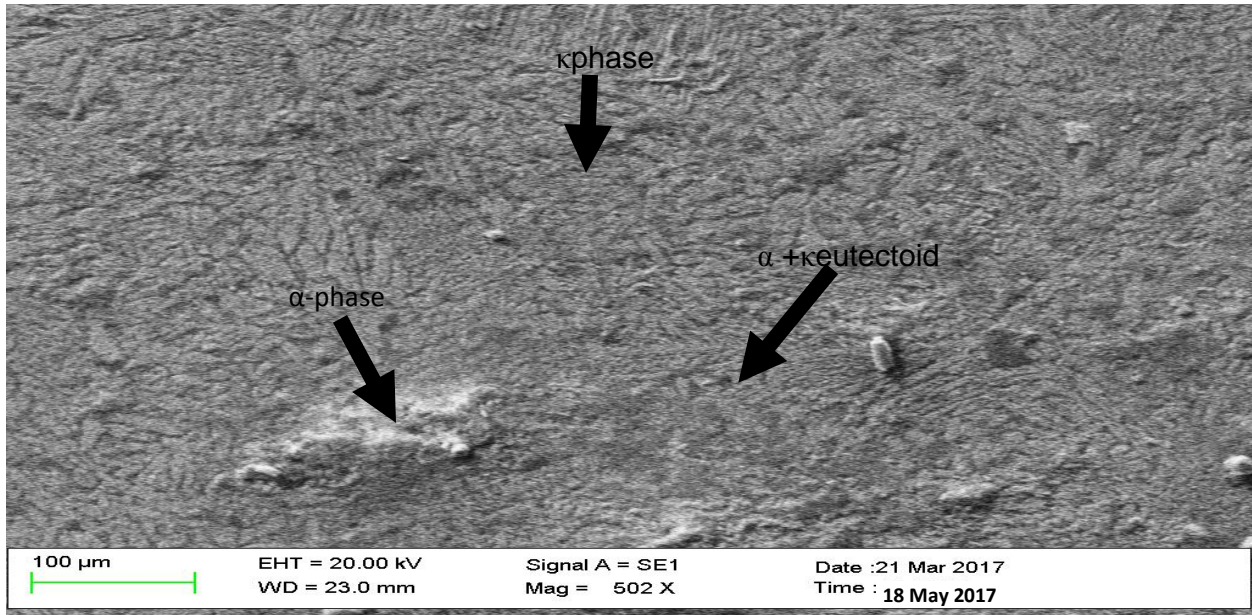


Plate 4.84: Scanning Electron Microscope of Cu-Al alloy with Mo

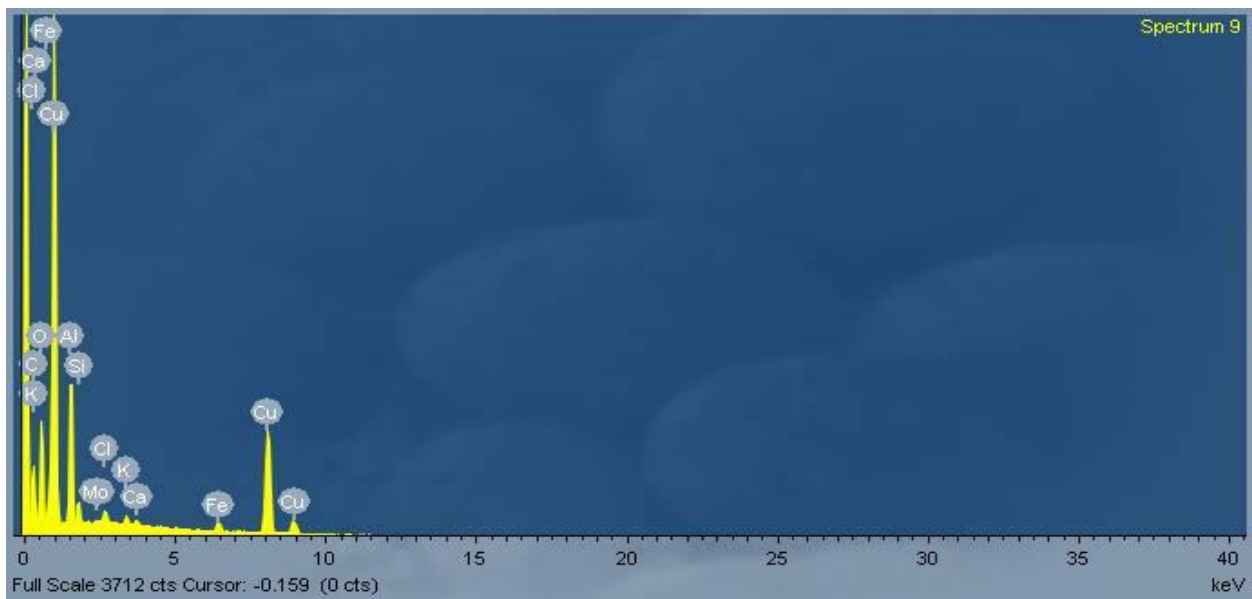


Figure 4.42; Energy dispersive spectrum (EDS) of Cu-10Al alloy with Mo

Plate 4.84 and Figure 4.42 show the scanning electron microscopy and energy dispersive spectroscopy of copper-aluminum modified with molybdenum. The structure consists of  $\alpha$  and martensitic  $\beta$  and some primary  $\kappa$  phases. It has shown that the phases and distribution of the  $\kappa$ -phase precipitates in bronze microstructure significantly affect its mechanical properties. Molybdenum refined and modified structure of the alloy and, instead of the formation of  $\gamma_2$ , a new phase  $\kappa$  (kappa) is created, which is more beneficial.

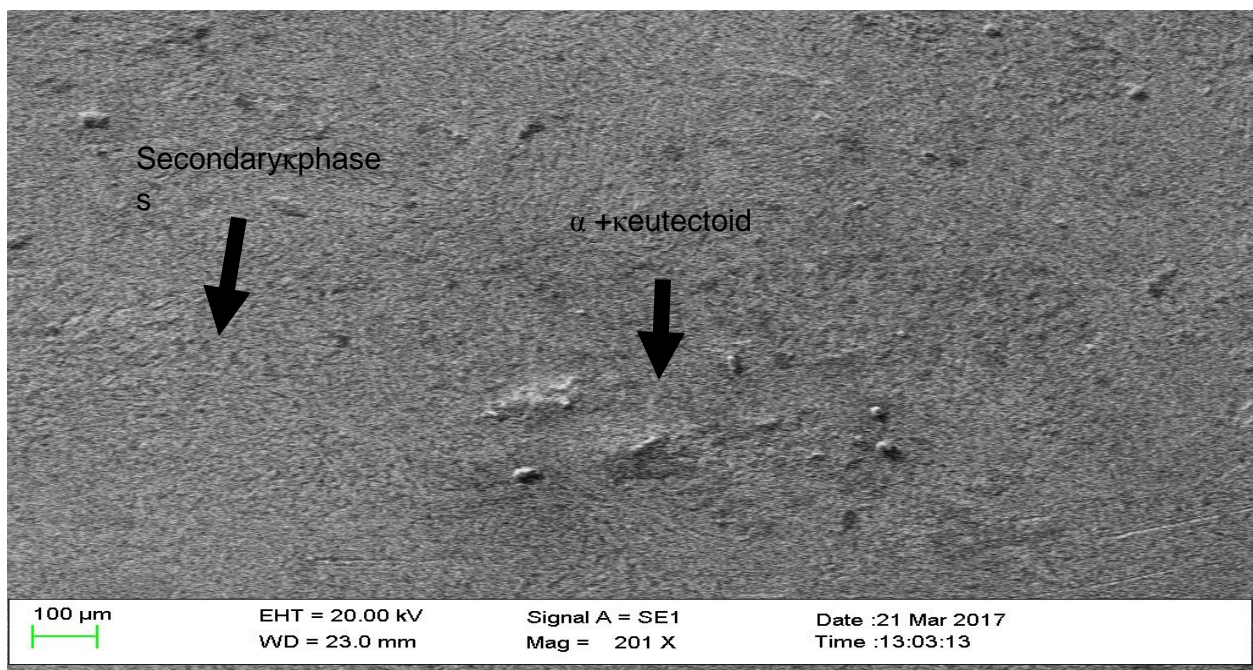


Plate 4.85: Scanning Electron Microscope of Cu-Al alloy with Ni

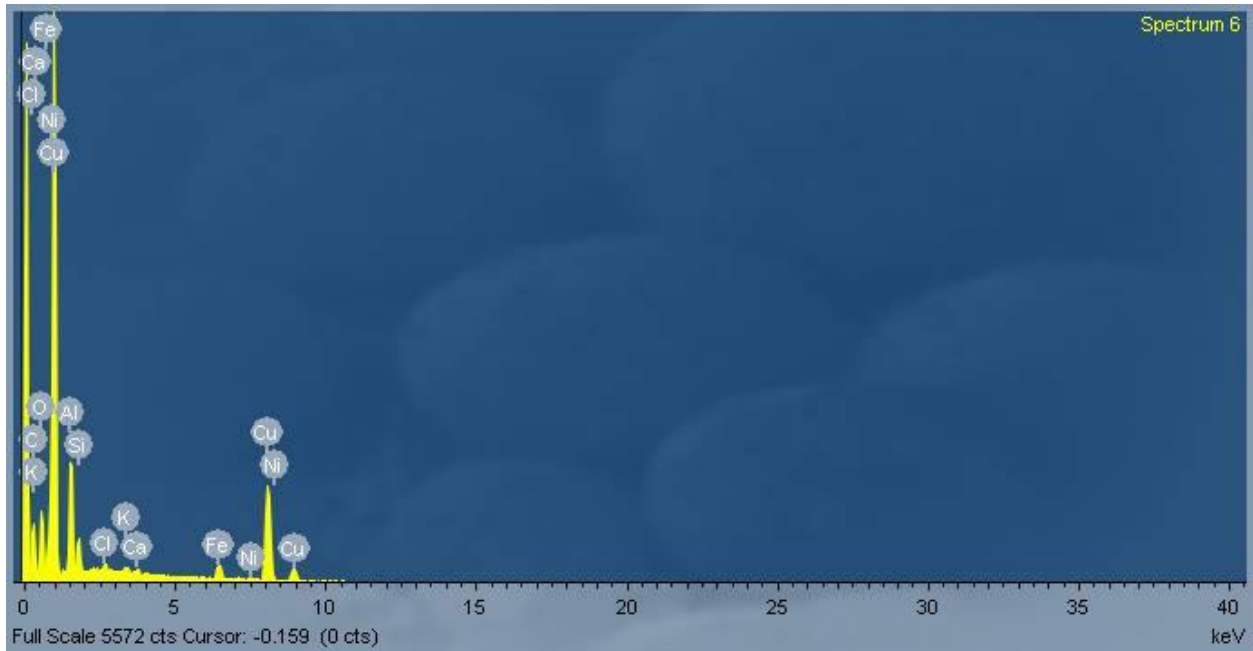


Figure 4.43: Energy dispersive spectrum (EDS) of Cu-10Al ally with Ni

Analysis of the scanning electron microscopy and energy dispersive spectroscopy of copper-aluminum modified with nickel are shown at Plate 4.85 and Figure 4.43 respectively. The  $\beta$  transforms to  $\alpha + \kappa$  eutectoid and further secondary  $\kappa$  precipitates from the structure. An increase in the number of small  $\kappa$ -phase precipitates increases the tensile strength, hardness and impact strength while the presence of large globular precipitates improves ductility of the modified alloy. According to Łabanowski et al, (2014) nickel additions suppress the  $\gamma_2$  phase, when both iron and nickel are present at nominally 5%, the structure of 9-10% aluminum alloys is modified and, instead of the formation of  $\gamma_2$ , a new phase  $\kappa$  (kappa) is created, which is more desirable.



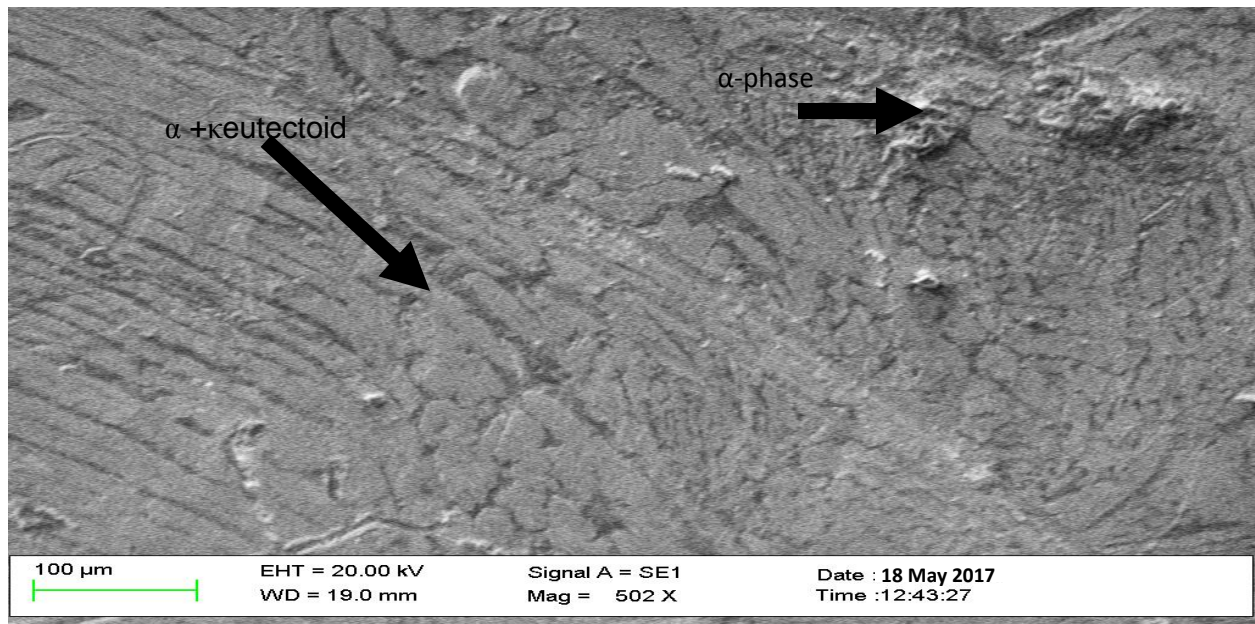


Plate 4.86: Scanning Electron Microscope of Cu-Al alloy with Ti

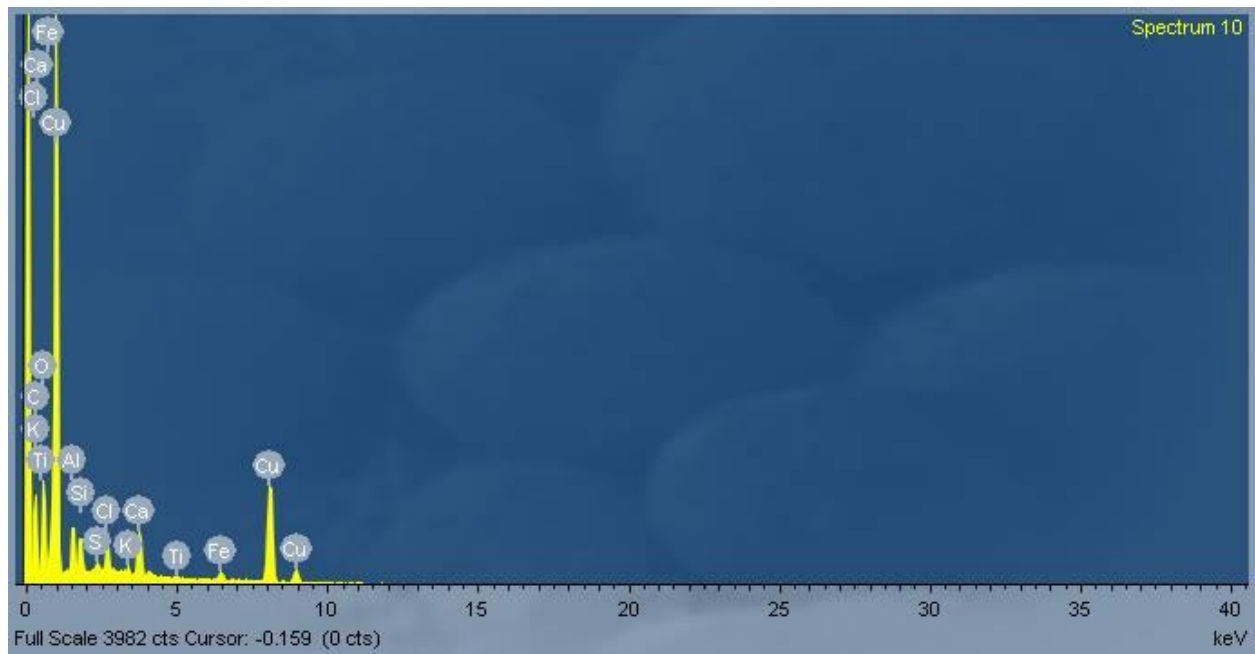


Figure 4.44: Energy dispersive spectrum (EDS) of Cu-10Al alloy with Ti

Plate 4.86 and Figure 4.44 show results of the scanning electron microscopy and energy dispersive spectroscopy respectively of copper-aluminum modified with

Titanium. The results of the analysis revealed the presence intermetallic phases. An increase in the number of small  $\kappa$ -phase precipitates and  $\alpha + \kappa$  eutectoid increases the mechanical and physical properties of the alloy. These particles are precipitated in the  $\beta$ -phase and cause a refinement of the microstructure by providing sites for the nucleation of the  $\beta$ -phase, to some extent, by impeding the growth of the  $\alpha$ -phase.

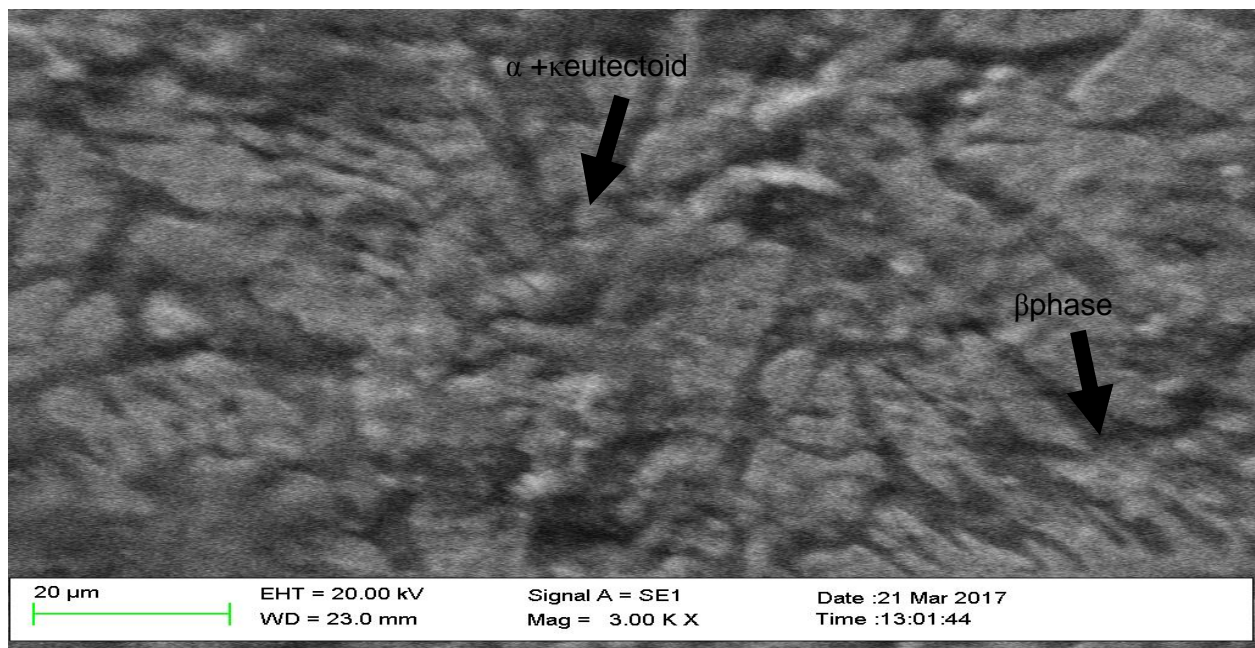


Plate 4.87: Scanning Electron Microscope of Cu-Al alloy with V

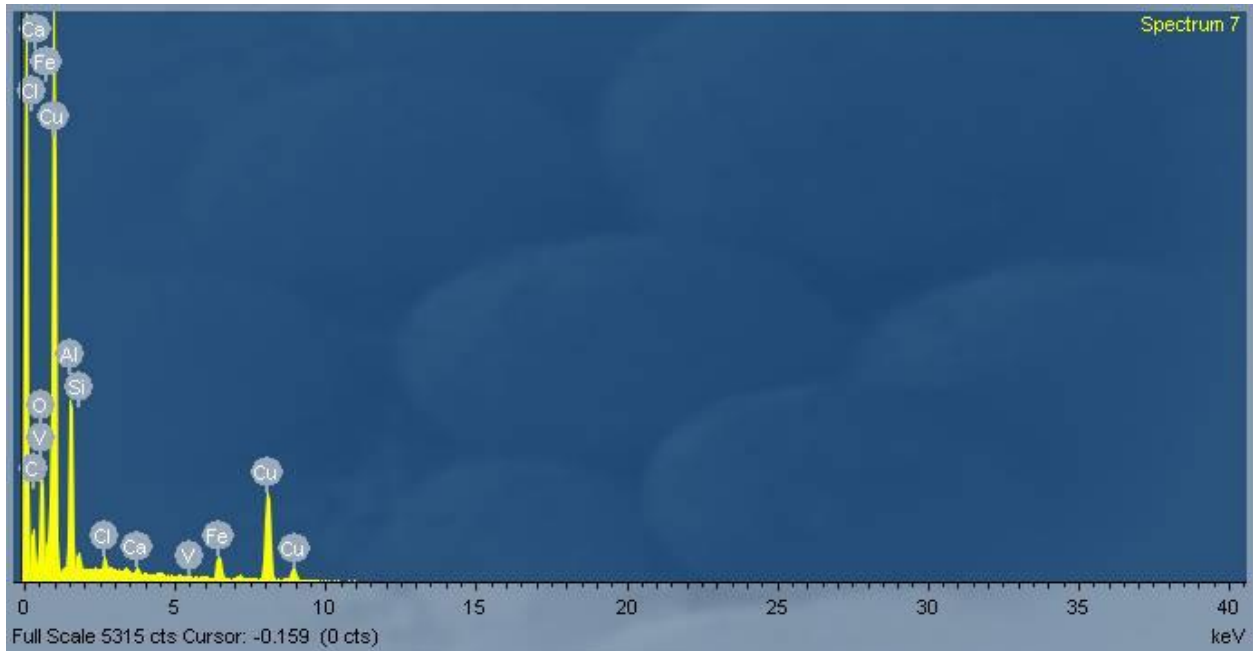


Figure 4.45: Energy dispersive spectrum (EDS) of Cu-10Al alloy with V

Plate 4.87 and Figure 4.45 indicate the scanning electron microscopy and energy dispersive spectroscopy respectively of copper-aluminum modified with vanadium. A second phase known as  $\beta$  (beta) appears in the metal structure and is stronger and harder. However, the  $\beta$  phase becomes unstable and decomposes to a finely divided structure (eutectoid) containing  $\alpha$  and another stronger. An increase in the number of small  $k$ -phase precipitates increases the tensile strength of castings, while the presence of large globular precipitates improves ductility. This shows that addition of vanadium refined and modified the intermetallic compound respectively, thereby caused improvement in the properties of Cu-Al alloy.

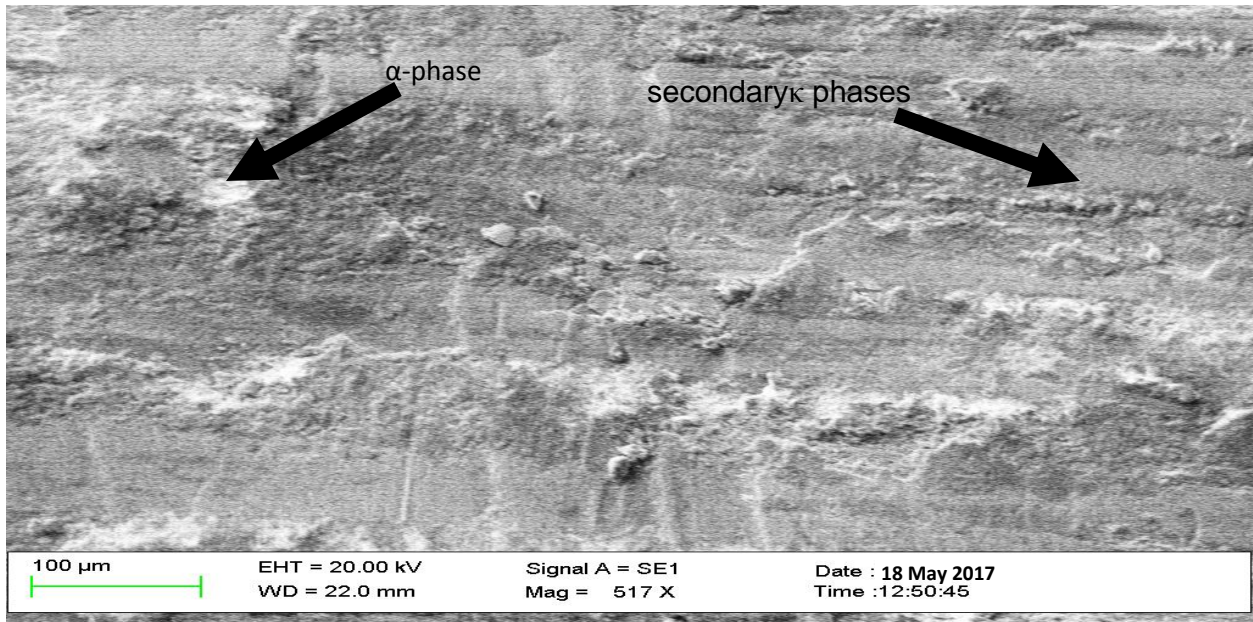


Plate 4.88: Scanning Electron Microscope of Cu-Al alloy with W

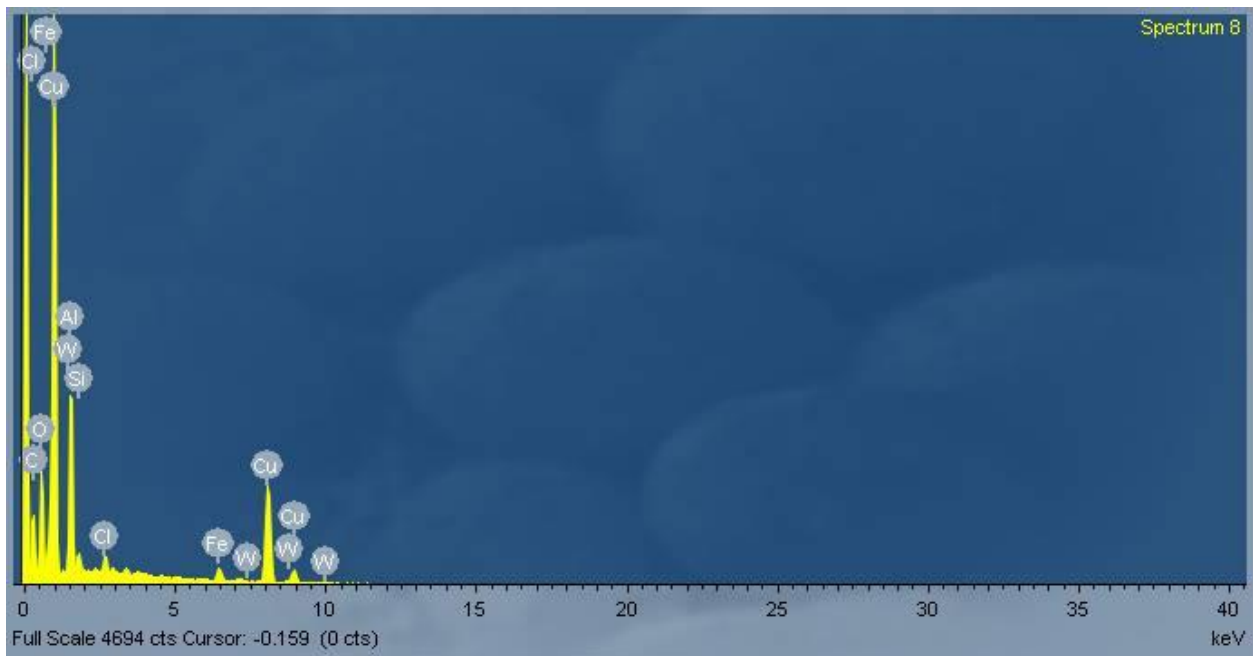


Figure 4.46: Energy dispersive spectrum (EDS) of Cu-10Al alloy with W



The scanning electron microscopy and energy dispersive spectroscopy of copper-aluminum modified with tungsten is shown in Plate 4.88 and Figure 4.46 respectively. They revealed the shape, size and distribution of the secondary k-phase precipitate in bronze microstructure. An increase in the number of small k-phase precipitates increases the tensile strength, hardness strength while the presence of large globular precipitates improves ductility of Cu-Al alloy. It shows that addition of tungsten refined and modified intermetallic compound, thereby caused improvement in properties of Cu-Al alloy.

## 4.5 Design Expert Analysis

### 4.5.1 Design Expert Analysis for Titanium

**Table 4.3: Represents Design Expert data for Titanium**

Std	Run	Factor 1 A:% Ti	Response 1 Yield Strength MPa	Response 2 UTS MPa	Response 3 Hardness BHN	Response 4 Elongation %	Response 5 Impact Strength J	Response 6 Resistivity mm	Response 7 Conductivity mm
1	1	0.5	189	383	113	25.61	38.94	6.06	8.84
11	2	1.5	245	440	165	23.12	35.23	7.21	7.35
13	3	2.5	336	483	236	21.41	32.63	8.43	5.1
5	4	3.5	391	532	296	18.14	29.67	9.37	4.35
4	5	4.5	450	562	345	16.48	27.05	10.45	3.93
3	6	5	463	592	362	15.83	26.87	12.32	3.46
7	7	6	501	624	378	15.39	26.07	12.82	3.1
12	8	6.5	558	658	410	15.12	25.23	13.21	3.02
2	9	7.5	466	573	346	17.21	27.83	11.43	3.9
9	10	8.5	435	532	326	19.41	29.67	10.87	4.85
6	11	9	423	532	315	20.36	30.92	10.37	5.14
8	12	9.5	412	528	310	21.78	31.05	9.85	5.63
10	13	10	405	520	302	22.83	32.87	9.42	5.86

**ANOVA for Response Surface Quadratic model**

**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	1.128E+005	2	56414.90	76.39	< 0.0001	Significant
A-% Ti	62992.23	1	62992.23	85.30	< 0.0001	
A <sup>2</sup>	64424.42	1	64424.42	87.24	< 0.0001	
Residual	7384.98	10	738.50			
Cor Total	1.202E+005	12				

The Model F-value of 76.39 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant.

$$\text{Yield Strength} = +109.42925 + 116.47608\% \text{Ti} - 8.86734\% \text{Ti}^2 \quad (4.1)$$

$$\text{UTS} = +325.50081 + 88.00125\% \text{Ti} - 7.03464\% \text{Ti}^2 \quad (4.2)$$

$$\text{Hardness} = +107.01490 + 1.10888\% \text{Ti} + 31.97697\% \text{Ti}^2 - 5.68969\% \text{Ti}^3 + 0.26767\% \text{Ti}^4 \quad (4.3)$$

$$\text{Elongation} = +25.73192 + 7.28985\text{E-}003\% \text{Ti} - 1.25171\% \text{Ti}^2 + 0.21758\% \text{Ti}^3 - 9.53343\text{E-}003\% \text{Ti}^4 \quad (4.4)$$

$$\text{Impact Strength} = +39.78763 - 1.70937\% \text{Ti} - 0.90408\% \text{Ti}^2 + 0.18528\% \text{Ti}^3 - 8.48694\text{E-}003\% \text{Ti}^4 \quad (4.5)$$

$$\text{Resistivity} = +4.06266 + 2.49279\% \text{Ti} - 0.19608\% \text{Ti}^2 \quad (4.6)$$

$$\text{Conductivity} = +9.91449 - 2.21852\% \text{Ti} + 0.18543\% \text{Ti}^2 \quad (4.7)$$

These equations 4.1 to 4.7 in terms of actual factors can be used to make predictions about the response for given levels of the factor (Ti).

## 4.5.2 Design Expert Analysis for Zirconium

**Table 4.4: Represents Design Expert data for Zirconium**

Std	Run	Factor 1 A:% Zr	Response 1 Yield Strength MPa	Response 2 UTS MPa	Response 3 Hardness BHN	Response 4 Elongation %	Response 5 Impact Strength J	Response 6 Resistivity mm	Response 7 Conductivity Mm
13	1	0.5	207	369	118	25.21	38.57	6.29	8.61
6	2	1.5	254	408	173	23.46	36.45	7.18	7.31
11	3	2.5	324	472	228	22.76	32.42	8.56	5.21
9	4	3.5	389	514	309	20.24	28.87	9.87	4.03
3	5	4.5	435	565	347	16.55	27.16	12.13	3.65
1	6	5	467	580	362	15.78	26.66	13.79	3.21
12	7	6	487	603	376	15.25	26.09	13.83	3.01
8	8	6.5	504	618	385	14.76	25.45	14.08	2.81
10	9	7.5	544	626	394	14.36	24.82	14.86	2.21
7	10	8.5	549	514	376	16.24	26.87	14.87	2.43
4	11	9	423	506	365	17.76	28.82	13.69	2.86
5	12	9.5	415	485	350	18.55	29.16	12.13	4.35
2	13	10	407	480	342	20.78	31.66	11.32	4.78

### ANOVA for Response Surface Cubic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	1.222E+005	3	40746.88	53.71	< 0.0001 Significant
A-% Zr	30030.08	1	30030.08	39.58	0.0001
A <sup>2</sup>	51282.95	1	51282.95	67.60	< 0.0001
A <sup>3</sup>	5725.89	1	5725.89	7.55	0.0226
Residual	6827.67	9	758.63		
Cor Total	1.291E+005	12			

The Model F-value of 53.71 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant.

$$\text{Yield Strength} = +187.13741 + 35.74624 * \% \text{ Zr} + 9.47955 * \% \text{ Zr}^2 - 1.10453 * \% \text{ Zr}^3 \quad (4.8)$$

$$\text{UTS} = +292.88623 + 98.56070 * \% \text{ Zr} - 8.10343 * \% \text{ Zr}^2 \quad (4.9)$$

$$\text{Hardness} = +92.26115 + 42.36420 * \% \text{ Zr} + 11.69772 * \% \text{ Zr}^2 - 2.41464 * \% \text{ Zr}^3 + 0.10711 * \% \text{ Zr}^4 \quad (4.10)$$

$$\text{Elongation} = +25.73810 - 0.59601 * \% \text{ Zr} - 0.51498 * \% \text{ Zr}^2 + 0.052785 * \% \text{ Zr}^3 \quad (4.11)$$

$$\text{Impact Strength} = +40.74234 - 3.52987 * \% \text{ Zr} + 0.015802 * \% \text{ Zr}^2 + 0.024532 * \% \text{ Zr}^3 \quad (4.12)$$

$$\text{Resistivity} = +6.05981 + 0.17083 * \% \text{ Zr} + 0.43947 * \% \text{ Zr}^2 - 0.040556 * \% \text{ Zr}^3 \quad (4.13)$$

$$\text{Conductivity} = +7.27639 + 5.02029 * \% \text{ Zr} - 5.67365 * \% \text{ Zr}^2 + 2.01722 * \% \text{ Zr}^3 - 0.33468 * \% \text{ Zr}^4 + 0.026306 * \% \text{ Zr}^5 - 7.86252 \text{E-}004 * \% \text{ Zr}^6 \quad (4.14)$$

These equations 4.8 to 4.14 in terms of actual factors can be used to make predictions about the response for given levels of the factor (Zr).

### 4.5.3 Design Expert Analysis for Tungsten

**Table 4.5: Represents Design Expert data for Tungsten**

Std	Run	Factor 1 A:%	Response 1 W Yield Strength MPa	Response 2 UTS MPa	Response 3 Hardness BHN	Response 4 Elongation %	Response 5 Impact Strength J	Response 6 Resistivity mm	Response 7 Conductivity Mm
8	1	0.5	201	384	113	25.58	37.57	6.13	8.64
5	2	1.5	266	434	162	23.22	34.64	7.51	7.71
9	3	2.5	342	492	223	21.51	31.26	8.89	5.23
7	4	3.5	381	521	289	18.34	28.63	9.56	4.29
4	5	4.5	415	561	318	16.44	26.86	10.27	4.03
3	6	5	432	582	335	15.64	26.56	12.75	3.86
10	7	6	483	607	357	15.38	26.13	13.83	3.23
6	8	6.5	496	614	362	15.22	25.74	14.21	3.05
12	9	7.5	538	632	383	14.81	24.76	15.39	2.23
11	10	8.5	496	561	359	17.84	27.83	13.56	2.89
13	11	9	478	555	346	18.46	28.38	12.83	3.14
1	12	9.5	465	551	338	19.84	29.86	12.27	3.83
2	13	10	452	549	335	20.64	30.58	11.75	4.26

**ANOVA for Response Surface Sixth model**

**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	1.149E+005	6	19153.94	312.26	< 0.0001	Significant
A-% W	9362.41	1	9362.41	152.63	< 0.0001	
A <sup>2</sup>	276.67	1	276.67	4.51	0.0779	
A <sup>3</sup>	607.23	1	607.23	9.90	0.0199	
A <sup>4</sup>	1027.98	1	1027.98	16.76	0.0064	
A <sup>5</sup>	312.13	1	312.13	5.09	0.0649	
A <sup>6</sup>	888.04	1	888.04	14.48	0.0089	
Residual	368.04	6	61.34			
Cor Total	1.153E+005	12				

$$\text{Yield Strength} = +209.02518 - 74.63545 * \% W + 142.31283 * \% W^2 - 58.88080 * \% W^3 + 11.14588 * \% W^4 - 0.98413 * \% W^5 + 0.032596 * \% W^6 \quad (4.15)$$

$$\text{UTS} = +323.95845 + 141.34433 * \% W - 61.03379 * \% W^2 + 16.09655 * \% W^3 - 1.86960 * \% W^4 + 0.07514 * \% W^5 - 0.00000 * \% W^6 \quad (4.16)$$

$$\text{Hardness} = +135.75857 - 97.15878 * \% W + 125.62177 * \% W^2 - 43.26278 * \% W^3 + 7.15814 * \% W^4 - 0.57772 * \% W^5 + 0.018063 * \% W^6 \quad (4.17)$$

$$\text{Elongation} = +25.92784 - 0.48652 * \% W - 0.93568 * \% W^2 + 0.15698 * \% W^3 - 6.36252E-003 * \% W^4 + 0.00000 * \% W^5 - 0.00000 * \% W^6 \quad (4.18)$$

$$\text{Impact Strength} = +40.29289 - 4.61250 * \% W + 0.36495 * \% W^2 + 0.00000 * \% W^3 - 0.00000 * \% W^4 + 0.00000 * \% W^5 - 0.00000 * \% W^6 \quad (4.19)$$

$$\text{Resistivity} = +3.71816 + 6.09385 * \% W - 3.32812 * \% W^2 + 0.87805 * \% W^3 - 0.095892 * \% W^4 + 3.60757E-003 * \% W^5 + 0.00000 * \% W^6 \quad (4.20)$$

$$\text{Conductivity} = +6.64349 + 7.11241 * \% W - 7.38071 * \% W^2 + 2.64177 * \% W^3 - 0.44617 * \% W^4 + 0.035828 * \% W^5 - 1.09839E-003 * \% W^6 \quad (4.21)$$

These equations 4.15 to 4.21 in terms of actual factors can be used to make predictions about the response for given levels of the factor (W).

#### 4.5.4 Design Expert Analysis for Vanadium

**Table 4.6: Represents Design Expert data for Vanadium**

	Factor 1	Response 1	Response 2	Response 3	Response 4	Response 5	Response 6	Response 7	
Std	Run	A:% V	Yield Strength	UTS	Hardness	Elongation	Impact Strength	Resistivity	Conductivity
		%	MPa	MPa	BHN	%	J	mm	Mm
5	1	0.5	203	378	109	25.46	37.57	6.02	8.67
4	2	1	222	391	128	24.63	35.63	6.76	8.32
2	3	1.5	268	407	157	23.43	34.64	7.37	7.38
10	4	2	318	433	195	22.96	32.45	8.11	6.78
8	5	2.5	335	473	243	22.46	31.26	8.84	5.03
1	6	3	372	498	297	21.35	30.42	9.37	4.78
9	7	3.5	420	512	325	19.28	28.63	9.89	4.39
3	8	4	467	561	363	16.46	26.88	8.56	4.14
7	9	4.5	428	532	341	16.58	27.06	7.78	3.65
6	10	5	402	506	316	17.64	27.43	7.07	3.36

#### ANOVA for Response Surface Cubic model

##### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	71778.10	3	23926.03	88.27	< 0.0001	significant
A-% V	21830.77	1	21830.77	80.54	0.0001	
A <sup>2</sup>	5434.92	1	5434.92	20.05	0.0042	
A <sup>3</sup>	3027.90	1	3027.90	11.17	0.0156	
Residual	1626.40	6	271.07			
Cor Total	73404.50	9				

The Model F-value of 88.27 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant.

$$\text{Yield Strength} = +205.50000 - 24.70280 * \% V + 52.51282 * \% V^2 - 7.92075 * \% V^3 \quad (4.22)$$

$$\text{UTS} = +397.60000 - 56.86247 * \% V + 52.59441 * \% V^2 - 7.36131 * \% V^3 \quad (4.23)$$

$$\text{Hardness} = +126.73333 - 61.40482 * \% V + 67.78555 * \% V^2 - 9.59751 * \% V^3 \quad (4.24)$$

$$\text{Elongation} = +29.47083 - 11.07068 * \% V + 7.97048 * \% V^2 - 2.48510 * \% V^3 + 0.24790 * \% V^4 \quad (4.25)$$

$$\text{Impact Strength} = +39.93367 - 4.44439 * \% V + 0.36212 * \% V^2 \quad (4.26)$$

$$\text{Resistivity} = +4.10083 + 3.27353 * \% V - 0.53258 * \% V^2 \quad (4.27)$$

$$\text{Conductivity} = +10.10750 - 2.24932 * \% V + 0.17955 * \% V^2 \quad (4.28)$$

These equations 4.22 to 4.28 in terms of actual factors can be used to make predictions about the response for given levels of the factor (V).

#### 4.5.5 Design Expert Analysis for Nickel

**Table 4.7: Represents Design Expert data for Nickel**

Std	Run	Factor 1 A:% Ni	Response 1 Yield Strength MPa	Response 2 UTS MPa	Response 3 Hardness BHN	Response 4 Elongation %	Response 5 Impact Strength J	Response 6 Resistivity mm	Response 7 Conductivity Mm
7	1	0.5	205	373	115	25.32	38.48	6.08	8.91
1	2	1	229	386	138	24.64	37.66	6.75	8.2
5	3	1.5	267	410	156	23.38	36.53	7.16	7.48
10	4	2	288	428	193	23.02	34.21	7.88	6.79
4	5	2.5	327	463	229	22.68	32.42	8.37	5.03
9	6	3	361	488	239	21.44	30.36	8.82	4.58
8	7	3.5	383	533	284	19.62	28.56	9.28	4.39
3	8	4	456	564	326	17.18	26.89	9.83	4.14
2	9	4.5	426	532	308	17.45	27.08	10.18	4.03
6	10	5	384	517	293	17.64	27.48	12.83	3.86

**ANOVA for Response Surface Cubic model**

**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	61141.09	3	20380.36	57.21	< 0.0001	significant
A-% Ni	20201.73	1	20201.73	56.71	0.0003	
A <sup>2</sup>	3141.94	1	3141.94	8.82	0.0250	
A <sup>3</sup>	3161.22	1	3161.22	8.87	0.0247	
Residual	2137.31	6	356.22			
Cor Total	63278.40	9				

The Model F-value of 57.21 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant.

$$\text{Yield Strength} = +223.93333 - 48.74359 * \% \text{ Ni} + 57.01166 * \% \text{ Ni}^2 - 8.09324 * \% \text{ Ni}^3 \quad (4.29)$$

$$\text{UTS} = +397.06667 - 64.11189 * \% \text{ Ni} + 55.99767 * \% \text{ Ni}^2 - 7.67832 * \% \text{ Ni}^3 \quad (4.30)$$

$$\text{Hardness} = +123.43333 - 27.53963 * \% \text{ Ni} + 41.91142 * \% \text{ Ni}^2 - 5.89744 * \% \text{ Ni}^3 \quad (4.31)$$

$$\text{Elongation} = +16.69633 + 37.42913 * \% \text{ Ni} - 56.09123 * \% \text{ Ni}^2 + 36.94037 * \% \text{ Ni}^3 - 12.06482 * \% \text{ Ni}^4 + 1.89406 * \% \text{ Ni}^5 - 0.11391 * \% \text{ Ni}^6 \quad (4.32)$$

$$\text{Impact Strength} = +38.11867 + 1.82630 * \% \text{ Ni} - 2.52473 * \% \text{ Ni}^2 + 0.34715 * \% \text{ Ni}^3 \quad (4.33)$$

$$\text{Resistivity} = +6.57917 - 1.96585 * \% \text{ Ni} + 2.70960 * \% \text{ Ni}^2 - 0.88051 * \% \text{ Ni}^3 + 0.093217 * \% \text{ Ni}^4 \quad (4.34)$$

$$\text{Conductivity} = +10.49733 - 2.65655 * \% \text{ Ni} + 0.26485 * \% \text{ Ni}^2 \quad (4.35)$$

These equations 4.29 to 4.35 in terms of actual factors can be used to make predictions about the response for given levels of the factor (Ni).



### 4.5.6 Design Expert Analysis for Molybdenum

**Table 4.8: Represents Design Expert data for Molybdenum**

Std	Run	Factor 1 A:% Mo	Response 1 Yield Strength MPa	Response 2 UTS MPa	Response 3 Hardness BHN	Response 4 Elongation %	Response 5 Impact Strength J	Response 6 Resistivity mm	Response 7 Conductivity Mm
6	1	0.5	201	373	110	25.58	36.75	6.95	8.68
12	2	1.5	266	434	156	23.32	33.56	8.21	7.45
5	3	2.5	346	481	215	21.32	30.56	9.34	5.03
3	4	3.5	393	522	302	18.24	27.67	10.79	4.31
4	5	4.5	434	554	345	16.42	26.41	12.62	3.71
8	6	5	457	576	365	15.53	26.25	13.36	3.46
7	7	6	494	592	388	15.2	25.84	13.83	3.06
11	8	6.5	516	608	396	14.12	25.36	14.11	2.85
2	9	7.5	512	591	385	14.32	26.56	13.84	2.86
9	10	8.5	463	562	362	16.74	28.67	12.29	3.91
10	11	9	451	546	351	17.16	29.21	11.81	4.05
1	12	9.5	434	534	345	18.42	30.71	11.62	4.31
13	13	10	427	536	336	20.53	31.25	11.26	4.86

#### ANOVA for Response Surface Sixth model

Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	1.050E+005	6	17500.44	327.84	< 0.0001	significant
A-% Mo	7618.94	1	7618.94	142.73	< 0.0001	
A <sup>2</sup>	85.70	1	85.70	1.61	0.2521	
A <sup>3</sup>	741.02	1	741.02	13.88	0.0098	
A <sup>4</sup>	267.18	1	267.18	5.01	0.0666	
A <sup>5</sup>	502.58	1	502.58	9.41	0.0220	
A <sup>6</sup>	337.81	1	337.81	6.33	0.0456	
Residual	320.29	6	53.38			
Cor Total	1.053E+005	12				

The Model F-value of 327.84 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob

> F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>3</sup>, A<sup>5</sup>, A<sup>6</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant.

$$\text{Yield Strength} = +186.82151 - 3.98282 * \% \text{ Mo} + 75.72026 * \% \text{ Mo}^2 - 32.21469 \% \text{ Mo}^3 + 6.27380 * \% \text{ Mo}^4 - 0.57916 * \% \text{ Mo}^5 + 0.020104 \% \text{ Mo}^6 \quad (4.36)$$

$$\text{UTS} = +320.73388 + 122.67634 * \% \text{ Mo} - 44.52137 * \% \text{ Mo}^2 + 11.32041 * \% \text{ Mo}^3 - 1.32849 * \% \text{ Mo}^4 + 0.054043 * \% \text{ Mo}^5 \quad (4.37)$$

$$\text{Hardness} = +101.55447 + 1.90808 * \% \text{ Mo} + 29.79911 * \% \text{ Mo}^2 - 5.05595 * \% \text{ Mo}^3 + 0.22932 * \% \text{ Mo}^4 \quad (4.38)$$

$$\text{Elongation} = +26.85836 - 2.20784 * \% \text{ Mo} - 0.16242 * \% \text{ Mo}^2 + 0.032024 * \% \text{ Mo}^3 \quad (4.39)$$

$$\text{Impact Strength} = +38.35746 - 3.09729 * \% \text{ Mo} - 0.21421 * \% \text{ Mo}^2 + 0.088269 * \% \text{ Mo}^3 - 4.28106 \text{E} - 003 * \% \text{ Mo}^4 \quad (4.40)$$

$$\text{Resistivity} = +6.30123 + 1.55142 * \% \text{ Mo} - 0.49087 * \% \text{ Mo}^2 + 0.22028 * \% \text{ Mo}^3 - 0.033488 * \% \text{ Mo}^4 + 1.53229 \text{E} - 003 * \% \text{ Mo}^5 \quad (4.41)$$

$$\text{Conductivity} = +9.78150 - 2.10533 * \% \text{ Mo} + 0.16211 * \% \text{ Mo}^2 \quad (4.42)$$

These equations 4.36 to 4.42 in terms of actual factors can be used to make predictions about the response for given levels of the factor (Cr).

### 4.5.7 Design Expert Analysis for Chromium

**Table 4.8: Represents Design Expert data for Chromium**

	Factor 1	Response 1	Response 2	Response 3	Response 4	Response 5	Response 6	Response 7	
Std	Run	A:% Cr	Yield Strength	UTS	Hardness	Elongation	Impact Strength	Resistivity	Conductivity
	%	MPa	MPa	BHN	%	J	mm	Mm	
1	1	0.5	189	362	112	25.36	37.57	5.73	8.71
6	2	1.5	267	411	144	23.76	34.64	6.98	7.38
4	3	2.5	367	466	229	22.64	31.26	8.23	5.23
2	4	3.5	397	508	266	20.64	28.63	9.32	4.39
5	5	4.5	429	556	317	16.58	26.86	10.4	4.03
3	6	5	441	586	330	15.64	26.43	11.68	3.76
12	7	6	478	605	349	15.14	26.03	12.41	3.11
11	8	6.5	497	611	354	15.06	25.64	12.98	2.88
8	9	7.5	507	586	379	16.44	26.86	12.23	2.93
13	10	8.5	467	568	356	18.84	28.33	11.22	3.89
7	11	9	456	549	343	20.46	29.88	10.89	4.14
9	12	9.5	449	526	337	21.68	30.46	10.3	4.63
10	13	10	441	516	330	16.84	31.93	9.68	4.86

#### ANOVA for Response Surface Sixth model

Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	Mean Square	F Value	p-value Prob > F
Model	1.013E+005	6	16883.15	185.06 < 0.0001 significant
A-% Cr	4707.20	1	4707.20	51.60 0.0004
A <sup>2</sup>	84.68	1	84.68	0.93 0.3725
A <sup>3</sup>	217.56	1	217.56	2.38 0.1735
A <sup>4</sup>	831.61	1	831.61	9.12 0.0234
A <sup>5</sup>	245.32	1	245.32	2.69 0.1522
A <sup>6</sup>	774.31	1	774.31	8.49 0.0269
Residual	547.38	6	91.23	
Cor Total	1.018E+005	12		

The Model F-value of 185.06 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than

0.0500 indicate model terms are significant. In this case A, A<sup>4</sup>, A<sup>6</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant.

$$\text{Yield Strength} = +181.62599 - 41.06409 * \% \text{Cr} + 131.41364 * \% \text{Cr}^2 - 55.82904 * \% \text{Cr}^3 + 10.50217 * \% \text{Cr}^4 - 0.92096 * \% \text{Cr}^5 + 0.030437 * \% \text{Cr}^6 \quad (4.43)$$

$$\text{UTS} = +355.40566 + 9.53983 * \% \text{Cr} + 20.91348 * \% \text{Cr}^2 - 3.48929 * \% \text{Cr}^3 + 0.14595 * \% \text{Cr}^4 \quad (4.44)$$

$$\text{Hardness} = +56.06850 + 81.33828 * \% \text{Cr} - 5.40669 * \% \text{Cr}^2 \quad (4.45)$$

$$\text{Elongation} = +27.23824 - 5.19406 * \% \text{Cr} + 3.38025 * \% \text{Cr}^2 - 1.16997 * \% \text{Cr}^3 + 0.15719 * \% \text{Cr}^4 - 6.97756 \text{E-}003 * \% \text{Cr}^5 \quad (4.46)$$

$$\text{Impact Strength} = +39.12243 - 2.84089 * \% \text{Cr} - 0.31800 * \% \text{Cr}^2 + 0.096120 * \% \text{Cr}^3 - 4.31622 \text{E-}003 * \% \text{Cr}^4 \quad (4.47)$$

$$\text{Resistivity} = +4.62821 + 2.68700 * \% \text{Cr} - 1.15366 * \% \text{Cr}^2 + 0.35026 * \% \text{Cr}^3 - 0.043360 * \% \text{Cr}^4 + 1.76997 \text{E-}003 * \% \text{Cr}^5 \quad (4.48)$$

$$\text{Conductivity} = +8.22597 + 2.64688 * \% \text{Cr} - 3.93585 * \% \text{Cr}^2 + 1.51176 * \% \text{Cr}^3 - 0.26814 * \% \text{Cr}^4 + 0.022582 * \% \text{Cr}^4 + 0.022582 * \% \text{Cr}^5 - 7.24812 \text{E-}004 * \% \text{Cr}^6 \quad (4.49)$$

#### 4.5.8 Design Expert Analysis for Manganese

**Table 4.8: Represents Design Expert data for Manganese**

	Factor 1	Response 1	Response 2	Response 3	Response 4	Response 5	Response 6	Response 7	
Std Run	A:% Mn	Yield Strength	UTS	Hardness	Elongation	Impact Strength	Resistivity	Conductivity	
	%	MPa	MPa	BHN	%	J	mm	Mm	
7	1	0.5	192	381	111	25.68	36.75	6.06	8.86
6	2	1	238	405	137	24.41	34.14	6.82	8.15
1	3	1.5	276	437	161	23.64	33.56	7.31	7.48
10	4	2	313	463	199	22.48	32.16	7.84	6.71
3	5	2.5	358	493	237	21.58	30.56	8.38	5.13
5	6	3	397	507	263	20.36	28.76	8.86	4.93
4	7	3.5	425	522	284	18.34	27.67	9.16	4.19
8	8	4	452	549	296	17.66	26.81	9.24	4.04
9	9	4.5	483	562	310	16.47	26.41	9.46	3.83
2	10	5	497	586	341	15.63	26.25	9.93	3.66

### ANOVA for Response Surface Sixth model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	1.002E+005	6	16703.90	3715.25	< 0.0001	Significant
A-% Mn	4874.62	1	4874.62	1084.20	< 0.0001	
A <sup>2</sup>	104.28	1	104.28	23.19	0.0171	
A <sup>3</sup>	8.76	1	8.76	1.95	0.2570	
A <sup>4</sup>	55.77	1	55.77	12.40	0.0389	
A <sup>5</sup>	2.26	1	2.26	0.50	0.5293	
A <sup>6</sup>	58.21	1	58.21	12.95	0.0368	
Residual	13.49	3	4.50			
Cor Total	1.002E+005	9				

The Model F-value of 3715.25 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>4</sup>, A<sup>6</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant.

$$\text{Yield Strength} = +58.20000 + 444.80191 * \% \text{Mn} - 483.67652 * \% \text{Mn}^2 + 300.71958 * \% \text{Mn}^3 - 94.60342 * \% \text{Mn}^4 + 14.54564 * \% \text{Mn}^5 - 0.87111 * \% \text{Mn}^6 \quad (4.50)$$

$$\text{UTS} = +350.25000 + 61.87121 * \% \text{Mn} - 3.10606 * \% \text{Mn}^2 \quad (4.51)$$

$$\text{Hardness} = +124.00000 - 62.37102 * \% \text{Mn} + 93.15268 * \% \text{Mn}^2 - 25.85392 * \% \text{Mn}^3 + 2.28904 * \% \text{Mn}^4 \quad (4.52)$$

$$\text{Elongation} = +26.94400 - 2.29782 * \% \text{Mn} \quad (4.53)$$

$$\text{Impact Strength} = +47.57567 - 37.31468 * \% \text{Mn} + 41.27143 * \% \text{Mn}^2 - 22.97783 * \% \text{Mn}^3 + 6.48365 * \% \text{Mn}^4 - 0.90091 * \% \text{Mn}^5 + 0.049200 * \% \text{Mn}^6 \quad (4.54)$$

$$\text{Resistivity} = +4.65933 + 3.96426 * \% \text{Mn} - 2.91086 * \% \text{Mn}^2 + 1.35694 * \% \text{Mn}^3 - 0.29289 * \% \text{Mn}^4 + 0.022933 * \% \text{Mn}^5 \quad (4.55)$$

$$\text{Conductivity} = +10.31633 - 2.45470 * \% \text{Mn} + 0.22152 * \% \text{Mn}^2 \quad (4.56)$$

These equations 4.50 to 4.56 in terms of actual factors can be used to make predictions about the response for given levels of the factor (Mn).

#### 4.5.9 Optimization of Copper-10%Aluminium Alloy

Optimization of the copper-10%aluminium alloy was performed to determine the optimal operating conditions at which the maximum responses are achieved.

**Table 4.9: Optimization limits for the Copper-10%Aluminum Alloy**

Name	Goal	Lower limit	Upper limit	Lower weight	Lower weight	Importance
Dopants	Range	0.5	10	-1	+1	3
Yield Strength(MPa)	Maximize	410	600	-1	+1	3
UTS (MPa)	Maximize	428	750	-1	+1	3
Hardness (BHN)	Maximize	363	478	-1	+1	3
% E	Maximize	13.1	24.1	-1	+1	3
Impart Strength (J)	Maximize	15	38	-1	+1	3
Resistivity (mm)	Maximize	3	15	-1	+1	3
Conductivity (S/m)	Maximize	4	9	-1	+1	3

The variables (modifying elements) were set in the range between low and high, then responses (yield strength, ultimate tensile strength, %E, hardness, impact strength, resistivity and conductivity) were set at maximum to achieve maximum responses.

**Table 4.10: Optimization Values for Copper-10%Aluminum Alloy**

No	Dopants	Yield Strength(MPa)	UTS (MPa)	Hardness (BHN)	%E	Impact Strength (J)	Resistivity (mm)	Conductivity (S/m)	Desirability
1	6.2%Ti	568.23	691.89	392.66	18.43	28.35	9.63	6.01	0.998
2	7.1%Zr	539.49	669.02	382.40	17.99	26.49	11.20	4.87	0.998
3	6.9%Cr	510.52	635.03	363.26	15.51	25.44	10.74	3.68	0.995
4	7.8%Mo	530.35	645.21	357.82	16.56	26.43	13.35	4.04	0.984
5	7.0%W	523.34	649.16	385.38	15.84	24.66	12.75	4.21	0.998
6	3.8%Mn	454.83	533.56	273.12	17.29	25.66	8.33	5.62	0.997
7	4.0%V	467.17	557.77	361.81	16.89	26.53	9.06	4.32	0.994
8	4.6%Ni	465.63	578.47	335.97	19.49	27.63	10.62	5.30	0.998

**Table 4.11: Experimental Data for Copper-aluminum alloy**

No	Dopants	Yield Strength(MPa)	UTS (MPa)	Hardness (BHN)	%E	Impact Strength (J)	Resistivity (mm)	Conductivity (S/m)
1	6.2%Ti	561.68	685.32	379.34	18.71	27.94	10.53	6.55
2	7.1%Zr	545.21	672.86	372.48	16.67	26.65	11.34	4.65
3	6.9%Cr	508.34	635.46	366.81	15.43	26.75	11.13	3.61
4	7.8%Mo	526.06	639.24	357.84	16.37	24.53	12.45	4.13
5	7.0%W	543.09	643.54	375.96	15.32	24.32	11.64	3.96
6	3.8%Mn	446.43	524.73	282.68	14.45	26.42	9.89	4.63
7	4.0%V	468.01	546.32	361.45	17.75	26.83	10.52	4.31
8	4.6%Ni	460.40	580.03	335.58	18.73	25.64	11.54	4.58

The responses of the variables in Table 4.10 were generated by Design Expert 10.0.6 software for the optimization based on the model obtained and Table 4.11 showed the experimental data. After optimization process, an experiment was carried out with optimization parameters for the purpose of comparing the both results. It was revealed that optimum values based on the run order gave the values of mechanical and physical properties that are relatively close to the experimental results.

**Table 4.2: Comparison of Experimental Data with Sekunowo, et al (2013) and Adeyemiet al (2013)**

S/No	Sekunowo, et al (2013) with Fe	Adeyemi et al (2013) with Mg	Nwambu et al (2017) With Ti	Nwambu et al (2017) with Zr	Nwambu et al (2017) with Mo	Nwambu et al (2017) with W	Nwambu et al (2017) with Cr	Nwambu et al (2017) with Mn	Nwambu et al (2017) with V	Nwambu et al (2017) with Ni
UTS (MPa)	643	325	694	656	642	676	653	549	561	564
Yield Strength (MPa)		173	558	544	538	528	518	452	467	456
Hardness (BHN)	83.9HRB	53HRc	410	394	383	407	381	296	363	326
% E	21.7	60	15.12	14.36	14.81	14.02	14.86	17.66	16.46	17.18
Impact Strength (J)	83.9		25.23	24.83	24.76	24.16	25.45	26.81	26.88	26.89

#### **4.6 Comparison between Experimental Results obtained in this work and Sekunowo (2013) and Adeyemi(2013).**

It was noted from the experiment carried out that addition of Ti, Zr, W, Mo and Cr to aluminum bronze increases the mechanical properties better than Fe, Mg, Ni, Mn and V. Therefore the experimental results agreed with Adeyemi (2013) that the high value of mechanical properties of aluminum bronze reduces the cause of failure in engineering designs and constructions. Addition of these modifying elements will increase the mechanical properties of aluminum bronze which can be used in a substitute for propeller of a sea-going vessel.



## CHAPTER FIVE

### CONCLUSIONS AND RECOMMENDATIONS

#### 5.1 Conclusions

The study on the performance enhancement of the structural sensitive properties of copper-10%aluminum alloys using carbide forming elements was conducted and the relationship between the microstructure, physical and mechanical properties investigated. From the research, the following conclusions were drawn based on the experimental results:

1. It was established that carbide forming elements (Ti, Zr, V, Cr, Mn, Mo, W and Ni) used to modify Cu-10%Al alloy suppressed the formation of  $(\alpha+\gamma_2)$  phase, stabilized  $\beta$ -phase and precipitates the  $\alpha + \kappa$  eutectoid which has a better combination of properties.
2. It was also established that the structural sensitive properties (mechanical and physical) of copper-10% aluminum alloys have been enhanced by altering the structure.
3. It was revealed that titanium maximally enhanced the structural sensitive properties of copper-10%aluminium alloy and at 6.5% composition.
4. Also it was noted that the transformation in structure led to improvement of the UTS by 52%, yield strength by 30%, hardness by 25% and resistivity by 39%. However, there was 41.9% and 60% reduction in %E and impact strength respectively.
5. Mathematical equations were developed which will help to predict the effects of these modifying elements on the alloys at any level/parameter.

6. Finally, the optimization values obtained are relatively close to the experimental results and that these modifying elements are significant factors that affected the experimental process.

## 5.2 Contributions to Knowledge

The research work on the performance enhancement of the structural sensitive properties of copper-10% aluminum alloys using carbide forming elements has shown that:

1. The structural sensitive properties such as coarse intermetallic compound in the structure, instability of beta phase and the formation of  $\gamma^2$  phase in copper-aluminum alloys were enhanced, thereby improved the mechanical and physical properties of the alloys.
2. The mathematical equations developed will help to predict the effects of these modifying elements on the alloys at any level without carrying out the experiment thereby saving cost and time.
3. The optimization was used for finding the best level of the process factors for producing aluminum bronze. The result obtained from the optimization agrees with the experimental results.
4. These developed alloys modified with Ti, Zr, W, Mo and Cr should be used as a substitute for making component (propeller) in sea-going vessel as against alloy modified with nickel, Fe, Mg, Mn and V because of their high values of mechanical properties as obtained in this study.

### 5.3 Recommendations

The following recommendations are hereby made:

1. These developed alloys modified with Ti, Zr, W, Mo and Cr should be used as a substitute for making component (propeller) in sea-going vessel as against alloy modified with nickel, Fe, Mg, Mn and V because of their high values of mechanical properties as obtained in this study.
2. The improved copper-aluminum alloys have high tensile strength, hardness and yield strength, they are recommended for use in military application and aerospace industries.
3. It is further recommended to investigate the effects of modification of Cu-10%Al alloys with rare earth metals (cerium, lanthanum, etc) on the structure and mechanical properties.

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## APPENDIX

### Design Expert Analysis for Titanium (Design Expert 10.0.6)

Std	Run	Factor 1 A:% Ti	Response 1 Yield Strength MPa	Response 2 UTS MPa	Response 3 Hardness BHN	Response 4 Elongation %	Response 5 Impact Strength J	Response 6 Resistivity mm	Response 7 Conductivity mm
1	1	0.5	189	383	113	25.61	38.94	6.06	8.84
11	2	1.5	245	440	165	23.12	35.23	7.21	7.35
13	3	2.5	336	483	236	21.41	32.63	8.43	5.1
5	4	3.5	391	532	296	18.14	29.67	9.37	4.35
4	5	4.5	450	562	345	16.48	27.05	10.45	3.93
3	6	5	463	592	362	15.83	26.87	12.32	3.46
7	7	6	501	624	378	15.39	26.07	12.82	3.1
12	8	6.5	558	658	410	15.12	25.23	13.21	3.02
2	9	7.5	466	573	346	17.21	27.83	11.43	3.9
9	10	8.5	435	532	326	19.41	29.67	10.87	4.85
6	11	9	423	532	315	20.36	30.92	10.37	5.14
8	12	9.5	412	528	310	21.78	31.05	9.85	5.63
10	13	10	405	520	302	22.83	32.87	9.42	5.86

### Response 1: Yield Strength

Response 1      Yield Strength Transform:    None

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0198		0.3484	0.1204
Quadratic	<u>&lt; 0.0001</u>		<u>0.9263</u>	<u>0.8874</u> <u>Suggested</u>
Cubic	0.4860		0.9226	0.8394
Quartic	0.0552		0.9466	0.9101
Fifth	0.2444		0.9504	0.8075
Sixth	0.8971		0.9423	-0.6430

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	p-value	
Mean vs Total	2.140E+006	1	2.140E+006			
Linear vs Mean	48405.37	1	48405.37	7.41	0.0198	
<u>Quadratic vs Linear</u>	<u>64424.42</u>	<u>1</u>	<u>64424.42</u>	<u>87.24</u>	<u>&lt; 0.0001</u>	<u>Suggested</u>
Cubic vs Quadratic	409.06	1	409.06	0.53	0.4860	
Quartic vs Cubic	2693.14	1	2693.14	5.03	0.0552	
Fifth vs Quartic	802.77	1	802.77	1.61	0.2444	
Sixth vs Fifth	10.52	1	10.52	0.018	0.8971	
Residual	3469.49	6	578.25			
Total	2.260E+006	13	1.738E+005			

**Model Summary Statistics**

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS	
Linear	80.80	0.4027	0.3484	0.1204	1.057E+005	
<u>Quadratic</u>	<u>27.18</u>	<u>0.9386</u>	<u>0.9263</u>	<u>0.8874</u>	<u>13534.78</u>	<u>Suggested</u>
Cubic	27.84	0.9420	0.9226	0.8394	19303.91	
Quartic	23.14	0.9644	0.9466	0.9101	10805.10	
Fifth	22.30	0.9711	0.9504	0.8075	23136.23	
Sixth	24.05	0.9711	0.9423	-0.6430	1.975E+005	

**ANOVA for Response Surface Quadratic model**

**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value
Model	1.128E+005	2	56414.90	76.39	< 0.0001 significant
A-% Ti	62992.23	1	62992.23	85.30	< 0.0001
A <sup>2</sup>	64424.42	1	64424.42	87.24	< 0.0001
Residual	7384.98	10	738.50		
Cor Total	1.202E+005	12			

The Model F-value of 76.39 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	27.18	R-Squared	0.9386
Mean	405.69	Adj R-Squared	0.9263
C.V. %	6.70	Pred R-Squared	0.8874
PRESS	13534.78	Adeq Precision	25.005
-2 Log Likelihood	119.34	BIC	127.04
		AICc	128.01

The "Pred R-Squared" of 0.8874 is in reasonable agreement with the "Adj R-Squared" of 0.9263; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 25.005 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard 95% CI 95% CI			High VIF
	Estimate	df	Error	Low	High	
Intercept	476.52	1	11.52	450.85	502.19	
A-% Ti	111.00	1	12.02	84.22	137.78	1.02
A <sup>2</sup>	-200.07	1	21.42	-247.80	-152.34	1.02

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Yield Strength} = & \\ & +476.52 \\ & +111.00 * A \\ & -200.07 * A^2 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

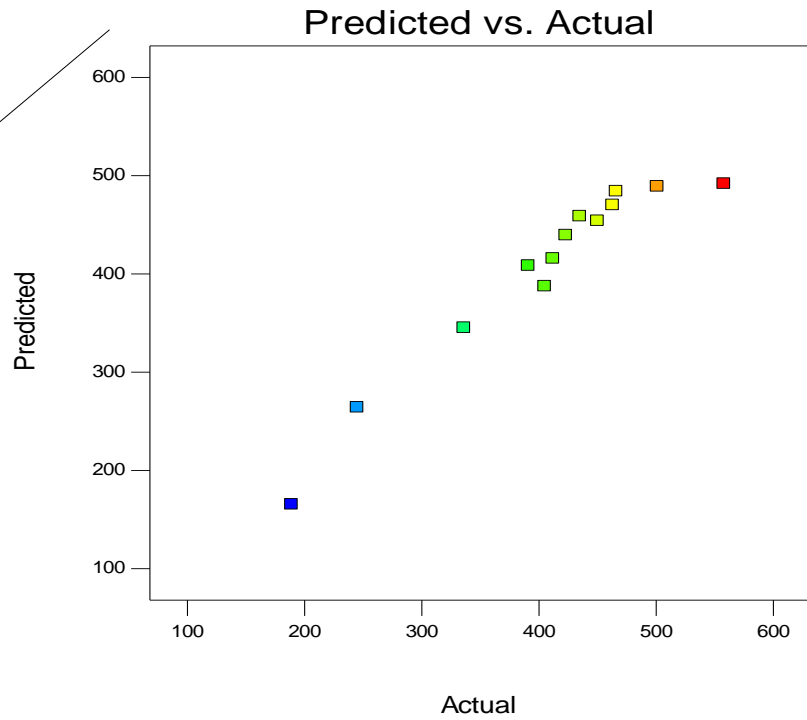
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Yield Strength} = & \\ & +109.42925 \\ & +116.47608 * \% \text{ Ti} \\ & -8.86734 * \% \text{ Ti}^2 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Yield Strength

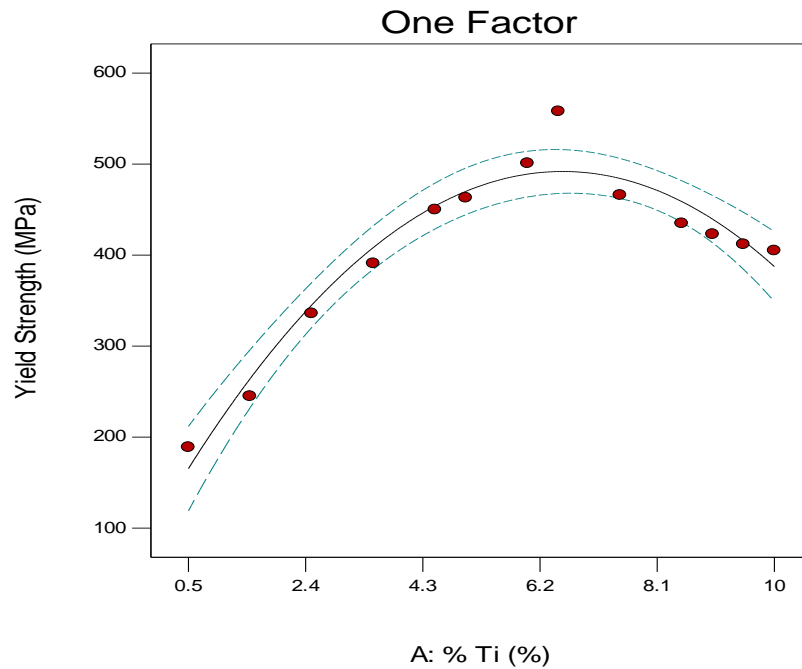
Color points by value of  
Yield Strength:





Design-Expert® Software  
 Factor Coding: Actual  
 Yield Strength (MPa)  
 • Design Points  
 --- 95% CI Bands

X1 = A: % Ti



## Response 2: Ultimate Tensile Strength

Response 2      UTS      Transform:   None

Summary (detailed tables shown below)

	Sequential Lack of Fit	Adjusted R-Squared	Predicted R-Squared
Source	p-value	p-value	R-Squared
Linear	0.0797	0.1851	-0.0830
<u>Quadratic</u>	<u>&lt; 0.0001</u>	<u>0.8686</u>	<u>0.8219</u> <u>Suggested</u>
Cubic	0.5772	0.8593	0.7610
Quartic	0.0609	0.9007	0.6373
Fifth	0.0725	0.9307	0.8241
Sixth	0.4993	0.9255	-1.1540

### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F	p-value
			Value		Prob > F
Mean vs Total	3.725E+006	1	3.725E+006		
Linear vs Mean	16095.17	1	16095.17	3.73	0.0797
<u>Quadratic vs Linear</u>	<u>40546.01</u>	<u>1</u>	<u>40546.01</u>	<u>58.23</u>	<u>&lt; 0.0001</u> <u>Suggested</u>
Cubic vs Quadratic	249.53	1	249.53	0.33	0.5772
Quartic vs	2501.89	1	2501.89	4.75	0.0609

Cubic					
Fifth vs Quartic	1640.50	1	1640.50	4.47	0.0725
Sixth vs Fifth	203.92	1	203.92	0.52	0.4993
Residual	2367.75	6	394.63		

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 3.789E+006 13 2.914E+005

additional terms are significant and the model is not aliased.

### Model Summary Statistics

Source	Std. Dev.	Adjusted R-Squared	Predicted R-Squared	Adjusted R-Squared	PRESS
Linear	65.72	0.2530	0.1851	-0.0830	68884.93
<u>Quadratic</u>	<u>26.39</u>	<u>0.8905</u>	<u>0.8686</u>	<u>0.8219</u>	<u>11330.50</u>
Cubic	27.31	0.8944	0.8593	0.7610	15201.48
Quartic	22.95	0.9338	0.9007	0.6373	23068.11
Fifth	19.17	0.9596	0.9307	0.8241	11185.25

Suggested

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

Sixth 19.87 0.9628 0.9255 -1.1540 1.370E+005

and the "Predicted R-Squared".

### ANOVA for Response Surface Quadratic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value
Model	56641.18	2	28320.59	40.67	< 0.0001 Significant
A-% Ti	23054.26	1	23054.26	33.11	0.0002
A <sup>2</sup>	40546.01	1	40546.01	58.23	< 0.0001
Residual	6963.59	10	696.36		
Cor Total	63604.77	12			

The Model F-value of 40.67 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup> are significant model terms. Values greater than

0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	26.39	R-Squared	0.8905
Mean	535.31	Adj R-Squared	0.8686
C.V. %	4.93	Pred R-Squared	0.8219
PRESS	11330.50	Adeq Precision	18.345
-2 Log Likelihood	118.58	BIC	126.27
		AICc	127.24

The "Pred R-Squared" of 0.8219 is in reasonable agreement with the "Adj R-Squared" of 0.8686; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 18.345 indicates an adequate signal. This model can be used to navigate the design space.

	<b>Coefficient</b>	<b>Standard Error</b>	<b>95% CI Low</b>	<b>95% CI High</b>	<b>VIF</b>
<b>Factor</b>	<b>Estimate</b>	<b>df</b>	<b>Error</b>	<b>Low</b>	<b>High</b>
Intercept	593.62	1	11.19	568.69	618.54
A-% Ti	67.15	1	11.67	41.15	93.16 1.02
A <sup>2</sup>	-158.72	1	20.80	-205.07	-112.37 1.02

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned}
 \text{UTS} = & \\
 & +593.62 \\
 & +67.15 * A \\
 & -158.72 * A^2
 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

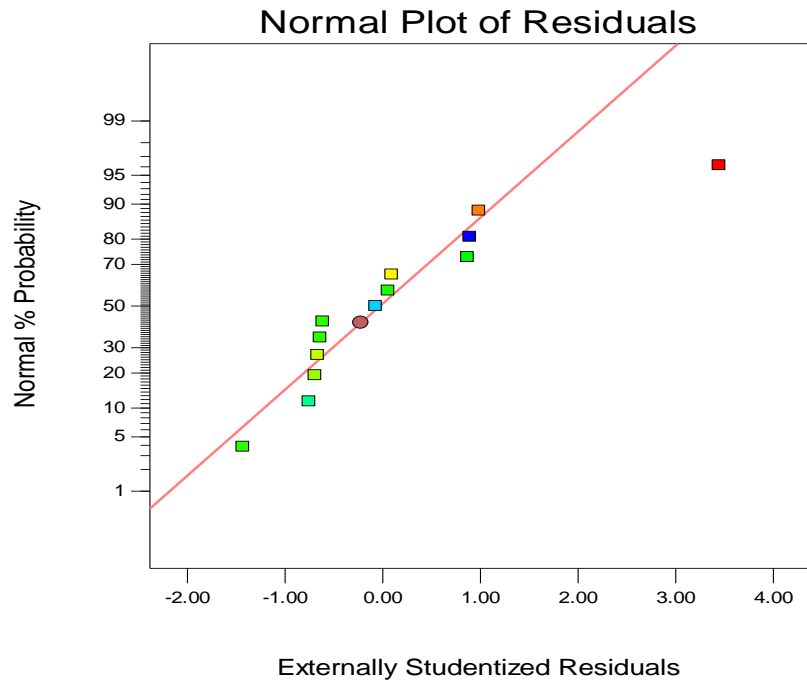
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned}
 \text{UTS} = & \\
 & +325.50081 \\
 & +88.00125 * \% \text{ Ti} \\
 & -7.03464 * \% \text{ Ti}^2
 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
UTS

Color points by value of  
UTS:

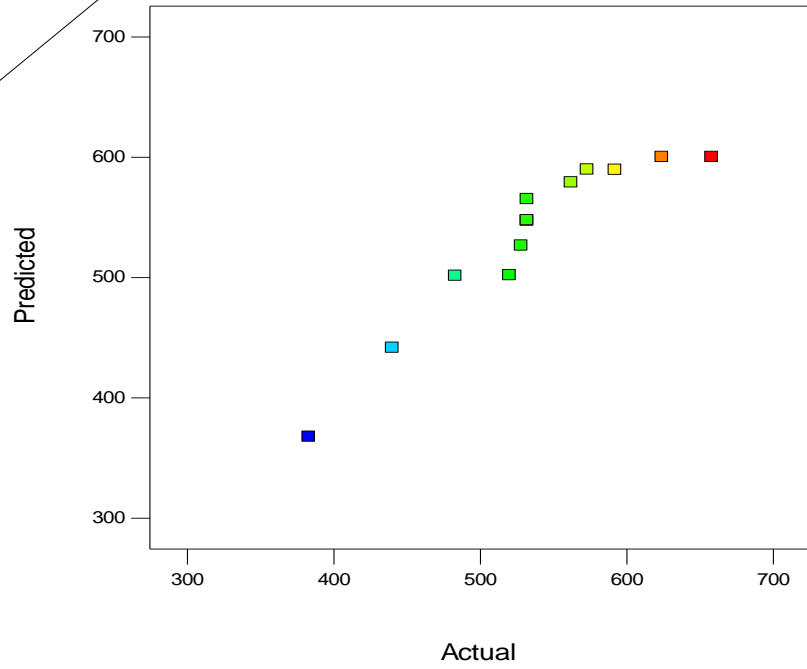


Design-Expert® Software  
UTS

Color points by value of  
UTS:



### Predicted vs. Actual

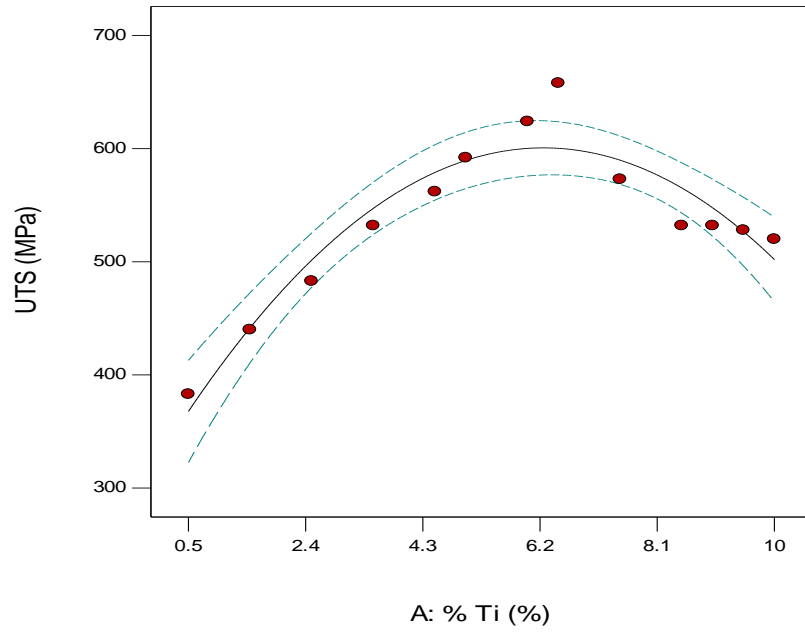


Design-Expert® Software  
Factor Coding: Actual  
UTS (MPa)

● Design Points  
--- 95% CI Bands

X1 = A: % Ti

### One Factor



### Response 3: Hardness

Response 3      Hardness   Transform:   None

Summary (detailed tables shown below)

Source	Sequential Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0182	0.3576	0.1231
Quadratic	< 0.0001	0.9507	0.9219
Cubic	0.8604	0.9454	0.8547
<u>Quartic</u>	<u>0.0056</u>	<u>0.9778</u>	<u>0.9663</u> <u>Suggested</u>
Fifth	0.4702	0.9765	0.9527
Sixth	0.7390	0.9732	0.4549

#### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob > F
Mean vs Total	1.172E+006	1	1.172E+006		
Linear vs Mean	34734.75	1	34734.75	7.68	0.0182
Quadratic vs Linear	46272.58	1	46272.58	133.30	< 0.0001
Cubic vs Quadratic	12.59	1	12.59	0.033	0.8604
<u>Quartic vs Cubic</u>	<u>2206.97</u>	<u>1</u>	<u>2206.97</u>	<u>14.10</u>	<u>0.0056</u> <u>Suggested</u>
Fifth vs Quartic	96.18	1	96.18	0.58	0.4702
Sixth vs Fifth	23.00	1	23.00	0.12	0.7390
Residual	1132.70	6	188.78		
Total	1.257E+006	13	96683.08		

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

	Std.		Adjusted	Predicted		
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS	
Linear	67.25	0.4112	0.3576	0.1231	74081.82	
Quadratic	18.63	0.9589	0.9507	0.9219	6599.42	
Cubic	19.60	0.9591	0.9454	0.8547	12278.72	
<u>Quartic</u>	<u>12.51</u>	<u>0.9852</u>	<u>0.9778</u>	<u>0.9663</u>	<u>2848.94</u>	<u>Suggested</u>
Fifth	12.85	0.9863	0.9765	0.9527	3993.60	
						"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"
Sixth	13.74	0.9866	0.9732	0.4549	46053.55	and the "Predicted R-Squared".

**ANOVA for Response Surface Quartic model**

**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	83226.89	4	20806.72	132.96	< 0.0001	significant
A-% Ti	7048.66	1	7048.66	45.04	0.0002	
A <sup>2</sup>	10652.76	1	10652.76	68.08	< 0.0001	
A <sup>3</sup>	22.14	1	22.14	0.14	0.7166	
A <sup>4</sup>	2206.97	1	2206.97	14.10	0.0056	
Residual	1251.88	8	156.48			
Cor Total	84478.77	12				

The Model F-value of 132.96 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>4</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	12.51	R-Squared	0.9852
Mean	300.31	Adj R-Squared	0.9778
C.V. %	4.17	Pred R-Squared	0.9663
PRESS	2848.94	Adeq Precision	34.531
-2 Log Likelihood	96.27	BIC	109.09
		AICc	114.84

The "Pred R-Squared" of 0.9663 is in reasonable agreement with the "Adj R-Squared" of 0.9778; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 34.531 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	374.23	1	6.48	359.30	389.16	
A-% Ti	101.31	1	15.10	66.50	136.13	7.57
A <sup>2</sup>	-301.67	1	36.56	-385.98	-217.36	13.98
A <sup>3</sup>	-7.36	1	19.58	-52.50	37.78	7.56
A <sup>4</sup>	136.26	1	36.28	52.59	219.93	13.89

#### Final Equation in Terms of Coded Factors:

$$\begin{aligned} \text{Hardness} = & \\ & +374.23 \\ & +101.31 * A \\ & -301.67 * A^2 \\ & -7.36 * A^3 \\ & +136.26 * A^4 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

#### Final Equation in Terms of Actual Factors:

$$\begin{aligned} \text{Hardness} = & \\ & +107.01490 \\ & +1.10888 * \% \text{ Ti} \\ & +31.97697 * \% \text{ Ti}^2 \end{aligned}$$



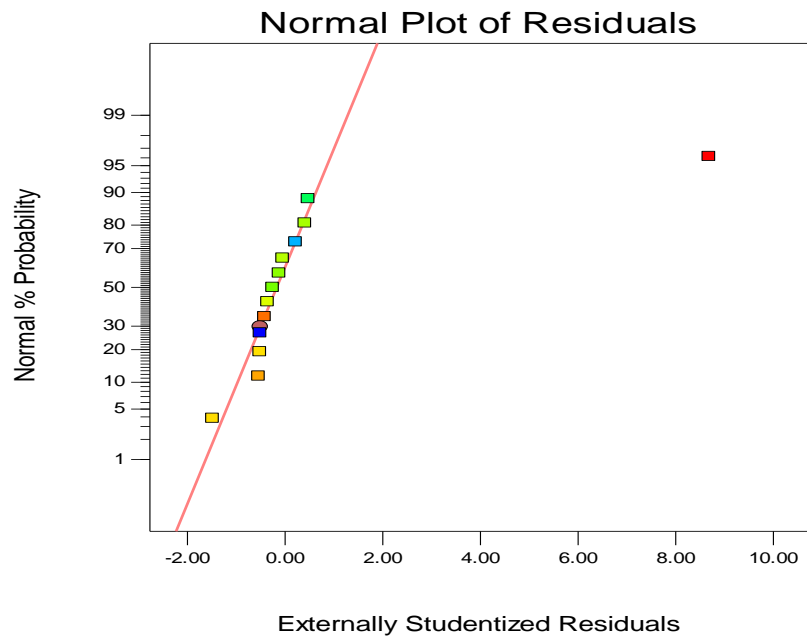
$$-5.68969 * \% \text{ Ti}^3$$

$$+0.26767 * \% \text{ Ti}^4$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Hardness

Color points by value of  
Hardness:

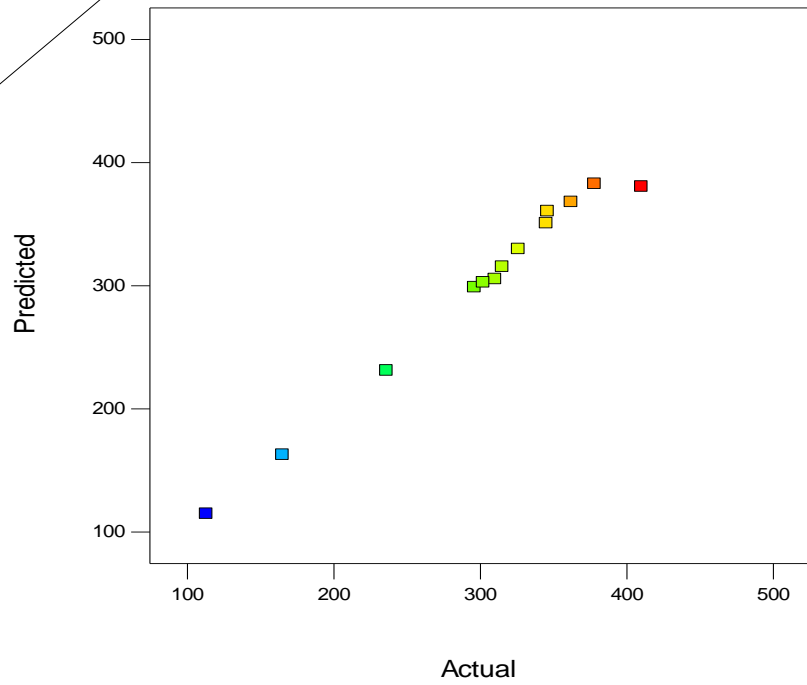


Design-Expert® Software  
Hardness

Color points by value of  
Hardness:



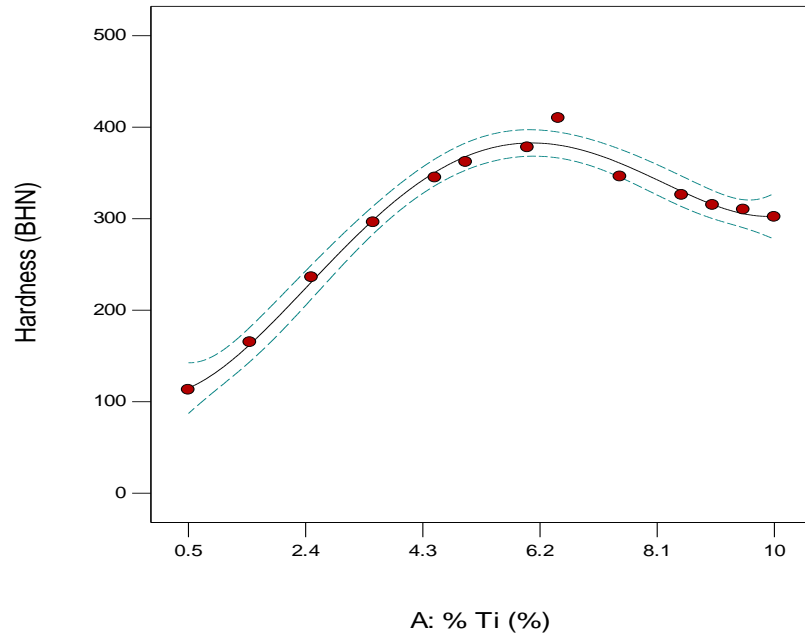
### Predicted vs. Actual



Design-Expert® Software  
Factor Coding: Actual  
Hardness (BHN)  
● Design Points  
--- 95% CI Bands

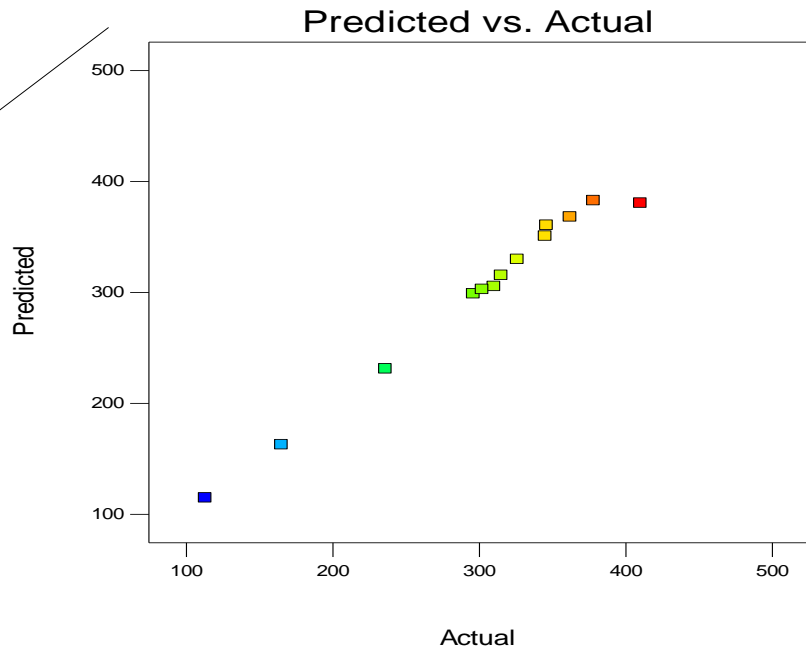
X1 = A: % Ti

### One Factor



Design-Expert® Software  
Hardness

Color points by value of  
Hardness:



## Response 4: Elongation

Response 4      Elongation Transform:    None

Summary (detailed tables shown below)

	Sequential Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.4817	-0.0407	-0.4350
Quadratic	< 0.0001	0.9559	0.9166
Cubic	0.1067	0.9639	0.9050
<u>Quartic</u>	<u>0.0011</u>	<u>0.9901</u>	<u>0.9533</u> <u>Suggested</u>
Fifth	0.2852	0.9905	0.9119
Sixth	0.3095	0.9908	0.8397

### Sequential Model Sum of Squares [Type I]

	Sum of Squares	df	Mean Square	F Value	Prob > F
Mean vs Total	4911.71	1	4911.71		
Linear vs Mean	6.29	1	6.29	0.53	0.4817
Quadratic vs Linear	125.51	1	125.51	249.82	< 0.0001

Cubic vs Quadratic	1.32	1	1.32	3.21	0.1067
<u>Quartic vs Cubic</u>	<u>2.80</u>	<u>1</u>	<u>2.80</u>	<u>24.80</u>	<u>0.0011</u>
Fifth vs Quartic	0.15	1	0.15	1.34	0.2852
Sixth vs Fifth	0.13	1	0.13	1.23	0.3095
Residual	0.63	6	0.10		

Suggested

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 5048.54 13 388.35

additional terms are significant and the model is not aliased.

### Model Summary Statistics

	Std.		Adjusted	Predicted	
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS
Linear	3.44	0.0460	-0.0407	-0.4350	196.36
Quadratic	0.71	0.9633	0.9559	0.9166	11.41
Cubic	0.64	0.9729	0.9639	0.9050	12.99
<u>Quartic</u>	<u>0.34</u>	<u>0.9934</u>	<u>0.9901</u>	<u>0.9533</u>	<u>6.38</u>
Fifth	0.33	0.9945	0.9905	0.9119	12.06
Sixth	0.32	0.9954	0.9908	0.8397	21.93

Suggested

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

and the "Predicted R-Squared".

### ANOVA for Response Surface Quartic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	135.93	4	33.98	301.00	< 0.0001	Significant
A-% Ti	6.80	1	6.80	60.22	< 0.0001	

$A^2$	21.35	1	21.35	189.15	< 0.0001
$A^3$	1.42	1	1.42	12.55	0.0076
$A^4$	2.80	1	2.80	24.80	0.0011
Residual	0.90	8	0.11		
Cor Total	136.83	12			

The Model F-value of 301.00 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A,  $A^2$ ,  $A^3$ ,  $A^4$  are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.34	R-Squared	0.9934
Mean	19.44	Adj R-Squared	0.9901
C.V. %	1.73	Pred R-Squared	0.9533
PRESS	6.38	Adeq Precision	48.435
-2 Log Likelihood	2.22	BIC	15.05
		AICc	20.80

The "Pred R-Squared" of 0.9533 is in reasonable agreement with the "Adj R-Squared" of 0.9901; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 48.435 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	15.51	1	0.17	15.11	15.91	
A-% Ti	-3.15	1	0.41	-4.08	-2.21	7.57
$A^2$	13.51	1	0.98	11.24	15.77	13.98
$A^3$	1.86	1	0.53	0.65	3.08	7.56
$A^4$	-4.85	1	0.97	-7.10	-2.61	13.89

#### Final Equation in Terms of Coded Factors:

$$\begin{aligned} \text{Elongation} = & \\ & +15.51 \\ & -3.15 * A \end{aligned}$$

$$\begin{aligned}
&+13.51 * A^2 \\
&+1.86 * A^3 \\
&-4.85 * A^4
\end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

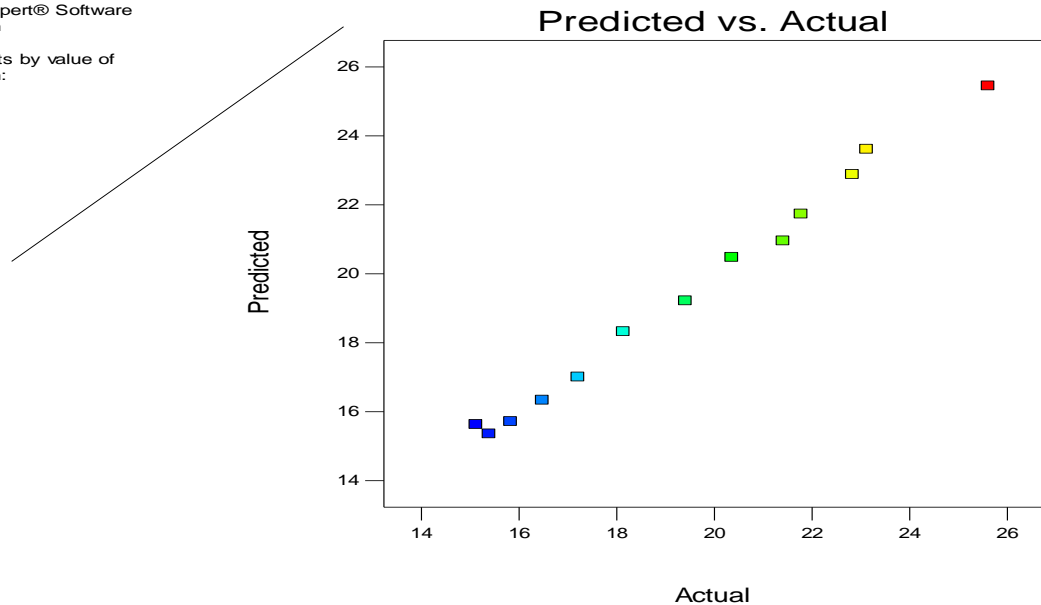
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned}
\text{Elongation} = & \\
&+25.73192 \\
&+7.28985\text{E-}003 * \% \text{ Ti} \\
&-1.25171 * \% \text{ Ti}^2 \\
&+0.21758 * \% \text{ Ti}^3 \\
&-9.53343\text{E-}003 * \% \text{ Ti}^4
\end{aligned}$$

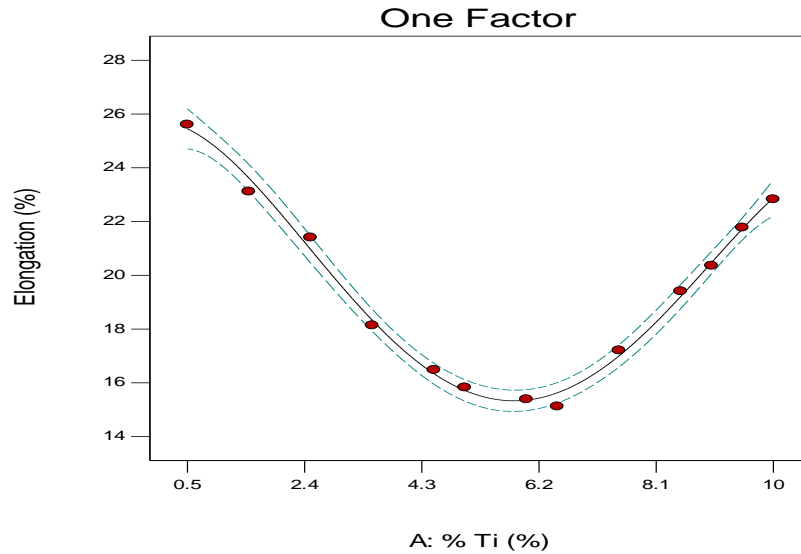
The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Elongation

Color points by value of  
Elongation:  
25.61  
15.12



Design-Expert® Software  
 Factor Coding: Actual  
 Elongation (%)  
 ● Design Points  
 --- 95% CI Bands  
 X1 = A: % Ti



## Response 5: Impact Strength

Response 5      Impact Strength Transform:    None

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.1593		0.0964	-0.2651
<u>Quadratic</u>	<u>&lt; 0.0001</u>		<u>0.9689</u>	<u>0.9556</u> <u>Suggested</u>
Cubic	0.5470		0.9669	0.9409
<u>Quartic</u>	<u>0.0256</u>		<u>0.9808</u>	<u>0.9331</u> <u>Suggested</u>
Fifth	0.3324		0.9810	0.8724
Sixth	0.2202		0.9831	0.8889

### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	P-value	Prob > F
Mean vs Total	11943.05	1	11943.05			
Linear vs Mean	31.71	1	31.71	2.28	0.1593	
<u>Quadratic vs Linear</u>	<u>148.25</u>	<u>1</u>	<u>148.25</u>	<u>309.67</u>	<u>0.0001</u>	<u>≤</u> <u>Suggested</u>

Cubic vs Quadratic	0.20	1	0.20	0.39	0.5470
<u>Quartic vs Cubic</u>	<u>2.22</u>	<u>1</u>	<u>2.22</u>	<u>7.49</u>	<u>0.0256</u>
Fifth vs Quartic	0.32	1	0.32	1.08	0.3324
Sixth vs Fifth	0.49	1	0.49	1.87	0.2202
Residual	1.56	6	0.26		

Suggested

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 12127.80 13 932.91

additional terms are significant and the model is not aliased.

### Model Summary Statistics

	Std.		Adjusted	Predicted	
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS
Linear	3.73	0.1717	0.0964	-0.2651	233.74
<u>Quadratic</u>	<u>0.69</u>	<u>0.9741</u>	<u>0.9689</u>	<u>0.9556</u>	<u>8.21</u>
Cubic	0.71	0.9752	0.9669	0.9409	10.93
<u>Quartic</u>	<u>0.54</u>	<u>0.9872</u>	<u>0.9808</u>	<u>0.9331</u>	<u>12.36</u>
Fifth	0.54	0.9889	0.9810	0.8724	23.58
Sixth	0.51	0.9915	0.9831	0.8889	20.53

Suggested

Suggested

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

and the "Predicted R-Squared".

### ANOVA for Response Surface Quartic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	182.38	4	45.60	153.98	< 0.0001	significant
A-% Ti	9.77	1	9.77	32.98	0.0004	



$A^2$	22.22	1	22.22	75.03	< 0.0001
$A^3$	0.23	1	0.23	0.79	0.4004
$A^4$	2.22	1	2.22	7.49	0.0256
Residual	2.37	8	0.30		
Cor Total	184.75	12			

The Model F-value of 153.98 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A,  $A^2$ ,  $A^4$  are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.54	R-Squared	0.9872
Mean	30.31	Adj R-Squared	0.9808
C.V. %	1.80	Pred R-Squared	0.9331
PRESS	12.36	Adeq Precision	37.700
-2 Log Likelihood	14.76	BIC	27.58
		AICc	33.33

The "Pred R-Squared" of 0.9331 is in reasonable agreement with the "Adj R-Squared" of 0.9808; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 37.700 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	26.26	1	0.28	25.61	26.91	
A-% Ti	-3.77	1	0.66	-5.29	-2.26	7.57
$A^2$	13.78	1	1.59	10.11	17.44	13.98
$A^3$	0.76	1	0.85	-1.21	2.72	7.56
$A^4$	-4.32	1	1.58	-7.96	-0.68	13.89

#### Final Equation in Terms of Coded Factors:

$$\begin{aligned} \text{Impact Strength} = & \\ & +26.26 \\ & -3.77 * A \\ & +13.78 * A^2 \\ & +0.76 * A^3 \end{aligned}$$

$$-4.32 * A^4$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

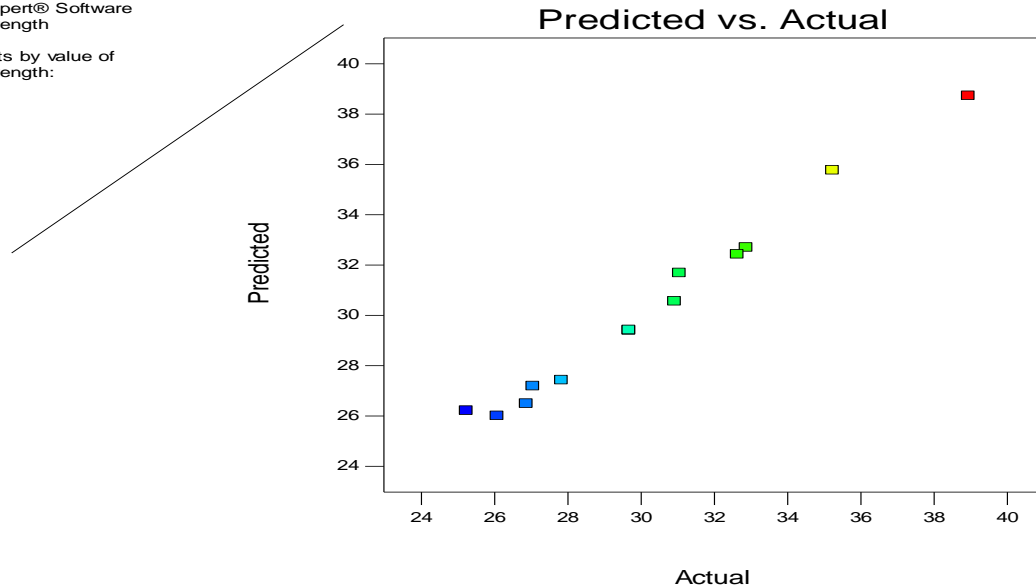
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Impact Strength} = & \\ & +39.78763 \\ & -1.70937 * \% \text{ Ti} \\ & -0.90408 * \% \text{ Ti}^2 \\ & +0.18528 * \% \text{ Ti}^3 \\ & -8.48694\text{E-}003 * \% \text{ Ti}^4 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

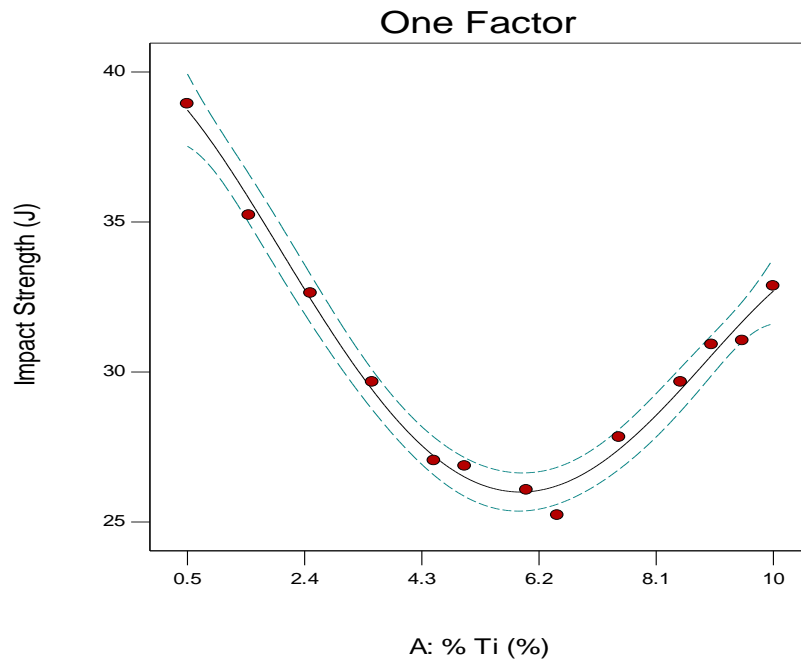
Design-Expert® Software  
Impact Strength

Color points by value of  
Impact Strength:



Design-Expert® Software  
 Factor Coding: Actual  
 Impact Strength (J)  
 • Design Points  
 --- 95% CI Bands

X1 = A: % Ti



## Response 6: Resistivity

Response 6      Resistivity    Transform:    None

Summary (detailed tables shown below)

Source	p-value	Sequential Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0542		0.2328	-0.0077
<u>Quadratic</u>	<u>&lt; 0.0001</u>		<u>0.8686</u>	<u>0.7885</u> <u>Suggested</u>
Cubic	0.0797		0.8981	0.8281
Quartic	0.0625		0.9277	0.7332
Fifth	0.1233		0.9426	0.8534
Sixth	0.2457		0.9475	-1.0532

### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	Df	Mean Square	F	p-value
Mean vs Total	1336.45	1	1336.45		
Linear vs Mean	15.75	1	15.75	4.64	0.0542
<u>Quadratic vs</u>	<u>31.50</u>	<u>1</u>	<u>31.50</u>	<u>54.23</u>	<u>&lt;0.0001</u>

Suggested

<u>Linear</u>					
Cubic vs Quadratic	1.76	1	1.76	3.90	0.0797
Quartic vs Cubic	1.50	1	1.50	4.68	0.0625
Fifth vs Quartic	0.78	1	0.78	3.07	0.1233
Sixth vs Fifth	0.38	1	0.38	1.65	0.2457
Residual	1.39	6	0.23		

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 1389.51 13 106.89

additional terms are significant and the model is not aliased.

### Model Summary Statistics

	Std.	Adjusted		Predicted		
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS	
Linear	1.84	0.2968	0.2328	-0.0077	53.46	
<u>Quadratic</u>	<u>0.76</u>	<u>0.8905</u>	<u>0.8686</u>	<u>0.7885</u>	<u>11.22</u>	<u>Suggested</u>
Cubic	0.67	0.9236	0.8981	0.8281	9.12	
Quartic	0.57	0.9518	0.9277	0.7332	14.15	
Fifth	0.50	0.9665	0.9426	0.8534	7.78	
Sixth	0.48	0.9737	0.9475	-1.0532	108.94	

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

### ANOVA for Response Surface Quadratic model

Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	47.25	2	23.62	40.67	< 0.0001 Significant

A-% Ti	21.72	1	21.72	37.39	0.0001
A <sup>2</sup>	31.50	1	31.50	54.23	< 0.0001
Residual	5.81	10	0.58		
Cor Total	53.06	12			

The Model F-value of 40.67 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.76	R-Squared	0.8905
Mean	10.14	Adj R-Squared	0.8686
C.V. %	7.52	Pred R-Squared	0.7885
PRESS	11.22	Adeq Precision	18.357
-2 Log Likelihood	26.42	BIC	34.12
		AICc	35.09

The "Pred R-Squared" of 0.7885 is in reasonable agreement with the "Adj R-Squared" of 0.8686; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 18.357 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard 95% CI 95% CI		
	Estimate	df	Error	Low	High VIF
Intercept	11.75	1	0.32	11.03	12.47
A-% Ti	2.06	1	0.34	1.31	2.81 1.02
A <sup>2</sup>	-4.42	1	0.60	-5.76	-3.09 1.02

#### Final Equation in Terms of Coded Factors:

$$\begin{aligned} \text{Resistivity} = & \\ & +11.75 \\ & +2.06 * A \\ & -4.42 * A^2 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

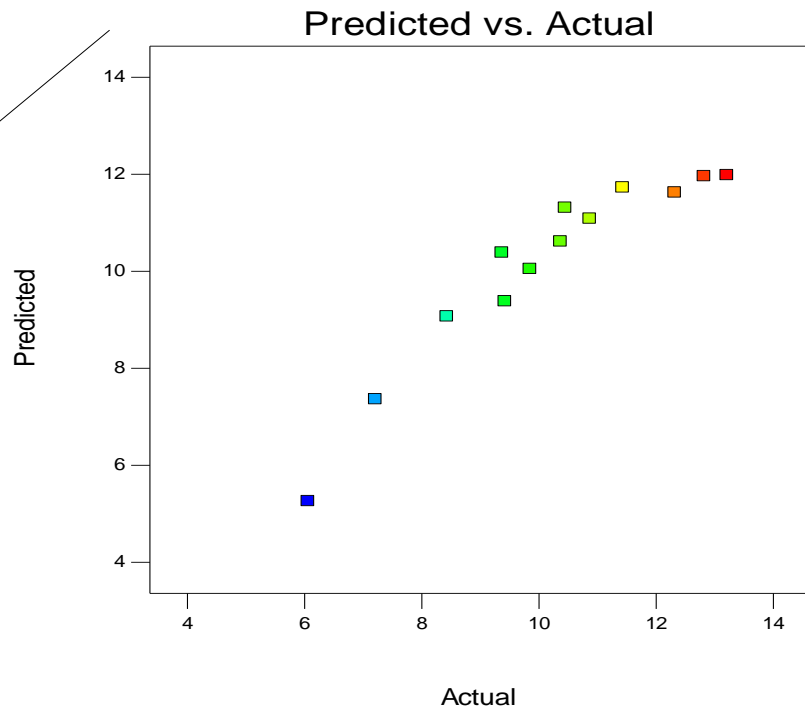
### Final Equation in Terms of Actual Factors:

$$\begin{aligned} \text{Resistivity} = & \\ & +4.06266 \\ & +2.49279 * \% \text{ Ti} \\ & -0.19608 * \% \text{ Ti}^2 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

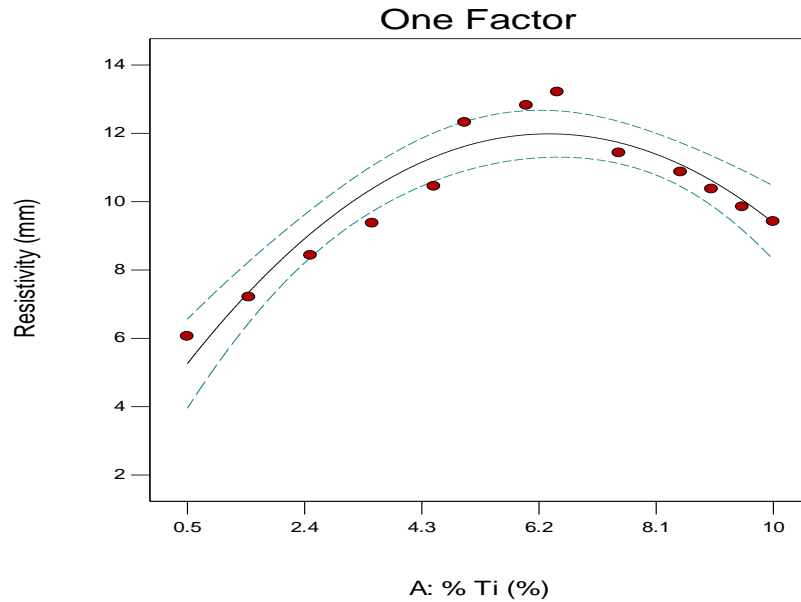
Design-Expert® Software  
Resistivity

Color points by value of  
Resistivity:  
13.21  
6.06



Design-Expert® Software  
 Factor Coding: Actual  
 Resistivity (mm)  
 • Design Points  
 --- 95% CI Bands

X1 = A: % Ti



## Response 7: Conductivity

Response 7      Conductivity Transform:    None

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.1940		0.0707	-0.3357
<u>Quadratic</u>	<u>&lt; 0.0001</u>		<u>0.9687</u>	<u>0.9543</u> <u>Suggested</u>
Cubic	0.2709		0.9698	0.9327
Quartic	0.1857		0.9731	0.9507
Fifth	0.3726		0.9728	0.5100
Sixth	0.2454		0.9751	-1.1054

### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Mean vs Total	320.32	1	320.32			
Linear vs Mean	5.06	1	5.06	1.91	0.1940	
<u>Quadratic vs</u>	<u>28.17</u>	<u>1</u>	<u>28.17</u>	<u>316.17</u>	<u>&lt;0.0001</u>	

Suggested

<u>Linear</u>					
Cubic vs Quadratic	0.12	1	0.12	1.38	0.2709
Quartic vs Cubic	0.16	1	0.16	2.10	0.1857
Fifth vs Quartic	0.070	1	0.070	0.91	0.3726
Sixth vs Fifth	0.12	1	0.12	1.66	0.2454
Residual	0.42	6	0.071		

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 354.44 13 27.26

additional terms are significant and the model is not aliased.

### Model Summary Statistics

	Std.	Adjusted		Predicted		
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS	
Linear	1.63	0.1482	0.0707	-0.3357	45.57	
<u>Quadratic</u>	<u>0.30</u>	<u>0.9739</u>	<u>0.9687</u>	<u>0.9543</u>	<u>1.56</u>	<u>Suggested</u>
Cubic	0.29	0.9773	0.9698	0.9327	2.30	
Quartic	0.28	0.9820	0.9731	0.9507	1.68	
Fifth	0.28	0.9841	0.9728	0.5100	16.72	
Sixth	0.27	0.9875	0.9751	-1.1054	71.83	

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

### ANOVA for Response Surface Quadratic model

Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	33.23	2	16.61	186.45	< 0.0001 significant



A-% Ti	8.50	1	8.50	95.41	< 0.0001
A <sup>2</sup>	28.17	1	28.17	316.17	< 0.0001
Residual	0.89	10	0.089		
Cor Total	34.12	12			

The Model F-value of 186.45 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.30	R-Squared	0.9739
Mean	4.96	Adj R-Squared	0.9687
C.V. %	6.01	Pred R-Squared	0.9543
PRESS	1.56	Adeq Precision	38.861
-2 Log Likelihood	2.05	BIC	9.74
		AICc	10.72

The "Pred R-Squared" of 0.9543 is in reasonable agreement with the "Adj R-Squared" of 0.9687; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 38.861 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	3.38	1	0.13	3.10	3.66	
A-% Ti	-1.29	1	0.13	-1.58	-1.00	1.02
A <sup>2</sup>	4.18	1	0.24	3.66	4.71	1.02

#### Final Equation in Terms of Coded Factors:

$$\begin{aligned} \text{Conductivity} = & \\ & +3.38 \\ & -1.29 * A \\ & +4.18 * A^2 \end{aligned}$$

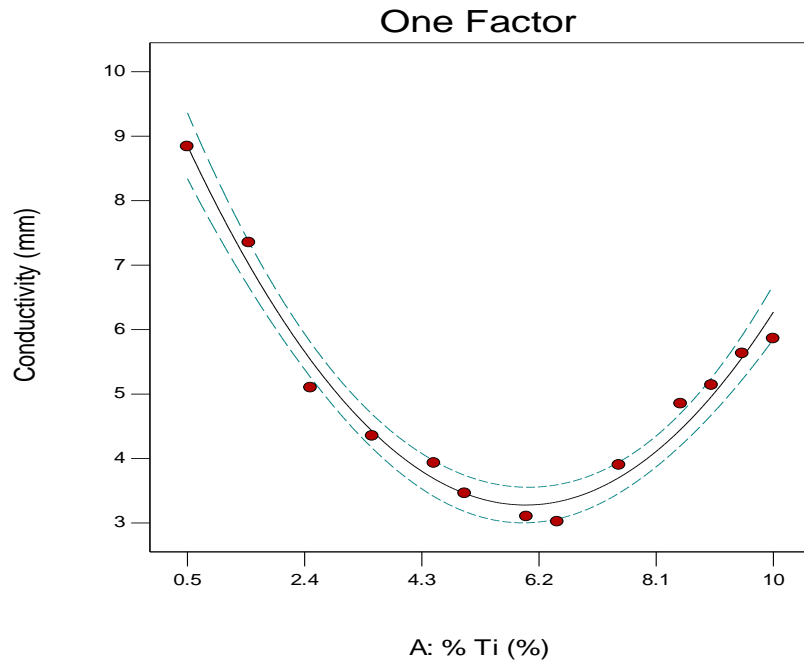
The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

### Final Equation in Terms of Actual Factors:

$$\begin{aligned} \text{Conductivity} = & \\ & +9.91449 \\ & -2.21852 * \% \text{ Ti} \\ & +0.18543 * \% \text{ Ti}^2 \end{aligned}$$

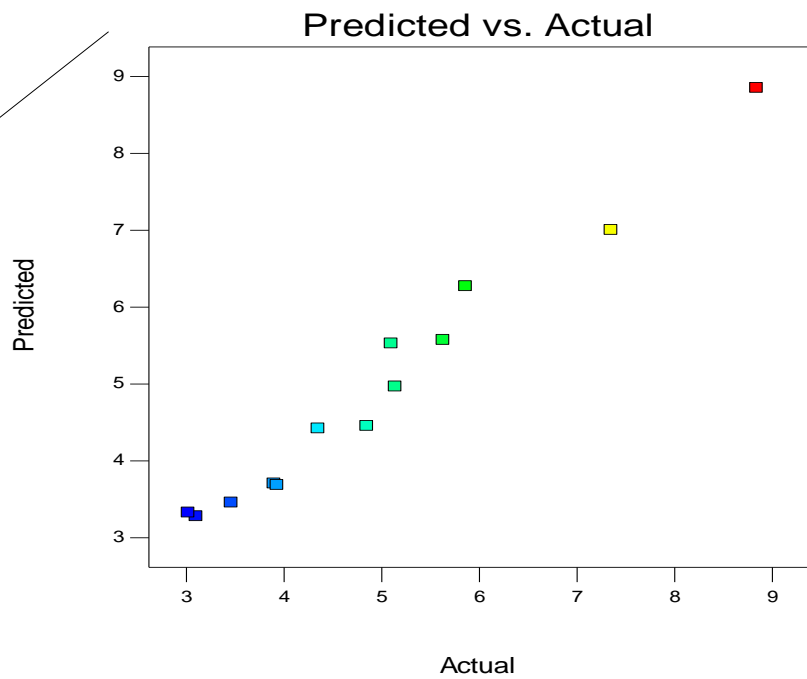
The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Factor Coding: Actual  
Conductivity (mm)  
● Design Points  
--- 95% CI Bands  
X1 = A: % Ti



Design-Expert® Software  
Conductivity

Color points by value of  
Conductivity:



**Factor Name Level Low Level High Level Std. Dev. Coding**

A % Ti 9.82 0.50 10.00 0.000 Actual

Response	Predicted		Observed	Std Dev	SE Mean	CI for Mean		99% of Population	
	Mean	Median <sup>1</sup>				95% CI low	95% CI high	95% TI low	95% TI high
Yield Strength	398.15	398.15	-	27.1753	16.0046	362.49	433.811	257.629	538.671
UTS	511.327	511.327	-	26.3886	15.5413	476.699	545.955	374.874	647.78
Hardness	302.661	302.661	-	12.5094	8.69004	282.622	322.701	231.37	373.953
Elongation	22.4869	22.4869	-	0.335996	0.23341	21.9487	23.0252	20.572	24.4018
Impact Strength	32.353	32.353	-	0.544175	0.378028	31.4813	33.2247	29.2517	35.4543
Resistivity	9.6338	9.6338	-	0.762176	0.448876	8.63364	10.634	5.69266	13.5749
Conductivity	6.00967	6.00967	-	0.298509	0.175804	5.61795	6.40139	4.4661	7.55323

**Confirmation Report**

Two-sided Confidence = 95% n = 1

Factor	Name	Level Low	Level High	Level Std. Dev.	Coding
A	% Ti	9.82	0.50	10.00	0.000 Actual

		Predicted Predicted							
Response	Mean	Median <sup>1</sup>	Observed	Std Dev	n	SE Pred	95% PI low	Data Mean	95% PI high
Yield Strength	398.15	398.15	-	27.1753	1	31.54	327.88		468.42
UTS	511.327	511.327	-	26.3886	1	30.63	443.09		579.56
Hardness	302.661	302.661	-	12.5094	1	15.23	267.54		337.79
Elongation	22.4869	22.4869	-	0.335996	1	0.41	21.54		23.43
Impact Strength	32.353	32.353	-	0.544175	1	0.66	30.83		33.88
Resistivity	9.6338	9.6338	-	0.762176	1	0.88	7.66		11.60
Conductivity	6.00967	6.00967	-	0.298509	1	0.35	5.24		6.78

Response	Intercept	A	A <sup>2</sup>	A <sup>3</sup>	A <sup>4</sup>
Yield Strength	476.523	111.003	-200.069		
p=		< 0.0001	< 0.0001		
UTS	593.615	67.1532	-158.719		
p=		0.0002	< 0.0001		
Hardness	374.23	101.315	-301.668	-7.36271	136.26
p=		0.0002	< 0.0001	0.7166	0.0056
Elongation	15.5123	-3.14642	13.5064	1.86273	-4.85315
p=		< 0.0001	< 0.0001	0.0076	0.0011
Impact Strength	26.2583	-3.77135	13.7766	0.756294	-4.32042
p=		0.0004	< 0.0001	0.4004	0.0256
Resistivity	11.7453	2.06118	-4.4241		
p=		0.0001	< 0.0001		
Conductivity	3.37823	-1.28955	4.18381		
p=		< 0.0001	< 0.0001		
Legend		p < .01	.01 <= p < .05	.05 <= p < .10	p >= .10

## Design Expert Analysis for Zirconium

### Response 1: Yield Strength

Response 1      Yield Strength Transform:    None

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0073		0.4491	0.2431
Quadratic	< 0.0001		0.8833	0.8182
<u>Cubic</u>	<u>0.0226</u>		<u>0.9295</u>	<u>0.9019</u> <u>Suggested</u>
Quartic	0.8672		0.9209	0.8356

Fifth	0.7268		0.9113	0.4954
Sixth	0.1316		0.9314	-0.5653

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Mean vs Total	2.247E+006	1	2.247E+006		
Linear vs Mean	63891.46	1	63891.46	10.78	0.0073
Quadratic vs Linear	52623.29	1	52623.29	41.92	< 0.0001
<u>Cubic vs Quadratic</u>	<u>5725.89</u>	<u>1</u>	<u>5725.89</u>	<u>7.55</u>	<u>0.0226</u>
Quartic vs Cubic	25.34	1	25.34	0.030	0.8672
Fifth vs Quartic	126.15	1	126.15	0.13	0.7268
Sixth vs Fifth	2247.15	1	2247.15	3.04	0.1316
Residual	4429.02	6	738.17		

Suggested

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 2.376E+006 13 1.828E+005

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS
Linear	76.98	0.4950	0.4491	0.2431	97694.36
Quadratic	35.43	0.9027	0.8833	0.8182	23463.20
<u>Cubic</u>	<u>27.54</u>	<u>0.9471</u>	<u>0.9295</u>	<u>0.9019</u>	<u>12663.86</u>
Quartic	29.16	0.9473	0.9209	0.8356	21217.53
Fifth	30.88	0.9483	0.9113	0.4954	65129.93
Sixth	27.17	0.9657	0.9314	-0.5653	2.020E+005

Suggested

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

and the "Predicted R-Squared".

**ANOVA for Response Surface Cubic model**

**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value
Model	1.222E+005	3	40746.88	53.71	< 0.0001 significant
A-% Zr	30030.08	1	30030.08	39.58	0.0001
A <sup>2</sup>	51282.95	1	51282.95	67.60	< 0.0001
A <sup>3</sup>	5725.89	1	5725.89	7.55	0.0226
Residual	6827.67	9	758.63		
Cor Total	1.291E+005	12			

The Model F-value of 53.71 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	27.54	R-Squared	0.9471
Mean	415.77	Adj R-Squared	0.9295
C.V. %	6.62	Pred R-Squared	0.9019
PRESS	12663.86	Adeq Precision	20.633
-2 Log Likelihood	118.32	BIC	128.58
		AICc	131.32

The "Pred R-Squared" of 0.9019 is in reasonable agreement with the "Adj R-Squared" of 0.9295; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 20.633 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard 95% CI 95% CI			VIF
	Estimate	df	Error	Low	High	
Intercept	476.26	1	11.68	449.83	502.69	
A-% Zr	208.77	1	33.18	133.70	283.83	7.55
A <sup>2</sup>	-178.62	1	21.73	-227.77	-129.48	1.02
A <sup>3</sup>	-118.37	1	43.09	-215.85	-20.90	7.55

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned}
 \text{Yield Strength} = & \\
 & +476.26 \\
 & +208.77 * A \\
 & -178.62 * A^2
 \end{aligned}$$

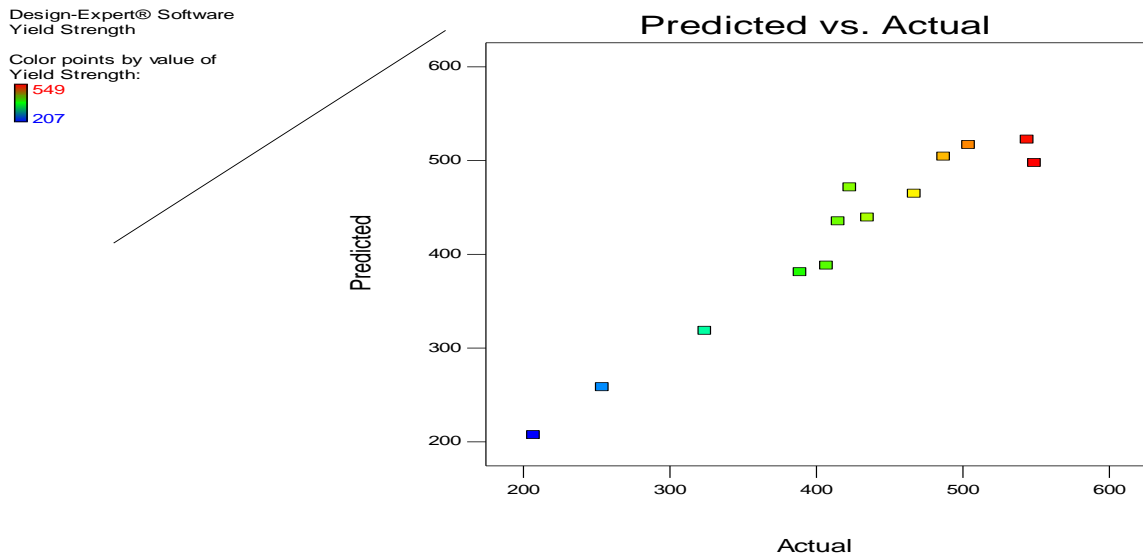
$$-118.37 * A^3$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

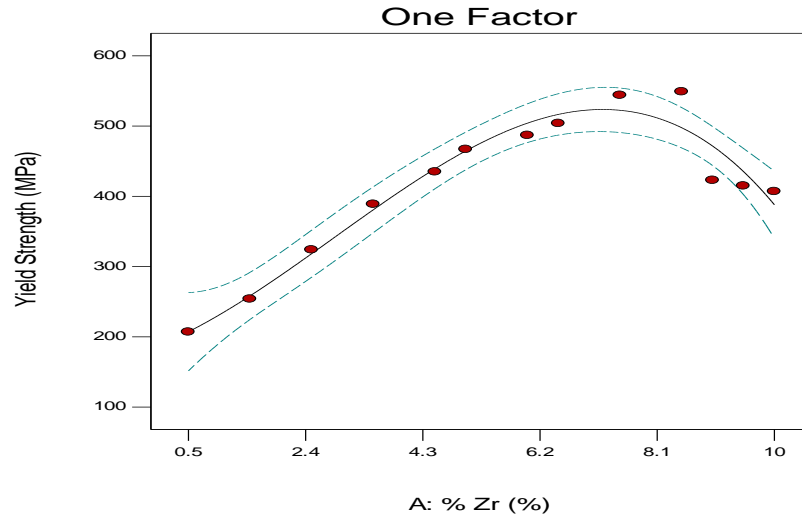
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Yield Strength} = & \\ & +187.13741 \\ & +35.74624 * \% \text{Zr} \\ & +9.47955 * \% \text{Zr}^2 \\ & -1.10453 * \% \text{Zr}^3 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.



Design-Expert® Software  
 Factor Coding: Actual  
 Yield Strength (MPa)  
 ● Design Points  
 --- 95% CI Bands  
 X1 = A: % Zr



## Response 2: Ultimate Tensile Strength

Response 2      UTS      Transform:   None

Summary (detailed tables shown below)

Source	p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.1476	0.1063	-0.2022
<u>Quadratic</u>	<u>&lt;0.0001</u>	<u>0.8886</u>	<u>0.8162</u> <u>Suggested</u>
Cubic	0.1032	0.9093	0.8068
Quartic	0.0881	0.9307	0.8141
Fifth	0.0718	0.9517	0.5965
Sixth	0.1658	0.9602	0.8171

### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	p-value
Mean vs Total	3.494E+006	1	3.494E+006		
Linear vs Mean	13386.15	1	13386.15	2.43	0.1476
<u>Quadratic vs Linear</u>	<u>53802.46</u>	<u>1</u>	<u>53802.46</u>	<u>78.24</u>	<u>0.0001</u> <u>Suggested</u>
Cubic vs Quadratic	1840.35	1	1840.35	3.29	0.1032
Quartic vs Cubic	1613.52	1	1613.52	3.77	0.0881
Fifth vs	1338.00	1	1338.00	4.49	0.0718



Quartic					
Sixth vs Fifth	610.95	1	610.95	2.49	0.1658
Residual	1473.79	6	245.63		

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 3.568E+006 13 2.745E+005

additional terms are significant and the model is not aliased.

### Model Summary Statistics

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS	
Linear	74.27	0.1807	0.1063	-0.2022	89040.74	
Quadratic	<u>26.22</u>	<u>0.9072</u>	<u>0.8886</u>	<u>0.8162</u>	<u>13613.78</u>	<u>Suggested</u>
Cubic	23.66	0.9320	0.9093	0.8068	14308.60	
Quartic	20.68	0.9538	0.9307	0.8141	13766.52	
Fifth	17.26	0.9719	0.9517	0.5965	29885.39	
Sixth	15.67	0.9801	0.9602	0.8171	13549.47	

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

### ANOVA for Response Surface Quadratic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	67188.61	2	33594.30	48.85	< 0.0001	significant
A-% Zr	20943.09	1	20943.09	30.46	0.0003	
A <sup>2</sup>	53802.46	1	53802.46	78.24	< 0.0001	
Residual	6876.63	10	687.66			
Cor Total	74065.23	12				

The Model F-value of 48.85 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	26.22	R-Squared	0.9072
Mean	518.46	Adj R-Squared	0.8886
C.V. %	5.06	Pred R-Squared	0.8162
PRESS	13613.78	Adeq Precision	20.035
-2 Log Likelihood	118.41	BIC	126.11
		AICc	127.08

The "Pred R-Squared" of 0.8162 is in reasonable agreement with the "Adj R-Squared" of 0.8886; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 20.035 indicates an adequate signal. This model can be used to navigate the design space.

	<b>Coefficient</b>	<b>Standard Error</b>	<b>95% CI</b>	<b>95% CI</b>		
<b>Factor</b>	<b>Estimate</b>	<b>df</b>	<b>Low</b>	<b>High</b>	<b>VIF</b>	
Intercept	586.98	1	11.12	562.21	611.75	
A-% Zr	64.00	1	11.60	38.16	89.85	1.02
A <sup>2</sup>	-182.83	1	20.67	-228.89	-136.78	1.02

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned}
 \text{UTS} = & \\
 & +586.98 \\
 & +64.00 * A \\
 & -182.83 * A^2
 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

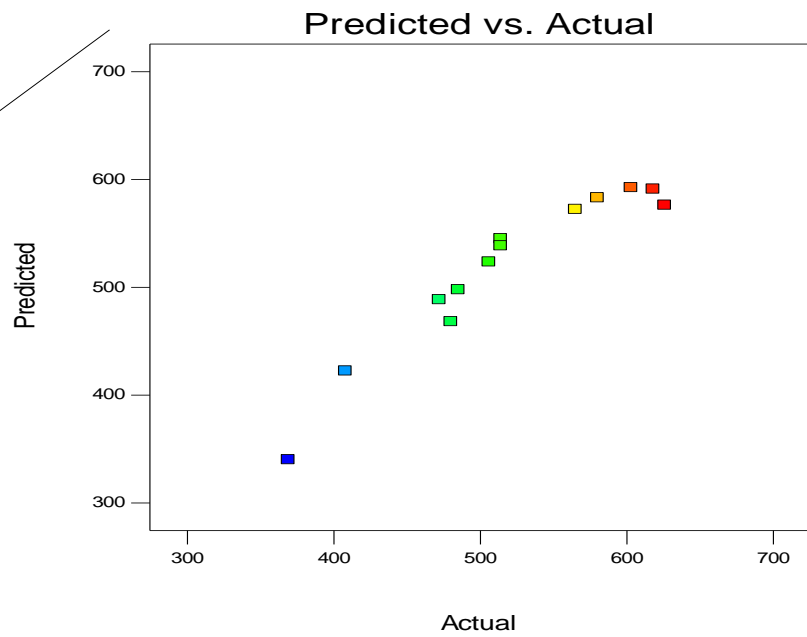
$$\begin{aligned}
 \text{UTS} = & \\
 & +292.88623 \\
 & +98.56070 * \% \text{ Zr} \\
 & -8.10343 * \% \text{ Zr}^2
 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because

the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
UTS

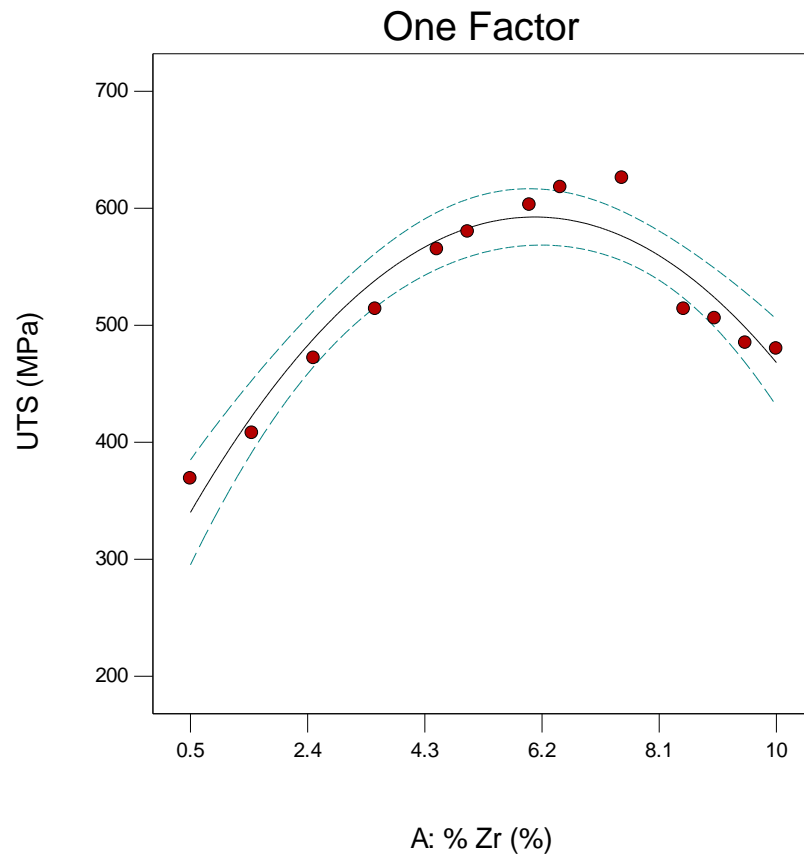
Color points by value of  
UTS:



Design-Expert® Software  
 Factor Coding: Actual  
 UTS (MPa)

● Design Points  
 --- 95% CI Bands

X1 = A: % Zr



### Response 3: Hardness

Response 3      Hardness    Transform:    None

Summary (detailed tables shown below)

	Sequential Lack of Fit	Adjusted	Predicted	
Source	p-value	R-Squared	R-Squared	
Linear	0.0010	0.6085	0.4511	
Quadratic	< 0.0001	0.9885	0.9767	
Cubic	0.2716	0.9889	0.9640	
<u>Quartic</u>	<u>0.0320</u>	<u>0.9932</u>	<u>0.9839</u>	<u>Suggested</u>
Fifth	0.2174	0.9938	0.9836	
Sixth	0.3817	0.9937	0.4719	

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	p-value
Mean vs Total	1.309E+006	1	1.309E+006		
Linear vs Mean	59483.98	1	59483.98	19.65	0.0010
Quadratic vs Linear	32402.39	1	32402.39	363.10	< 0.0001
Cubic vs Quadratic	118.02	1	118.02	1.37	0.2716
<u>Quartic vs Cubic</u>	<u>353.40</u>	<u>1</u>	<u>353.40</u>	<u>6.72</u>	<u>0.0320</u>
Fifth vs Quartic	87.51	1	87.51	1.84	0.2174
Sixth vs Fifth	43.11	1	43.11	0.89	0.3817
Residual	290.36	6	48.39		

Suggested

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 1.402E+006 13 1.078E+005

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS
Linear	55.02	0.6411	0.6085	0.4511	50925.53
Quadratic	9.45	0.9904	0.9885	0.9767	2161.98
Cubic	9.28	0.9917	0.9889	0.9640	3335.54
<u>Quartic</u>	<u>7.25</u>	<u>0.9955</u>	<u>0.9932</u>	<u>0.9839</u>	<u>1492.16</u>
Fifth	6.90	0.9964	0.9938	0.9836	1523.27
Sixth	6.96	0.9969	0.9937	0.4719	48998.23

Suggested

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

**ANOVA for Response Surface Quartic model**

**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	92357.79	4	23089.45	438.78	< 0.0001	significant
A-% Zr	11739.42	1	11739.42	223.09	< 0.0001	
A <sup>2</sup>	4427.39	1	4427.39	84.14	< 0.0001	
A <sup>3</sup>	128.22	1	128.22	2.44	0.1572	
A <sup>4</sup>	353.40	1	353.40	6.72	0.0320	
Residual	420.98	8	52.62			
Cor Total	92778.77	12				

The Model F-value of 438.78 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>4</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	7.25	R-Squared	0.9955
Mean	317.31	Adj R-Squared	0.9932
C.V. %	2.29	Pred R-Squared	0.9839
PRESS	1492.16	Adeq Precision	60.874
-2 Log Likelihood	82.10	BIC	94.93
		AICc	100.67

The "Pred R-Squared" of 0.9839 is in reasonable agreement with the "Adj R-Squared" of 0.9932; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 60.874 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard 95% CI			VIF
	Estimate	df	Error	Low	High	
Intercept	369.06	1	3.76	360.40	377.72	
A-% Zr	130.75	1	8.75	110.56	150.94	7.57
A <sup>2</sup>	-194.48	1	21.20	-243.37	-145.59	13.98
A <sup>3</sup>	-17.72	1	11.35	-43.90	8.46	7.56
A <sup>4</sup>	54.53	1	21.04	6.01	103.05	13.89

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned}\text{Hardness} = & \\ & +369.06 \\ & +130.75 * A \\ & -194.48 * A^2 \\ & -17.72 * A^3 \\ & +54.53 * A^4\end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned}\text{Hardness} = & \\ & +92.26115 \\ & +42.36420 * \% Zr \\ & +11.69772 * \% Zr^2 \\ & -2.41464 * \% Zr^3 \\ & +0.10711 * \% Zr^4\end{aligned}$$

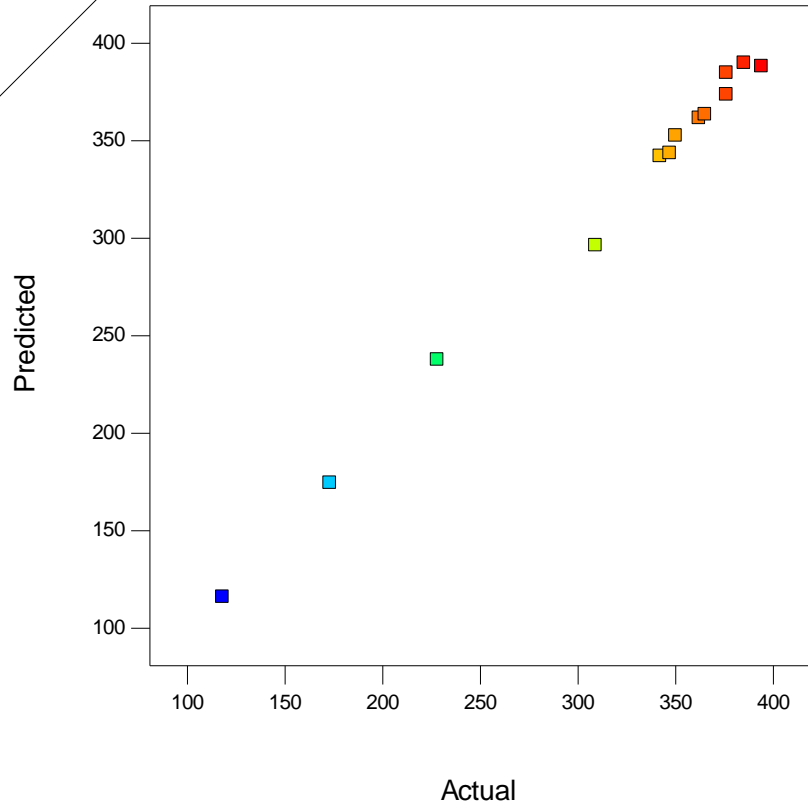
The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Hardness

Color points by value of  
Hardness:



Predicted vs. Actual

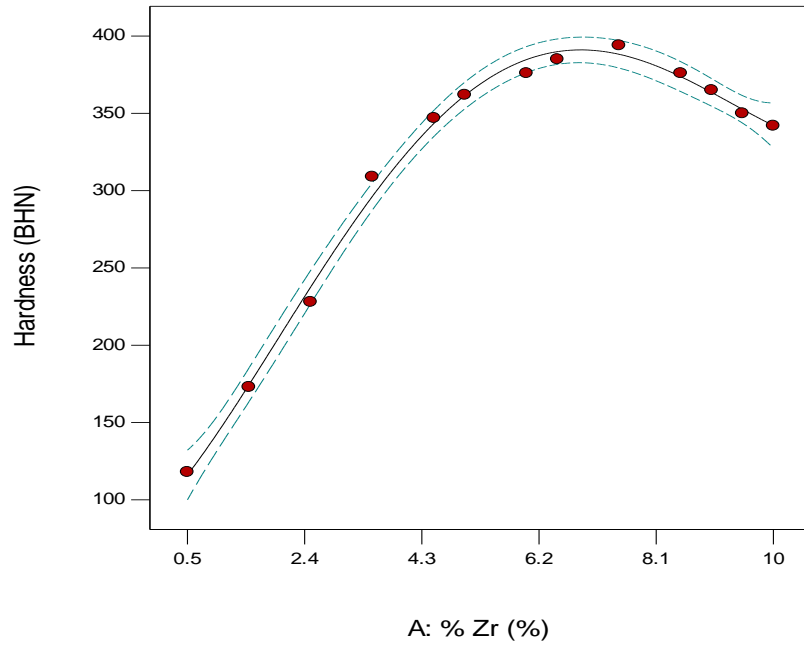


Design-Expert® Software  
Factor Coding: Actual  
Hardness (BHN)

● Design Points  
--- 95% CI Bands

X1 = A: % Zr

One Factor





## Response 4: Elongation

Response 4 Elongation Transform: None

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0345		0.2865	0.0411
Quadratic	< 0.0001		0.8713	0.7440
<u>Cubic</u>	<u>0.0003</u>		<u>0.9699</u>	<u>0.9569</u> <u>Suggested</u>
Quartic	0.1392		0.9747	0.9100
Fifth	0.7408		0.9716	0.5893
Sixth	0.1938		0.9756	0.0864

### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Mean vs Total	4493.76	1	4493.76			
Linear vs Mean	53.43	1	53.43	5.82	0.0345	
Quadratic vs Linear	84.44	1	84.44	50.99	< 0.0001	
<u>Cubic vs Quadratic</u>	<u>13.08</u>	<u>1</u>	<u>13.08</u>	<u>33.80</u>	<u>0.0003</u>	<u>Suggested</u>
Quartic vs Cubic	0.88	1	0.88	2.70	0.1392	
Fifth vs Quartic	0.043	1	0.043	0.12	0.7408	
Sixth vs Fifth	0.67	1	0.67	2.14	0.1938	
Residual	1.89	6	0.31			
Total	4648.20	13	357.55			

"Sequential Model Sum of Squares [Type I]":  
Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

	Std.		Adjusted	Predicted	
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS
Linear	3.03	0.3460	0.2865	0.0411	148.09
Quadratic	1.29	0.8928	0.8713	0.7440	39.54
<u>Cubic</u>	<u>0.62</u>	<u>0.9775</u>	<u>0.9699</u>	<u>0.9569</u>	<u>6.65</u>
Quartic	0.57	0.9831	0.9747	0.9100	13.90
Fifth	0.60	0.9834	0.9716	0.5893	63.43
Sixth	0.56	0.9878	0.9756	0.0864	141.09

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

**ANOVA for Response Surface Cubic model**

**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value	
Model	150.95	3	50.32	130.04	< 0.0001	Significant
A-% Zr	41.74	1	41.74	107.88	< 0.0001	
A <sup>2</sup>	81.90	1	81.90	211.66	< 0.0001	
A <sup>3</sup>	13.08	1	13.08	33.80	0.0003	
Residual	3.48	9	0.39			
Cor Total	154.44	12				

The Model F-value of 130.04 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.62	R-Squared	0.9775
Mean	18.59	Adj R-Squared	0.9699
C.V. %	3.35	Pred R-Squared	0.9569
PRESS	6.65	Adeq Precision	31.151
-2 Log Likelihood	19.77	BIC	30.03
		AICc	32.77

The "Pred R-Squared" of 0.9569 is in reasonable agreement with the "Adj R-Squared" of 0.9699; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 31.151 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		High VIF
	Estimate	df		Low	High	
Intercept	16.05	1	0.26	15.46	16.65	
A-% Zr	-7.78	1	0.75	-9.48	-6.09	7.55
A <sup>2</sup>	7.14	1	0.49	6.03	8.25	1.02
A <sup>3</sup>	5.66	1	0.97	3.46	7.86	7.55

#### Final Equation in Terms of Coded Factors:

$$\begin{aligned} \text{Elongation} = & \\ & +16.05 \\ & -7.78 * A \\ & +7.14 * A^2 \\ & +5.66 * A^3 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

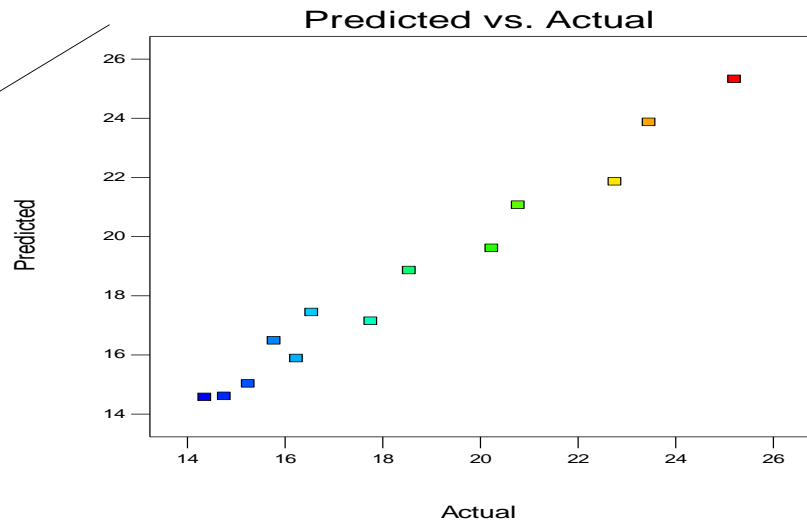
#### Final Equation in Terms of Actual Factors:

$$\begin{aligned} \text{Elongation} = & \\ & +25.73810 \\ & -0.59601 * \% Zr \\ & -0.51498 * \% Zr^2 \\ & +0.052785 * \% Zr^3 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

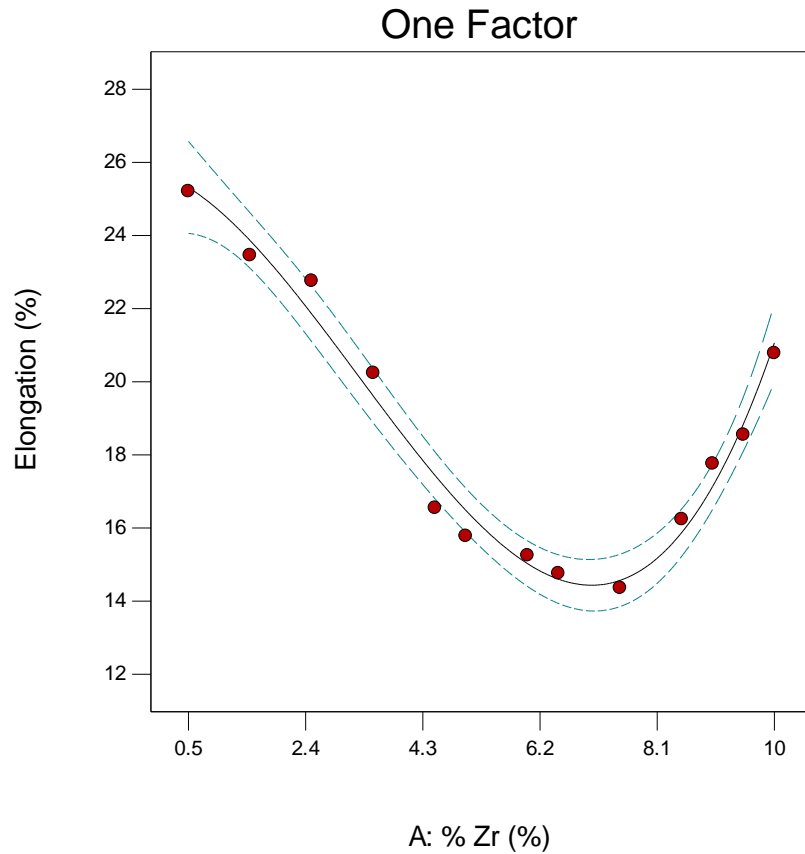
Design-Expert® Software  
Elongation

Color points by value of  
Elongation:  
25.21  
14.36



Design-Expert® Software  
 Factor Coding: Actual  
 Elongation (%)  
 ● Design Points  
 --- 95% CI Bands

X1 = A: % Zr



## Response 5: Impact Strength

**Response 5**      **Impact Strength Transform: None**

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0336		0.2895	-0.0043
Quadratic	< 0.0001		0.9657	0.9235
<u>Cubic</u>	<u>0.0218</u>		<u>0.9794</u>	<u>0.9538</u> <b>Suggested</b>
Quartic	0.6952		0.9773	0.8863
Fifth	0.2052		0.9797	0.8330
Sixth	0.0742		0.9867	0.8926

**Sequential Model Sum of Squares [Type I]**

	Sum of	Mean	F	p-value
Source	Squares	df	Square	Value
Mean vs Total	11283.77	1	11283.77	
Linear vs Mean	75.00	1	75.00	5.89
Quadratic vs Linear	133.96	1	133.96	218.04
<u>Cubic vs Quadratic</u>	<u>2.82</u>	<u>1</u>	<u>2.82</u>	<u>7.66</u>
Quartic vs Cubic	0.067	1	0.067	0.17
Fifth vs Quartic	0.71	1	0.71	1.95
Sixth vs Fifth	1.11	1	1.11	4.66
Residual	1.43	6	0.24	

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 11498.88 13 884.53

additional terms are significant and the model is not aliased.

Suggested

**Model Summary Statistics**

	Std. Source Dev.	Adjusted R-Squared	Predicted R-Squared	R-PRESS
Linear	3.57	0.3487	0.2895	-0.0043
Quadratic	0.78	0.9714	0.9657	0.9235
<u>Cubic</u>	<u>0.61</u>	<u>0.9846</u>	<u>0.9794</u>	<u>0.9538</u>
Quartic	0.64	0.9849	0.9773	0.8863
Fifth	0.60	0.9882	0.9797	0.8330
Sixth	0.49	0.9933	0.9867	0.8926

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

and the "Predicted R-Squared".

Suggested

**ANOVA for Response Surface Cubic model**

**Analysis of variance table [Partial sum of squares - Type III]**

<b>Source</b>	<b>Sum of Squares</b>	<b>df</b>	<b>Mean Square</b>	<b>F Value</b>	<b>p-value Prob &gt; F</b>	
Model	211.79	3	70.60	191.43	< 0.0001	significant
A-% Zr	27.72	1	27.72	75.18	< 0.0001	
A <sup>2</sup>	132.35	1	132.35	358.89	< 0.0001	
A <sup>3</sup>	2.82	1	2.82	7.66	0.0218	
Residual	3.32	9	0.37			
Cor Total	215.11	12				

The Model F-value of 191.43 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.61	R-Squared	0.9846
Mean	29.46	Adj R-Squared	0.9794
C.V. %	2.06	Pred R-Squared	0.9538
PRESS	9.93	Adeq Precision	40.912
-2 Log Likelihood	19.14	BIC	29.40
		AICc	32.14

The "Pred R-Squared" of 0.9538 is in reasonable agreement with the "Adj R-Squared" of 0.9794; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 40.912 indicates an adequate signal. This model can be used to navigate the design space.

<b>Factor</b>	<b>Coefficient Estimate</b>	<b>df</b>	<b>Standard Error</b>	<b>95% CI Low</b>	<b>95% CI High</b>	<b>VIF</b>
Intercept	26.20	1	0.26	25.61	26.78	
A-% Zr	-6.34	1	0.73	-8.00	-4.69	7.55
A <sup>2</sup>	9.07	1	0.48	7.99	10.16	1.02
A <sup>3</sup>	2.63	1	0.95	0.48	4.78	7.55

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned}\text{Impact Strength} = & \\ & +26.20 \\ & -6.34 * A \\ & +9.07 * A^2 \\ & +2.63 * A^3\end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned}\text{Impact Strength} = & \\ & +40.74234 \\ & -3.52987 * \% Zr \\ & +0.015802 * \% Zr^2 \\ & +0.024532 * \% Zr^3\end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

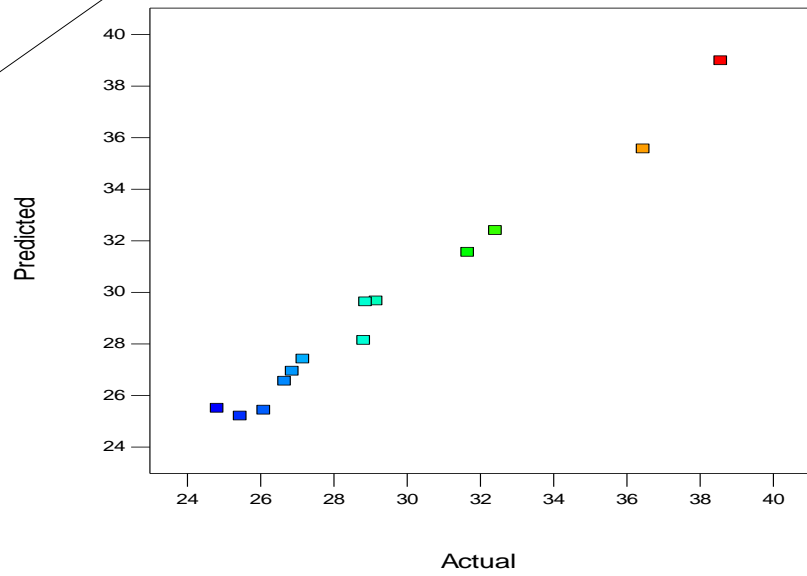


Design-Expert® Software  
Impact Strength

Color points by value of  
Impact Strength:



### Predicted vs. Actual



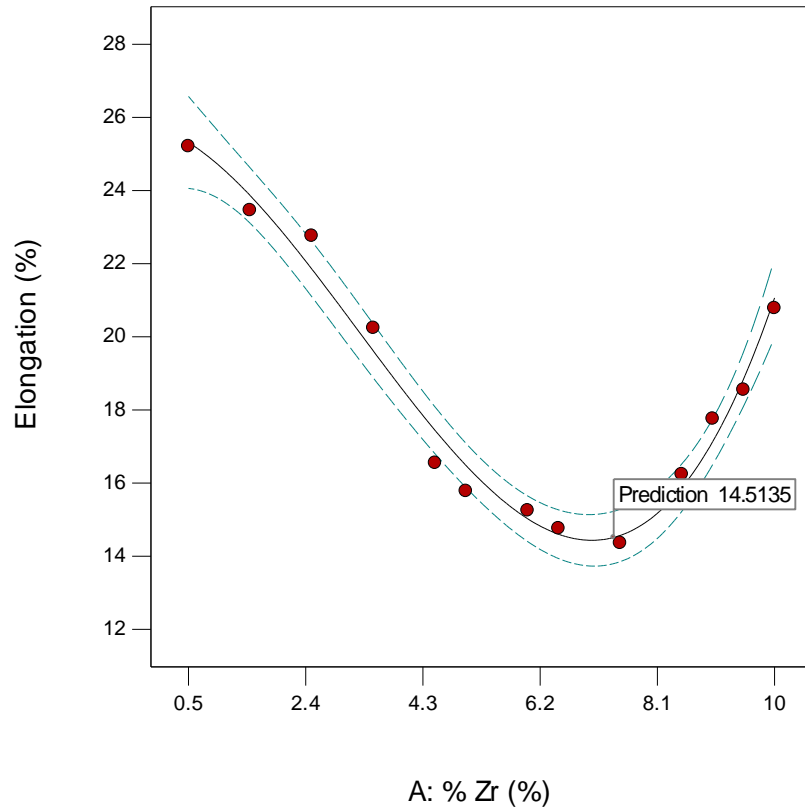
Design-Expert® Software  
Factor Coding: Actual

Elongation (%)

● Design Points  
--- 95% CI Bands

X1 = A: % Zr

### One Factor



## Response 6: Resistivity

Response 6                  Resistivity Transform:    None

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0027		0.5354	0.3709
Quadratic	0.0002		0.8858	0.7728
<u>Cubic</u>	<u>0.0002</u>		<u>0.9740</u>	<u>0.9669</u> <u>Suggested</u>
Quartic	0.8312		0.9709	0.9609
Fifth	0.6673		0.9677	0.8861
Sixth	0.7581		0.9630	0.5312

### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Mean vs Total	1791.29	1	1791.29			
Linear vs Mean	58.54	1	58.54	14.83	0.0027	
Quadratic vs Linear	33.72	1	33.72	34.74	0.0002	
<u>Cubic vs Quadratic</u>	<u>7.72</u>	<u>1</u>	<u>7.72</u>	<u>34.97</u>	<u>0.0002</u>	<u>Suggested</u>
Quartic vs Cubic	0.012	1	0.012	0.048	0.8312	
Fifth vs Quartic	0.055	1	0.055	0.20	0.6673	
Sixth vs Fifth	0.033	1	0.033	0.10	0.7581	
Residual	1.89	6	0.31			
Total	1893.26	13	145.64			

"Sequential Model Sum of Squares [Type I]":  
Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

	Std.		Adjusted	Predicted	
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS
Linear	1.99	0.5741	0.5354	0.3709	64.15
Quadratic	0.99	0.9048	0.8858	0.7728	23.16
<u>Cubic</u>	<u>0.47</u>	<u>0.9805</u>	<u>0.9740</u>	<u>0.9669</u>	<u>3.37</u>
Quartic	0.50	0.9806	0.9709	0.9609	3.98
Fifth	0.52	0.9812	0.9677	0.8861	11.62
Sixth	0.56	0.9815	0.9630	0.5312	47.80

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

**ANOVA for Response Surface Cubic model**

**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value	
					Prob > F	
Model	99.98	3	33.33	150.97	< 0.0001	significant
A-% Zr	31.87	1	31.87	144.38	< 0.0001	
A <sup>2</sup>	32.50	1	32.50	147.21	< 0.0001	
A <sup>3</sup>	7.72	1	7.72	34.97	0.0002	
Residual	1.99	9	0.22			
Cor Total	101.97	12				

The Model F-value of 150.97 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.47	R-Squared	0.9805
Mean	11.74	Adj R-Squared	0.9740
C.V. %	4.00	Pred R-Squared	0.9669
PRESS	3.37	Adeq Precision	33.389

-2 Log Likelihood 12.47 BIC 22.73  
 AICc 25.47

The "Pred R-Squared" of 0.9669 is in reasonable agreement with the "Adj R-Squared" of 0.9740; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 33.389 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient Estimate	df	Standard Error	95% CI Low	95% CI High	VIF
Intercept	13.20	1	0.20	12.75	13.65	
A-% Zr	6.80	1	0.57	5.52	8.08	7.55
A <sup>2</sup>	-4.50	1	0.37	-5.33	-3.66	1.02
A <sup>3</sup>	-4.35	1	0.73	-6.01	-2.68	7.55

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Resistivity} = & \\ & +13.20 \\ & +6.80 * A \\ & -4.50 * A^2 \\ & -4.35 * A^3 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

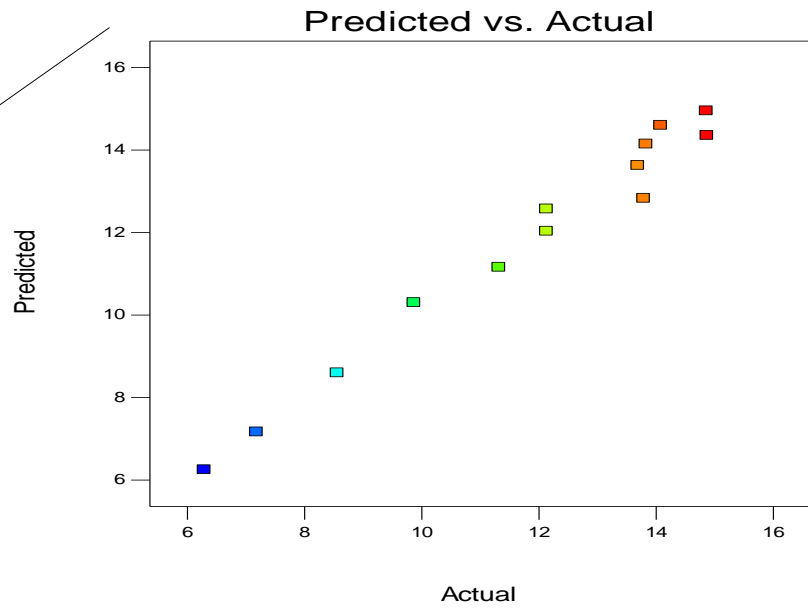
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Resistivity} = & \\ & +6.05981 \\ & +0.17083 * \% \text{ Zr} \\ & +0.43947 * \% \text{ Zr}^2 \\ & -0.040556 * \% \text{ Zr}^3 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Resistivity

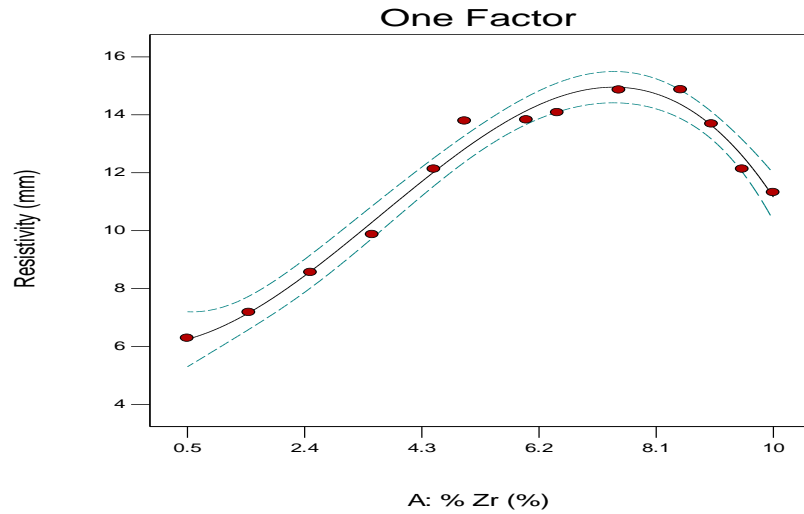
Color points by value of  
Resistivity:



Design-Expert® Software  
Factor Coding: Actual  
Resistivity (mm)

● Design Points  
--- 95% CI Bands

X1 = A: % Zr



## Response 7: Conductivity

Response 7      Conductivity Transform:    None

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0106		0.4131	0.1395
<u>Quadratic</u>	<u>&lt; 0.0001</u>		<u>0.9458</u>	<u>0.9212</u> <u>Suggested</u>
Cubic	0.2021		0.9503	0.9306
Quartic	0.0768		0.9631	0.7755
Fifth	0.1609		0.9688	0.5216
<u>Sixth</u>	<u>0.0165</u>		<u>0.9870</u>	<u>0.8331</u> <u>Suggested</u>

### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Mean vs Total	228.23	1	228.23		
Linear vs Mean	20.35	1	20.35	9.45	0.0106
<u>Quadratic vs Linear</u>	<u>21.71</u>	<u>1</u>	<u>21.71</u>	<u>109.16</u>	<u>&lt; 0.0001</u>
Cubic vs Quadratic	0.35	1	0.35	1.89	0.2021
Quartic vs Cubic	0.56	1	0.56	4.12	0.0768
Fifth vs Quartic	0.28	1	0.28	2.46	0.1609
<u>Sixth vs Fifth</u>	<u>0.52</u>	<u>1</u>	<u>0.52</u>	<u>10.85</u>	<u>0.0165</u>
Residual	0.29	6	0.048		

"Sequential Model Sum of Squares [Type I]":  
Select the highest order polynomial where the

Total    272.28    13    20.94

additional terms are significant and the model is not aliased.

### Model Summary Statistics

Std.      Adjusted Predicted

Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS	
Linear	1.47	0.4620	0.4131	0.1395	37.90	
<u>Quadratic</u>	<u>0.45</u>	<u>0.9549</u>	<u>0.9458</u>	<u>0.9212</u>	<u>3.47</u>	<u>Suggested</u>
Cubic	0.43	0.9627	0.9503	0.9306	3.06	
Quartic	0.37	0.9754	0.9631	0.7755	9.89	
Fifth	0.34	0.9818	0.9688	0.5216	21.07	
						<u>Suggested</u> "Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"
<u>Sixth</u>	<u>0.22</u>	<u>0.9935</u>	<u>0.9870</u>	<u>0.8331</u>	<u>7.35</u>	and the "Predicted R-Squared".

### ANOVA for Response Surface Sixth model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	
Model	43.76	6	7.29	153.10	< 0.0001	significant
A-% Zr	0.63	1	0.63	13.28	0.0108	
A <sup>2</sup>	0.18	1	0.18	3.88	0.0965	
A <sup>3</sup>	0.26	1	0.26	5.48	0.0578	
A <sup>4</sup>	0.66	1	0.66	13.95	0.0097	
A <sup>5</sup>	0.41	1	0.41	8.53	0.0266	
A <sup>6</sup>	0.52	1	0.52	10.85	0.0165	
Residual	0.29	6	0.048			
Cor Total	44.05	12				

The Model F-value of 153.10 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>4</sup>, A<sup>5</sup>, A<sup>6</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.22	R-Squared	0.9935
Mean	4.19	Adj R-Squared	0.9870
C.V. %	5.21	Pred R-Squared	0.8331

PRESS 7.35 Adeq Precision 40.121  
 -2 Log Likelihood -12.73 BIC 5.22  
 AICc 23.67

The "Pred R-Squared" of 0.8331 is in reasonable agreement with the "Adj R-Squared" of 0.9870; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 40.121 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error		95% CI		VIF
	Estimate	df	Error	Low	High		
Intercept	3.35	1	0.13	3.03	3.68		
A-% Zr	-1.73	1	0.47	-2.88	-0.57	24.47	
A <sup>2</sup>	-3.25	1	1.65	-7.29	0.79	93.52	
A <sup>3</sup>	-3.85	1	1.64	-7.87	0.18	175.22	
A <sup>4</sup>	15.67	1	4.20	5.40	25.94	610.47	
A <sup>5</sup>	3.72	1	1.27	0.60	6.84	85.49	
A <sup>6</sup>	-9.03	1	2.74	-15.74	-2.32	257.04	

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Conductivity} = & \\ & +3.35 \\ & -1.73 * A \\ & -3.25 * A^2 \\ & -3.85 * A^3 \\ & +15.67 * A^4 \\ & +3.72 * A^5 \\ & -9.03 * A^6 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Conductivity} = & \\ & +7.27639 \\ & +5.02029 * \% \text{ Zr} \end{aligned}$$

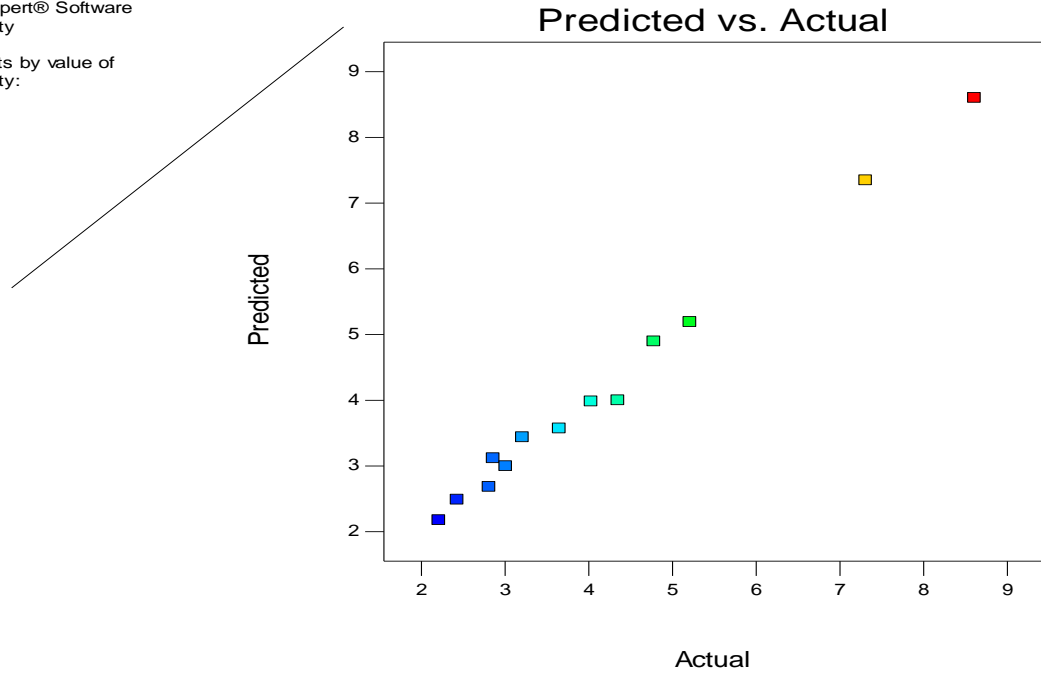


$$\begin{aligned}
& -5.67365 * \% Zr^2 \\
& +2.01722 * \% Zr^3 \\
& -0.33468 * \% Zr^4 \\
& +0.026306 * \% Zr^5 \\
& -7.86252E-004 * \% Zr^6
\end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

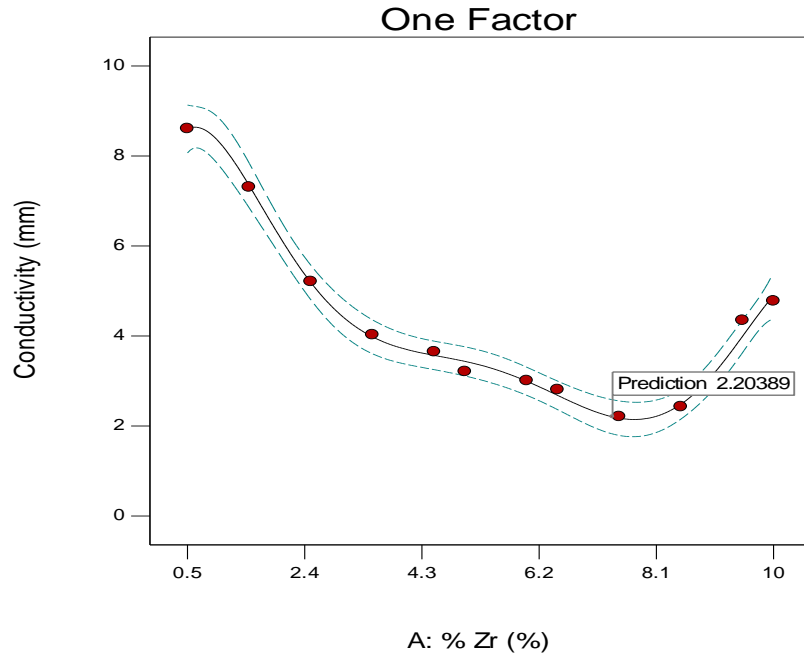
Design-Expert® Software  
Conductivity

Color points by value of  
Conductivity:



Design-Expert® Software  
 Factor Coding: Actual  
 Conductivity (mm)  
 ● Design Points  
 --- 95% CI Bands

X1 = A: % Zr



## Design Expert Analysis for Molybdenum

### Response 1: Yield Strength

Response 1      Yield Strength Transform:      None

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0036		0.5114	0.3178
<u>Quadratic</u>	<u>&lt; 0.0001</u>		<u>0.9748</u>	<u>0.9606</u> Suggested
Cubic	0.0768		0.9806	0.9641
Quartic	0.2854		0.9813	0.9499
Fifth	0.0332		0.9893	0.7814
<u>Sixth</u>	<u>0.0456</u>		<u>0.9939</u>	<u>0.6957</u> Suggested

#### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Mean vs Total	2.238E+006	1	2.238E+006			
Linear vs Mean	58147.40	1	58147.40	13.56	0.0036	

<u>Quadratic vs Linear</u>	<u>44965.81</u>	<u>1</u>	<u>44965.81</u>	<u>203.49</u>	<u>0.0001</u>	<u>≤</u>	<u>Suggested</u>
Cubic vs Quadratic	678.84	1	678.84	3.99	0.0768		
Quartic vs Cubic	215.45	1	215.45	1.31	0.2854		
Fifth vs Quartic	657.31	1	657.31	6.99	0.0332		
<u>Sixth vs Fifth</u>	<u>337.81</u>	<u>1</u>	<u>337.81</u>	<u>6.33</u>	<u>0.0456</u>		<u>Suggested</u>
Residual	320.29	6	53.38				

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 2.343E+006 13 1.803E+005

additional terms are significant and the model is not aliased.

### Model Summary Statistics

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS	
Linear	65.49	0.5521	0.5114	0.3178	71852.43	
<u>Quadratic</u>	<u>14.87</u>	<u>0.9790</u>	<u>0.9748</u>	<u>0.9606</u>	<u>4145.88</u>	<u>Suggested</u>
Cubic	13.04	0.9855	0.9806	0.9641	3783.21	
Quartic	12.82	0.9875	0.9813	0.9499	5271.43	
Fifth	9.70	0.9938	0.9893	0.7814	23022.15	
<u>Sixth</u>	<u>7.31</u>	<u>0.9970</u>	<u>0.9939</u>	<u>0.6957</u>	<u>32047.57</u>	and the "Predicted R-Squared".

Suggested"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

### ANOVA for Response Surface Sixth model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	1.050E+005	6	17500.44	327.84	< 0.0001	significant
A-% Mo	7618.94	1	7618.94	142.73	< 0.0001	
A <sup>2</sup>	85.70	1	85.70	1.61	0.2521	
A <sup>3</sup>	741.02	1	741.02	13.88	0.0098	

$A^4$	267.18	1	267.18	5.01	0.0666
$A^5$	502.58	1	502.58	9.41	0.0220
$A^6$	337.81	1	337.81	6.33	0.0456
Residual	320.29	6	53.38		
Cor Total	1.053E+005	12			

The Model F-value of 327.84 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A,  $A^3$ ,  $A^5$ ,  $A^6$  are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	7.31	R-Squared	0.9970
Mean	414.92	Adj R-Squared	0.9939
C.V. %	1.76	Pred R-Squared	0.6957
PRESS	32047.57	Adeq Precision	57.929
-2 Log Likelihood	78.55	BIC	96.50
		AICc	114.95

The "Pred R-Squared" of 0.6957 is not as close to the "Adj R-Squared" of 0.9939 as one might normally expect; i.e. the difference is more than 0.2. This may indicate a large block effect or a possible problem with your model and/or data. Things to consider are model reduction, response transformation, outliers, etc. All empirical models should be tested by doing confirmation runs. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 57.929 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error		95% CI		VIF
	Estimate	df	Error	Low	High		
Intercept	468.59	1	4.47	457.65	479.52		
A-% Mo	189.35	1	15.85	150.57	228.13	24.47	
$A^2$	-69.97	1	55.23	-205.11	65.16	93.52	
$A^3$	-205.08	1	55.04	-339.77	-70.40	175.22	
$A^4$	-314.30	1	140.49	-658.05	29.46	610.47	
$A^5$	130.86	1	42.65	26.50	235.21	85.49	
$A^6$	230.91	1	91.79	6.30	455.52	257.04	

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned}\text{Yield Strength} = & \\ & +468.59 \\ & +189.35 * A \\ & -69.97 * A^2 \\ & -205.08 * A^3 \\ & -314.30 * A^4 \\ & +130.86 * A^5 \\ & +230.91 * A^6\end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned}\text{Yield Strength} = & \\ & +186.82151 \\ & -3.98282 * \% \text{ Mo} \\ & +75.72026 * \% \text{ Mo}^2 \\ & -32.21469 * \% \text{ Mo}^3 \\ & +6.27380 * \% \text{ Mo}^4 \\ & -0.57916 * \% \text{ Mo}^5 \\ & +0.020104 * \% \text{ Mo}^6\end{aligned}$$

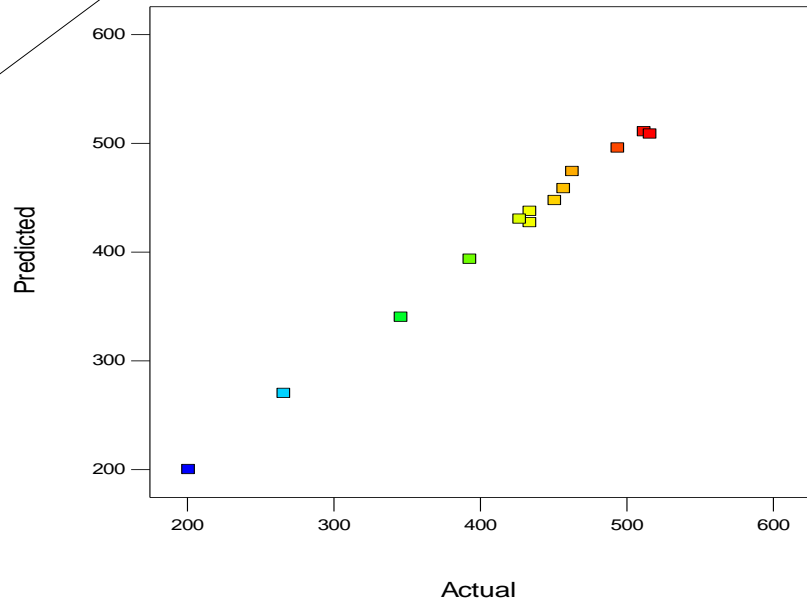
The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Yield Strength

Color points by value of  
Yield Strength:



Predicted vs. Actual

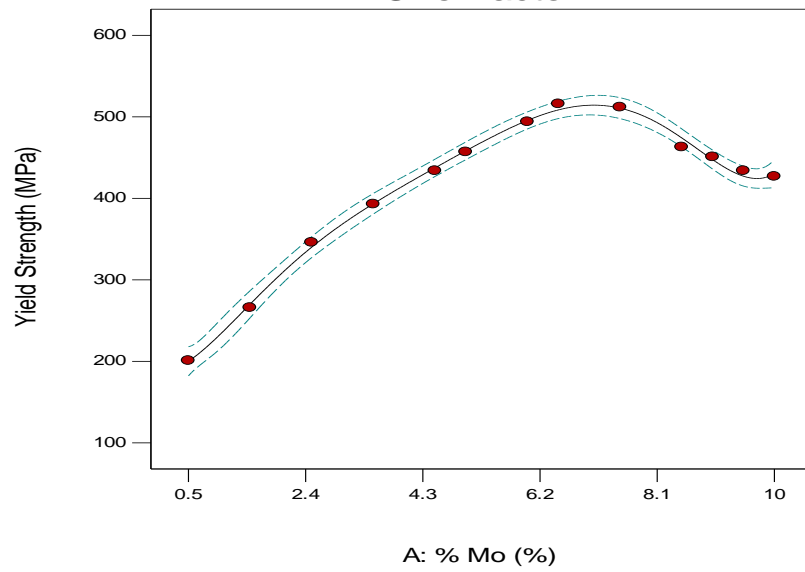


Design-Expert® Software  
Factor Coding: Actual  
Yield Strength (MPa)

● Design Points  
--- 95% CI Bands

X1 = A: % Mo

One Factor



## Response 2: Ultimate Tensile Strength

Response 2            UTS            Transform:    None

Summary (detailed tables shown below)

	Sequential Lack of Fit	Adjusted R-Squared	Predicted R-Squared
Source	p-value	p-value	R-Squared
Linear	0.0108	0.4109	0.1747
Quadratic	< 0.0001	0.9768	0.9652
Cubic	0.3805	0.9765	0.9497
Quartic	0.0741	0.9827	0.8947
<u>Fifth</u>	<u>0.0007</u>	<u>0.9965</u>	<u>0.9767</u> <u>Suggested</u>
Sixth	0.2800	0.9967	0.9876

### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	P-value	Prob > F
Mean vs Total	3.672E+006	1	3.672E+006			
Linear vs Mean	24828.18	1	24828.18	9.37	0.0108	
Quadratic vs Linear	28105.40	1	28105.40	269.82	< 0.0001	
Cubic vs Quadratic	89.92	1	89.92	0.85	0.3805	
Quartic vs Cubic	328.49	1	328.49	4.22	0.0741	
<u>Fifth vs Quartic</u>	<u>514.00</u>	<u>1</u>	<u>514.00</u>	<u>32.93</u>	<u>0.0007</u>	<u>Suggested</u>
Sixth vs Fifth	20.78	1	20.78	1.41	0.2800	
Residual	88.47	6	14.74			

Total 3.726E+006 13 2.866E+005

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

	Std.	Adjusted		Predicted	
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS
Linear	51.48	0.4600	0.4109	0.1747	44545.77
Quadratic	10.21	0.9807	0.9768	0.9652	1877.20
Cubic	10.28	0.9824	0.9765	0.9497	2712.74
Quartic	8.83	0.9885	0.9827	0.8947	5684.88
<u>Fifth</u>	<u>3.95</u>	<u>0.9980</u>	<u>0.9965</u>	<u>0.9767</u>	<u>1258.58</u>
					<u>Suggested</u>
					"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".
Sixth	3.84	0.9984	0.9967	0.9876	667.07

**ANOVA for Response Surface Fifth model**

**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value	
Model	53865.98	5	10773.20	690.29	< 0.0001	significant
A-% Mo	3698.89	1	3698.89	237.00	< 0.0001	
A <sup>2</sup>	3492.92	1	3492.92	223.81	< 0.0001	
A <sup>3</sup>	588.21	1	588.21	37.69	0.0005	
A <sup>4</sup>	247.77	1	247.77	15.88	0.0053	
A <sup>5</sup>	514.00	1	514.00	32.93	0.0007	
Residual	109.25	7	15.61			
Cor Total	53975.23	12				

The Model F-value of 690.29 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup>, A<sup>4</sup>, A<sup>5</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	3.95	R-Squared	0.9980
Mean	531.46	Adj R-Squared	0.9965



C.V. %	0.74	Pred R-Squared	0.9767
PRESS	1258.58	Adeq Precision	85.444
-2 Log Likelihood	64.57	BIC	79.95
		AICc	90.57

The "Pred R-Squared" of 0.9767 is in reasonable agreement with the "Adj R-Squared" of 0.9965; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 85.444 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	582.07	1	2.05	577.22	586.92	
A-% Mo	131.07	1	8.51	110.94	151.21	24.15
A <sup>2</sup>	-174.21	1	11.64	-201.75	-146.68	14.22
A <sup>3</sup>	-180.28	1	29.37	-249.72	-110.84	170.57
A <sup>4</sup>	45.89	1	11.52	18.66	73.12	14.03
A <sup>5</sup>	130.68	1	22.77	76.83	184.53	83.36

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned}
 \text{UTS} = & \\
 & +582.07 \\
 & +131.07 * A \\
 & -174.21 * A^2 \\
 & -180.28 * A^3 \\
 & +45.89 * A^4 \\
 & +130.68 * A^5
 \end{aligned}$$

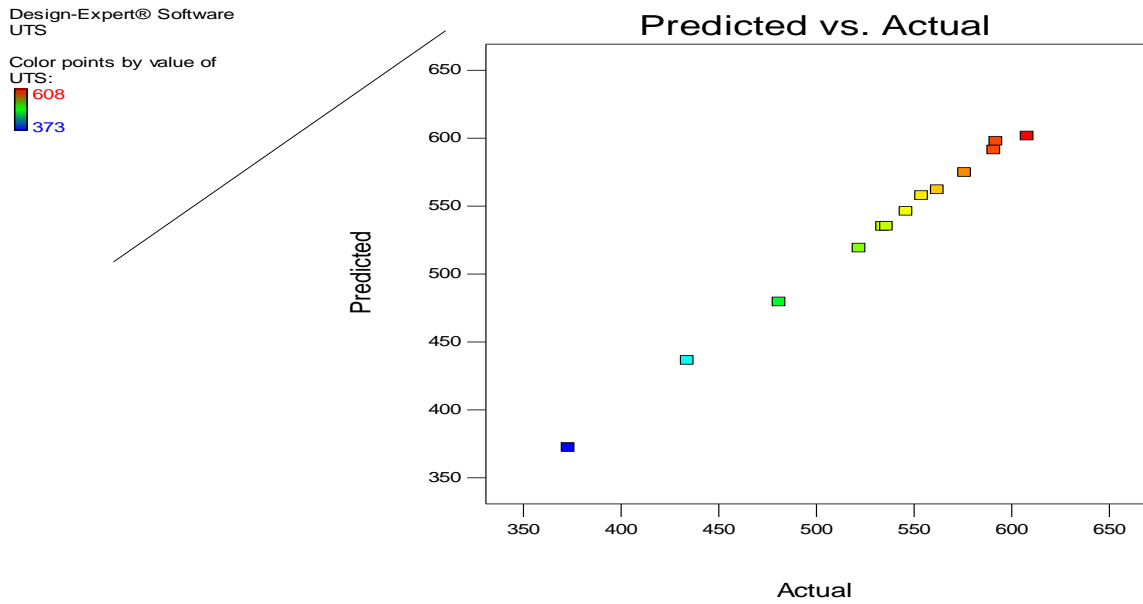
The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

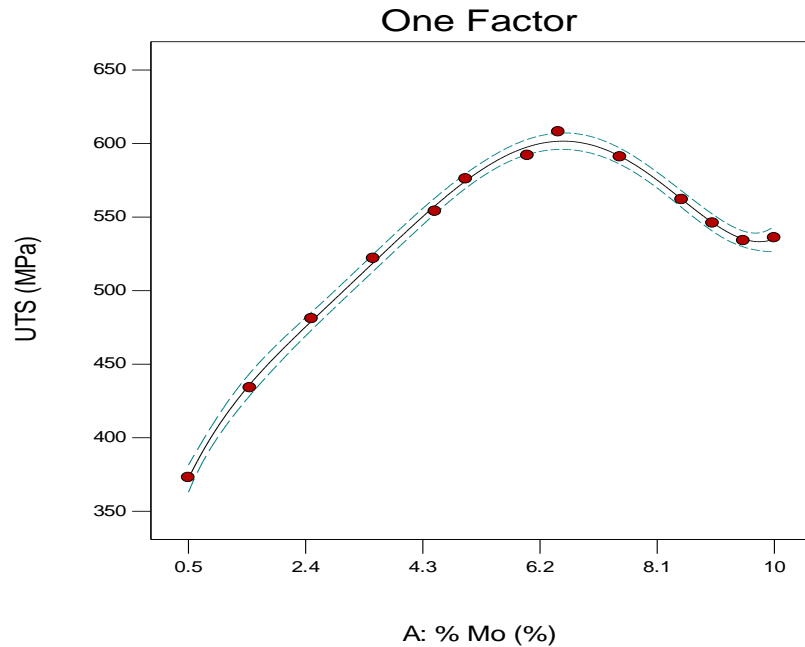
$$\begin{aligned}
 \text{UTS} = & \\
 & +320.73388 \\
 & +122.67634 * \% \text{ Mo} \\
 & -44.52137 * \% \text{ Mo}^2
 \end{aligned}$$

$$\begin{aligned}
&+11.32041 * \% Mo^3 \\
&-1.32849 * \% Mo^4 \\
&+0.054043 * \% Mo^5
\end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.



Design-Expert® Software  
 Factor Coding: Actual  
 UTS (MPa)  
 ● Design Points  
 --- 95% CI Bands  
 X1 = A: % Mo



### Response 3: Hardness

Response 3      Hardness    Transform:    None

Summary (detailed tables shown below)

	Sequential Lack of Fit	Adjusted	Predicted
Source	p-value	R-Squared	R-Squared
Linear	0.0019	0.5614	0.4007
Quadratic	< 0.0001	0.9757	0.9478
Cubic	0.3071	0.9761	0.9101
<u>Quartic</u>	<u>&lt; 0.0001</u>	<u>0.9968</u>	<u>0.9942</u> <u>Suggested</u>
Fifth	0.4738	0.9967	0.9930
Sixth	0.8139	0.9961	0.9101

#### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	P-value
					Prob > F
Mean vs Total	1.265E+006	1	1.265E+006		
Linear vs	61227.02	1	61227.02	16.36	0.0019

Mean						
Quadratic vs Linear	39092.40	1	39092.40	188.43	0.0001	<
Cubic vs Quadratic	239.10	1	239.10	1.17	0.3071	
<u>Quartic vs Cubic</u>	<u>1619.95</u>	<u>1</u>	<u>1619.95</u>	<u>60.13</u>	<u>0.0001</u>	<u>≤</u>
Fifth vs Quartic	16.31	1	16.31	0.57	0.4738	
Sixth vs Fifth	1.99	1	1.99	0.060	0.8139	
Residual	197.24	6	32.87			

Suggested

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 1.368E+006 13 1.052E+005

additional terms are significant and the model is not aliased.

### Model Summary Statistics

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS	
Linear	61.18	0.5980	0.5614	0.4007	61367.75	
Quadratic	14.40	0.9797	0.9757	0.9478	5341.50	
Cubic	14.28	0.9821	0.9761	0.9101	9200.31	
<u>Quartic</u>	<u>5.19</u>	<u>0.9979</u>	<u>0.9968</u>	<u>0.9942</u>	<u>589.12</u>	<u>Suggested</u>
Fifth	5.33	0.9981	0.9967	0.9930	714.92	
Sixth	5.73	0.9981	0.9961	0.9101	9203.08	

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

### ANOVA for Response Surface Quartic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
--------	----------------	----	-------------	---------	------------------

Model	1.022E+005	4	25544.61	948.11	< 0.0001	significant
A-% Mo	13455.33	1	13455.33	499.41	< 0.0001	
A <sup>2</sup>	8449.88	1	8449.88	313.62	< 0.0001	
A <sup>3</sup>	270.59	1	270.59	10.04	0.0132	
A <sup>4</sup>	1619.95	1	1619.95	60.13	< 0.0001	
Residual	215.54	8	26.94			
Cor Total	1.024E+005	12				

The Model F-value of 948.11 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup>, A<sup>4</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	5.19	R-Squared	0.9979
Mean	312.00	Adj R-Squared	0.9968
C.V. %	1.66	Pred R-Squared	0.9942
PRESS	589.12	Adeq Precision	88.377
-2 Log Likelihood	73.40	BIC	86.22
		AICc	91.97

The "Pred R-Squared" of 0.9942 is in reasonable agreement with the "Adj R-Squared" of 0.9968; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 88.377 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error		95% CI		VIF
	Estimate	df	Error	Low	High		
Intercept	375.51	1	2.69	369.32	381.71		
A-% Mo	139.98	1	6.26	125.54	154.42	7.57	
A <sup>2</sup>	-268.67	1	15.17	-303.66	-233.69	13.98	
A <sup>3</sup>	-25.74	1	8.12	-44.47	-7.01	7.56	
A <sup>4</sup>	116.74	1	15.06	82.02	151.46	13.89	

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Hardness} = & \\ & +375.51 \\ & +139.98 * A \end{aligned}$$

$$\begin{aligned} & -268.67 * A^2 \\ & -25.74 * A^3 \\ & +116.74 * A^4 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Hardness} = & \\ & +101.55447 \\ & +1.90808 * \% \text{ Mo} \\ & +29.79911 * \% \text{ Mo}^2 \\ & -5.05595 * \% \text{ Mo}^3 \\ & +0.22932 * \% \text{ Mo}^4 \end{aligned}$$

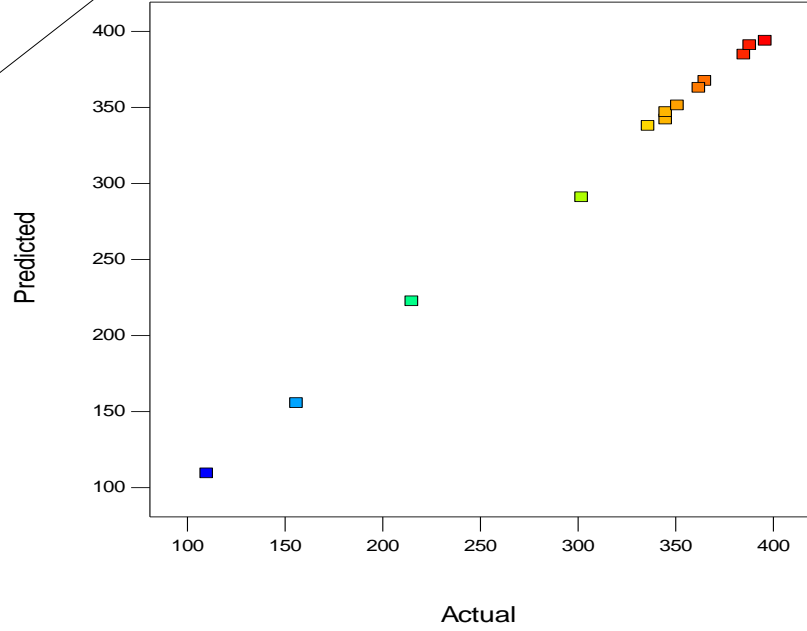
The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Hardness

Color points by value of  
Hardness:



### Predicted vs. Actual

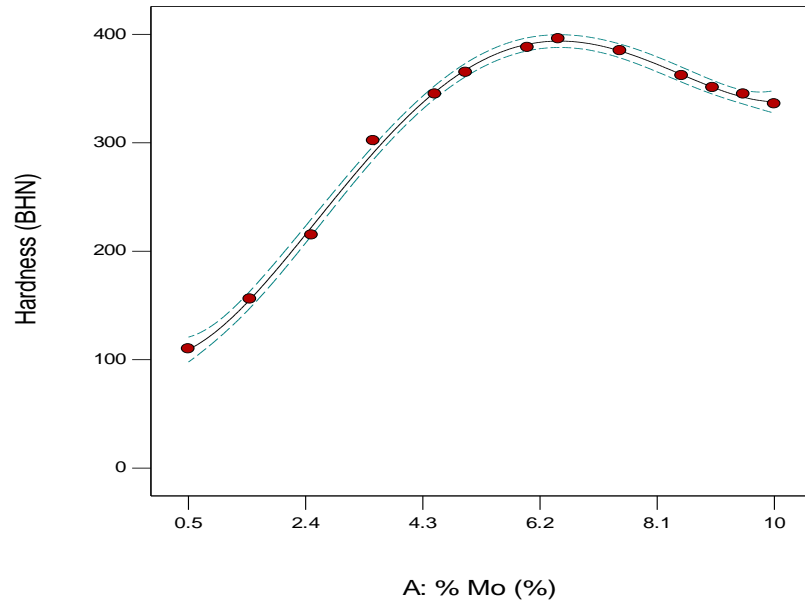


Design-Expert® Software  
Factor Coding: Actual  
Hardness (BHN)

● Design Points  
--- 95% CI Bands

X1 = A: % Mo

### One Factor



## Response 4: Elongation

Response 4 Elongation Transform: None

Summary (detailed tables shown below)

Source	Sequential Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0485	0.2463	-0.0494
Quadratic	< 0.0001	0.9496	0.8939
<u>Cubic</u>	<u>0.0004</u>	<u>0.9867</u>	<u>0.9801</u> <u>Suggested</u>
Quartic	0.3317	0.9868	0.9797
Fifth	0.8374	0.9851	0.9623
Sixth	0.9198	0.9826	0.8152

### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Mean vs Total	4317.05	1	4317.05		
Linear vs Mean	46.39	1	46.39	4.92	0.0485
Quadratic vs Linear	97.40	1	97.40	154.48	< 0.0001
<u>Cubic vs Quadratic</u>	<u>4.81</u>	<u>1</u>	<u>4.81</u>	<u>29.04</u>	<u>0.0004</u> <u>Suggested</u>
Quartic vs Cubic	0.18	1	0.18	1.07	0.3317
Fifth vs Quartic	8.474E-003	1	8.474E-003	0.045	0.8374
Sixth vs Fifth	2.396E-003	1	2.396E-003	0.011	0.9198
Residual	1.31	6	0.22		
Total	4467.14	13	343.63		

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.



**Model Summary Statistics**

	Std.	Adjusted R-Squared	Predicted R-Squared	R-PRESS		
Linear	3.07	0.3091	0.2463	-0.0494	157.51	
Quadratic	0.79	0.9580	0.9496	0.8939	15.92	
<u>Cubic</u>	<u>0.41</u>	<u>0.9901</u>	<u>0.9867</u>	<u>0.9801</u>	<u>2.99</u>	<u>Suggested</u>
Quartic	0.41	0.9912	0.9868	0.9797	3.04	
Fifth	0.43	0.9913	0.9851	0.9623	5.66	
Sixth	0.47	0.9913	0.9826	0.8152	27.73	

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

**ANOVA for Response Surface Cubic model**

**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	148.60	3	49.53	298.85	< 0.0001	significant
A-% Mo	24.89	1	24.89	150.15	< 0.0001	
A <sup>2</sup>	95.68	1	95.68	577.27	< 0.0001	
A <sup>3</sup>	4.81	1	4.81	29.04	0.0004	
Residual	1.49	9	0.17			
Cor Total	150.10	12				

The Model F-value of 298.85 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.41	R-Squared	0.9901
Mean	18.22	Adj R-Squared	0.9867
C.V. %	2.23	Pred R-Squared	0.9801
PRESS	2.99	Adeq Precision	49.940

-2 Log Likelihood 8.75 BIC 19.01  
 AICc 21.75

The "Pred R-Squared" of 0.9801 is in reasonable agreement with the "Adj R-Squared" of 0.9867; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 49.940 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient Estimate	df	Standard Error	95% CI Low	95% CI High	VIF
Intercept	15.42	1	0.17	15.03	15.82	
A-% Mo	-6.01	1	0.49	-7.12	-4.90	7.55
A <sup>2</sup>	7.72	1	0.32	6.99	8.44	1.02
A <sup>3</sup>	3.43	1	0.64	1.99	4.87	7.55

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Elongation} = & \\ & +15.42 \\ & -6.01 * A \\ & +7.72 * A^2 \\ & +3.43 * A^3 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Elongation} = & \\ & +26.85836 \\ & -2.20784 * \% \text{ Mo} \\ & -0.16242 * \% \text{ Mo}^2 \\ & +0.032024 * \% \text{ Mo}^3 \end{aligned}$$

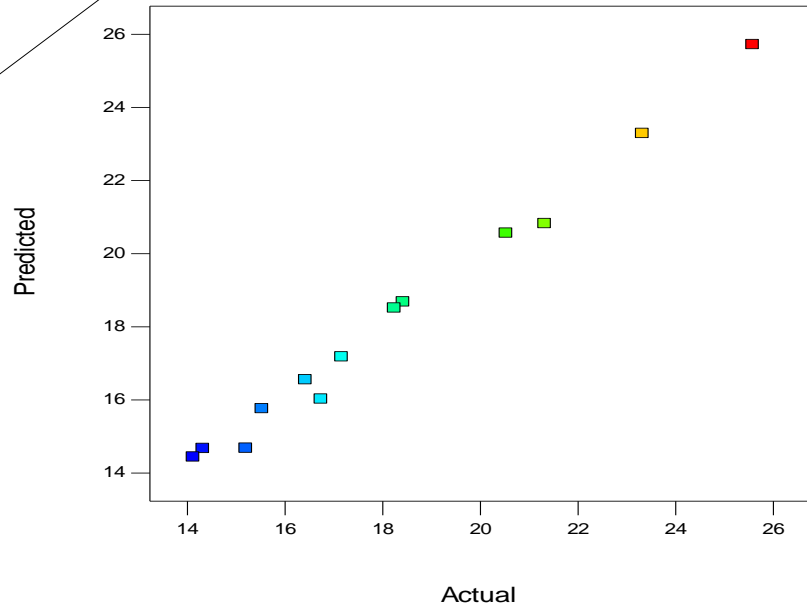
The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Elongation

Color points by value of  
Elongation:



Predicted vs. Actual

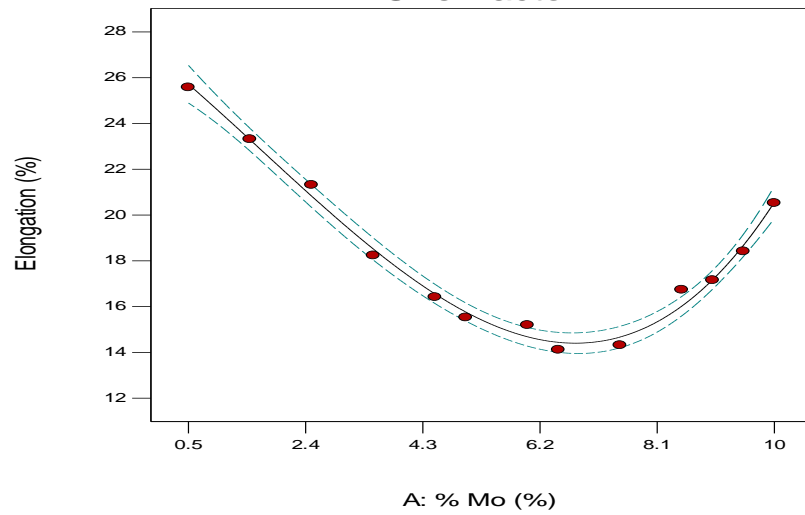


Design-Expert® Software  
Factor Coding: Actual  
Elongation (%)

● Design Points  
--- 95% CI Bands

X1 = A: % Mo

One Factor



**Response 5                      Impact Strength Transform:    None**

**Summary (detailed tables shown below)**

<b>Source</b>	<b>Sequential p-value</b>	<b>Lack of Fit p-value</b>	<b>Adjusted R-Squared</b>	<b>Predicted R-Squared</b>
Linear	0.1925		0.0718	-0.3275
Quadratic	< 0.0001		0.9880	0.9806
Cubic	0.7440		0.9868	0.9623
<u>Quartic</u>	<u>0.0433</u>		<u>0.9914</u>	<u>0.9854 Suggested</u>
Fifth	0.6695		0.9904	0.9467
Sixth	0.1697		0.9920	0.9414

**Sequential Model Sum of Squares [Type I]**

<b>Source</b>	<b>Sum of Squares</b>	<b>df</b>	<b>Mean Square</b>	<b>F Value</b>	<b>P-value Prob &gt; F</b>
Mean vs Total	11037.65	1	11037.65		
Linear vs Mean	20.34	1	20.34	1.93	0.1925
Quadratic vs Linear	114.70	1	114.70	839.18	< 0.0001
Cubic vs Quadratic	0.017	1	0.017	0.11	0.7440
<u>Quartic vs Cubic</u>	<u>0.56</u>	<u>1</u>	<u>0.56</u>	<u>5.75</u>	<u>0.0433</u>
Fifth vs Quartic	0.022	1	0.022	0.20	0.6695
Sixth vs Fifth	0.22	1	0.22	2.43	0.1697
Residual	0.54	6	0.091		

Total 11174.06 13 859.54

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

	Std.		Adjusted	Predicted	
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS
Linear	3.25	0.1491	0.0718	-0.3275	181.09
Quadratic	0.37	0.9900	0.9880	0.9806	2.64
Cubic	0.39	0.9901	0.9868	0.9623	5.14
<u>Quartic</u>	<u>0.31</u>	<u>0.9942</u>	<u>0.9914</u>	<u>0.9854</u>	<u>1.99</u>
Fifth	0.33	0.9944	0.9904	0.9467	7.27
Sixth	0.30	0.9960	0.9920	0.9414	7.99

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

Suggested

**Response 5 Impact Strength**

**ANOVA for Response Surface Quartic model**  
**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	135.62	4	33.91	345.44	< 0.0001	significant
<i>A-% Mo</i>	4.28	1	4.28	43.62	0.0002	
<i>A<sup>2</sup></i>	13.05	1	13.05	133.00	< 0.0001	
<i>A<sup>3</sup></i>	0.013	1	0.013	0.13	0.7303	
<i>A<sup>4</sup></i>	0.56	1	0.56	5.75	0.0433	
Residual	0.79	8	0.098			
Cor Total	136.41	12				

The Model F-value of 345.44 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>4</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.31	R-Squared	0.9942
Mean	29.14	Adj R-Squared	0.9914
C.V. %	1.08	Pred R-Squared	0.9854
PRESS	1.99	Adeq Precision	57.572
-2 Log Likelihood	0.40	BIC	13.23
		AICc	18.98

The "Pred R-Squared" of 0.9854 is in reasonable agreement with the "Adj R-Squared" of 0.9914; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 57.572 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	25.71	1	0.16	25.34	26.09	
A-% Mo	-2.50	1	0.38	-3.37	-1.63	7.57
A <sup>2</sup>	10.56	1	0.92	8.45	12.67	13.98
A <sup>3</sup>	-0.18	1	0.49	-1.31	0.96	7.56
A <sup>4</sup>	-2.18	1	0.91	-4.27	-0.084	13.89

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Impact Strength} = & \\ & +25.71 \\ & -2.50 * A \\ & +10.56 * A^2 \\ & -0.18 * A^3 \\ & -2.18 * A^4 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Impact Strength} = & \\ & +38.35746 \\ & -3.09729 * \% \text{ Mo} \\ & -0.21421 * \% \text{ Mo}^2 \end{aligned}$$

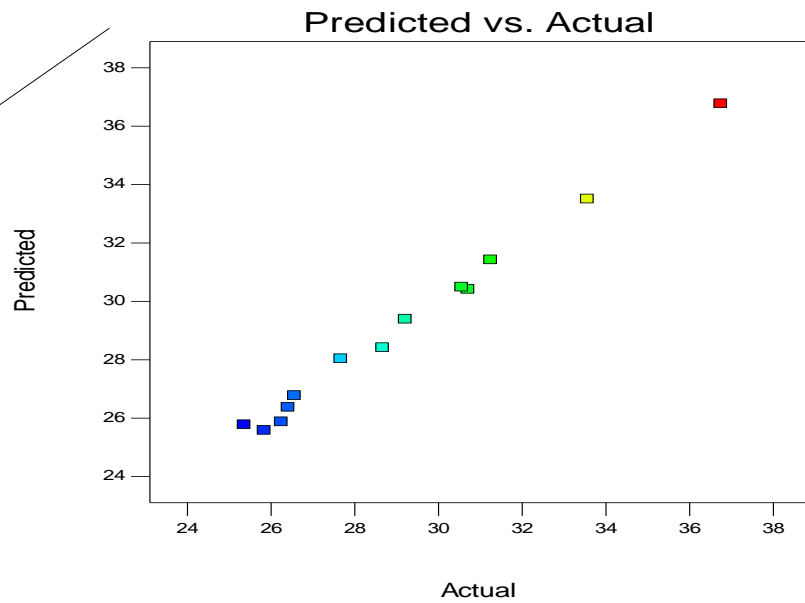
$$+0.088269 * \% Mo^3$$

$$-4.28106E-003 * \% Mo^4$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

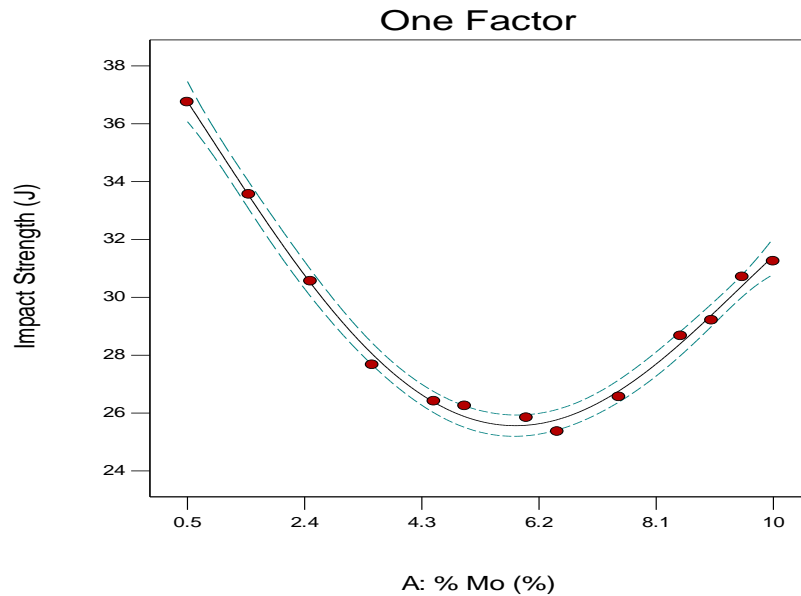
Design-Expert® Software  
Impact Strength

Color points by value of  
Impact Strength:



Design-Expert® Software  
 Factor Coding: Actual  
 Impact Strength (J)  
 ● Design Points  
 --- 95% CI Bands

X1 = A: % Mo



**Response 6 Resistivity Transform: None**

**Summary (detailed tables shown below)**

Source	p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0206	0.3443	0.1249
Quadratic	< 0.0001	0.9296	0.8704
Cubic	0.0477	0.9506	0.8840
Quartic	0.0025	0.9834	0.8842
<u>Fifth</u>	<u>0.0115</u>	<u>0.9928</u>	<u>0.9550</u> <u>Suggested</u>
Sixth	0.3502	0.9929	0.9724

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	Df	Mean Square	F Value	Prob > F
Mean vs Total	1731.46	1	1731.46		
Linear vs Mean	23.90	1	23.90	7.30	0.0206
Quadratic vs Linear	32.49	1	32.49	92.47	< 0.0001
Cubic vs	1.29	1	1.29	5.25	0.0477



Quadratic					
Quartic vs Cubic	1.56	1	1.56	18.75	0.0025
<u>Fifth vs Quartic</u>	<u>0.41</u>	<u>1</u>	<u>0.41</u>	<u>11.55</u>	<u>0.0115</u>
Sixth vs Fifth	0.037	1	0.037	1.03	0.3502
Residual	0.21	6	0.036		

Suggested

"Sequential Model Sum of Squares [Type I]":  
Select the highest order polynomial where the

Total 1791.36 13 137.80

additional terms are significant and the model  
is not aliased.

### Model Summary Statistics

	Std.	Adjusted		Predicted	
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS
Linear	1.81	0.3990	0.3443	0.1249	52.42
Quadratic	0.59	0.9413	0.9296	0.8704	7.76
Cubic	0.50	0.9629	0.9506	0.8840	6.95
Quartic	0.29	0.9889	0.9834	0.8842	6.94
<u>Fifth</u>	<u>0.19</u>	<u>0.9958</u>	<u>0.9928</u>	<u>0.9550</u>	<u>2.70</u>
Sixth	0.19	0.9964	0.9929	0.9724	1.65

Suggested

"Model Summary Statistics": Focus on  
the model maximizing the "Adjusted R-  
Squared"  
and the "Predicted R-Squared".

## Response 6 Resistivity

### ANOVA for Response Surface Fifth model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	59.65	5	11.93	333.40	< 0.0001	significant
A-% Mo	5.34	1	5.34	149.23	< 0.0001	
A <sup>2</sup>	6.87	1	6.87	191.90	< 0.0001	
A <sup>3</sup>	0.76	1	0.76	21.36	0.0024	
A <sup>4</sup>	1.38	1	1.38	38.64	0.0004	
A <sup>5</sup>	0.41	1	0.41	11.55	0.0115	
Residual	0.25	7	0.036			
Cor Total	59.90	12				

The Model F-value of 333.40 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup>, A<sup>4</sup>, A<sup>5</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.19	R-Squared	0.9958
Mean	11.54	Adj R-Squared	0.9928
C.V. %	1.64	Pred R-Squared	0.9550
PRESS	2.70	Adeq Precision	55.722
-2 Log Likelihood	-14.45	BIC	0.94
		AICc	11.55

The "Pred R-Squared" of 0.9550 is in reasonable agreement with the "Adj R-Squared" of 0.9928; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 55.722 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient Estimate	Standard Error	95% CI Low	95% CI High	VIF
Intercept	13.46	0.098	13.23	13.69	
A-% Mo	4.98	0.41	4.02	5.94	24.15
A <sup>2</sup>	-7.72	0.56	-9.04	-6.41	14.22

A <sup>3</sup>	-6.50	1	1.41	-9.82	-3.17	170.57
A <sup>4</sup>	3.43	1	0.55	2.12	4.73	14.03
A <sup>5</sup>	3.71	1	1.09	1.13	6.28	83.36

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned}
 \text{Resistivity} = & \\
 & +13.46 \\
 & +4.98 * A \\
 & -7.72 * A^2 \\
 & -6.50 * A^3 \\
 & +3.43 * A^4 \\
 & +3.71 * A^5
 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned}
 \text{Resistivity} = & \\
 & +6.30123 \\
 & +1.55142 * \% \text{ Mo} \\
 & -0.49087 * \% \text{ Mo}^2 \\
 & +0.22028 * \% \text{ Mo}^3 \\
 & -0.033488 * \% \text{ Mo}^4 \\
 & +1.53229\text{E-}003 * \% \text{ Mo}^5
 \end{aligned}$$

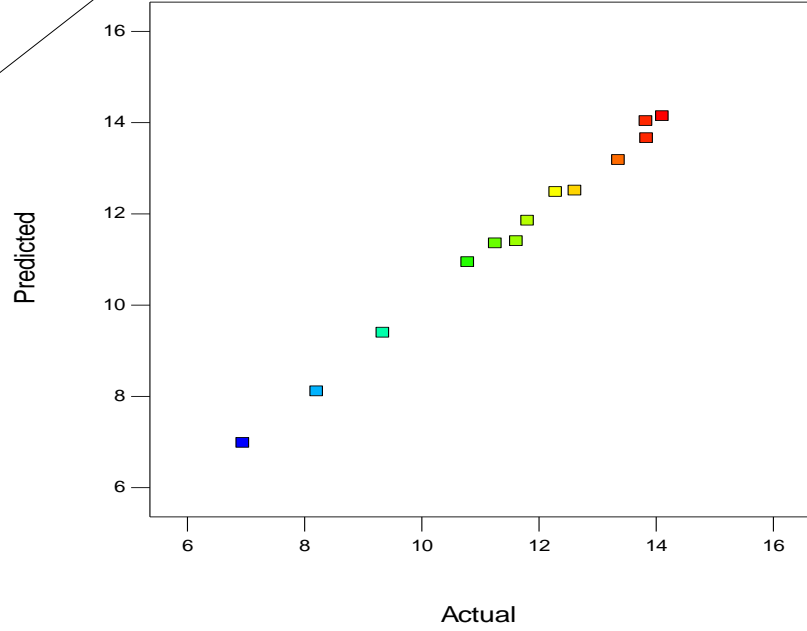
The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Resistivity

Color points by value of  
Resistivity:



### Predicted vs. Actual

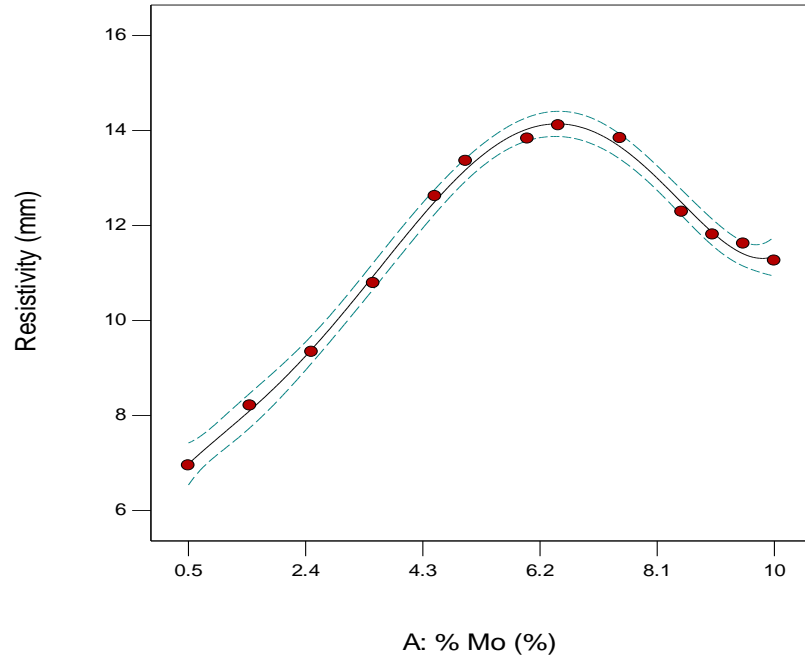


Design-Expert® Software  
Factor Coding: Actual  
Resistivity (mm)

● Design Points  
--- 95% CI Bands

X1 = A: % Mo

### One Factor



Response 7 Conductivity Transform: None

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0228		0.3332	0.0339
<u>Quadratic</u>	<u>&lt; 0.0001</u>		<u>0.9764</u>	<u>0.9679</u> <u>Suggested</u>
Cubic	0.6873		0.9743	0.9510
Quartic	0.8325		0.9712	0.8864
Fifth	0.9215		0.9672	0.3006
Sixth	0.0797		0.9780	-0.6131

Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob > F
Mean vs Total	263.61	1	263.61		
Linear vs Mean	14.15	1	14.15	7.00	0.0228
<u>Quadratic vs Linear</u>	<u>21.53</u>	<u>1</u>	<u>21.53</u>	<u>300.65</u>	<u>≤ 0.0001</u> <u>Suggested</u>
Cubic vs Quadratic	0.014	1	0.014	0.17	0.6873
Quartic vs Cubic	4.167E-003	1	4.167E-003	0.048	0.8325
Fifth vs Quartic	1.039E-003	1	1.039E-003	0.010	0.9215
Sixth vs Fifth	0.30	1	0.30	4.44	0.0797
Residual	0.40	6	0.067		
Total	300.01	13	23.08		

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

	Std.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS	
Linear	1.42	0.3888	0.3332	0.0339	35.16	
<u>Quadratic</u>	<u>0.27</u>	<u>0.9803</u>	<u>0.9764</u>	<u>0.9679</u>	<u>1.17</u>	<u>Suggested</u>
Cubic	0.28	0.9807	0.9743	0.9510	1.78	
Quartic	0.30	0.9808	0.9712	0.8864	4.14	
Fifth	0.32	0.9808	0.9672	0.3006	25.46	
Sixth	0.26	0.9890	0.9780	-0.6131	58.72	"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

**Response 7 Conductivity**

**ANOVA for Response Surface Quadratic model**

**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	35.68	2	17.84	249.13	< 0.0001	significant
A-% Mo	18.75	1	18.75	261.82	< 0.0001	
A <sup>2</sup>	21.53	1	21.53	300.65	< 0.0001	
Residual	0.72	10	0.072			
Cor Total	36.40	12				

The Model F-value of 249.13 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.27	R-Squared	0.9803
Mean	4.50	Adj R-Squared	0.9764
C.V. %	5.94	Pred R-Squared	0.9679

PRESS	1.17	Adeq Precision	45.298
-2 Log Likelihood	-0.79	BIC	6.90
		AICc	7.87

The "Pred R-Squared" of 0.9679 is in reasonable agreement with the "Adj R-Squared" of 0.9764; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 45.298 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error		95% CI		High VIF
	Estimate	df	Error	Low	High		
Intercept	3.20	1	0.11	2.94	3.45		
A-% Mo	-1.92	1	0.12	-2.18	-1.65	1.02	
A <sup>2</sup>	3.66	1	0.21	3.19	4.13	1.02	

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Conductivity} = & \\ & +3.20 \\ & -1.92 * A \\ & +3.66 * A^2 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Conductivity} = & \\ & +9.78150 \\ & -2.10533 * \% \text{ Mo} \\ & +0.16211 * \% \text{ Mo}^2 \end{aligned}$$

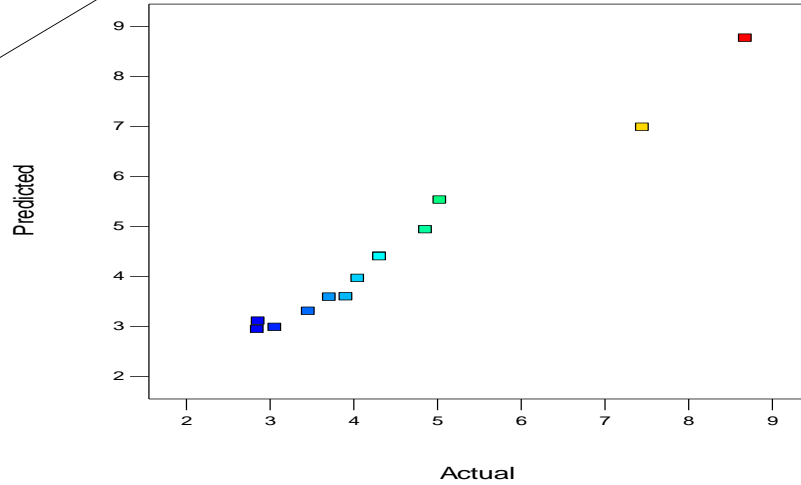
The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Conductivity

Color points by value of  
Conductivity:



Predicted vs. Actual

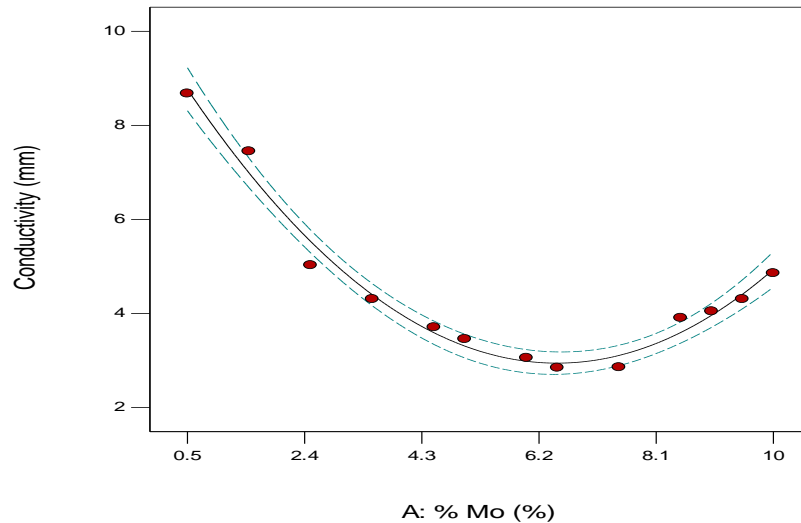


Design-Expert® Software  
Factor Coding: Actual  
Conductivity (mm)

● Design Points  
--- 95% CI Bands

X1 = A: % Mo

One Factor



## Design Expert Analysis for Tungsten

Response 1      Yield Strength Transform:      None

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0002		0.6963	0.5708
Quadratic	< 0.0001		0.9649	0.9532
Cubic	<u>0.0339</u>		<u>0.9769</u>	<u>0.9610</u> <u>Suggested</u>



Quartic	0.3433		0.9770	0.9490
Fifth	0.1347		0.9813	0.6474
<u>Sixth</u>	<u>0.0089</u>		<u>0.9936</u>	<u>0.9155 Suggested</u>

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	P-value	Prob > F
Mean vs Total	2.281E+006	1	2.281E+006			
Linear vs Mean	83192.63	1	83192.63	28.51	0.0002	
Quadratic vs Linear	28722.13	1	28722.13	85.05	< 0.0001	
<u>Cubic vs Quadratic</u>	<u>1383.38</u>	<u>1</u>	<u>1383.38</u>	<u>6.25</u>	<u>0.0339</u>	<u>Suggested</u>
Quartic vs Cubic	224.41	1	224.41	1.01	0.3433	
Fifth vs Quartic	513.06	1	513.06	2.86	0.1347	
<u>Sixth vs Fifth</u>	<u>888.04</u>	<u>1</u>	<u>888.04</u>	<u>14.48</u>	<u>0.0089</u>	<u>Suggested</u>
Residual	368.04	6	61.34			

Total 2.396E+006 13 1.843E+005

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS	
Linear	54.02	0.7216	0.6963	0.5708	49479.26	
Quadratic	18.38	0.9707	0.9649	0.9532	5395.39	
<u>Cubic</u>	<u>14.88</u>	<u>0.9827</u>	<u>0.9769</u>	<u>0.9610</u>	<u>4492.78</u>	<u>Suggested</u>
Quartic	14.87	0.9847	0.9770	0.9490	5880.91	

Fifth 13.40 0.9891 0.9813 0.6474 40647.42

Suggested"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

Sixth 7.83 0.9968 0.9936 0.9155 9737.18

and the "Predicted R-Squared".

### Response 1 Yield Strength

#### ANOVA for Response Surface Sixth model

##### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	1.149E+005	6	19153.94	312.26	< 0.0001	significant
A-% W	9362.41	1	9362.41	152.63	< 0.0001	
A <sup>2</sup>	276.67	1	276.67	4.51	0.0779	
A <sup>3</sup>	607.23	1	607.23	9.90	0.0199	
A <sup>4</sup>	1027.98	1	1027.98	16.76	0.0064	
A <sup>5</sup>	312.13	1	312.13	5.09	0.0649	
A <sup>6</sup>	888.04	1	888.04	14.48	0.0089	
Residual	368.04	6	61.34			
Cor Total	1.153E+005	12				

The Model F-value of 312.26 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>3</sup>, A<sup>4</sup>, A<sup>6</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	7.83	R-Squared	0.9968
Mean	418.85	Adj R-Squared	0.9936
C.V. %	1.87	Pred R-Squared	0.9155
PRESS	9737.18	Adeq Precision	56.951
-2 Log Likelihood	80.35	BIC	98.31

AICc 116.75

The "Pred R-Squared" of 0.9155 is in reasonable agreement with the "Adj R-Squared" of 0.9936; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 56.951 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	444.34	1	4.79	432.62	456.06	
A-% W	209.90	1	16.99	168.32	251.47	24.47
A <sup>2</sup>	125.73	1	59.20	-19.13	270.58	93.52
A <sup>3</sup>	-185.65	1	59.01	-330.03	-41.27	175.22
A <sup>4</sup>	-616.49	1	150.59	-984.98	-248.00	610.47
A <sup>5</sup>	103.13	1	45.72	-8.74	214.99	85.49
A <sup>6</sup>	374.39	1	98.40	133.62	615.16	257.04

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned}
 \text{Yield Strength} = & \\
 & +444.34 \\
 & +209.90 * A \\
 & +125.73 * A^2 \\
 & -185.65 * A^3 \\
 & -616.49 * A^4 \\
 & +103.13 * A^5 \\
 & +374.39 * A^6
 \end{aligned}$$

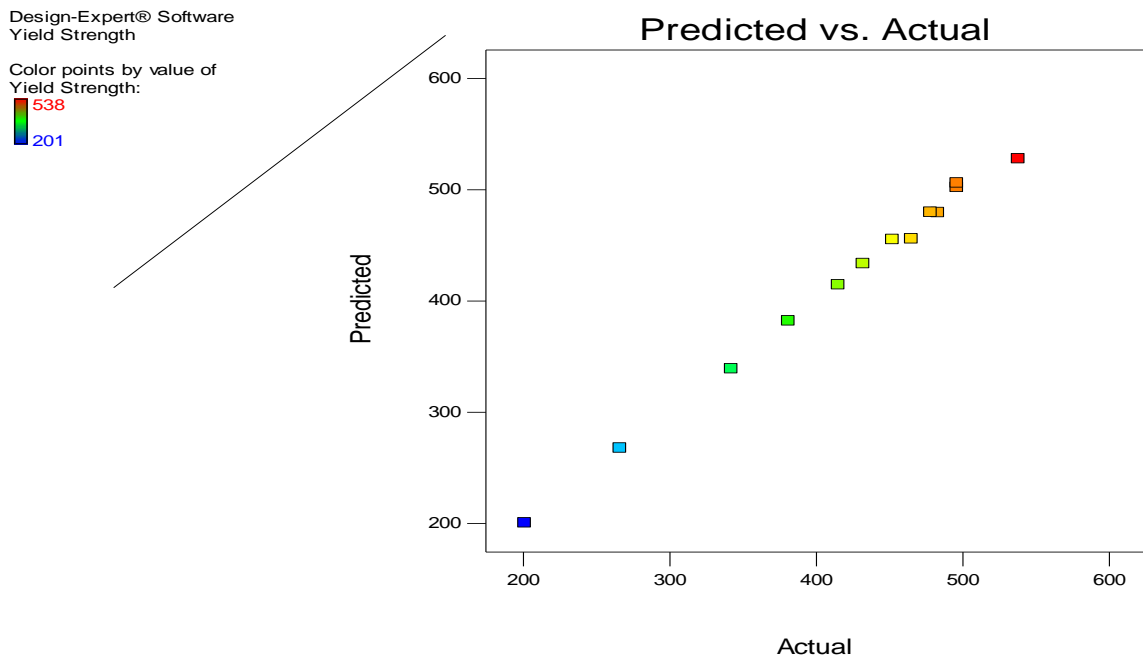
The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned}
 \text{Yield Strength} = & \\
 & +209.02518 \\
 & -74.63545 * \% W \\
 & +142.31283 * \% W^2 \\
 & -58.88080 * \% W^3
 \end{aligned}$$

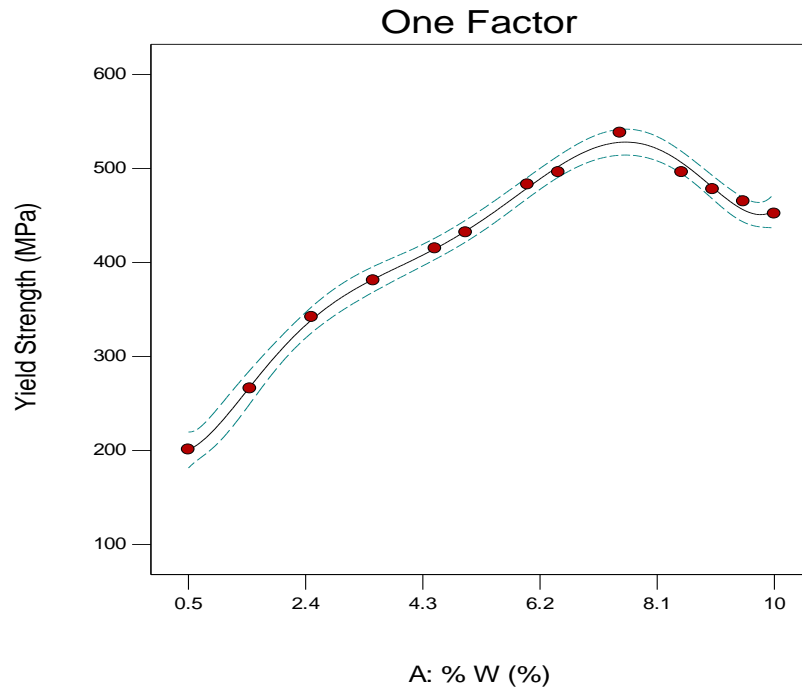
$$\begin{aligned}
 &+11.14588 * \% W^4 \\
 &-0.98413 * \% W^5 \\
 &+0.032596 * \% W^6
 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.



Design-Expert® Software  
 Factor Coding: Actual  
 Yield Strength (MPa)  
 ● Design Points  
 --- 95% CI Bands

X1 = A: % W



**Response 2          UTS          Transform:   None**

**Summary (detailed tables shown below)**

Source	p-value	Sequential Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0088		0.4309	0.2205
<u>Quadratic</u>	<u>&lt; 0.0001</u>		<u>0.9409</u>	<u>0.9175</u> <u>Suggested</u>
Cubic	0.2373		0.9442	0.9003
Quartic	0.2198		0.9487	0.8433
<u>Fifth</u>	<u>0.0368</u>		<u>0.9699</u>	<u>0.7245</u> <u>Suggested</u>
Sixth	0.2135		0.9734	0.6940

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F	P-value
Mean vs Total	3.816E+006	1	3.816E+006		
Linear vs Mean	28554.00	1	28554.00	10.09	0.0088

<u>Quadratic vs Linear</u>	<u>28203.33</u>	<u>1</u>	<u>28203.33</u>	<u>95.90</u>	<u>0.0001</u>	<u>≤</u>	<u>Suggested</u>
Cubic vs Quadratic	444.61	1	444.61	1.60	0.2373		
Quartic vs Cubic	452.70	1	452.70	1.77	0.2198		
<u>Fifth vs Quartic</u>	<u>993.79</u>	<u>1</u>	<u>993.79</u>	<u>6.63</u>	<u>0.0368</u>		<u>Suggested</u>
Sixth vs Fifth	256.09	1	256.09	1.94	0.2135		
Residual	793.80	6	132.30				

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 3.875E+006 13 2.981E+005

additional terms are significant and the model is not aliased.

### Model Summary Statistics

	Std.		Adjusted	Predicted		
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS	
Linear	53.21	0.4783	0.4309	0.2205	46533.14	
<u>Quadratic</u>	<u>17.15</u>	<u>0.9507</u>	<u>0.9409</u>	<u>0.9175</u>	<u>4927.30</u>	<u>Suggested</u>
Cubic	16.65	0.9582	0.9442	0.9003	5951.04	
Quartic	15.98	0.9658	0.9487	0.8433	9353.48	
<u>Fifth</u>	<u>12.25</u>	<u>0.9824</u>	<u>0.9699</u>	<u>0.7245</u>	<u>16447.47</u>	<u>Suggested</u>
Sixth	11.50	0.9867	0.9734	0.6940	18269.41	

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

## Response 2 UTS

### ANOVA for Response Surface Fifth model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	58648.42	5	11729.68	78.21	< 0.0001	significant
A-% W	5870.45	1	5870.45	39.14	0.0004	
A <sup>2</sup>	3712.04	1	3712.04	24.75	0.0016	
A <sup>3</sup>	1250.74	1	1250.74	8.34	0.0234	
A <sup>4</sup>	323.38	1	323.38	2.16	0.1855	
A <sup>5</sup>	993.79	1	993.79	6.63	0.0368	
Residual	1049.88	7	149.98			
Cor Total	59698.31	12				

The Model F-value of 78.21 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup>, A<sup>5</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	12.25	R-Squared	0.9824
Mean	541.77	Adj R-Squared	0.9699
C.V. %	2.26	Pred R-Squared	0.7245
PRESS	16447.47	Adeq Precision	28.584
-2 Log Likelihood	93.98	BIC	109.37
		AICc	119.98

The "Pred R-Squared" of 0.7245 is not as close to the "Adj R-Squared" of 0.9699 as one might normally expect; i.e. the difference is more than 0.2. This may indicate a large block effect or a possible problem with your model and/or data. Things to consider are model reduction, response transformation, outliers, etc. All empirical models should be tested by doing confirmation runs. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 28.584 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	592.38	1	6.36	577.35	607.42	
A-% W	165.13	1	26.39	102.71	227.54	24.15
A <sup>2</sup>	-179.59	1	36.10	-264.95	-94.23	14.22
A <sup>3</sup>	-262.89	1	91.03	-478.15	-47.62	170.57
A <sup>4</sup>	52.43	1	35.70	-32.00	136.85	14.03
A <sup>5</sup>	181.71	1	70.59	14.79	348.63	83.36

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned}
 \text{UTS} = & \\
 & +592.38 \\
 & +165.13 * A \\
 & -179.59 * A^2 \\
 & -262.89 * A^3 \\
 & +52.43 * A^4 \\
 & +181.71 * A^5
 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

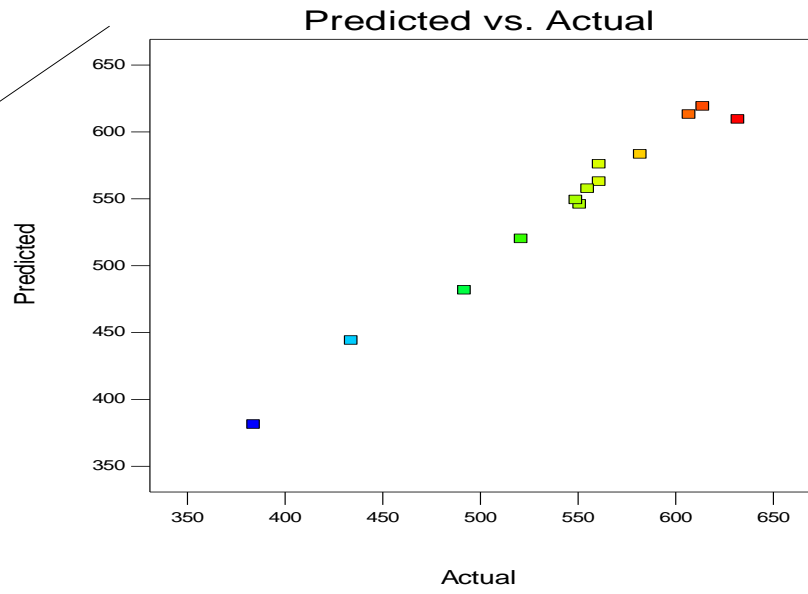
$$\begin{aligned}
 \text{UTS} = & \\
 & +323.95845 \\
 & +141.34433 * \% W \\
 & -61.03379 * \% W^2 \\
 & +16.09655 * \% W^3 \\
 & -1.86960 * \% W^4 \\
 & +0.075146 * \% W^5
 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.



Design-Expert® Software  
UTS

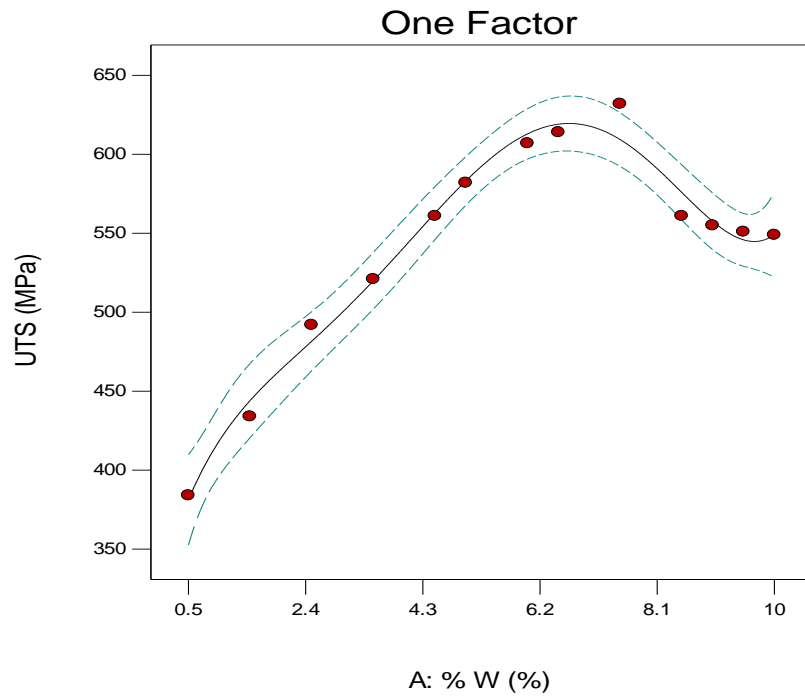
Color points by value of  
UTS:  
632  
384



Design-Expert® Software  
Factor Coding: Actual  
UTS (MPa)

• Design Points  
--- 95% CI Bands

X1 = A: % W



Response 3                      Hardness   Transform:   None

Summary (detailed tables shown below)

	Sequential Lack of Fit	Adjusted	Predicted
Source	p-value	R-Squared	R-Squared
Linear	0.0004	0.6613	0.5237
<u>Quadratic</u>	<u>&lt; 0.0001</u>	<u>0.9892</u>	<u>0.9791</u> <u>Suggested</u>
Cubic	0.2372	0.9898	0.9702
Quartic	0.1143	0.9918	0.9802
Fifth	0.9024	0.9906	0.9193
<u>Sixth</u>	<u>0.0254</u>	<u>0.9955</u>	<u>0.8323</u> <u>Suggested</u>

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	P-value	Prob > F
Mean vs Total	1.182E+006	1	1.182E+006			
Linear vs Mean	57798.55	1	57798.55	24.42	0.0004	
<u>Quadratic vs Linear</u>	<u>25274.17</u>	<u>1</u>	<u>25274.17</u>	<u>334.09</u>	<u>&lt; 0.0001</u>	<u>Suggested</u>
Cubic vs Quadratic	114.41	1	114.41	1.60	0.2372	
Quartic vs Cubic	181.05	1	181.05	3.14	0.1143	
Fifth vs Quartic	1.06	1	1.06	0.016	0.9024	
<u>Sixth vs Fifth</u>	<u>272.69</u>	<u>1</u>	<u>272.69</u>	<u>8.74</u>	<u>0.0254</u>	<u>Suggested</u>
Residual	187.29	6	31.22			

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 1.266E+006 13 97373.85

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS	
Linear	48.65	0.6895	0.6613	0.5237	39931.43	
<u>Quadratic</u>	<u>8.70</u>	<u>0.9910</u>	<u>0.9892</u>	<u>0.9791</u>	<u>1749.89</u>	<u>Suggested</u>
Cubic	8.45	0.9923	0.9898	0.9702	2495.58	
Quartic	7.59	0.9945	0.9918	0.9802	1657.22	
Fifth	8.11	0.9945	0.9906	0.9193	6764.10	
<u>Sixth</u>	<u>5.59</u>	<u>0.9978</u>	<u>0.9955</u>	<u>0.8323</u>	<u>14056.40</u>	

Suggested"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

and the "Predicted R-Squared".

**Response 3 Hardness**

**ANOVA for Response Surface Sixth model**

**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	83641.94	6	13940.32	446.58	< 0.0001	significant
A-% W	3139.12	1	3139.12	100.56	< 0.0001	
A <sup>2</sup>	40.88	1	40.88	1.31	0.2961	
A <sup>3</sup>	1.94	1	1.94	0.062	0.8114	
A <sup>4</sup>	203.83	1	203.83	6.53	0.0432	
A <sup>5</sup>	13.12	1	13.12	0.42	0.5407	
A <sup>6</sup>	272.69	1	272.69	8.74	0.0254	
Residual	187.29	6	31.22			
Cor Total	83829.23	12				

The Model F-value of 446.58 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>4</sup>, A<sup>6</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model

terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	5.59	R-Squared	0.9978
Mean	301.54	Adj R-Squared	0.9955
C.V. %	1.85	Pred R-Squared	0.8323
PRESS	14056.40	Adeq Precision	63.938
-2 Log Likelihood	71.57	BIC	89.53
		AICc	107.97

The "Pred R-Squared" of 0.8323 is in reasonable agreement with the "Adj R-Squared" of 0.9955; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 63.938 indicates an adequate signal. This model can be used to navigate the design space.

	<b>Coefficient</b>	<b>Standard Error</b>	<b>95% CI</b>	<b>95% CI</b>	
<b>Factor</b>	<b>Estimate</b>	<b>df</b>	<b>Low</b>	<b>High</b>	<b>VIF</b>
Intercept	339.88	1	3.42	331.51 348.24	
A-% W	121.54	1	12.12	91.88 151.19	24.47
A <sup>2</sup>	-48.33	1	42.23	-151.66 55.01	93.52
A <sup>3</sup>	10.50	1	42.09	-92.50 113.49	175.22
A <sup>4</sup>	-274.52	1	107.43	-537.39 -11.65	610.47
A <sup>5</sup>	-21.14	1	32.61	-100.94 58.65	85.49
A <sup>6</sup>	207.47	1	70.19	35.71 379.22	257.04

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned}
 \text{Hardness} = & \\
 & +339.88 \\
 & +121.54 * A \\
 & -48.33 * A^2 \\
 & +10.50 * A^3 \\
 & -274.52 * A^4 \\
 & -21.14 * A^5 \\
 & +207.47 * A^6
 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

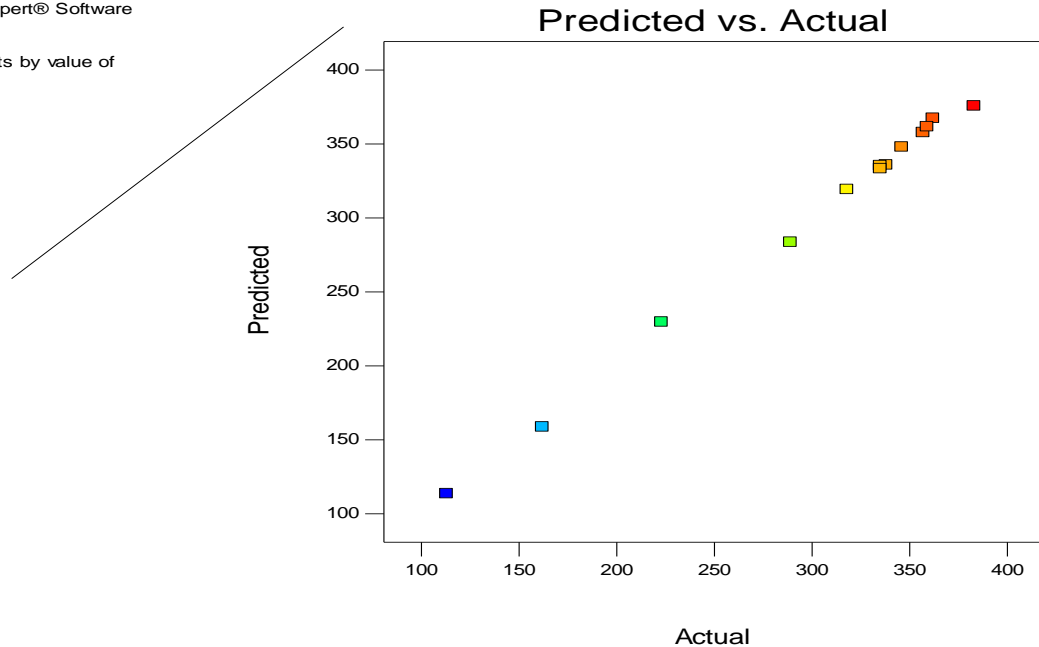
### Final Equation in Terms of Actual Factors:

$$\begin{aligned} \text{Hardness} = & \\ & +135.75857 \\ & -97.15878 * \% W \\ & +125.62177 * \% W^2 \\ & -43.26278 * \% W^3 \\ & +7.15814 * \% W^4 \\ & -0.57772 * \% W^5 \\ & +0.018063 * \% W^6 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

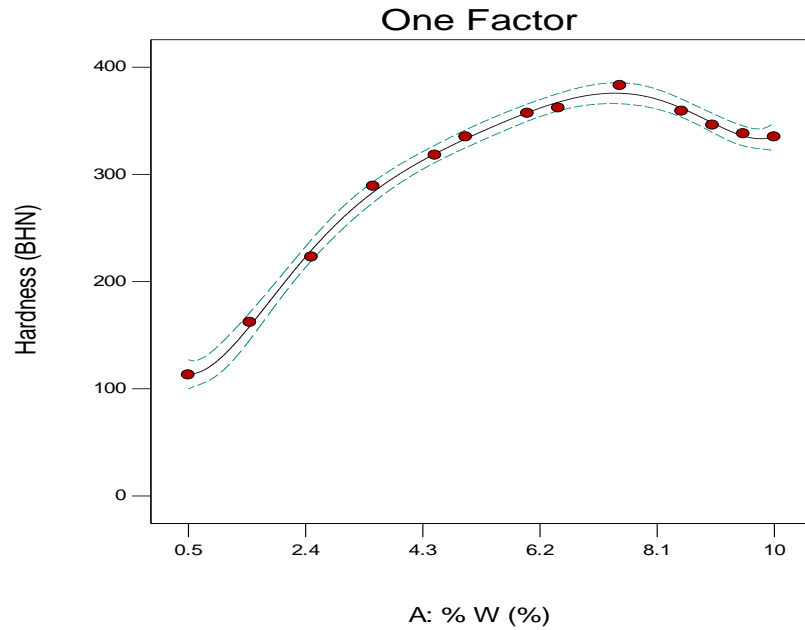
Design-Expert® Software  
Hardness

Color points by value of  
Hardness:



Design-Expert® Software  
 Factor Coding: Actual  
 Hardness (BHN)  
 ● Design Points  
 --- 95% CI Bands

X1 = A: % W



**Response 4 Elongation Transform: None**

**Summary (detailed tables shown below)**

Source	p-value	Sequential Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.1025		0.1533	-0.1675
Quadratic	< 0.0001		0.9509	0.9084
Cubic	0.0238		0.9700	0.9330
<u>Quartic</u>	<u>0.0452</u>		<u>0.9802</u>	<u>0.9521</u> <u>Suggested</u>
Fifth	0.5041		0.9789	0.9528
Sixth	0.6727		0.9761	-0.0470

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	Prob > F
Mean vs Total	4539.24	1	4539.24		
Linear vs Mean	30.09	1	30.09	3.17	0.1025
Quadratic vs Linear	98.80	1	98.80	179.76	< 0.0001
Cubic vs Quadratic	2.48	1	2.48	7.37	0.0238

<u>Quartic vs</u> <u>Cubic</u>	<u>1.25</u>	<u>1</u>	<u>1.25</u>	<u>5.62</u>	<u>0.0452</u>	<u>Suggested</u>
Fifth vs Quartic	0.12	1	0.12	0.50	0.5041	
Sixth vs Fifth	0.053	1	0.053	0.20	0.6727	
Residual	1.60	6	0.27			

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 4673.63 13 359.51

additional terms are significant and the model is not aliased.

### Model Summary Statistics

	Std.		Adjusted	Predicted		
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS	
Linear	3.08	0.2239	0.1533	-0.1675	156.90	
Quadratic	0.74	0.9591	0.9509	0.9084	12.30	
Cubic	0.58	0.9775	0.9700	0.9330	9.00	
<u>Quartic</u>	<u>0.47</u>	<u>0.9868</u>	<u>0.9802</u>	<u>0.9521</u>	<u>6.44</u>	<u>Suggested</u>
Fifth	0.49	0.9877	0.9789	0.9528	6.35	
Sixth	0.52	0.9881	0.9761	-0.0470	140.70	

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

and the "Predicted R-Squared".

## Response 4 Elongation

### ANOVA for Response Surface Quartic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	132.61	4	33.15	149.49	< 0.0001	significant
A-% W	15.92	1	15.92	71.80	< 0.0001	
A <sup>2</sup>	13.99	1	13.99	63.08	< 0.0001	
A <sup>3</sup>	2.56	1	2.56	11.55	0.0094	
A <sup>4</sup>	1.25	1	1.25	5.62	0.0452	
Residual	1.77	8	0.22			
Cor Total	134.38	12				

The Model F-value of 149.49 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup>, A<sup>4</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.47	R-Squared	0.9868
Mean	18.69	Adj R-Squared	0.9802
C.V. %	2.52	Pred R-Squared	0.9521
PRESS	6.44	Adeq Precision	35.898
-2 Log Likelihood	11.00	BIC	23.83
		AICc	29.57

The "Pred R-Squared" of 0.9521 is in reasonable agreement with the "Adj R-Squared" of 0.9802; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 35.898 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient Estimate	df	Standard Error	95% CI Low	95% CI High	VIF
Intercept	15.47	1	0.24	14.90	16.03	
A-% W	-4.82	1	0.57	-6.13	-3.50	7.57
A <sup>2</sup>	10.93	1	1.38	7.76	14.11	13.98
A <sup>3</sup>	2.50	1	0.74	0.80	4.20	7.56



$$A^4 \quad -3.24 \quad 1 \quad 1.37 \quad -6.39 \quad -0.089 \quad 13.89$$

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Elongation} = & \\ & +15.47 \\ & -4.82 * A \\ & +10.93 * A^2 \\ & +2.50 * A^3 \\ & -3.24 * A^4 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

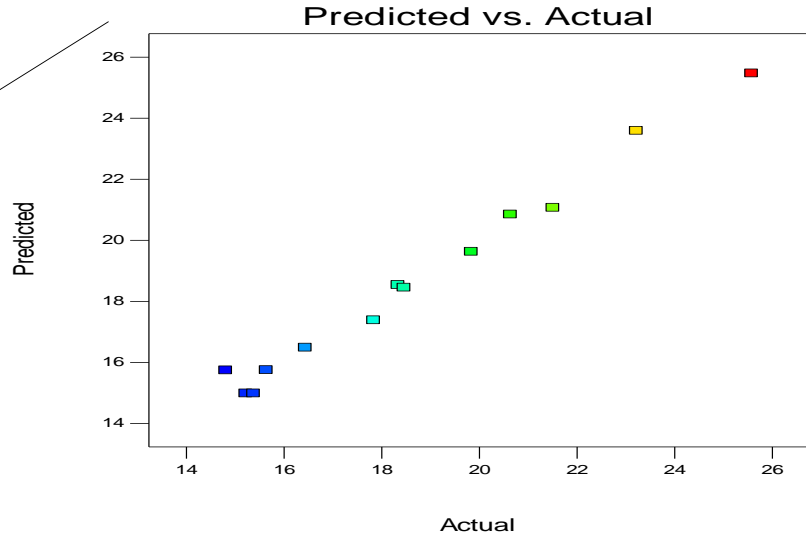
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Elongation} = & \\ & +25.92784 \\ & -0.48652 * \% W \\ & -0.93568 * \% W^2 \\ & +0.15698 * \% W^3 \\ & -6.36252E-003 * \% W^4 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

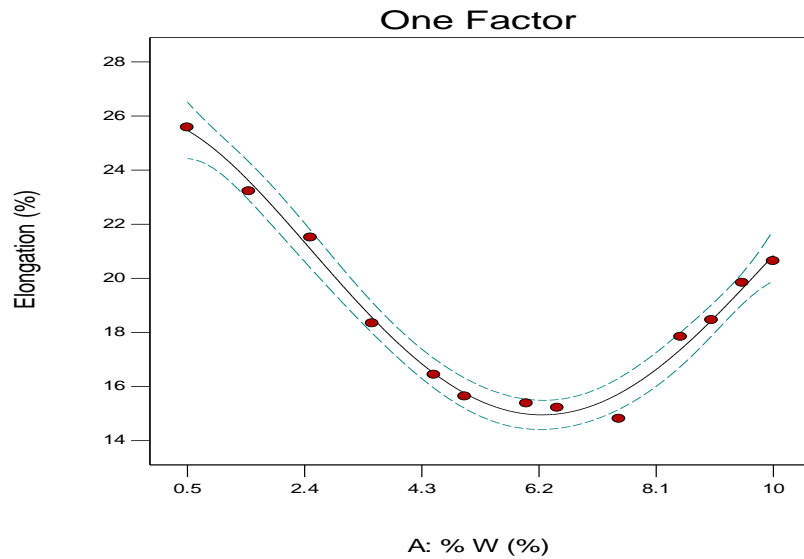
Design-Expert® Software  
Elongation

Color points by value of  
Elongation:  
25.58  
14.81



Design-Expert® Software  
Factor Coding: Actual  
Elongation (%)  
● Design Points  
--- 95% CI Bands

X1 = A: % W



**Response 5**      **Impact Strength Transform: None**

**Summary (detailed tables shown below)**

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0484		0.2465	-0.0728
<u>Quadratic</u>	<u>&lt; 0.0001</u>		<u>0.9764</u>	<u>0.9650</u> <u>Suggested</u>
Cubic	0.2081		0.9782	0.9630
Quartic	0.6657		0.9761	0.9537

Fifth	0.8273		0.9729	0.7970
Sixth	0.0717		0.9824	0.7579

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob > F	
Mean vs Total	11037.65	1	11037.65			
Linear vs Mean	50.29	1	50.29	4.93	0.0484	
<u>Quadratic vs Linear</u>	<u>109.13</u>	<u>1</u>	<u>109.13</u>	<u>341.57</u>	<u>0.0001</u>	<u>Suggested</u>
Cubic vs Quadratic	0.54	1	0.54	1.84	0.2081	
Quartic vs Cubic	0.065	1	0.065	0.20	0.6657	
Fifth vs Quartic	0.019	1	0.019	0.051	0.8273	
Sixth vs Fifth	1.14	1	1.14	4.77	0.0717	
Residual	1.43	6	0.24			
Total	11200.26	13	861.56			

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	R-PRESS	
Linear	3.20	0.3093	0.2465	-0.0728	174.45	
<u>Quadratic</u>	<u>0.57</u>	<u>0.9804</u>	<u>0.9764</u>	<u>0.9650</u>	<u>5.69</u>	<u>Suggested</u>
Cubic	0.54	0.9837	0.9782	0.9630	6.02	
Quartic	0.57	0.9841	0.9761	0.9537	7.53	
Fifth	0.61	0.9842	0.9729	0.7970	33.01	
Sixth	0.49	0.9912	0.9824	0.7579	39.37	"Model Summary Statistics": Focus on

the model maximizing the "Adjusted R-Squared"

and the "Predicted R-Squared".

## Response 5 Impact Strength

### ANOVA for Response Surface Quadratic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	159.42	2	79.71	249.49	< 0.0001	significant
A-% W	70.27	1	70.27	219.96	< 0.0001	
A <sup>2</sup>	109.13	1	109.13	341.57	< 0.0001	
Residual	3.19	10	0.32			
Cor Total	162.61	12				

The Model F-value of 249.49 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.57	R-Squared	0.9804
Mean	29.14	Adj R-Squared	0.9764
C.V. %	1.94	Pred R-Squared	0.9650
PRESS	5.69	Adeq Precision	45.473
-2 Log Likelihood	18.65	BIC	26.34
		AICc	27.31

The "Pred R-Squared" of 0.9650 is in reasonable agreement with the "Adj R-Squared" of 0.9764; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 45.473 indicates an adequate signal. This model can be used to navigate the design space.

**Coefficient    Standard 95% CI    95% CI**

Factor	Estimate	df	Error	Low	High	VIF
Intercept	26.14	1	0.24	25.60	26.67	
A-% W	-3.71	1	0.25	-4.26	-3.15	1.02
A <sup>2</sup>	8.23	1	0.45	7.24	9.23	1.02

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Impact Strength} = & \\ & +26.14 \\ & -3.71 * A \\ & +8.23 * A^2 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Impact Strength} = & \\ & +40.29289 \\ & -4.61250 * \% W \\ & +0.36495 * \% W^2 \end{aligned}$$

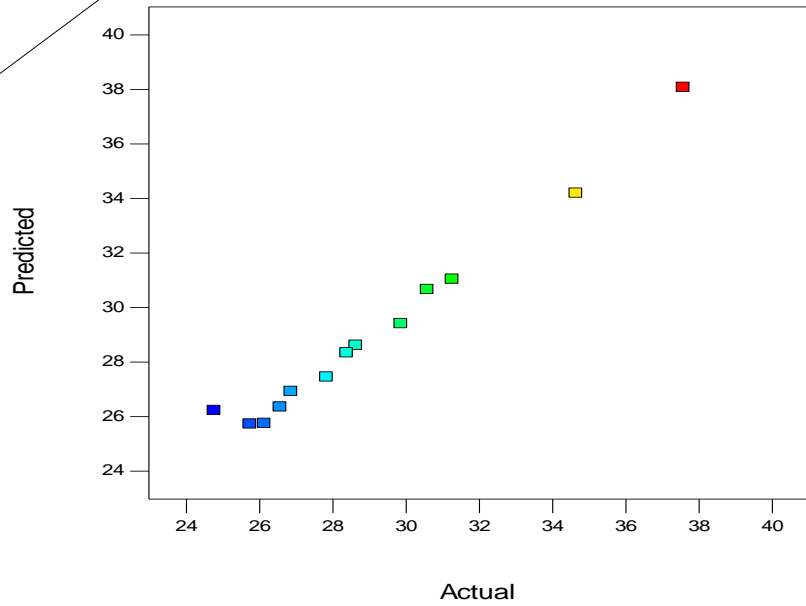
The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Impact Strength

Color points by value of  
Impact Strength:



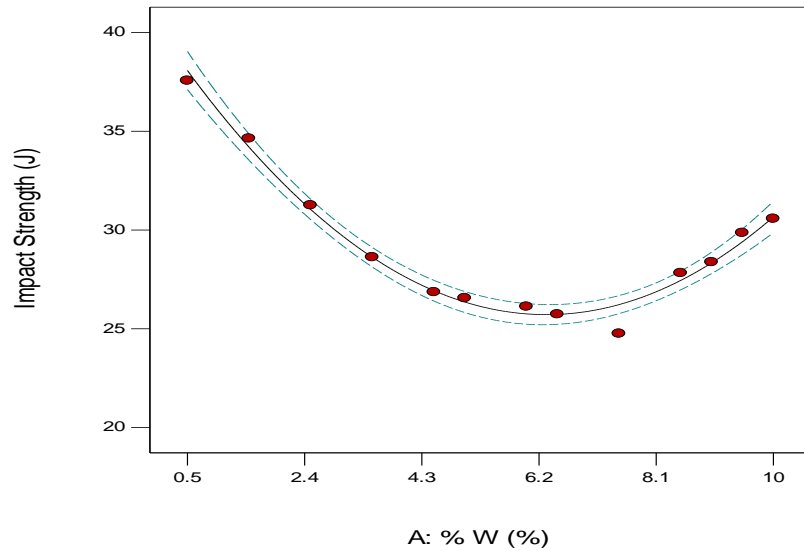
Predicted vs. Actual



Design-Expert® Software  
Factor Coding: Actual  
Impact Strength (J)  
● Design Points  
--- 95% CI Bands

X1 = A: % W

One Factor



**Response 6      Resistivity Transform:    None**

**Summary (detailed tables shown below)**

Source	p-value	Sequential Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0015		0.5809	0.4518

Quadratic	0.0009		0.8543	0.7798
<u>Cubic</u>	<u>0.0038</u>		<u>0.9391</u>	<u>0.8925 Suggested</u>
Quartic	0.9409		0.9316	0.7469
<u>Fifth</u>	<u>0.0241</u>		<u>0.9641</u>	<u>0.7770 Suggested</u>
Sixth	0.4336		0.9625	0.5308

### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob > F	
Mean vs Total	1706.62	1	1706.62			
Linear vs Mean	57.25	1	57.25	17.63	0.0015	
Quadratic vs Linear	24.42	1	24.42	21.64	0.0009	
<u>Cubic vs Quadratic</u>	<u>7.05</u>	<u>1</u>	<u>7.05</u>	<u>14.95</u>	<u>0.0038</u>	<u>Suggested</u>
Quartic vs Cubic	3.105E-003	1	3.105E-003	5.860E-003	0.9409	
<u>Fifth vs Quartic</u>	<u>2.29</u>	<u>1</u>	<u>2.29</u>	<u>8.23</u>	<u>0.0241</u>	<u>Suggested</u>
Sixth vs Fifth	0.20	1	0.20	0.70	0.4336	
Residual	1.74	6	0.29			

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 1799.58 13 138.43

additional terms are significant and the model is not aliased.

### Model Summary Statistics

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	Adjusted R-PRESS
Linear	1.80	0.6158	0.5809	0.4518	50.96
Quadratic	1.06	0.8786	0.8543	0.7798	20.47

Cubic	0.69	0.9544	0.9391	0.8925	10.00	<u>Suggested</u>
Quartic	0.73	0.9544	0.9316	0.7469	23.52	
Fifth	0.53	0.9790	0.9641	0.7770	20.73	<u>Suggested</u>
Sixth	0.54	0.9812	0.9625	0.5308	43.61	"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

## Response 6 Resistivity

### ANOVA for Response Surface Fifth model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	
Model	91.01	5	18.20	65.37	< 0.0001	significant
A-% W	18.52	1	18.52	66.50	< 0.0001	
A <sup>2</sup>	1.09	1	1.09	3.93	0.0879	
A <sup>3</sup>	4.15	1	4.15	14.91	0.0062	
A <sup>4</sup>	0.043	1	0.043	0.16	0.7047	
A <sup>5</sup>	2.29	1	2.29	8.23	0.0241	
Residual	1.95	7	0.28			
Cor Total	92.96	12				

The Model F-value of 65.37 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>3</sup>, A<sup>5</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.53	R-Squared	0.9790
Mean	11.46	Adj R-Squared	0.9641
C.V. %	4.61	Pred R-Squared	0.7770
PRESS	20.73	Adeq Precision	24.571
-2 Log Likelihood	12.22	BIC	27.61



AICc 38.22

The "Pred R-Squared" of 0.7770 is in reasonable agreement with the "Adj R-Squared" of 0.9641; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 24.571 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	12.58	1	0.27	11.93	13.22	
A-% W	9.27	1	1.14	6.58	11.96	24.15
A <sup>2</sup>	-3.08	1	1.56	-6.76	0.59	14.22
A <sup>3</sup>	-15.15	1	3.92	-24.42	-5.87	170.57
A <sup>4</sup>	-0.61	1	1.54	-4.25	3.03	14.03
A <sup>5</sup>	8.72	1	3.04	1.53	15.92	83.36

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Resistivity} = & \\ & +12.58 \\ & +9.27 * A \\ & -3.08 * A^2 \\ & -15.15 * A^3 \\ & -0.61 * A^4 \\ & +8.72 * A^5 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

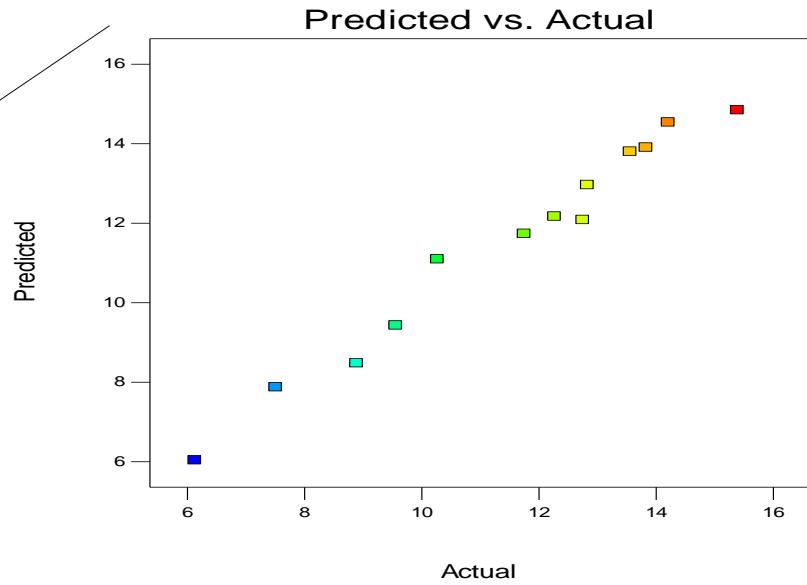
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Resistivity} = & \\ & +3.71816 \\ & +6.09385 * \% W \\ & -3.32812 * \% W^2 \\ & +0.87805 * \% W^3 \\ & -0.095892 * \% W^4 \\ & +3.60757E-003 * \% W^5 \end{aligned}$$

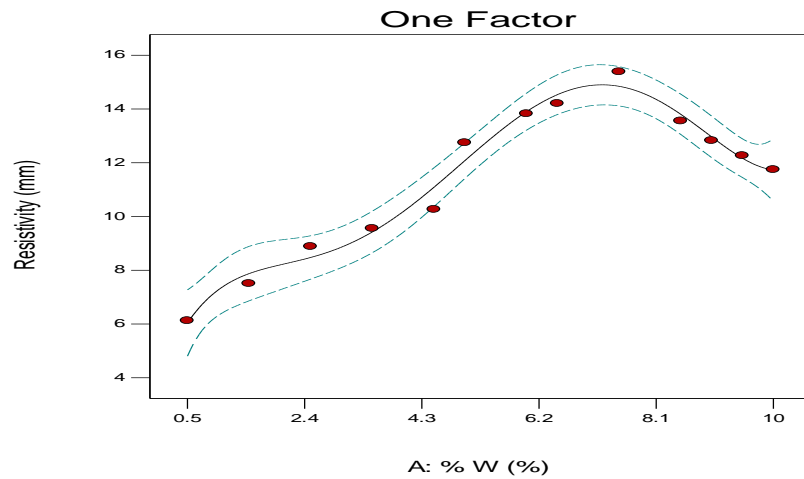
The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Resistivity

Color points by value of Resistivity:  
■ 15.39  
■ 6.13



Design-Expert® Software  
Factor Coding: Actual  
Resistivity (mm)  
● Design Points  
 --- 95% CI Bands  
 X1 = A: % W



Response 7 Conductivity Transform: None

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0022		0.5530	0.3476
<u>Quadratic</u>	<u>&lt; 0.0001</u>		<u>0.9534</u>	<u>0.9351</u> <u>Suggested</u>
Cubic	0.4928		0.9510	0.9293
Quartic	0.2367		0.9542	0.7061
Fifth	0.4118		0.9528	-0.1685
<u>Sixth</u>	<u>0.0007</u>		<u>0.9928</u>	<u>0.7001</u> <u>Suggested</u>

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Mean vs Total	244.60	1	244.60			
Linear vs Mean	24.87	1	24.87	15.85	0.0022	
<u>Quadratic vs Linear</u>	<u>15.62</u>	<u>1</u>	<u>15.62</u>	<u>95.47</u>	<u>0.0001</u>	<u>Suggested</u>
Cubic vs Quadratic	0.088	1	0.088	0.51	0.4928	
Quartic vs Cubic	0.26	1	0.26	1.64	0.2367	
Fifth vs Quartic	0.13	1	0.13	0.76	0.4118	
<u>Sixth vs Fifth</u>	<u>1.01</u>	<u>1</u>	<u>1.01</u>	<u>40.03</u>	<u>0.0007</u>	<u>Suggested</u>
Residual	0.15	6	0.025			
Total	286.73	13	22.06			

"Sequential Model Sum of Squares [Type I]":  
Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

Source	Std. Dev.	Adjusted R-Squared	Predicted R-Squared	R-PRESS

Linear	1.25	0.5903	0.5530	0.3476	27.48	
<u>Quadratic</u>	<u>0.40</u>	<u>0.9612</u>	<u>0.9534</u>	<u>0.9351</u>	<u>2.73</u>	<u>Suggested</u>
Cubic	0.41	0.9632	0.9510	0.9293	2.98	
Quartic	0.40	0.9695	0.9542	0.7061	12.38	
Fifth	0.41	0.9725	0.9528	-0.1685	49.22	
						<u>Suggested</u> "Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"
<u>Sixth</u>	<u>0.16</u>	<u>0.9964</u>	<u>0.9928</u>	<u>0.7001</u>	<u>12.63</u>	and the "Predicted R-Squared".

## Response 7 Conductivity

### ANOVA for Response Surface Sixth model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	41.97	6	7.00	277.73	< 0.0001	significant
A-% W	0.71	1	0.71	28.16	0.0018	
A <sup>2</sup>	0.47	1	0.47	18.71	0.0049	
A <sup>3</sup>	0.20	1	0.20	7.94	0.0305	
A <sup>4</sup>	1.13	1	1.13	44.98	0.0005	
A <sup>5</sup>	0.26	1	0.26	10.29	0.0184	
A <sup>6</sup>	1.01	1	1.01	40.03	0.0007	
Residual	0.15	6	0.025			
Cor Total	42.13	12				

The Model F-value of 277.73 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup>, A<sup>4</sup>, A<sup>5</sup>, A<sup>6</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.16	R-Squared	0.9964
Mean	4.34	Adj R-Squared	0.9928

C.V. %	3.66	Pred R-Squared	0.7001
PRESS	12.63	Adeq Precision	54.224
-2 Log Likelihood	-21.02	BIC	-3.06
		AICc	15.38

The "Pred R-Squared" of 0.7001 is not as close to the "Adj R-Squared" of 0.9928 as one might normally expect; i.e. the difference is more than 0.2. This may indicate a large block effect or a possible problem with your model and/or data. Things to consider are model reduction, response transformation, outliers, etc. All empirical models should be tested by doing confirmation runs. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 54.224 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	3.77	1	0.097	3.53	4.01	
A-% W	-1.83	1	0.34	-2.67	-0.98	24.47
A <sup>2</sup>	-5.19	1	1.20	-8.13	-2.25	93.52
A <sup>3</sup>	-3.37	1	1.20	-6.29	-0.44	175.22
A <sup>4</sup>	20.47	1	3.05	13.00	27.94	610.47
A <sup>5</sup>	2.97	1	0.93	0.70	5.24	85.49
A <sup>6</sup>	-12.62	1	1.99	-17.49	-7.74	257.04

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Conductivity} = & \\ & +3.77 \\ & -1.83 * A \\ & -5.19 * A^2 \\ & -3.37 * A^3 \\ & +20.47 * A^4 \\ & +2.97 * A^5 \\ & -12.62 * A^6 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

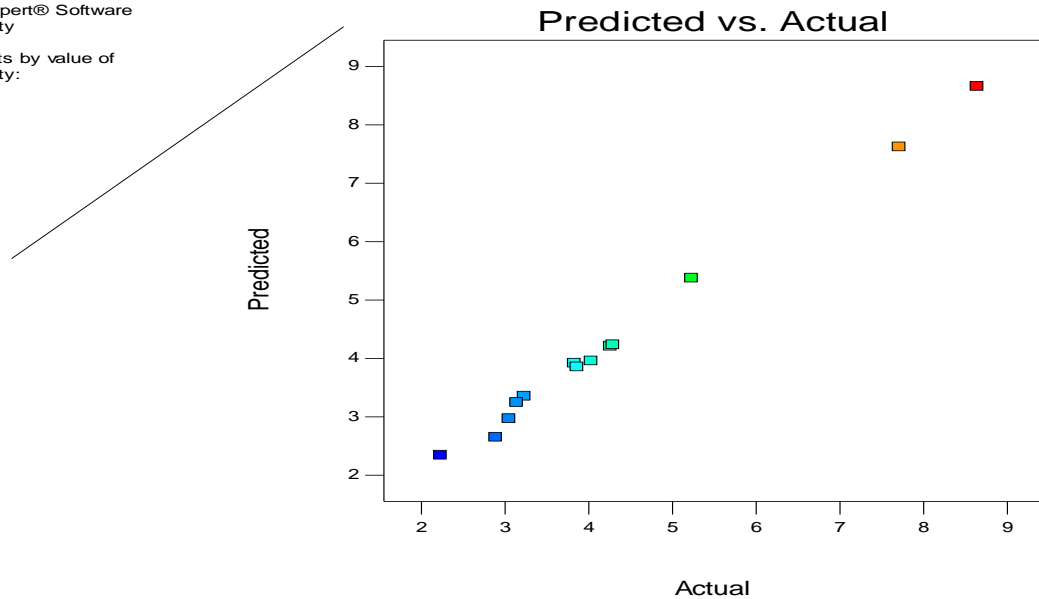
$$\text{Conductivity} =$$

$$\begin{aligned}
&+6.64349 \\
&+7.11241 * \% W \\
&-7.38071 * \% W^2 \\
&+2.64177 * \% W^3 \\
&-0.44617 * \% W^4 \\
&+0.035828 * \% W^5 \\
&-1.09839E-003 * \% W^6
\end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

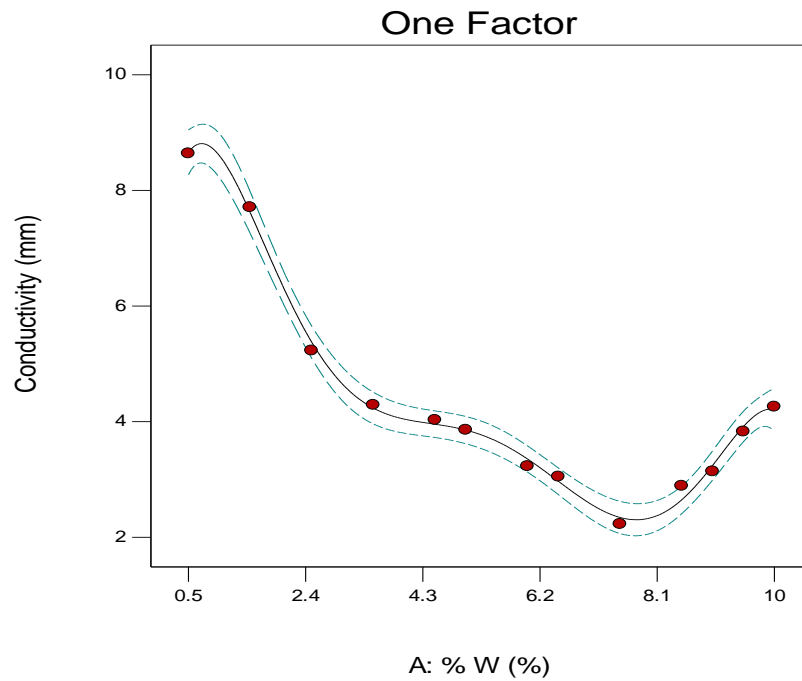
Design-Expert® Software  
Conductivity

Color points by value of  
Conductivity:



Design-Expert® Software  
 Factor Coding: Actual  
 Conductivity (mm)  
 ● Design Points  
 --- 95% CI Bands

X1 = A: % W



Factor	Name	Level	Low Level	High Level	Std. Dev.	Coding
A	% W	10.00	0.50	10.00	0.000	Actual

Response	Predicted		Observed	Std Dev	SE Mean	CI for Mean		99% of Population	
	Mean	Median <sup>1</sup>				95% CI low	95% CI high	95% TI low	95% TI high
Yield Strength	455.338	455.338	-	7.832	7.5711	436.812	473.864	401.984	508.691
UTS	549.166	549.166	-	12.2468	11.4249	522.15	576.182	470.829	627.504
Hardness	335.386	335.386	-	5.58708	5.40096	322.171	348.602	297.326	373.447
Elongation	20.8474	20.8474	-	0.470934	0.410522	19.9008	21.7941	18.0085	23.6864
Impact Strength	30.6628	30.6628	-	0.565232	0.358952	29.863	31.4626	27.6928	33.6328
Resistivity	11.7354	11.7354	-	0.527667	0.492258	10.5714	12.8994	8.36015	15.1107
Conductivity	4.21031	4.21031	-	0.15871	0.153423	3.8349	4.58572	3.12913	5.29148

### Confirmation Report

Two-sided Confidence = 95%      n = 1

Factor	Name	Level	Low Level	High Level	Std. Dev.	Coding
--------	------	-------	-----------	------------	-----------	--------

A      % W      10.00      0.50      10.00      0.000      Actual

Response	Predicted		Observed	Std Dev	n	SE Pred	95% PI low	Data Mean	95% PI high
	Mean	Median <sup>1</sup>							
Yield Strength	455.338	455.338	-	7.832	1	10.89	428.68		481.99
UTS	549.166	549.166	-	12.2468	1	16.75	509.56		588.77
Hardness	335.386	335.386	-	5.58708	1	7.77	316.37		354.40
Elongation	20.8474	20.8474	-	0.470934	1	0.62	19.41		22.29
Impact Strength	30.6628	30.6628	-	0.565232	1	0.67	29.17		32.15
Resistivity	11.7354	11.7354	-	0.527667	1	0.72	10.03		13.44
Conductivity	4.21031	4.21031	-	0.15871	1	0.22	3.67		4.75

### Design Expert Analysis for Chromium

Response 1      Yield Strength Transform:      None

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0013		0.5883	0.3998
<u>Quadratic</u>	<u>&lt; 0.0001</u>		<u>0.9790</u>	<u>0.9740</u> <u>Suggested</u>
Cubic	0.9282		0.9767	0.9666
Quartic	0.6576		0.9745	0.9495
Fifth	0.1830		0.9778	0.4744
<u>Sixth</u>	<u>0.0269</u>		<u>0.9893</u>	<u>0.2336</u> <u>Suggested</u>

### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	P-value	Prob > F
Mean vs Total	2.231E+006	1	2.231E+006			
Linear vs Mean	63407.67	1	63407.67	18.15	0.0013	
<u>Quadratic vs Linear</u>	<u>36657.03</u>	<u>1</u>	<u>36657.03</u>	<u>205.75</u>	<u>≤ 0.0001</u>	<u>Suggested</u>
Cubic vs Quadratic	1.70	1	1.70	8.577E-003	0.9282	
Quartic vs	45.90	1	45.90	0.21	0.6576	



Cubic						
Fifth vs Quartic	412.32	1	412.32	2.18	0.1830	
<u>Sixth vs Fifth</u>	<u>774.31</u>	<u>1</u>	<u>774.31</u>	<u>8.49</u>	<u>0.0269</u>	<u>Suggested</u>
Residual	547.38	6	91.23			

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 2.332E+006 13 1.794E+005

additional terms are significant and the model is not aliased.

### Model Summary Statistics

Source	Std. Dev.	Adjusted R-Squared	Predicted R-Squared	Adjusted R-Squared	PRESS	
Linear	59.11	0.6226	0.5883	0.3998	61128.61	
<u>Quadratic</u>	<u>13.35</u>	<u>0.9825</u>	<u>0.9790</u>	<u>0.9740</u>	<u>2647.87</u>	<u>Suggested</u>
Cubic	14.06	0.9825	0.9767	0.9666	3396.60	
Quartic	14.72	0.9830	0.9745	0.9495	5138.26	
Fifth	13.74	0.9870	0.9778	0.4744	53527.19	
<u>Sixth</u>	<u>9.55</u>	<u>0.9946</u>	<u>0.9893</u>	<u>0.2336</u>	<u>78054.61</u>	

Suggested"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

and the "Predicted R-Squared".

### Response 1 Yield Strength

#### ANOVA for Response Surface Sixth model

Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value
Model	1.013E+005	6	16883.15	185.06	< 0.0001 significant

A-% Cr	4707.20	1	4707.20	51.60	0.0004
A <sup>2</sup>	84.68	1	84.68	0.93	0.3725
A <sup>3</sup>	217.56	1	217.56	2.38	0.1735
A <sup>4</sup>	831.61	1	831.61	9.12	0.0234
A <sup>5</sup>	245.32	1	245.32	2.69	0.1522
A <sup>6</sup>	774.31	1	774.31	8.49	0.0269
Residual	547.38	6	91.23		
Cor Total	1.018E+005	12			

The Model F-value of 185.06 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>4</sup>, A<sup>6</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	9.55	R-Squared	0.9946
Mean	414.23	Adj R-Squared	0.9893
C.V. %	2.31	Pred R-Squared	0.2336
PRESS	78054.61	Adeq Precision	45.226
-2 Log Likelihood	85.51	BIC	103.47
		AICc	121.91

The "Pred R-Squared" of 0.2336 is not as close to the "Adj R-Squared" of 0.9893 as one might normally expect; i.e. the difference is more than 0.2. This may indicate a large block effect or a possible problem with your model and/or data. Things to consider are model reduction, response transformation, outliers, etc. All empirical models should be tested by doing confirmation runs. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 45.226 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	452.07	1	5.84	437.77	466.37	
A-% Cr	148.83	1	20.72	98.13	199.53	24.47
A <sup>2</sup>	69.56	1	72.20	-107.10	246.21	93.52
A <sup>3</sup>	-111.12	1	71.96	-287.20	64.95	175.22
A <sup>4</sup>	-554.49	1	183.66	-1003.88	-105.10	610.47
A <sup>5</sup>	91.42	1	55.75	-45.00	227.85	85.49
A <sup>6</sup>	349.60	1	120.00	55.97	643.22	257.04

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Yield Strength} = & \\ & +452.07 \\ & +148.83 * A \\ & +69.56 * A^2 \\ & -111.12 * A^3 \\ & -554.49 * A^4 \\ & +91.42 * A^5 \\ & +349.60 * A^6 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

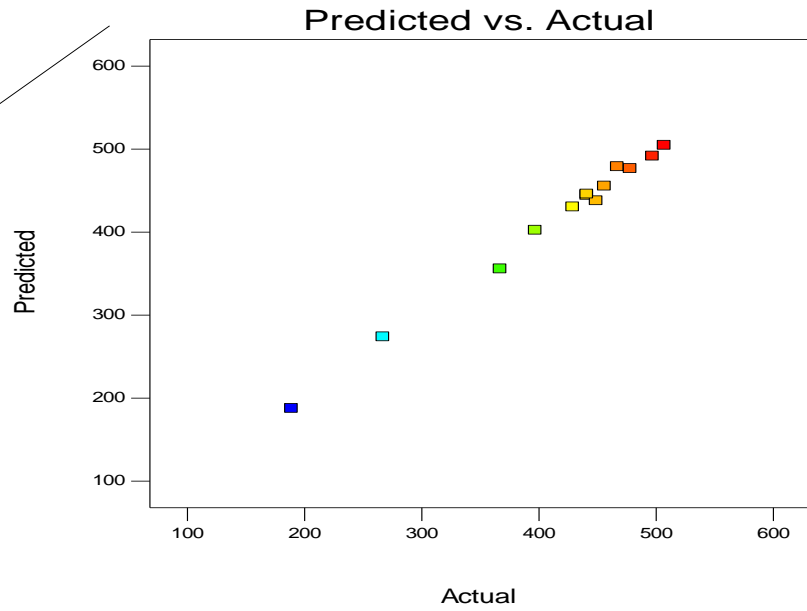
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Yield Strength} = & \\ & +181.62599 \\ & -41.06409 * \% Cr \\ & +131.41364 * \% Cr^2 \\ & -55.82904 * \% Cr^3 \\ & +10.50217 * \% Cr^4 \\ & -0.92096 * \% Cr^5 \\ & +0.030437 * \% Cr^6 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Yield Strength

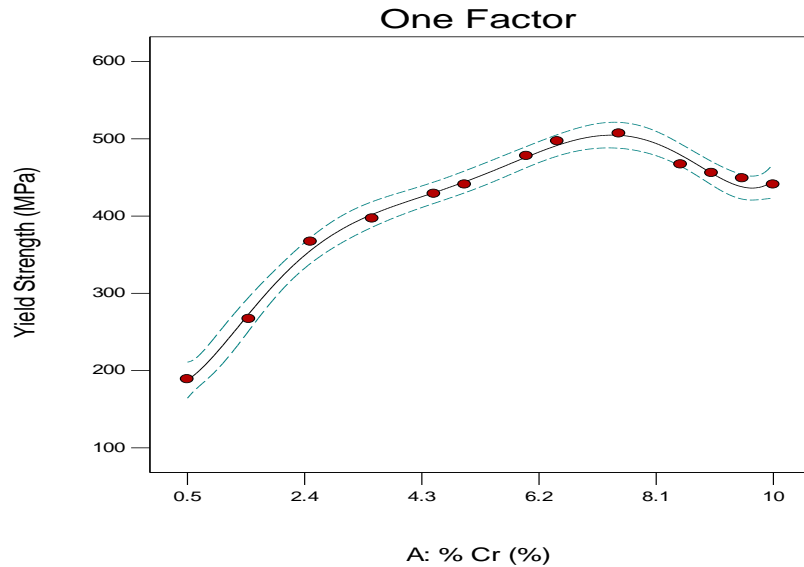
Color points by value of  
Yield Strength:



Design-Expert® Software  
Factor Coding: Actual  
Yield Strength (MPa)

● Design Points  
--- 95% CI Bands

X1 = A: % Cr



**Response 2      UTS      Transform:    None**

**Summary (detailed tables shown below)**

Source	p-value	Sequential Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0159		0.3718	0.1366

Quadratic	< 0.0001	0.9672	0.9360
Cubic	0.0272	0.9794	0.9492
<u>Quartic</u>	<u>0.0065</u>	<u>0.9913</u>	<u>0.9644</u> <u>Suggested</u>
Fifth	0.0896	0.9936	0.9900
Sixth	0.3937	0.9935	0.8257

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob > F	
Mean vs Total	3.609E+006	1	3.609E+006			
Linear vs Mean	28836.73	1	28836.73	8.10	0.0159	
Quadratic vs Linear	37298.03	1	37298.03	200.73	< 0.0001	
Cubic vs Quadratic	808.49	1	808.49	6.93	0.0272	
<u>Quartic vs Cubic</u>	<u>656.21</u>	<u>1</u>	<u>656.21</u>	<u>13.34</u>	<u>0.0065</u>	<u>Suggested</u>
Fifth vs Quartic	140.29	1	140.29	3.88	0.0896	
Sixth vs Fifth	31.22	1	31.22	0.84	0.3937	
Residual	221.96	6	36.99			

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 3.677E+006 13 2.829E+005

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS
Linear	59.66	0.4241	0.3718	0.1366	58704.12
Quadratic	13.63	0.9727	0.9672	0.9360	4350.80

Cubic	10.80	0.9846	0.9794	0.9492	3453.91	
<u>Quartic</u>	<u>7.01</u>	<u>0.9942</u>	<u>0.9913</u>	<u>0.9644</u>	<u>2420.66</u>	<u>Suggested</u>
Fifth	6.01	0.9963	0.9936	0.9900	681.83	
						"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"
Sixth	6.08	0.9967	0.9935	0.8257	11852.74	and the "Predicted R-Squared".

## Response 2 UTS

### ANOVA for Response Surface Quartic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	
Model	67599.46	4	16899.86	343.61	< 0.0001	Significant
A-% Cr	9753.74	1	9753.74	198.31	< 0.0001	
A <sup>2</sup>	5847.22	1	5847.22	118.89	< 0.0001	
A <sup>3</sup>	844.28	1	844.28	17.17	0.0032	
A <sup>4</sup>	656.21	1	656.21	13.34	0.0065	
Residual	393.47	8	49.18			
Cor Total	67992.92	12				

The Model F-value of 343.61 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup>, A<sup>4</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	7.01	R-Squared	0.9942
Mean	526.92	Adj R-Squared	0.9913
C.V. %	1.33	Pred R-Squared	0.9644
PRESS	2420.66	Adeq Precision	54.795
-2 Log Likelihood	81.22	BIC	94.05
		AICc	99.79

The "Pred R-Squared" of 0.9644 is in reasonable agreement with the "Adj R-Squared" of 0.9913; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 54.795 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error		95% CI		VIF
	Estimate	df	Error	Low	High		
Intercept	587.89	1	3.63	579.51	596.26		
A-% Cr	119.18	1	8.46	99.66	138.70	7.57	
A <sup>2</sup>	-223.50	1	20.50	-270.77	-176.23	13.98	
A <sup>3</sup>	-45.47	1	10.97	-70.78	-20.16	7.56	
A <sup>4</sup>	74.30	1	20.34	27.39	121.21	13.89	

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned}
 \text{UTS} = & \\
 & +587.89 \\
 & +119.18 * A \\
 & -223.50 * A^2 \\
 & -45.47 * A^3 \\
 & +74.30 * A^4
 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

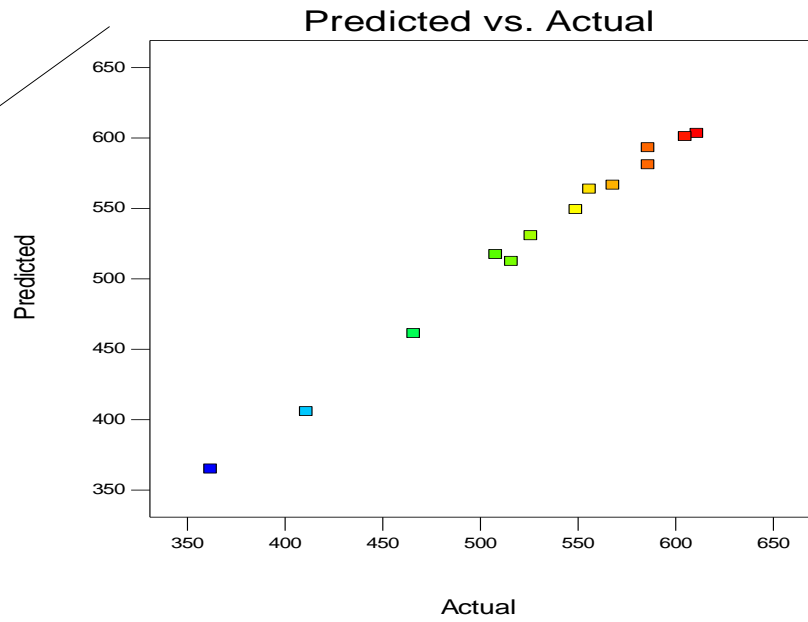
$$\begin{aligned}
 \text{UTS} = & \\
 & +355.40566 \\
 & +9.53983 * \% \text{ Cr} \\
 & +20.91348 * \% \text{ Cr}^2 \\
 & -3.48929 * \% \text{ Cr}^3 \\
 & +0.14595 * \% \text{ Cr}^4
 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because

the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
UTS

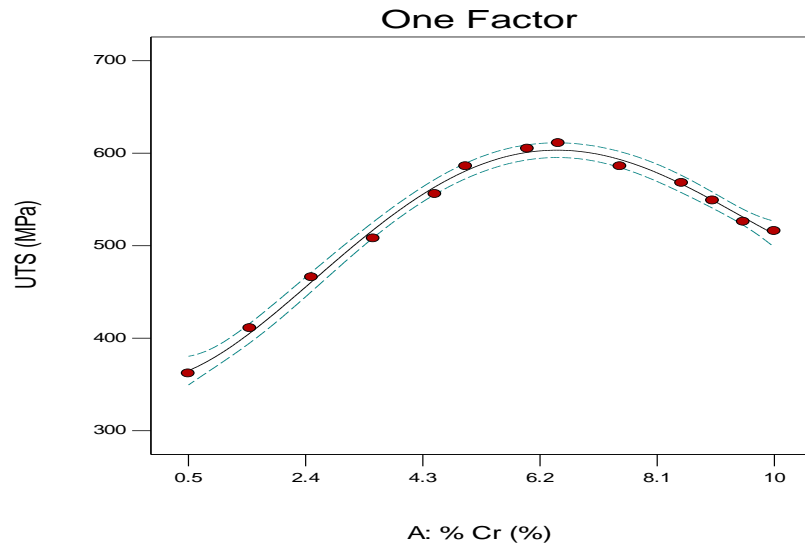
Color points by value of UTS:  
611  
362



Design-Expert® Software  
Factor Coding: Actual  
UTS (MPa)

● Design Points  
--- 95% CI Bands

X1 = A: % Cr





Response 3                      Hardness   Transform:   None

Summary (detailed tables shown below)

Source	p-value	Sequential Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0003		0.6793	0.5515
<u>Quadratic</u>	<u>&lt; 0.0001</u>		<u>0.9828</u>	<u>0.9625</u> <u>Suggested</u>
Cubic	0.1872		0.9844	0.9504
Quartic	0.1730		0.9863	0.9386
Fifth	0.5692		0.9851	0.7616
Sixth	0.0523		0.9912	0.8765

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob > F
Mean vs Total	1.138E+006	1	1.138E+006		
Linear vs Mean	60473.68	1	60473.68	26.42	0.0003
<u>Quadratic vs Linear</u>	<u>23951.20</u>	<u>1</u>	<u>23951.20</u>	<u>194.91</u>	<u>0.0001</u> $\leq$
Cubic vs Quadratic	226.82	1	226.82	2.04	0.1872
Quartic vs Cubic	219.02	1	219.02	2.24	0.1730
Fifth vs Quartic	37.96	1	37.96	0.36	0.5692
Sixth vs Fifth	367.05	1	367.05	5.83	0.0523
Residual	377.97	6	62.99		

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 1.223E+006 13 94113.69

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

	Std.	Adjusted		Predicted		
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS	
Linear	47.84	0.7060	0.6793	0.5515	38413.51	
<u>Quadratic</u>	<u>11.09</u>	<u>0.9857</u>	<u>0.9828</u>	<u>0.9625</u>	<u>3209.13</u>	<u>Suggested</u>
Cubic	10.55	0.9883	0.9844	0.9504	4247.11	
Quartic	9.89	0.9909	0.9863	0.9386	5261.62	
Fifth	10.32	0.9913	0.9851	0.7616	20418.27	
						"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".
Sixth	7.94	0.9956	0.9912	0.8765	10574.36	

**Response 3 Hardness**

**ANOVA for Response Surface Quadratic model**

**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	84424.88	2	42212.44	343.52	< 0.0001	significant
A-% Cr	69621.79	1	69621.79	566.58	< 0.0001	
A <sup>2</sup>	23951.20	1	23951.20	194.91	< 0.0001	
Residual	1228.81	10	122.88			
Cor Total	85653.69	12				

The Model F-value of 343.52 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	11.09	R-Squared	0.9857
Mean	295.85	Adj R-Squared	0.9828
C.V. %	3.75	Pred R-Squared	0.9625
PRESS	3209.13	Adeq Precision	50.063
-2 Log Likelihood	96.03	BIC	103.72
		AICc	104.69

The "Pred R-Squared" of 0.9625 is in reasonable agreement with the "Adj R-Squared" of 0.9828; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 50.063 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	334.07	1	4.70	323.60	344.54	
A-% Cr	116.70	1	4.90	105.77	127.62	1.02
A <sup>2</sup>	-121.99	1	8.74	-141.46	-102.52	1.02

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Hardness} = & \\ & +334.07 \\ & +116.70 * A \\ & -121.99 * A^2 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

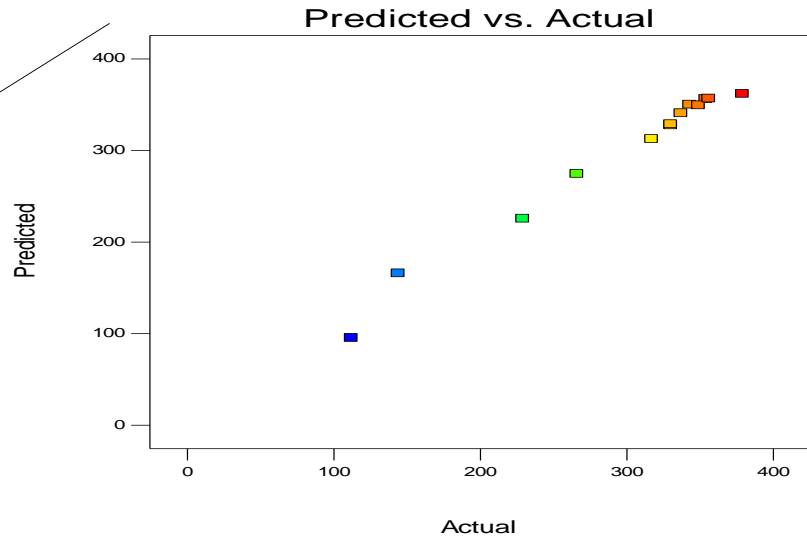
$$\begin{aligned} \text{Hardness} = & \\ & +56.06850 \\ & +81.33828 * \% \text{ Cr} \\ & -5.40669 * \% \text{ Cr}^2 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because

the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Hardness

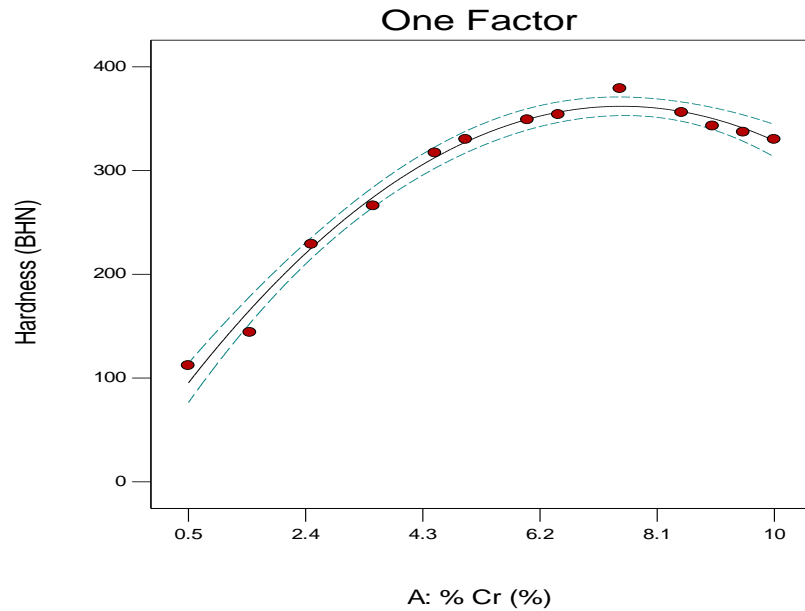
Color points by value of  
Hardness:



Design-Expert® Software  
Factor Coding: Actual  
Hardness (BHN)

● Design Points  
--- 95% CI Bands

X1 = A: % Cr



**Response 4                      Elongation Transform:    None**

**Summary (detailed tables shown below)**

	<b>Sequential Lack of Fit</b>	<b>Adjusted</b>	<b>Predicted</b>
<b>Source</b>	<b>p-value</b>	<b>R-Squared</b>	<b>R-Squared</b>
Linear	0.0758	0.1915	-0.0404
Quadratic	0.0013	0.6967	0.4632
Cubic	0.8039	0.6655	0.0340
Quartic	0.0055	0.8645	0.0099
<u>Fifth</u>	<u>0.0090</u>	<u>0.9452</u>	<u>0.7718</u> <u>Suggested</u>
Sixth	0.5453	0.9401	-1.1969

**Sequential Model Sum of Squares [Type I]**

	<b>Sum of</b>	<b>Mean</b>	<b>F</b>	<b>p-value</b>	
<b>Source</b>	<b>Squares</b>	<b>df</b>	<b>Square Value</b>	<b>Prob &gt;</b>	<b>F</b>
Mean vs Total	4772.37	1	4772.37		
Linear vs Mean	38.02	1	38.02	3.84	0.0758
Quadratic vs Linear	71.71	1	71.71	19.32	0.0013
Cubic vs Quadratic	0.27	1	0.27	0.065	0.8039
Quartic vs Cubic	23.58	1	23.58	14.22	0.0055
<u>Fifth vs Quartic</u>	<u>8.57</u>	<u>1</u>	<u>8.57</u>	<u>12.77</u>	<u>0.0090</u>
Sixth vs Fifth	0.30	1	0.30	0.41	0.5453
Residual	4.39	6	0.73		

Suggested

"Sequential Model Sum of Squares [Type I]":  
Select the highest order polynomial where the

Total 4919.21 13 378.40

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

<b>Source</b>	<b>Std. Dev.</b>	<b>Adjusted R-Squared</b>	<b>Predicted R-Squared</b>	<b>R-PRESS</b>
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Linear	3.15	0.2589	0.1915	-0.0404	152.78
Quadratic	1.93	0.7473	0.6967	0.4632	78.82
Cubic	2.02	0.7491	0.6655	0.0340	141.84
Quartic	1.29	0.9097	0.8645	0.0099	145.39
<u>Fifth</u>	<u>0.82</u>	<u>0.9680</u>	<u>0.9452</u>	<u>0.7718</u>	<u>33.51</u>

Suggested

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

Sixth	0.86	0.9701	0.9401	-1.1969	322.59
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and the "Predicted R-Squared".

## Response 4 Elongation

### ANOVA for Response Surface Fifth model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	142.15	5	28.43	42.38	< 0.0001	significant
A-% Cr	18.77	1	18.77	27.98	0.0011	
A <sup>2</sup>	42.46	1	42.46	63.30	< 0.0001	
A <sup>3</sup>	8.97	1	8.97	13.38	0.0081	
A <sup>4</sup>	20.57	1	20.57	30.67	0.0009	
A <sup>5</sup>	8.57	1	8.57	12.77	0.0090	
Residual	4.70	7	0.67			
Cor Total	146.84	12				

The Model F-value of 42.38 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup>, A<sup>4</sup>, A<sup>5</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.82	R-Squared	0.9680
Mean	19.16	Adj R-Squared	0.9452
C.V. %	4.27	Pred R-Squared	0.7718

PRESS	33.51	Adeq Precision	19.489
-2 Log Likelihood	23.65	BIC	39.04
		AICc	49.65

The "Pred R-Squared" of 0.7718 is in reasonable agreement with the "Adj R-Squared" of 0.9452; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 19.489 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error		95% CI		VIF
	Estimate	df	Error	Low	High		
Intercept	15.42	1	0.43	14.42	16.43		
A-% Cr	-9.34	1	1.77	-13.51	-5.16	24.15	
A <sup>2</sup>	19.21	1	2.41	13.50	24.92	14.22	
A <sup>3</sup>	22.27	1	6.09	7.87	36.66	170.57	
A <sup>4</sup>	-13.22	1	2.39	-18.87	-7.58	14.03	
A <sup>5</sup>	-16.87	1	4.72	-28.03	-5.71	83.36	

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Elongation} = & +15.42 \\ & -9.34 * A \\ & +19.21 * A^2 \\ & +22.27 * A^3 \\ & -13.22 * A^4 \\ & -16.87 * A^5 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

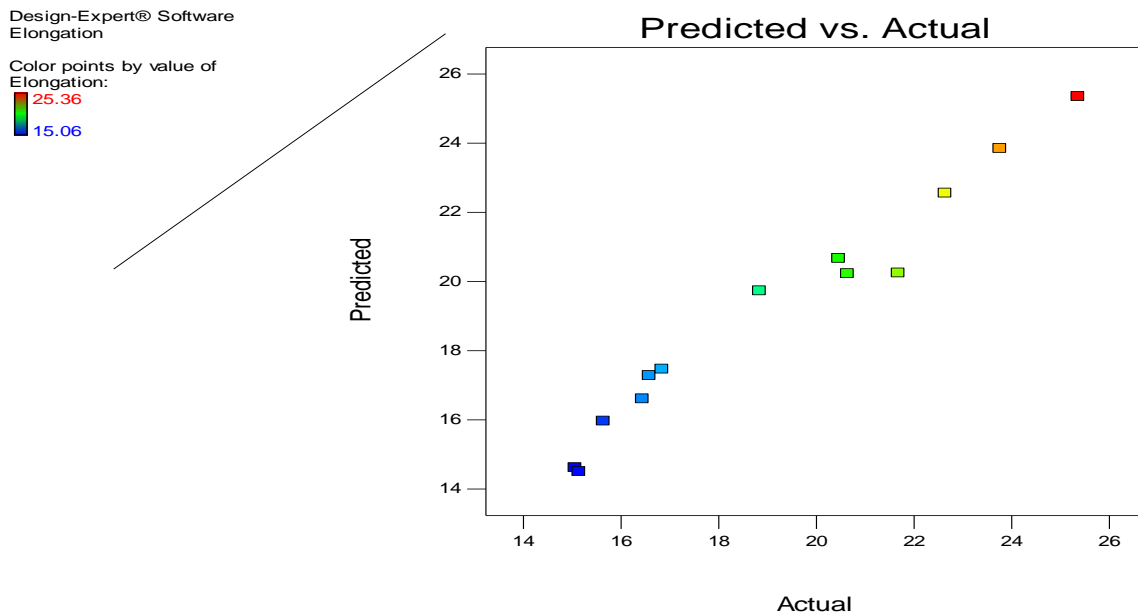
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Elongation} = & +27.23824 \\ & -5.19406 * \% \text{ Cr} \\ & +3.38025 * \% \text{ Cr}^2 \\ & -1.16997 * \% \text{ Cr}^3 \end{aligned}$$

$$+0.15719 * \% Cr^4$$

$$-6.97756E-003 * \% Cr^5$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.



**Response 5                      Impact Strength Transform:    None**

**Summary (detailed tables shown below)**

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.1349		0.1179	-0.2550
Quadratic	< 0.0001		0.9913	0.9815
Cubic	0.3097		0.9914	0.9709
<u>Quartic</u>	<u>0.0104</u>		<u>0.9960</u>	<u>0.9899</u> <u>Suggested</u>
Fifth	0.6951		0.9955	0.9728
Sixth	0.3531		0.9955	0.9650



**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob > F	
Mean vs Total	11373.51	1	11373.51			
Linear vs Mean	29.46	1	29.46	2.60	0.1349	
Quadratic vs Linear	123.38	1	123.38	1106.24	< 0.0001	
Cubic vs Quadratic	0.13	1	0.13	1.16	0.3097	
<u>Quartic vs Cubic</u>	<u>0.57</u>	<u>1</u>	<u>0.57</u>	<u>11.08</u>	<u>0.0104</u>	<u>Suggested</u>
Fifth vs Quartic	9.643E-003	1	9.643E-003	0.17	0.6951	
Sixth vs Fifth	0.058	1	0.058	1.01	0.3531	
Residual	0.35	6	0.058			
Total	11527.47	13	886.73			

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS	
Linear	3.36	0.1914	0.1179	-0.2550	193.22	
Quadratic	0.33	0.9928	0.9913	0.9815	2.85	
Cubic	0.33	0.9936	0.9914	0.9709	4.48	
<u>Quartic</u>	<u>0.23</u>	<u>0.9973</u>	<u>0.9960</u>	<u>0.9899</u>	<u>1.56</u>	<u>Suggested</u>
Fifth	0.24	0.9974	0.9955	0.9728	4.18	
Sixth	0.24	0.9978	0.9955	0.9650	5.38	

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

and the "Predicted R-Squared".

## Response 5 Impact Strength

### ANOVA for Response Surface Quartic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	153.55	4	38.39	741.34	< 0.0001	significant
<i>A-% Cr</i>	8.26	1	8.26	159.56	< 0.0001	
$A^2$	13.85	1	13.85	267.47	< 0.0001	
$A^3$	0.14	1	0.14	2.72	0.1377	
$A^4$	0.57	1	0.57	11.08	0.0104	
Residual	0.41	8	0.052			
Cor Total	153.96	12				

The Model F-value of 741.34 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A,  $A^2$ ,  $A^4$  are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.23	R-Squared	0.9973
Mean	29.58	Adj R-Squared	0.9960
C.V. %	0.77	Pred R-Squared	0.9899
PRESS	1.56	Adeq Precision	83.877
-2 Log Likelihood	-7.91	BIC	4.92
		AICc	10.66

The "Pred R-Squared" of 0.9899 is in reasonable agreement with the "Adj R-Squared" of 0.9960; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 83.877 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	26.07	1	0.12	25.80	26.34	
A-% Cr	-3.47	1	0.27	-4.10	-2.84	7.57
A <sup>2</sup>	10.88	1	0.67	9.34	12.41	13.98
A <sup>3</sup>	0.59	1	0.36	-0.23	1.41	7.56
A <sup>4</sup>	-2.20	1	0.66	-3.72	-0.68	13.89

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Impact Strength} = & \\ & +26.07 \\ & -3.47 * A \\ & +10.88 * A^2 \\ & +0.59 * A^3 \\ & -2.20 * A^4 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

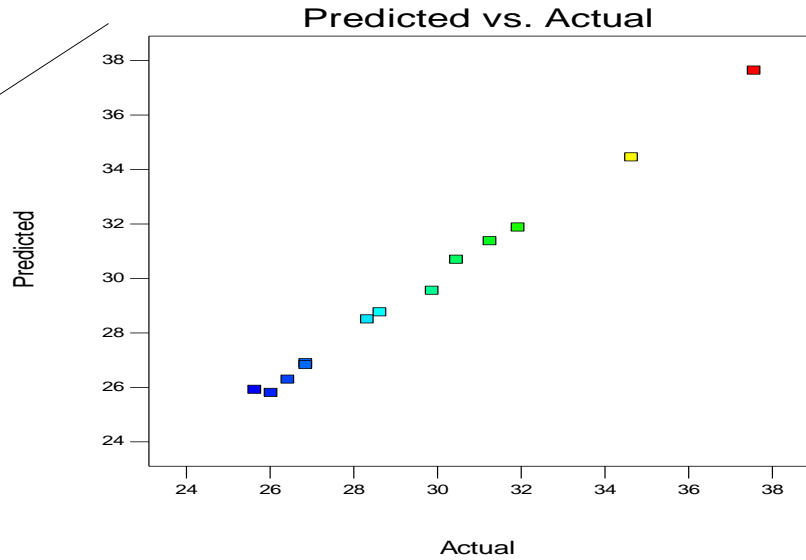
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Impact Strength} = & \\ & +39.12243 \\ & -2.84089 * \% \text{ Cr} \\ & -0.31800 * \% \text{ Cr}^2 \\ & +0.096120 * \% \text{ Cr}^3 \\ & -4.31622\text{E-}003 * \% \text{ Cr}^4 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Impact Strength

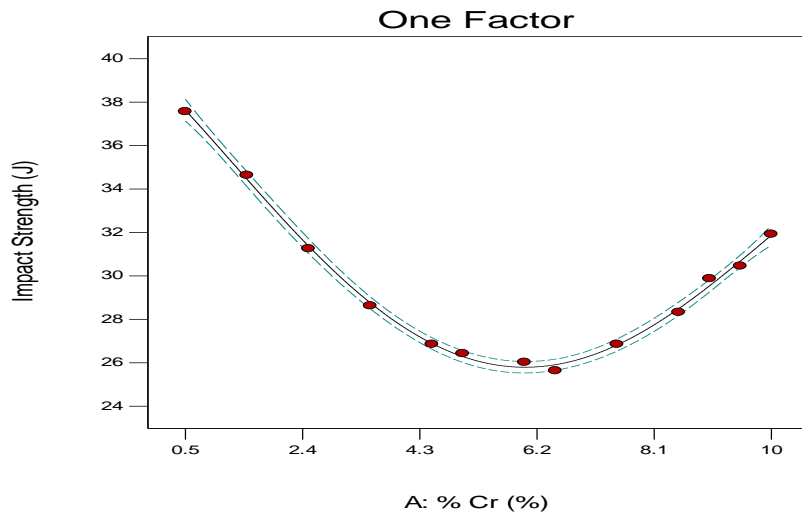
Color points by value of  
Impact Strength:



Design-Expert® Software  
Factor Coding: Actual  
Impact Strength (J)

● Design Points  
-- 95% CI Bands

X1 = A: % Cr



**Response 6      Resistivity Transform:    None**

**Summary (detailed tables shown below)**

	Sequential Lack of Fit	Adjusted	Predicted
Source	p-value	R-Squared	R-Squared
Linear	0.0136	0.3878	0.1769
Quadratic	< 0.0001	0.9215	0.8613
Cubic	0.0042	0.9665	0.9463
Quartic	0.1199	0.9727	0.8772

<u>Fifth</u>	<u>0.0234</u>		<u>0.9858</u>	<u>0.9663</u>	<u>Suggested</u>
Sixth	0.5046		0.9847	0.3353	

**Sequential Model Sum of Squares [Type I]**

	Sum of	Mean	F	p-value		
Source	Squares	df	Square	Value	Prob > F	
Mean vs Total	1341.32	1	1341.32			
Linear vs Mean	24.46	1	24.46	8.60	0.0136	
Quadratic vs Linear	27.64	1	27.64	75.82	< 0.0001	
Cubic vs Quadratic	2.25	1	2.25	14.46	0.0042	
Quartic vs Cubic	0.38	1	0.38	3.03	0.1199	
<u>Fifth vs Quartic</u>	<u>0.55</u>	<u>1</u>	<u>0.55</u>	<u>8.33</u>	<u>0.0234</u>	<u>Suggested</u>
Sixth vs Fifth	0.036	1	0.036	0.50	0.5046	
Residual	0.43	6	0.071			

"Sequential Model Sum of Squares [Type I]":  
Select the highest order polynomial where the

Total 1397.07 13 107.47

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

	Std.	Adjusted	Predicted			
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS	
Linear	1.69	0.4388	0.3878	0.1769	45.88	
Quadratic	0.60	0.9346	0.9215	0.8613	7.73	
Cubic	0.39	0.9749	0.9665	0.9463	2.99	
Quartic	0.36	0.9818	0.9727	0.8772	6.84	
<u>Fifth</u>	<u>0.26</u>	<u>0.9917</u>	<u>0.9858</u>	<u>0.9663</u>	<u>1.88</u>	<u>Suggested</u>
Sixth	0.27	0.9923	0.9847	0.3353	37.05	

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-

Squared"

and the "Predicted R-Squared".

## Response 6 Resistivity

### ANOVA for Response Surface Fifth model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	
Model	55.28	5	11.06	167.12	< 0.0001	significant
A-% Cr	6.56	1	6.56	99.11	< 0.0001	
A <sup>2</sup>	3.56	1	3.56	53.79	0.0002	
A <sup>3</sup>	1.09	1	1.09	16.49	0.0048	
A <sup>4</sup>	0.29	1	0.29	4.43	0.0732	
A <sup>5</sup>	0.55	1	0.55	8.33	0.0234	
Residual	0.46	7	0.066			
Cor Total	55.74	12				

The Model F-value of 167.12 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup>, A<sup>5</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.26	R-Squared	0.9917
Mean	10.16	Adj R-Squared	0.9858
C.V. %	2.53	Pred R-Squared	0.9663
PRESS	1.88	Adeq Precision	39.800
-2 Log Likelihood	-6.46	BIC	8.93
		AICc	19.54

The "Pred R-Squared" of 0.9663 is in reasonable agreement with the "Adj R-Squared" of 0.9858; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio

greater than 4 is desirable. Your ratio of 39.800 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	11.74	1	0.13	11.42	12.06	
A-% Cr	5.52	1	0.55	4.21	6.83	24.15
A <sup>2</sup>	-5.56	1	0.76	-7.35	-3.77	14.22
A <sup>3</sup>	-7.76	1	1.91	-12.29	-3.24	170.57
A <sup>4</sup>	1.58	1	0.75	-0.19	3.35	14.03
A <sup>5</sup>	4.28	1	1.48	0.77	7.79	83.36

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Resistivity} = & \\ & +11.74 \\ & +5.52 * A \\ & -5.56 * A^2 \\ & -7.76 * A^3 \\ & +1.58 * A^4 \\ & +4.28 * A^5 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.


**Final Equation in Terms of Actual Factors:**

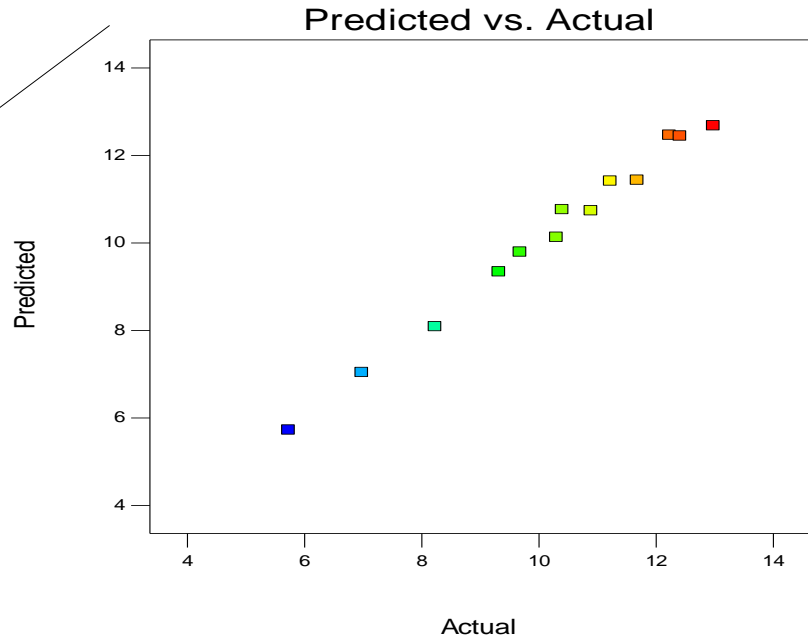
$$\begin{aligned} \text{Resistivity} = & \\ & +4.62821 \\ & +2.68700 * \% \text{ Cr} \\ & -1.15366 * \% \text{ Cr}^2 \\ & +0.35026 * \% \text{ Cr}^3 \\ & -0.043360 * \% \text{ Cr}^4 \\ & +1.76997\text{E-}003 * \% \text{ Cr}^5 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because

the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Resistivity

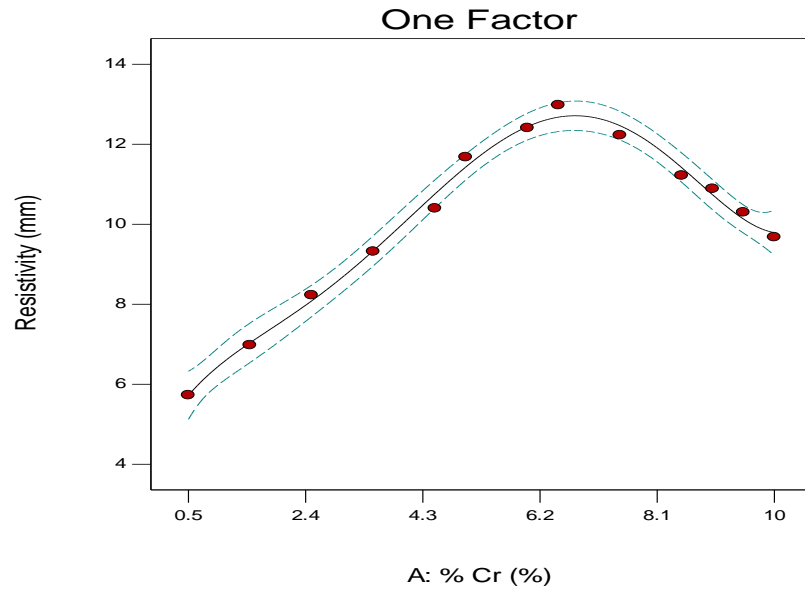
Color points by value of Resistivity:  

 12.98  
 5.73



Design-Expert® Software  
Factor Coding: Actual  
Resistivity (mm)

● Design Points  
 --- 95% CI Bands

X1 = A: % Cr





Response 7 Conductivity Transform: None

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0203		0.3455	0.0535
<u>Quadratic</u>	<u>&lt; 0.0001</u>		<u>0.9760</u>	<u>0.9683</u> <u>Suggested</u>
Cubic	0.9144		0.9734	0.9589
Quartic	0.9551		0.9701	0.9207
Fifth	0.5115		0.9680	0.1955
<u>Sixth</u>	<u>0.0123</u>		<u>0.9879</u>	<u>0.1127</u> <u>Suggested</u>

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob > F
Mean vs Total	276.37	1	276.37		
Linear vs Mean	13.94	1	13.94	7.34	0.0203
<u>Quadratic vs Linear</u>	<u>20.21</u>	<u>1</u>	<u>20.21</u>	<u>290.47</u>	<u>≤ 0.0001</u> <u>Suggested</u>
Cubic vs Quadratic	9.428E-004	1	9.428E-004	0.012	0.9144
Quartic vs Cubic	2.925E-004	1	2.925E-004	3.369E-003	0.9551
Fifth vs Quartic	0.044	1	0.044	0.48	0.5115
<u>Sixth vs Fifth</u>	<u>0.44</u>	<u>1</u>	<u>0.44</u>	<u>12.49</u>	<u>0.0123</u> <u>Suggested</u>
Residual	0.21	6	0.035		
Total	311.21	13	23.94		

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

	Std.	Adjusted Predicted				
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS	
Linear	1.38	0.4001	0.3455	0.0535	32.98	
<u>Quadratic</u>	<u>0.26</u>	<u>0.9800</u>	<u>0.9760</u>	<u>0.9683</u>	<u>1.10</u>	<u>Suggested</u>
Cubic	0.28	0.9801	0.9734	0.9589	1.43	
Quartic	0.29	0.9801	0.9701	0.9207	2.76	
Fifth	0.30	0.9813	0.9680	0.1955	28.03	
						<u>Suggested</u> "Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"
<u>Sixth</u>	<u>0.19</u>	<u>0.9939</u>	<u>0.9879</u>	<u>0.1127</u>	<u>30.91</u>	and the "Predicted R-Squared".

## Response 7 Conductivity

### ANOVA for Response Surface Sixth model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	34.63	6	5.77	164.17	< 0.0001	significant
A-% Cr	0.97	1	0.97	27.69	0.0019	
A <sup>2</sup>	0.025	1	0.025	0.71	0.4302	
A <sup>3</sup>	0.010	1	0.010	0.29	0.6073	
A <sup>4</sup>	0.44	1	0.44	12.42	0.0125	
A <sup>5</sup>	0.011	1	0.011	0.31	0.6007	
A <sup>6</sup>	0.44	1	0.44	12.49	0.0123	
Residual	0.21	6	0.035			
Cor Total	34.84	12				

The Model F-value of 164.17 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>4</sup>, A<sup>6</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.19	R-Squared	0.9939
Mean	4.61	Adj R-Squared	0.9879
C.V. %	4.07	Pred R-Squared	0.1127
PRESS	30.91	Adeq Precision	42.381
-2 Log Likelihood	-16.68	BIC	1.27
		AICc	19.72

The "Pred R-Squared" of 0.1127 is not as close to the "Adj R-Squared" of 0.9879 as one might normally expect; i.e. the difference is more than 0.2. This may indicate a large block effect or a possible problem with your model and/or data. Things to consider are model reduction, response transformation, outliers, etc. All empirical models should be tested by doing confirmation runs. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 42.381 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	3.58	1	0.11	3.30	3.86	
A-% Cr	-2.14	1	0.41	-3.14	-1.14	24.47
A <sup>2</sup>	-1.20	1	1.42	-4.67	2.27	93.52
A <sup>3</sup>	0.77	1	1.41	-2.69	4.22	175.22
A <sup>4</sup>	12.71	1	3.61	3.88	21.53	610.47
A <sup>5</sup>	-0.60	1	1.09	-3.28	2.07	85.49
A <sup>6</sup>	-8.33	1	2.36	-14.09	-2.56	257.04

**Final Equation in Terms of Coded Factors:**

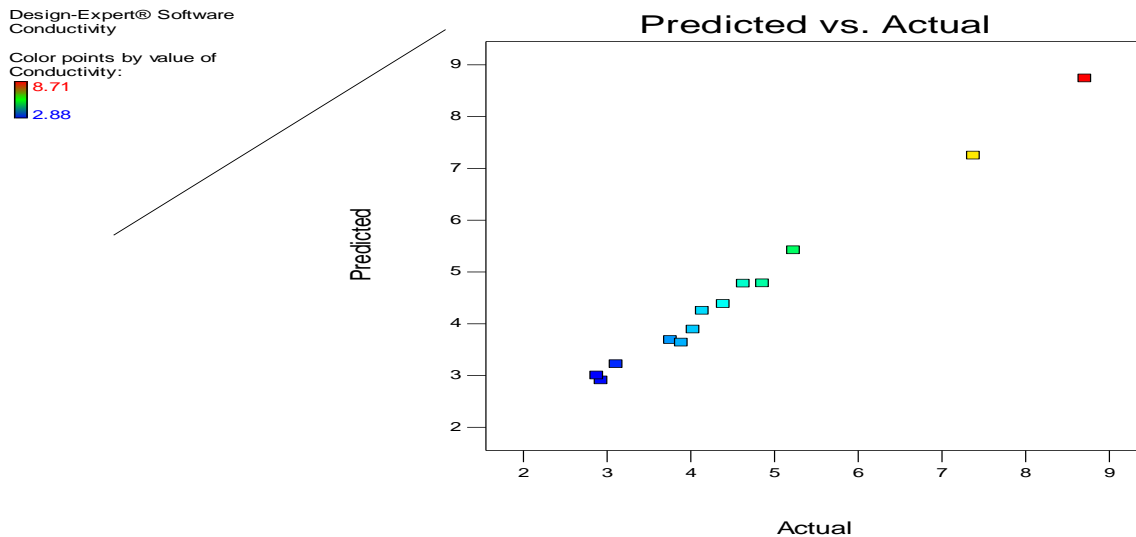
$$\begin{aligned}
 \text{Conductivity} = & \\
 & +3.58 \\
 & -2.14 * A \\
 & -1.20 * A^2 \\
 & +0.77 * A^3 \\
 & +12.71 * A^4 \\
 & -0.60 * A^5 \\
 & -8.33 * A^6
 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

### Final Equation in Terms of Actual Factors:

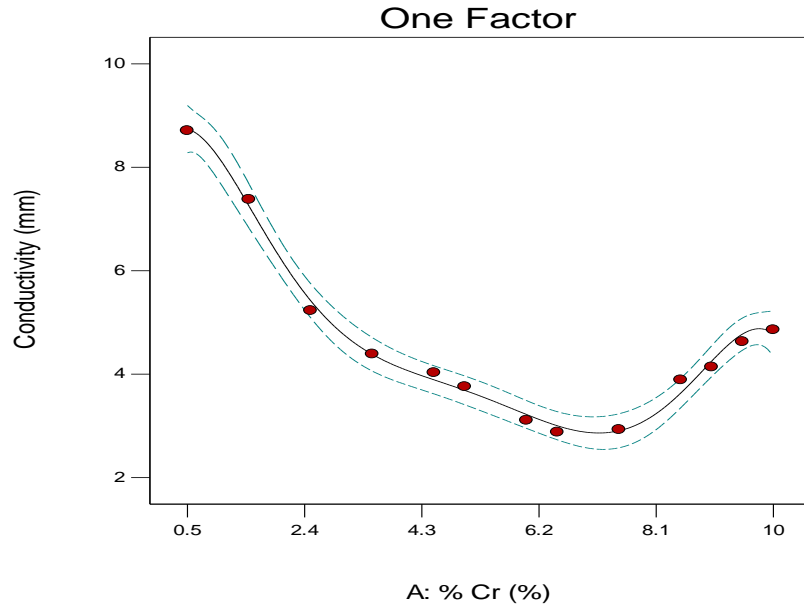
$$\begin{aligned} \text{Conductivity} = & \\ & +8.22597 \\ & +2.64688 * \% \text{ Cr} \\ & -3.93585 * \% \text{ Cr}^2 \\ & +1.51176 * \% \text{ Cr}^3 \\ & -0.26814 * \% \text{ Cr}^4 \\ & +0.022582 * \% \text{ Cr}^5 \\ & -7.24812\text{E-}004 * \% \text{ Cr}^6 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.



Design-Expert® Software  
 Factor Coding: Actual  
 Conductivity (mm)  
 ● Design Points  
 --- 95% CI Bands

X1 = A: % Cr



**Factor Name Level Low Level High Level Std. Dev. Coding**  
 A % Cr 9.38 0.50 10.00 0.000 Actual

Response	Predicted		Observed	Std Dev	SE Mean	CI for Mean		99% of Population	
	Mean	Median <sup>1</sup>				95% CI low	95% CI high	95% TI low	95% TI high
Yield Strength	440.517	440.517	-	9.55144	6.55843	424.469	456.565	380.648	500.387
UTS	535.03	535.03	-	7.01307	3.60345	526.72	543.339	497.42	572.639
Hardness	343.265	343.265	-	11.0852	5.46679	331.084	355.446	287.869	398.661
Elongation	20.5177	20.5177	-	0.81899	0.477882	19.3877	21.6478	15.8211	25.2144
Impact Strength	30.4161	30.4161	-	0.227554	0.116922	30.1465	30.6857	29.1958	31.6364
Resistivity	10.2576	10.2576	-	0.257209	0.150082	9.9027	10.6125	8.78258	11.7326
Conductivity	4.68022	4.68022	-	0.187503	0.128747	4.36518	4.99525	3.50493	5.8555

**Confirmation Report**

Two-sided Confidence = 95% n = 1

**Factor Name Level Low Level High Level Std. Dev. Coding**  
 A % Cr 9.38 0.50 10.00 0.000 Actual

Response	Predicted		Observed	Std Dev	n	SE Pred	95% PI low	Data Mean	95% PI high
	Mean	Median <sup>1</sup>							
Yield Strength	440.517	440.517	-	9.55144	1	11.59	412.17		468.87
UTS	535.03	535.03	-	7.01307	1	7.88	516.85		553.21
Hardness	343.265	343.265	-	11.0852	1	12.36	315.73		370.80
Elongation	20.5177	20.5177	-	0.81899	1	0.95	18.28		22.76
Impact Strength	30.4161	30.4161	-	0.227554	1	0.26	29.83		31.01
Resistivity	10.2576	10.2576	-	0.257209	1	0.30	9.55		10.96
Conductivity	4.68022	4.68022	-	0.187503	1	0.23	4.12		5.24

## Design Expert Analysis for Manganese

Response 1      Yield Strength Transform:      None

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	< 0.0001		0.9858	0.9764
<u>Quadratic</u>	<u>0.0001</u>		<u>0.9983</u>	<u>0.9965</u> <u>Suggested</u>
Cubic	0.0693		0.9989	0.9978
Quartic	0.8824		0.9987	0.9942
Fifth	0.7404		0.9984	0.9425
<u>Sixth</u>	<u>0.0368</u>		<u>0.9996</u>	<u>0.9417</u> <u>Suggested</u>

### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	P-value	Prob > F
Mean vs Total	1.318E+006	1	1.318E+006			
Linear vs Mean	98973.41	1	98973.41	626.67	< 0.0001	
<u>Quadratic vs Linear</u>	<u>1128.76</u>	<u>1</u>	<u>1128.76</u>	<u>58.64</u>	<u>0.0001</u>	<u>Suggested</u>
Cubic vs Quadratic	60.42	1	60.42	4.88	0.0693	
Quartic vs Cubic	0.36	1	0.36	0.024	0.8824	
Fifth vs Quartic	2.26	1	2.26	0.13	0.7404	

<u>Sixth vs Fifth</u>	<u>58.21</u>	<u>1</u>	<u>58.21</u>	<u>12.95</u>	<u>0.0368</u>	<u>Suggested</u>
Residual	13.49	3	4.50			

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 1.419E+006 10 1.419E+005

additional terms are significant and the model is not aliased.

### Model Summary Statistics

Source	Std. Dev.	Adjusted R-Squared	Predicted R-Squared	R-PRESS		
Linear	12.57	0.9874	0.9858	0.9764	2367.41	
<u>Quadratic</u>	<u>4.39</u>	<u>0.9987</u>	<u>0.9983</u>	<u>0.9965</u>	<u>351.68</u>	<u>Suggested</u>
Cubic	3.52	0.9993	0.9989	0.9978	218.19	
Quartic	3.85	0.9993	0.9987	0.9942	578.93	
Fifth	4.23	0.9993	0.9984	0.9425	5765.13	
<u>Sixth</u>	<u>2.12</u>	<u>0.9999</u>	<u>0.9996</u>	<u>0.9417</u>	<u>5840.80</u>	

Suggested"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

### Response 1 Yield Strength

#### ANOVA for Response Surface Sixth model

Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	1.002E+005	6	16703.90	3715.25	< 0.0001 significant
A-% Mn	4874.62	1	4874.62	1084.20	< 0.0001
A <sup>2</sup>	104.28	1	104.28	23.19	0.0171
A <sup>3</sup>	8.76	1	8.76	1.95	0.2570

A <sup>4</sup>	55.77	1	55.77	12.40	0.0389
A <sup>5</sup>	2.26	1	2.26	0.50	0.5293
A <sup>6</sup>	58.21	1	58.21	12.95	0.0368
Residual	13.49	3	4.50		
Cor Total	1.002E+005	9			

The Model F-value of 3715.25 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>4</sup>, A<sup>6</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	2.12	R-Squared	0.9999
Mean	363.10	Adj R-Squared	0.9996
C.V. %	0.58	Pred R-Squared	0.9417
PRESS	5840.80	Adeq Precision	172.107
-2 Log Likelihood	31.37	BIC	47.49
		AICc	101.37

The "Pred R-Squared" of 0.9417 is in reasonable agreement with the "Adj R-Squared" of 0.9996; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 172.107 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error		95% CI		VIF
	Estimate	df	Error	Low	High		
Intercept	378.05	1	1.55	373.11	382.99		
A-% Mn	168.15	1	5.11	151.90	184.40	23.63	
A <sup>2</sup>	-89.26	1	18.53	-148.25	-30.28	98.39	
A <sup>3</sup>	-25.42	1	18.21	-83.36	32.52	184.65	
A <sup>4</sup>	168.70	1	47.90	16.26	321.14	723.40	
A <sup>5</sup>	9.94	1	14.01	-34.65	54.52	94.63	
A <sup>6</sup>	-113.02	1	31.41	-212.99	-13.06	323.19	

**Final Equation in Terms of Coded Factors:**

$$\text{Yield Strength} = +378.05$$



$$\begin{aligned}
&+168.15 * A \\
&-89.26 * A^2 \\
&-25.42 * A^3 \\
&+168.70 * A^4 \\
&+9.94 * A^5 \\
&-113.02 * A^6
\end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

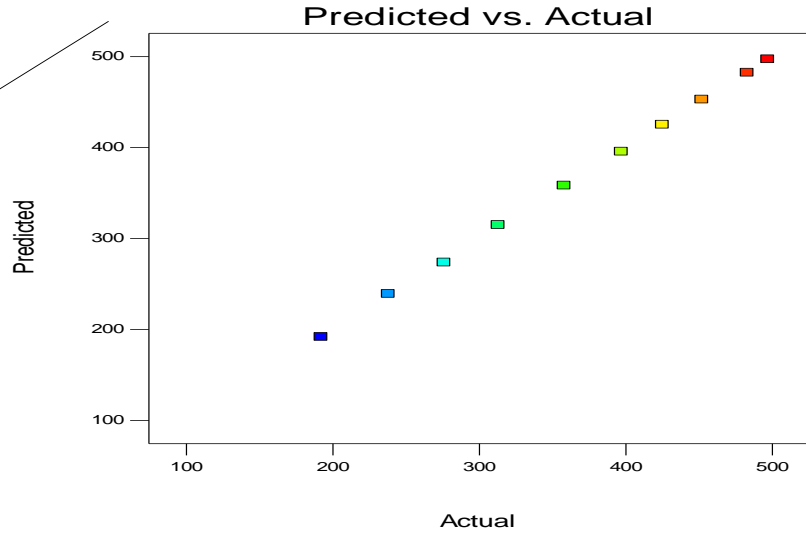
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned}
\text{Yield Strength} = & \\
&+58.20000 \\
&+444.80191 * \% \text{ Mn} \\
&-483.67652 * \% \text{ Mn}^2 \\
&+300.71958 * \% \text{ Mn}^3 \\
&-94.60342 * \% \text{ Mn}^4 \\
&+14.54564 * \% \text{ Mn}^5 \\
&-0.87111 * \% \text{ Mn}^6
\end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Yield Strength

Color points by value of  
Yield Strength:

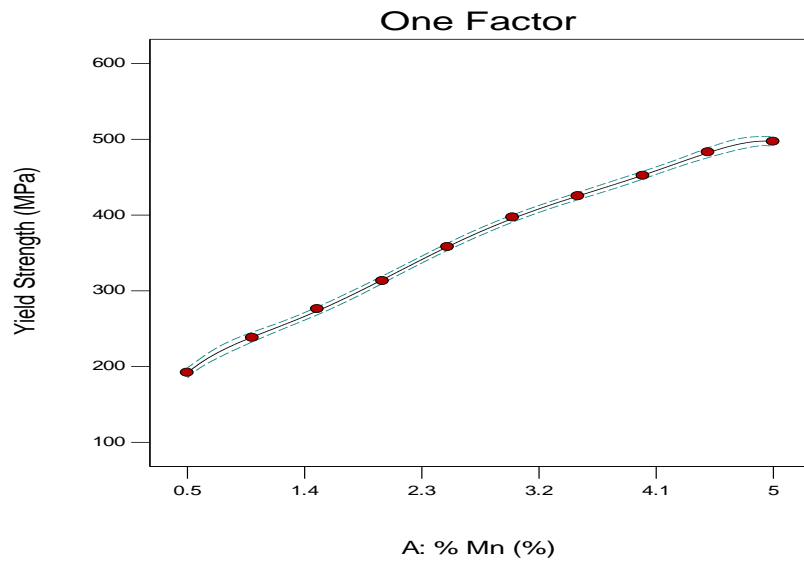


Design-Expert® Software  
Factor Coding: Actual

Yield Strength (MPa)

● Design Points  
--- 95% CI Bands

X1 = A: % Mn



**Response 2      UTS      Transform:   None**

**Summary (detailed tables shown below)**

	Sequential Lack of Fit	Adjusted	Predicted
Source	p-value	R-Squared	R-Squared
Linear	< 0.0001	0.9873	0.9819
<u>Quadratic</u>	<u>0.0066</u>	<u>0.9953</u>	<u>0.9918</u> Suggested

Cubic	0.2857		0.9955	0.9857
Quartic	0.2068		0.9962	0.9879
Fifth	0.3470		0.9963	0.9700
Sixth	0.7972		0.9952	0.6108

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob > F	
Mean vs Total	2.406E+006	1	2.406E+006			
Linear vs Mean	41372.80	1	41372.80	701.68	< 0.0001	
<u>Quadratic vs Linear</u>	<u>318.37</u>	<u>1</u>	<u>318.37</u>	<u>14.54</u>	<u>0.0066</u>	<u>Suggested</u>
Cubic vs Quadratic	28.56	1	28.56	1.37	0.2857	
Quartic vs Cubic	36.93	1	36.93	2.10	0.2068	
Fifth vs Quartic	19.40	1	19.40	1.13	0.3470	
Sixth vs Fifth	1.75	1	1.75	0.079	0.7972	
Residual	66.69	3	22.23			

Total 2.448E+006 10 2.448E+005

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS	
Linear	7.68	0.9887	0.9873	0.9819	757.23	
<u>Quadratic</u>	<u>4.68</u>	<u>0.9963</u>	<u>0.9953</u>	<u>0.9918</u>	<u>343.72</u>	<u>Suggested</u>
Cubic	4.56	0.9970	0.9955	0.9857	596.94	

Quartic	4.19	0.9979	0.9962	0.9879	504.29
Fifth	4.14	0.9984	0.9963	0.9700	1256.32
Sixth	4.71	0.9984	0.9952	0.6108	16287.55

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

## Response 2 UTS

### ANOVA for Response Surface Quadratic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	
Model	41691.17	2	20845.59	951.69	< 0.0001	significant
<i>A-% Mn</i>	41372.80	1	41372.80	1888.85	< 0.0001	
<i>A<sup>2</sup></i>	318.37	1	318.37	14.54	0.0066	
Residual	153.33	7	21.90			
Cor Total	41844.50	9				

The Model F-value of 951.69 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	4.68	R-Squared	0.9963
Mean	490.50	Adj R-Squared	0.9953
C.V. %	0.95	Pred R-Squared	0.9918
PRESS	343.72	Adeq Precision	78.624
-2 Log Likelihood	55.68	BIC	62.59
		AICc	65.68

The "Pred R-Squared" of 0.9918 is in reasonable agreement with the "Adj R-Squared" of 0.9953; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio

greater than 4 is desirable. Your ratio of 78.624 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	496.91	1	2.24	491.61	502.20	
A-% Mn	100.77	1	2.32	95.29	106.26	1.00
A <sup>2</sup>	-15.72	1	4.12	-25.48	-5.97	1.00

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned}
 \text{UTS} = & \\
 & +496.91 \\
 & +100.77 * A \\
 & -15.72 * A^2
 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

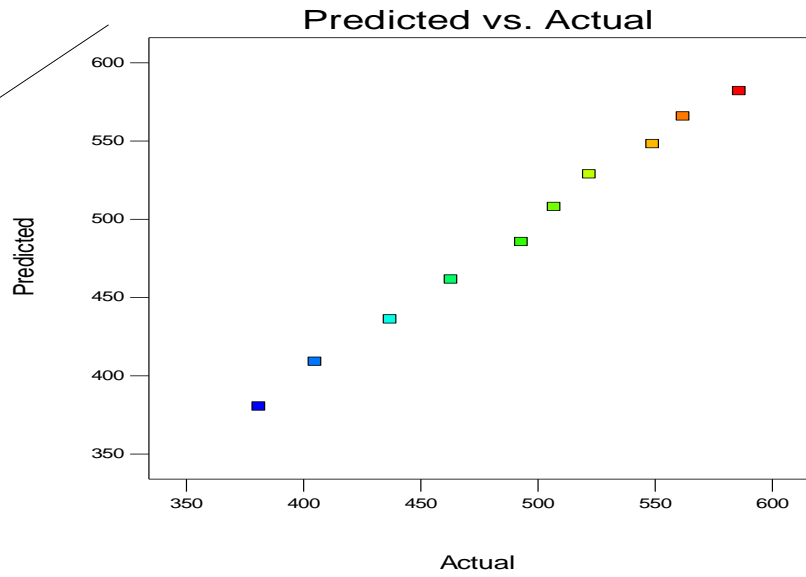
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned}
 \text{UTS} = & \\
 & +350.25000 \\
 & +61.87121 * \% \text{ Mn} \\
 & -3.10606 * \% \text{ Mn}^2
 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
UTS

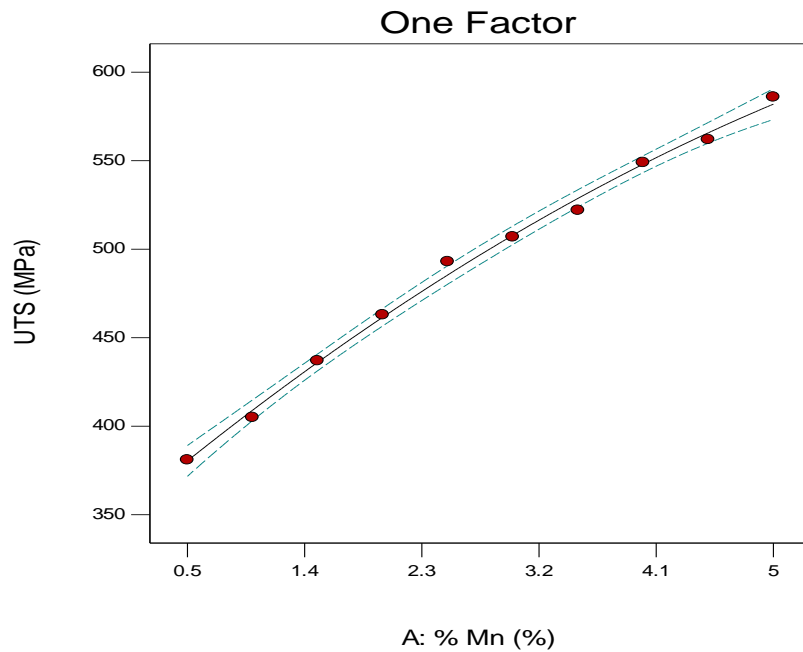
Color points by value of UTS:  
■ 586  
■ 500  
■ 381



Design-Expert® Software  
Factor Coding: Actual  
UTS (MPa)

● Design Points  
 --- 95% CI Bands

X1 = A: % Mn



**Response 3      Hardness   Transform:   None**

**Summary (detailed tables shown below)**

Source	p-value	Adjusted p-value	Predicted R-Squared
Linear	< 0.0001	0.9775	0.9699

<u>Quadratic</u>	<u>0.0125</u>	<u>0.9901</u>	<u>0.9810</u>	<u>Suggested</u>
Cubic	0.5899	0.9890	0.9468	
<u>Quartic</u>	<u>0.0044</u>	<u>0.9977</u>	<u>0.9721</u>	<u>Suggested</u>
Fifth	0.0612	0.9989	0.9691	
Sixth	0.1796	0.9993	0.9079	

### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob > F	
Mean vs Total	5.471E+005	1	5.471E+005			
Linear vs Mean	54400.51	1	54400.51	391.94	< 0.0001	
<u>Quadratic vs Linear</u>	<u>681.82</u>	<u>1</u>	<u>681.82</u>	<u>11.14</u>	<u>0.0125</u>	<u>Suggested</u>
Cubic vs Quadratic	21.95	1	21.95	0.32	0.5899	
<u>Quartic vs Cubic</u>	<u>337.18</u>	<u>1</u>	<u>337.18</u>	<u>24.28</u>	<u>0.0044</u>	<u>Suggested</u>
Fifth vs Quartic	43.41	1	43.41	6.67	0.0612	
Sixth vs Fifth	13.10	1	13.10	3.04	0.1796	
Residual	12.93	3	4.31			

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 6.026E+005 10 60260.30

additional terms are significant and the model is not aliased.

### Model Summary Statistics

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS	
Linear	11.78	0.9800	0.9775	0.9699	1671.84	
<u>Quadratic</u>	<u>7.82</u>	<u>0.9923</u>	<u>0.9901</u>	<u>0.9810</u>	<u>1054.74</u>	<u>Suggested</u>

Cubic	8.23	0.9927	0.9890	0.9468	2954.61	
<u>Quartic</u>	<u>3.73</u>	<u>0.9987</u>	<u>0.9977</u>	<u>0.9721</u>	<u>1547.66</u>	<u>Suggested</u>
Fifth	2.55	0.9995	0.9989	0.9691	1714.99	
						"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"
Sixth	2.08	0.9998	0.9993	0.9079	5109.99	and the "Predicted R-Squared".

### Response 3 Hardness

#### ANOVA for Response Surface Quartic model

##### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	
Model	55441.46	4	13860.36	998.00	< 0.0001	significant
A-% Mn	8876.33	1	8876.33	639.13	< 0.0001	
A <sup>2</sup>	604.22	1	604.22	43.51	0.0012	
A <sup>3</sup>	21.95	1	21.95	1.58	0.2642	
A <sup>4</sup>	337.18	1	337.18	24.28	0.0044	
Residual	69.44	5	13.89			
Cor Total	55510.90	9				

The Model F-value of 998.00 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>4</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	3.73	R-Squared	0.9987
Mean	233.90	Adj R-Squared	0.9977
C.V. %	1.59	Pred R-Squared	0.9721
PRESS	1547.66	Adeq Precision	86.090
-2 Log Likelihood	47.76	BIC	59.27
		AICc	72.76



The "Pred R-Squared" of 0.9721 is in reasonable agreement with the "Adj R-Squared" of 0.9977; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 86.090 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard 95% CI			VIF
	Estimate	df	Error	Low	High	
Intercept	250.18	1	2.27	244.35	256.01	
A-% Mn	121.11	1	4.79	108.80	133.43	6.73
A <sup>2</sup>	-82.40	1	12.49	-114.51	-50.29	14.47
A <sup>3</sup>	-7.68	1	6.11	-23.39	8.02	6.73
A <sup>4</sup>	58.67	1	11.91	28.06	89.27	14.47

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Hardness} = & \\ & +250.18 \\ & +121.11 * A \\ & -82.40 * A^2 \\ & -7.68 * A^3 \\ & +58.67 * A^4 \end{aligned}$$

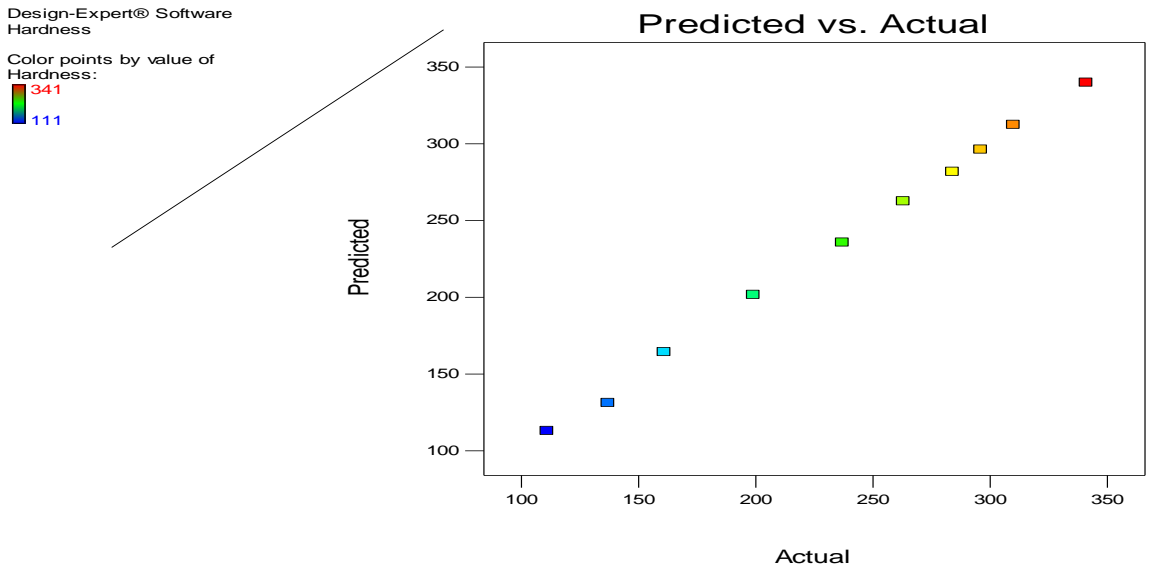
The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Hardness} = & \\ & +124.00000 \\ & -62.37102 * \% \text{ Mn} \\ & +93.15268 * \% \text{ Mn}^2 \\ & -25.85392 * \% \text{ Mn}^3 \\ & +2.28904 * \% \text{ Mn}^4 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because

the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.



**Response 4                      Elongation Transform:    None**

**Summary (detailed tables shown below)**

Source	p-value	Adjusted p-value	Predicted R-Squared	Adjusted R-Squared
<u>Linear</u>	<u>&lt; 0.0001</u>		<u>0.9926</u>	<u>0.9908</u> <u>Suggested</u>
Quadratic	0.6153		0.9919	0.9865
Cubic	0.1706		0.9932	0.9819
Quartic	0.2325		0.9941	0.9771
Fifth	0.3400		0.9943	0.9879
Sixth	0.6888		0.9928	0.8043

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Mean vs Total	4253.91	1	4253.91			
<u>Linear vs Mean</u>	<u>108.90</u>	<u>1</u>	<u>108.90</u>	<u>1210.16</u>	<u>0.0001</u>	<u>≤</u> <u>Suggested</u>

Quadratic vs Linear	0.027	1	0.027	0.28	0.6153
Cubic vs Quadratic	0.20	1	0.20	2.42	0.1706
Quartic vs Cubic	0.13	1	0.13	1.84	0.2325
Fifth vs Quartic	0.082	1	0.082	1.17	0.3400
Sixth vs Fifth	0.017	1	0.017	0.19	0.6888
Residual	0.26	3	0.087		

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 4363.53 10 436.35

additional terms are significant and the model is not aliased.

### Model Summary Statistics

	Std. Source Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS	
<u>Linear</u>	0.30	0.9934	0.9926	0.9908	1.01	<u>Suggested</u>
Quadratic	0.31	0.9937	0.9919	0.9865	1.48	
Cubic	0.29	0.9955	0.9932	0.9819	1.98	
Quartic	0.27	0.9967	0.9941	0.9771	2.51	
Fifth	0.26	0.9975	0.9943	0.9879	1.33	
Sixth	0.30	0.9976	0.9928	0.8043	21.45	

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

and the "Predicted R-Squared".

## Response 4 Elongation

### ANOVA for Response Surface Linear model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	108.90	1	108.90	1210.16	< 0.0001	significant
A-% Mn	108.90	1	108.90	1210.16	< 0.0001	
Residual	0.72	8	0.090			
Cor Total	109.62	9				

The Model F-value of 1210.16 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A is a significant model term. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.30	R-Squared	0.9934
Mean	20.62	Adj R-Squared	0.9926
C.V. %	1.45	Pred R-Squared	0.9908
PRESS	1.01	Adeq Precision	77.076
-2 Log Likelihood	2.07	BIC	6.67
		AICc	7.78

The "Pred R-Squared" of 0.9908 is in reasonable agreement with the "Adj R-Squared" of 0.9926; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 77.076 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard 95% CI 95% CI		
	Estimate	df	Error	Low	High VIF
Intercept	20.63	1	0.095	20.41	20.84
A-% Mn	-5.17	1	0.15	-5.51	-4.83 1.00

#### Final Equation in Terms of Coded Factors:

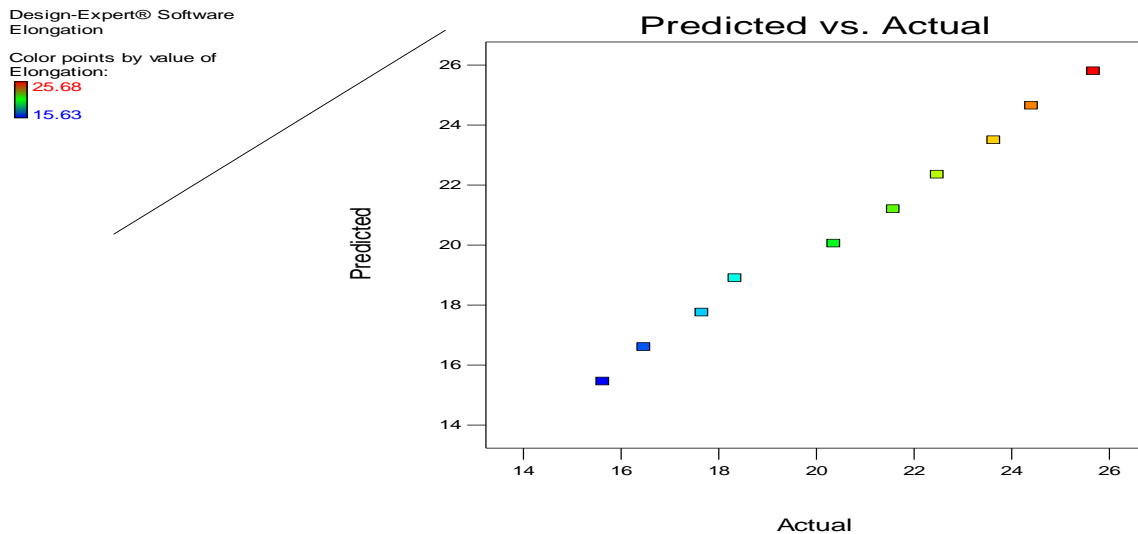
$$\begin{aligned} \text{Elongation} = & \\ & +20.63 \\ & -5.17 * A \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Elongation} = & \\ & +26.94400 \\ & -2.29782 * \% \text{ Mn} \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.



**Response 5      Impact Strength Transform:    None**

**Summary (detailed tables shown below)**

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	< 0.0001		0.9531	0.9208

<u>Quadratic</u>	<u>0.0044</u>		<u>0.9844</u>	<u>0.9748</u> <u>Suggested</u>
Cubic	0.1426		0.9877	0.9499
Quartic	0.3426		0.9879	0.8454
Fifth	0.0310		0.9959	0.8360
<u>Sixth</u>	<u>0.0350</u>		<u>0.9990</u>	<u>0.9008</u> <u>Suggested</u>

**Sequential Model Sum of Squares [Type I]**

	Sum of	Mean	F	p-value		
Source	Squares	df	Square	Value	Prob > F	
Mean vs Total	9185.14	1	9185.14			
Linear vs Mean	118.36	1	118.36	183.88	< 0.0001	
<u>Quadratic vs Linear</u>	<u>3.65</u>	<u>1</u>	<u>3.65</u>	<u>17.10</u>	<u>0.0044</u>	<u>Suggested</u>
Cubic vs Quadratic	0.48	1	0.48	2.85	0.1426	
Quartic vs Cubic	0.18	1	0.18	1.10	0.3426	
Fifth vs Quartic	0.60	1	0.60	10.66	0.0310	
<u>Sixth vs Fifth</u>	<u>0.19</u>	<u>1</u>	<u>0.19</u>	<u>13.47</u>	<u>0.0350</u>	<u>Suggested</u>
Residual	0.041	3	0.014			
Total	9308.65	10	930.86			

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

	Std. Source Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	Adjusted R-PRESS	
Linear	0.80	0.9583	0.9531	0.9208	9.79	
<u>Quadratic</u>	<u>0.46</u>	<u>0.9879</u>	<u>0.9844</u>	<u>0.9748</u>	<u>3.11</u>	<u>Suggested</u>
Cubic	0.41	0.9918	0.9877	0.9499	6.19	
Quartic	0.41	0.9933	0.9879	0.8454	19.10	

Fifth 0.24 0.9982 0.9959 0.8360 20.26

Suggested"Model Summary Statistics":  
Focus on the model maximizing the  
"Adjusted R-Squared"

Sixth 0.12 0.9997 0.9990 0.9008 12.25

and the "Predicted R-Squared".

### Response 5 Impact Strength

#### ANOVA for Response Surface Sixth model

##### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	123.46	6	20.58	1492.52	< 0.0001	significant
<i>A-% Mn</i>	10.48	1	10.48	760.17	0.0001	
$A^2$	0.19	1	0.19	13.57	0.0347	
$A^3$	0.80	1	0.80	58.21	0.0047	
$A^4$	0.13	1	0.13	9.72	0.0526	
$A^5$	0.60	1	0.60	43.87	0.0070	
$A^6$	0.19	1	0.19	13.47	0.0350	
Residual	0.041	3	0.014			
Cor Total	123.51	9				

The Model F-value of 1492.52 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case  $A$ ,  $A^2$ ,  $A^3$ ,  $A^5$ ,  $A^6$  are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.12	R-Squared	0.9997
Mean	30.31	Adj R-Squared	0.9990
C.V. %	0.39	Pred R-Squared	0.9008
PRESS	12.25	Adeq Precision	106.728
-2 Log Likelihood	-26.50	BIC	-10.38
		AICc	43.50

The "Pred R-Squared" of 0.9008 is in reasonable agreement with the "Adj R-Squared" of 0.9990; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 106.728 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	29.61	1	0.086	29.33	29.88	
A-% Mn	-7.80	1	0.28	-8.70	-6.90	23.63
A <sup>2</sup>	3.78	1	1.03	0.51	7.05	98.39
A <sup>3</sup>	7.69	1	1.01	4.48	10.90	184.65
A <sup>4</sup>	-8.27	1	2.65	-16.71	0.17	723.40
A <sup>5</sup>	-5.14	1	0.78	-7.61	-2.67	94.63
A <sup>6</sup>	6.38	1	1.74	0.85	11.92	323.19

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Impact Strength} = & \\ & +29.61 \\ & -7.80 * A \\ & +3.78 * A^2 \\ & +7.69 * A^3 \\ & -8.27 * A^4 \\ & -5.14 * A^5 \\ & +6.38 * A^6 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

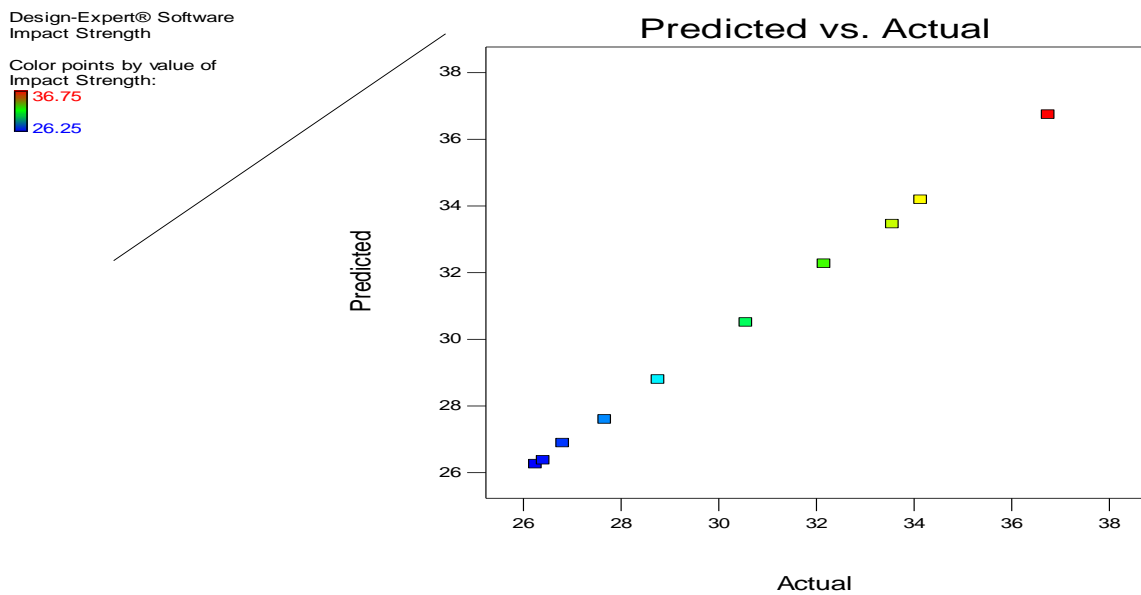
$$\begin{aligned} \text{Impact Strength} = & \\ & +47.57567 \\ & -37.31468 * \% \text{ Mn} \\ & +41.27143 * \% \text{ Mn}^2 \\ & -22.97783 * \% \text{ Mn}^3 \\ & +6.48365 * \% \text{ Mn}^4 \end{aligned}$$



$$-0.90091 * \% \text{ Mn}^5$$

$$+0.049200 * \% \text{ Mn}^6$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.



**Response 6      Resistivity Transform:    None**

**Summary (detailed tables shown below)**

	<b>Sequential Lack of Fit</b>	<b>Adjusted</b>	<b>Predicted</b>	
<b>Source</b>	<b>p-value</b>	<b>R-Squared</b>	<b>R-Squared</b>	
Linear	< 0.0001	0.9546	0.9302	
<u>Quadratic</u>	<u>0.0005</u>	<u>0.9917</u>	<u>0.9802</u>	<u>Suggested</u>
Cubic	0.3239	0.9918	0.9700	
Quartic	0.1180	0.9943	0.9298	
<u>Fifth</u>	<u>0.0058</u>	<u>0.9991</u>	<u>0.9681</u>	<u>Suggested</u>
Sixth	0.1039	0.9996	0.9507	

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob > F	
Mean vs Total	689.90	1	689.90			
Linear vs Mean	13.77	1	13.77	190.17	< 0.0001	
<u>Quadratic vs Linear</u>	<u>0.49</u>	<u>1</u>	<u>0.49</u>	<u>36.57</u>	<u>0.0005</u>	<u>Suggested</u>
Cubic vs Quadratic	0.015	1	0.015	1.15	0.3239	
Quartic vs Cubic	0.032	1	0.032	3.56	0.1180	
<u>Fifth vs Quartic</u>	<u>0.040</u>	<u>1</u>	<u>0.040</u>	<u>28.94</u>	<u>0.0058</u>	<u>Suggested</u>
Sixth vs Fifth	3.547E-003	1	3.547E-003	5.34	0.1039	
Residual	1.991E-003	3	6.637E-004			
Total	704.24	10	70.42			

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the additional terms are significant and the model is not aliased.

**Model Summary Statistics**

Source	Std. Dev.	Adjusted R-Squared	Predicted R-Squared	R-PRESS		
Linear	0.27	0.9596	0.9546	0.9302	1.00	
<u>Quadratic</u>	<u>0.12</u>	<u>0.9935</u>	<u>0.9917</u>	<u>0.9802</u>	<u>0.28</u>	<u>Suggested</u>
Cubic	0.11	0.9946	0.9918	0.9700	0.43	
Quartic	0.095	0.9968	0.9943	0.9298	1.01	
<u>Fifth</u>	<u>0.037</u>	<u>0.9996</u>	<u>0.9991</u>	<u>0.9681</u>	<u>0.46</u>	<u>Suggested</u>
Sixth	0.026	0.9999	0.9996	0.9507	0.71	"Model Summary Statistics": Focus on

the model maximizing the "Adjusted R-Squared"

and the "Predicted R-Squared".

## Response 6 Resistivity

### ANOVA for Response Surface Fifth model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	14.34	5	2.87	2071.51	< 0.0001 significant
A-% Mn	0.76	1	0.76	549.43	< 0.0001
A <sup>2</sup>	0.13	1	0.13	92.07	0.0007
A <sup>3</sup>	0.030	1	0.030	21.64	0.0097
A <sup>4</sup>	0.032	1	0.032	23.42	0.0084
A <sup>5</sup>	0.040	1	0.040	28.94	0.0058
Residual	5.538E-003	4	1.384E-003		
Cor Total	14.35	9			

The Model F-value of 2071.51 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup>, A<sup>4</sup>, A<sup>5</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.037	R-Squared	0.9996
Mean	8.31	Adj R-Squared	0.9991
C.V. %	0.45	Pred R-Squared	0.9681
PRESS	0.46	Adeq Precision	134.396
-2 Log Likelihood	-46.61	BIC	-32.79
		AICc	-6.61

The "Pred R-Squared" of 0.9681 is in reasonable agreement with the "Adj R-Squared" of 0.9991; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A

ratio greater than 4 is desirable. Your ratio of 134.396 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error		95% CI		VIF
	Estimate	df	Error	Low	High		
Intercept	8.62	1	0.023	8.56	8.69		
A-% Mn	2.10	1	0.090	1.85	2.35	23.63	
A <sup>2</sup>	-1.20	1	0.12	-1.54	-0.85	14.47	
A <sup>3</sup>	-1.49	1	0.32	-2.37	-0.60	184.65	
A <sup>4</sup>	0.58	1	0.12	0.25	0.91	14.47	
A <sup>5</sup>	1.32	1	0.25	0.64	2.01	94.63	

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Resistivity} = & \\ & +8.62 \\ & +2.10 * A \\ & -1.20 * A^2 \\ & -1.49 * A^3 \\ & +0.58 * A^4 \\ & +1.32 * A^5 \end{aligned}$$

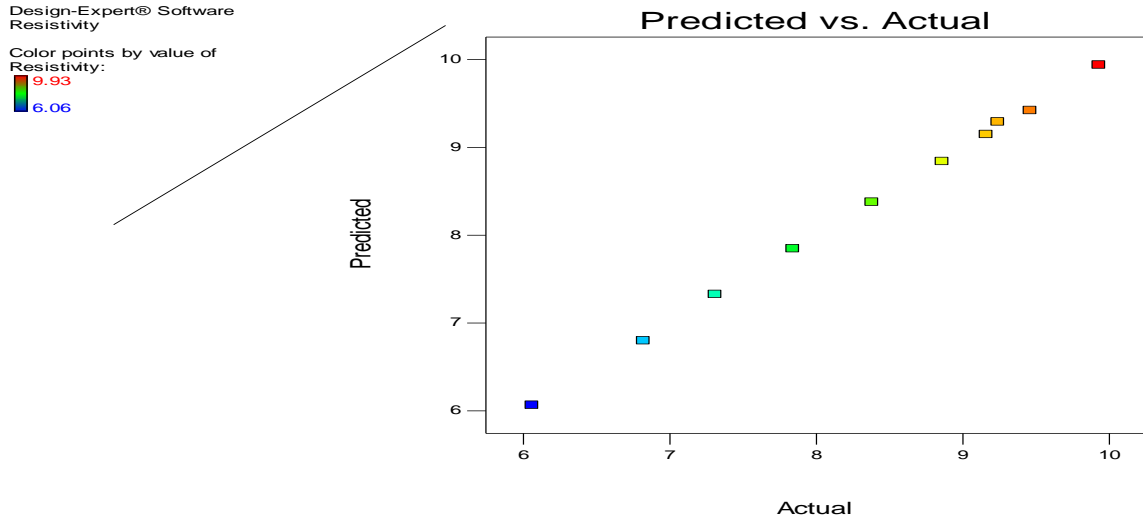
The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Resistivity} = & \\ & +4.65933 \\ & +3.96426 * \% \text{ Mn} \\ & -2.91086 * \% \text{ Mn}^2 \\ & +1.35694 * \% \text{ Mn}^3 \\ & -0.29289 * \% \text{ Mn}^4 \\ & +0.022933 * \% \text{ Mn}^5 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because

the coefficients are scaled to accommodate the units of each factor and the intercepts not at the center of the design space.



**Response 7 Conductivity Transform: None**

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	< 0.0001		0.9247	0.8904
<u>Quadratic</u>	<u>0.0040</u>		<u>0.9756</u>	<u>0.9563</u> <u>Suggested</u>
Cubic	0.1323		0.9811	0.9558
Quartic	0.1722		0.9849	0.9583
Fifth	0.6935		0.9820	0.8357
Sixth	0.4569		0.9806	0.8019

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Mean vs Total	324.67	1	324.67			
Linear vs Mean	31.53	1	31.53	111.54	< 0.0001	
<u>Quadratic vs</u>	<u>1.62</u>	<u>1</u>	<u>1.62</u>	<u>17.66</u>	<u>0.0040</u>	<u>Suggested</u>

<u>Linear</u>					
Cubic vs Quadratic	0.22	1	0.22	3.03	0.1323
Quartic vs Cubic	0.14	1	0.14	2.54	0.1722
Fifth vs Quartic	0.012	1	0.012	0.18	0.6935
Sixth vs Fifth	0.053	1	0.053	0.73	0.4569
Residual	0.22	3	0.073		

"Sequential Model Sum of Squares [Type I]":  
Select the highest order polynomial where the

Total 358.46 10 35.85

additional terms are significant and the model is not aliased.

### Model Summary Statistics

	Std.	Adjusted		Predicted		
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS	
Linear	0.53	0.9331	0.9247	0.8904	3.70	
<u>Quadratic</u>	<u>0.30</u>	<u>0.9810</u>	<u>0.9756</u>	<u>0.9563</u>	<u>1.48</u>	<u>Suggested</u>
Cubic	0.27	0.9874	0.9811	0.9558	1.49	
Quartic	0.24	0.9916	0.9849	0.9583	1.41	
Fifth	0.26	0.9920	0.9820	0.8357	5.55	
Sixth	0.27	0.9935	0.9806	0.8019	6.69	

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

and the "Predicted R-Squared".

## Response 7 Conductivity

### ANOVA for Response Surface Quadratic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	33.15	2	16.57	180.70	< 0.0001	significant
A-% Mn	31.53	1	31.53	343.75	< 0.0001	
A <sup>2</sup>	1.62	1	1.62	17.66	0.0040	
Residual	0.64	7	0.092			
Cor Total	33.79	9				

The Model F-value of 180.70 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.30	R-Squared	0.9810
Mean	5.70	Adj R-Squared	0.9756
C.V. %	5.31	Pred R-Squared	0.9563
PRESS	1.48	Adeq Precision	33.541
-2 Log Likelihood	0.92	BIC	7.83
		AICc	10.92

The "Pred R-Squared" of 0.9563 is in reasonable agreement with the "Adj R-Squared" of 0.9756; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 33.541 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard 95% CI 95% CI		
	Estimate	df	Error	Low	High VIF
Intercept	5.24	1	0.14	4.90	5.58
A-% Mn	-2.78	1	0.15	-3.14	-2.43 1.00
A <sup>2</sup>	1.12	1	0.27	0.49	1.75 1.00

### Final Equation in Terms of Coded Factors:

$$\begin{aligned} \text{Conductivity} = & \\ & +5.24 \\ & -2.78 * A \\ & +1.12 * A^2 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

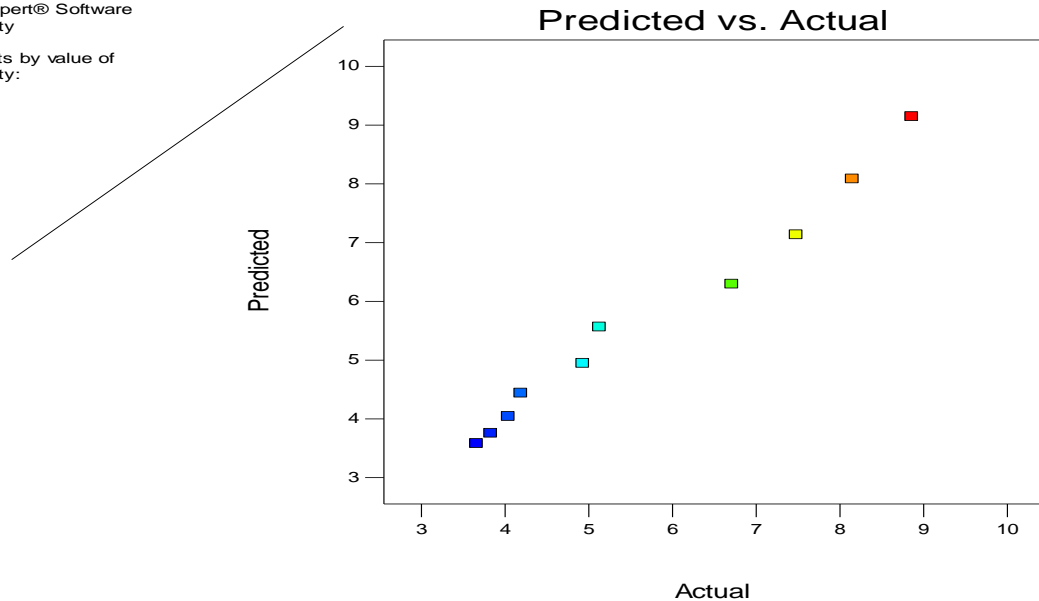
### Final Equation in Terms of Actual Factors:

$$\begin{aligned} \text{Conductivity} = & \\ & +10.31633 \\ & -2.45470 * \% \text{ Mn} \\ & +0.22152 * \% \text{ Mn}^2 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

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Color points by value of  
Conductivity:





**Factor Name Level Low Level High Level Std. Dev. Coding**  
 A % Mn 2.46 0.50 5.00 0.000 Actual

Response	Predicted Predicted		Observed	Std Dev	SE Mean	CI for Mean		99% of Population	
	Mean	Median <sup>1</sup>				95% CI low	95% CI high	95% TI low	95% TI high
Yield Strength	354.849	354.849	-	2.12039	1.47989	350.139	359.559	335.418	374.28
UTS	483.576	483.576	-	4.68014	2.20818	478.355	488.798	457.728	509.425
Hardness	233.125	233.125	-	3.72668	2.20229	227.464	238.787	208.632	257.618
Elongation	21.2953	21.2953	-	0.29998	0.0967991	21.0721	21.5185	19.7932	22.7974
Impact Strength	30.6613	30.6613	-	0.117418	0.08195	30.4005	30.9221	29.5853	31.7373
Resistivity	8.33493	8.33493	-	0.0372082	0.0238162	8.2688	8.40105	8.05678	8.61307
Conductivity	5.62065	5.62065	-	0.302846	0.142889	5.28277	5.95852	3.94801	7.29328

**Confirmation Report**

**Two-sided Confidence = 95% n = 1**

**Factor Name Level Low Level High Level Std. Dev. Coding**  
 A % Mn 2.46 0.50 5.00 0.000 Actual

Response	Predicted Predicted		Observed	Std Dev	n	SE Pred	95% PI low	Data Mean	95% PI high
	Mean	Median <sup>1</sup>							
Yield Strength	354.849	354.849	-	2.12039	1	2.59	346.62		363.08
UTS	483.576	483.576	-	4.68014	1	5.17	471.34		495.81
Hardness	233.125	233.125	-	3.72668	1	4.33	222.00		244.25
Elongation	21.2953	21.2953	-	0.29998	1	0.32	20.57		22.02
Impact Strength	30.6613	30.6613	-	0.117418	1	0.14	30.21		31.12
Resistivity	8.33493	8.33493	-	0.0372082	1	0.044	8.21		8.46
Conductivity	5.62065	5.62065	-	0.302846	1	0.33	4.83		6.41

**Design Expert Analysis for Nickel**

**Response 1 Yield Strength Transform: None**

**Summary (detailed tables shown below)**

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	< 0.0001		0.8499	0.7456

Quadratic	0.0810		0.8923	0.7402
<u>Cubic</u>	<u>0.0247</u>		<u>0.9493</u>	<u>0.8528 Suggested</u>
Quartic	0.0873		0.9680	0.9147
Fifth	0.6150		0.9628	0.4358
Sixth	0.5307		0.9575	-2.2477

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F	P-value	Prob > F
Mean vs Total	1.106E+006	1	1.106E+006			
Linear vs Mean	54837.93	1	54837.93	51.98	<	0.0001
Quadratic vs Linear	3141.94	1	3141.94	4.15	0.0810	
<u>Cubic vs Quadratic</u>	<u>3161.22</u>	<u>1</u>	<u>3161.22</u>	<u>8.87</u>	<u>0.0247</u>	<u>Suggested</u>
Quartic vs Cubic	1012.87	1	1012.87	4.50	0.0873	
Fifth vs Quartic	77.58	1	77.58	0.30	0.6150	
Sixth vs Fifth	149.39	1	149.39	0.50	0.5307	
Residual	897.47	3	299.16			

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 1.170E+006 10 1.170E+005

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS
Linear	32.48	0.8666	0.8499	0.7456	16101.11
Quadratic	27.51	0.9163	0.8923	0.7402	16440.95

Cubic	18.87	0.9662	0.9493	0.8528	9311.51	<u>Suggested</u>
Quartic	15.00	0.9822	0.9680	0.9147	5399.59	
Fifth	16.18	0.9835	0.9628	0.4358	35700.21	
Sixth	17.30	0.9858	0.9575	-2.2477	2.055E+005	"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

### Response 1 Yield Strength

#### ANOVA for Response Surface Cubic model

##### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	
Model	61141.09	3	20380.36	57.21	< 0.0001	significant
A-% Ni	20201.73	1	20201.73	56.71	0.0003	
A <sup>2</sup>	3141.94	1	3141.94	8.82	0.0250	
A <sup>3</sup>	3161.22	1	3161.22	8.87	0.0247	
Residual	2137.31	6	356.22			
Cor Total	63278.40	9				

The Model F-value of 57.21 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	18.87	R-Squared	0.9662
Mean	332.60	Adj R-Squared	0.9493
C.V. %	5.67	Pred R-Squared	0.8528
PRESS	9311.51	Adeq Precision	17.624
-2 Log Likelihood	82.03	BIC	91.24
		AICc	98.03

The "Pred R-Squared" of 0.8528 is in reasonable agreement with the "Adj R-Squared" of 0.9493; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 17.624 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	352.72	1	9.03	330.63	374.82	
A-% Ni	182.71	1	24.26	123.34	242.08	6.73
A <sup>2</sup>	-49.40	1	16.63	-90.10	-8.70	1.00
A <sup>3</sup>	-92.19	1	30.95	-167.91	-16.47	6.73

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Yield Strength} = & \\ & +352.72 \\ & +182.71 * A \\ & -49.40 * A^2 \\ & -92.19 * A^3 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Yield Strength} = & \\ & +223.93333 \\ & -48.74359 * \% \text{ Ni} \\ & +57.01166 * \% \text{ Ni}^2 \\ & -8.09324 * \% \text{ Ni}^3 \end{aligned}$$

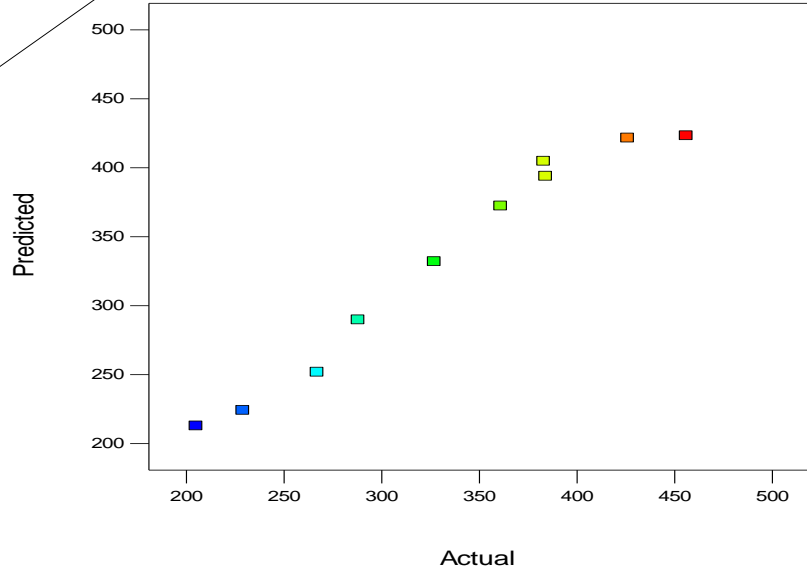
The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Yield Strength

Color points by value of  
Yield Strength:



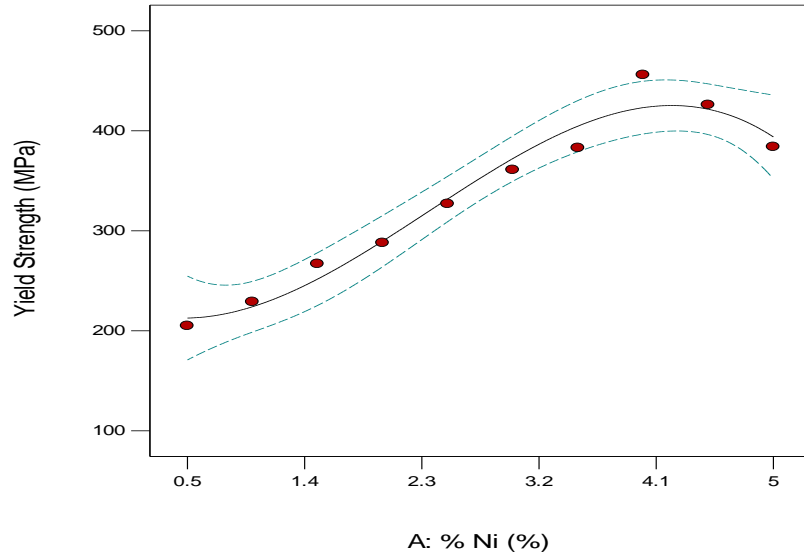
**Predicted vs. Actual**



Design-Expert® Software  
Factor Coding: Actual  
Yield Strength (MPa)  
● Design Points  
--- 95% CI Bands

X1 = A: % Ni

**One Factor**



**Response 2          UTS          Transform:   None**

**Summary (detailed tables shown below)**

	<b>Sequential Lack of Fit</b>	<b>Adjusted</b>	<b>Predicted</b>
<b>Source</b>	<b>p-value</b>	<b>R-Squared</b>	<b>R-Squared</b>
Linear	< 0.0001	0.8508	0.7637
Quadratic	0.1076	0.8852	0.7423

Cubic	0.0038	0.9701	0.9401	<u>Suggested</u>
Quartic	0.2770	0.9723	0.8012	
Fifth	0.4351	0.9709	-0.0126	
Sixth	0.1092	0.9856	-0.6254	

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F	P-value	
Mean vs Total	2.203E+006	1	2.203E+006			
Linear vs Mean	35609.65	1	35609.65	52.30	< 0.0001	
Quadratic vs Linear	1782.01	1	1782.01	3.40	0.1076	
<u>Cubic vs Quadratic</u>	<u>2845.39</u>	<u>1</u>	<u>2845.39</u>	<u>20.84</u>	<u>0.0038</u>	<u>Suggested</u>
Quartic vs Cubic	187.86	1	187.86	1.49	0.2770	
Fifth vs Quartic	99.80	1	99.80	0.75	0.4351	
Sixth vs Fifth	334.70	1	334.70	5.10	0.1092	
Residual	196.99	3	65.66			

Total 2.244E+006 10 2.244E+005

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS
Linear	26.09	0.8673	0.8508	0.7637	9701.90
Quadratic	22.88	0.9107	0.8852	0.7423	10580.25

Cubic	11.69	0.9800	0.9701	0.9401	2458.51	<u>Suggested</u>  "Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"  and the "Predicted R-Squared".
Quartic	11.24	0.9846	0.9723	0.8012	8162.28	
Fifth	11.53	0.9870	0.9709	-0.0126	41575.70	
Sixth	8.10	0.9952	0.9856	-0.6254	66734.31	

## Response 2 UTS

### ANOVA for Response Surface Cubic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	
Model	40237.05	3	13412.35	98.22	< 0.0001	significant
A-% Ni	14871.51	1	14871.51	108.90	< 0.0001	
A <sup>2</sup>	1782.01	1	1782.01	13.05	0.0112	
A <sup>3</sup>	2845.39	1	2845.39	20.84	0.0038	
Residual	819.35	6	136.56			
Cor Total	41056.40	9				

The Model F-value of 98.22 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	11.69	R-Squared	0.9800
Mean	469.40	Adj R-Squared	0.9701
C.V. %	2.49	Pred R-Squared	0.9401
PRESS	2458.51	Adeq Precision	22.612
-2 Log Likelihood	72.44	BIC	81.65

AICc 88.44

The "Pred R-Squared" of 0.9401 is in reasonable agreement with the "Adj R-Squared" of 0.9701; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 22.612 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	484.56	1	5.59	470.88	498.24	
A-% Ni	156.77	1	15.02	120.01	193.52	6.73
A <sup>2</sup>	-37.20	1	10.30	-62.40	-12.00	1.00
A <sup>3</sup>	-87.46	1	19.16	-134.34	-40.58	6.73

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned}
 \text{UTS} = & \\
 & +484.56 \\
 & +156.77 * A \\
 & -37.20 * A^2 \\
 & -87.46 * A^3
 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

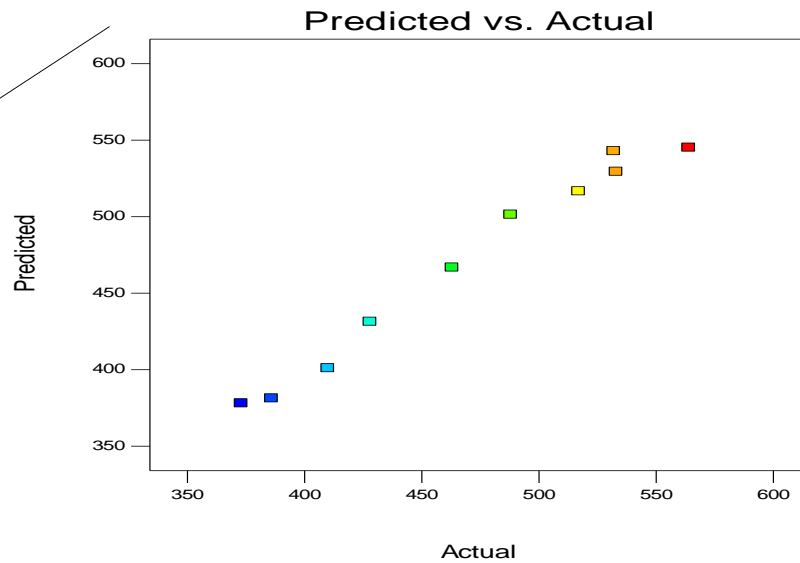
$$\begin{aligned}
 \text{UTS} = & \\
 & +397.06667 \\
 & -64.11189 * \% \text{ Ni} \\
 & +55.99767 * \% \text{ Ni}^2 \\
 & -7.67832 * \% \text{ Ni}^3
 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.



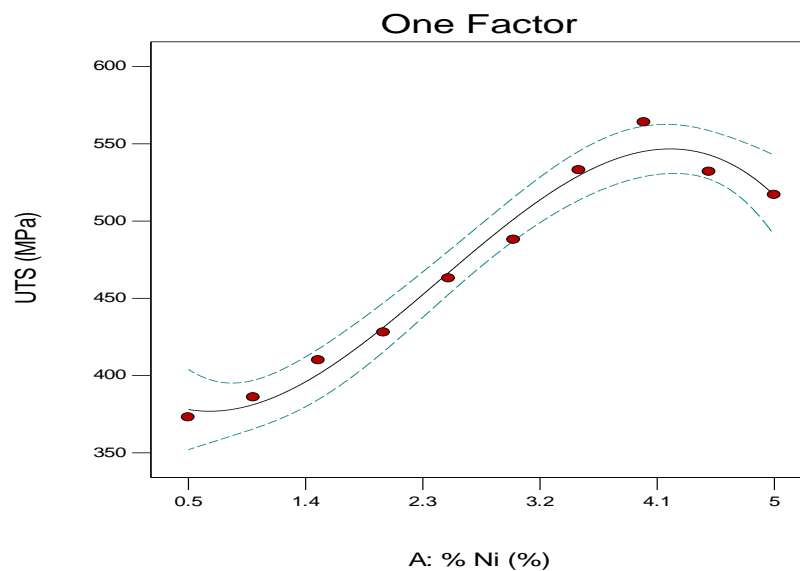
Design-Expert® Software  
UTS

Color points by value of  
UTS:  
564  
373



Design-Expert® Software  
Factor Coding: Actual  
UTS (MPa)  
● Design Points  
--- 95% CI Bands

X1 = A: % Ni



**Response 3      Hardness   Transform:   None**

**Summary (detailed tables shown below)**

	Sequential Lack of Fit	Adjusted	Predicted	
Source	p-value	R-Squared	R-Squared	
Linear	< 0.0001	0.9096	0.8526	
Quadratic	0.0835	0.9346	0.8509	
<u>Cubic</u>	<u>0.0156</u>	<u>0.9733</u>	<u>0.9468</u>	<u>Suggested</u>
Quartic	0.2506	0.9761	0.9462	

Fifth	0.8332	0.9705	0.5499
Sixth	0.5210	0.9665	-4.5102

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F	P-value	Prob > F
Mean vs Total	5.203E+005	1	5.203E+005			
Linear vs Mean	46683.71	1	46683.71	91.51	<	0.0001
Quadratic vs Linear	1500.19	1	1500.19	4.07	0.0835	
<u>Cubic vs Quadratic</u>	<u>1678.56</u>	<u>1</u>	<u>1678.56</u>	<u>11.16</u>	<u>0.0156</u>	<u>Suggested</u>
Quartic vs Cubic	227.71	1	227.71	1.69	0.2506	
Fifth vs Quartic	8.41	1	8.41	0.050	0.8332	
Sixth vs Fifth	99.30	1	99.30	0.53	0.5210	
Residual	567.02	3	189.01			
Total	5.711E+005	10	57106.10			

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

Source	Std. Dev.	Adjusted R-Squared	Predicted R-Squared	Adjusted R-Squared	PRESS
Linear	22.59	0.9196	0.9096	0.8526	7484.87
Quadratic	19.20	0.9492	0.9346	0.8509	7569.27
<u>Cubic</u>	<u>12.26</u>	<u>0.9822</u>	<u>0.9733</u>	<u>0.9468</u>	<u>2701.21</u>
Quartic	11.62	0.9867	0.9761	0.9462	2733.17

Suggested

Fifth 12.91 0.9869 0.9705 0.5499 22850.58

"Model Summary Statistics":  
Focus on the model maximizing  
the "Adjusted R-Squared"

Sixth 13.75 0.9888 0.9665 -4.5102 2.797E+005

and the "Predicted R-Squared".

### Response 3 Hardness

#### ANOVA for Response Surface Cubic model

##### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	
Model	49862.46	3	16620.82	110.51	< 0.0001	significant
A-% Ni	14659.57	1	14659.57	97.47	< 0.0001	
A <sup>2</sup>	1500.19	1	1500.19	9.97	0.0196	
A <sup>3</sup>	1678.56	1	1678.56	11.16	0.0156	
Residual	902.44	6	150.41			
Cor Total	50764.90	9				

The Model F-value of 110.51 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	12.26	R-Squared	0.9822
Mean	228.10	Adj R-Squared	0.9733
C.V. %	5.38	Pred R-Squared	0.9468
PRESS	2701.21	Adeq Precision	24.677
-2 Log Likelihood	73.40	BIC	82.61
		AICc	89.40

The "Pred R-Squared" of 0.9468 is in reasonable agreement with the "Adj R-Squared" of 0.9733; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 24.677 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		High VIF
	Estimate	df		Low	High	
Intercept	242.01	1	5.87	227.65	256.36	
A-% Ni	155.64	1	15.77	117.07	194.22	6.73
A <sup>2</sup>	-34.13	1	10.81	-60.58	-7.69	1.00
A <sup>3</sup>	-67.18	1	20.11	-116.38	-17.97	6.73

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Hardness} = & \\ & +242.01 \\ & +155.64 * A \\ & -34.13 * A^2 \\ & -67.18 * A^3 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

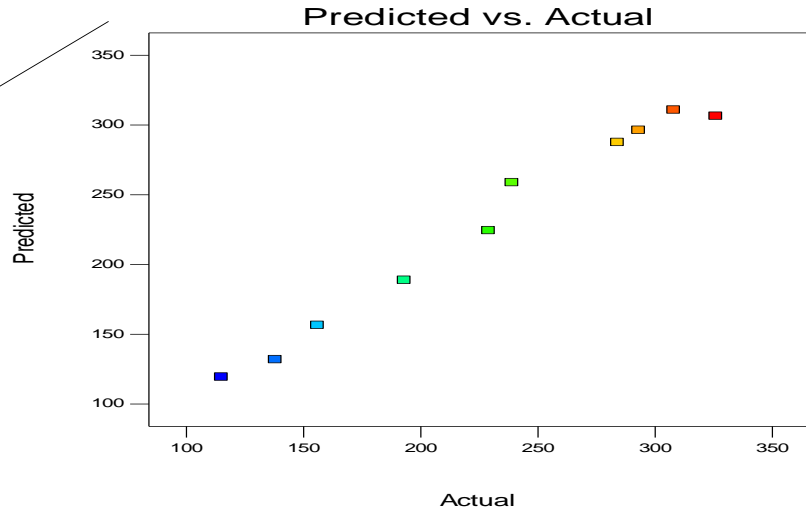
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Hardness} = & \\ & +123.43333 \\ & -27.53963 * \% \text{ Ni} \\ & +41.91142 * \% \text{ Ni}^2 \\ & -5.89744 * \% \text{ Ni}^3 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

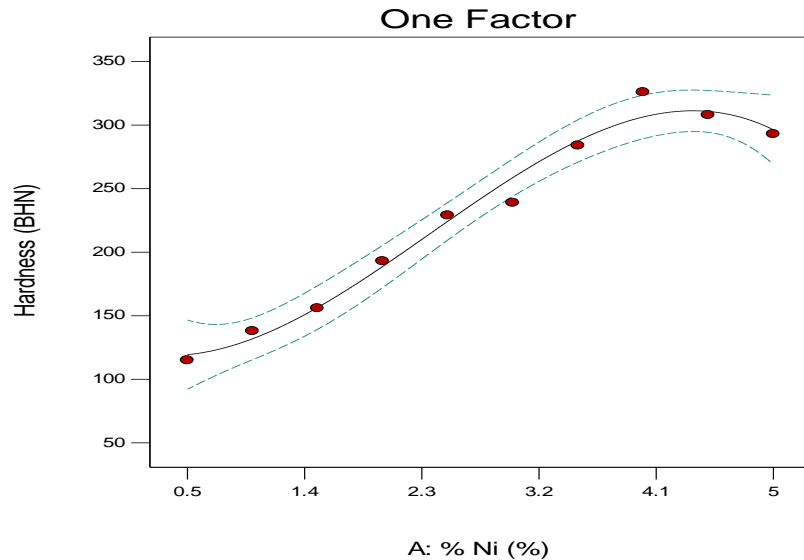
Design-Expert® Software  
Hardness

Color points by value of  
Hardness:  
326  
115



Design-Expert® Software  
Factor Coding: Actual  
Hardness (BHN)  
● Design Points  
--- 95% CI Bands

X1 = A: % Ni



**Response 4                      Elongation Transform:    None**

**Summary (detailed tables shown below)**

	<b>Sequential Lack of Fit</b>	<b>Adjusted</b>	<b>Predicted</b>	
<b>Source</b>	<b>p-value</b>	<b>p-value</b>	<b>R-Squared</b>	<b>R-Squared</b>
<u>Linear</u>	<u>&lt; 0.0001</u>		<u>0.9324</u>	<u>0.9071</u> <u>Suggested</u>
Quadratic	0.7651		0.9237	0.8481
Cubic	0.1521		0.9386	0.8132
Quartic	0.0297		0.9738	0.8656
Fifth	0.8293		0.9677	-0.3776
<u>Sixth</u>	<u>0.0344</u>		<u>0.9922</u>	<u>0.2022</u> <u>Suggested</u>

**Sequential Model Sum of Squares [Type I]**

	Sum of Squares	df	Mean Square	F Value	p-value	
Mean vs Total	4510.10	1	4510.10			
<u>Linear vs Mean</u>	<u>79.42</u>	<u>1</u>	<u>79.42</u>	<u>125.05</u>	<u>0.0001</u>	<u>Suggested</u>
Quadratic vs Linear	0.069	1	0.069	0.097	0.7651	
Cubic vs Quadratic	1.55	1	1.55	2.69	0.1521	
Quartic vs Cubic	2.23	1	2.23	9.06	0.0297	
Fifth vs Quartic	0.016	1	0.016	0.053	0.8293	
<u>Sixth vs Fifth</u>	<u>1.00</u>	<u>1</u>	<u>1.00</u>	<u>13.65</u>	<u>0.0344</u>	<u>Suggested</u>
Residual	0.22	3	0.073			

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 4594.60 10 459.46

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS	
<u>Linear</u>	<u>0.80</u>	<u>0.9399</u>	<u>0.9324</u>	<u>0.9071</u>	<u>7.85</u>	<u>Suggested</u>
Quadratic	0.85	0.9407	0.9237	0.8481	12.83	
Cubic	0.76	0.9591	0.9386	0.8132	15.79	
Quartic	0.50	0.9854	0.9738	0.8656	11.35	
Fifth	0.55	0.9856	0.9677	-0.3776	116.40	
<u>Sixth</u>	<u>0.27</u>	<u>0.9974</u>	<u>0.9922</u>	<u>0.2022</u>	<u>67.41</u>	<u>Suggested</u>

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

and the "Predicted R-Squared".

### Response 4 Elongation

#### ANOVA for Response Surface Sixth model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	84.28	6	14.05	192.66	0.0006	significant
A-% Ni	5.47	1	5.47	75.08	0.0032	
A <sup>2</sup>	2.25	1	2.25	30.88	0.0115	
A <sup>3</sup>	0.013	1	0.013	0.18	0.7027	
A <sup>4</sup>	1.44	1	1.44	19.71	0.0212	
A <sup>5</sup>	0.016	1	0.016	0.22	0.6708	
A <sup>6</sup>	1.00	1	1.00	13.65	0.0344	
Residual	0.22	3	0.073			
Cor Total	84.50	9				

The Model F-value of 192.66 implies the model is significant. There is only a 0.06% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>4</sup>, A<sup>6</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.27	R-Squared	0.9974
Mean	21.24	Adj R-Squared	0.9922
C.V. %	1.27	Pred R-Squared	0.2022
PRESS	67.41	Adeq Precision	35.371
-2 Log Likelihood	-9.85	BIC	6.27
		AICc	60.15

The "Pred R-Squared" of 0.2022 is not as close to the "Adj R-Squared" of 0.9922 as one might normally expect; i.e. the difference is more than 0.2. This may indicate a large block effector a possible problem with your model and/or data. Things to consider are model

reduction, response transformation, outliers, etc. All empirical models should be tested by doing confirmation runs. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 35.371 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error		95% CI		VIF
	Estimate	df	Error	Low	High		
Intercept	22.30	1	0.20	21.67	22.93		
A-% Ni	-5.63	1	0.65	-7.70	-3.57	23.63	
A <sup>2</sup>	-13.12	1	2.36	-20.63	-5.60	98.39	
A <sup>3</sup>	0.97	1	2.32	-6.40	8.35	184.65	
A <sup>4</sup>	27.08	1	6.10	7.67	46.49	723.40	
A <sup>5</sup>	0.84	1	1.78	-4.84	6.52	94.63	
A <sup>6</sup>	-14.78	1	4.00	-27.51	-2.05	323.19	

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Elongation} = & \\ & +22.30 \\ & -5.63 * A \\ & -13.12 * A^2 \\ & +0.97 * A^3 \\ & +27.08 * A^4 \\ & +0.84 * A^5 \\ & -14.78 * A^6 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

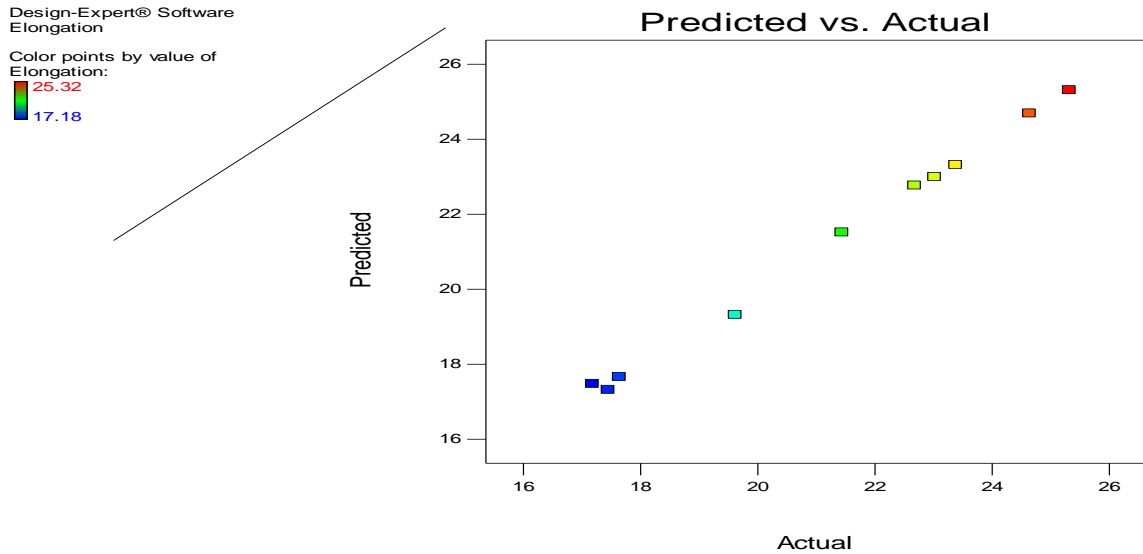
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Elongation} = & \\ & +16.69633 \\ & +37.42913 * \% \text{ Ni} \\ & -56.09123 * \% \text{ Ni}^2 \\ & +36.94037 * \% \text{ Ni}^3 \\ & -12.06482 * \% \text{ Ni}^4 \\ & +1.89406 * \% \text{ Ni}^5 \end{aligned}$$



$$-0.11391 * \% \text{ Ni}^6$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.



**Response 5                      Impact Strength Transform:    None**

**Summary (detailed tables shown below)**

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	< 0.0001		0.9396	0.9027
Quadratic	0.0758		0.9574	0.8854
<u>Cubic</u>	<u>&lt; 0.0001</u>		<u>0.9974</u>	<u>0.9958</u> <b>Suggested</b>
Quartic	0.8198		0.9970	0.9889
Fifth	0.8061		0.9963	0.9439
Sixth	0.5060		0.9958	0.2835

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob >
--------	----------------	----	-------------	---------	----------------

	<b>F</b>				
Mean vs Total	10218.89	1	10218.89		
Linear vs Mean	174.94	1	174.94	140.93	< 0.0001
Quadratic vs Linear	3.80	1	3.80	4.34	0.0758
<u>Cubic vs Quadratic</u>	<u>5.82</u>	<u>1</u>	<u>5.82</u>	<u>110.37</u>	<u>≤ 0.0001</u>
Quartic vs Cubic	3.603E-003	1	3.603E-003	0.058	0.8198
Fifth vs Quartic	5.283E-003	1	5.283E-003	0.069	0.8061
Sixth vs Fifth	0.049	1	0.049	0.57	0.5060
Residual	0.26	3	0.086		

Suggested

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 10403.76 10 1040.38

additional terms are significant and the model is not aliased.

### Model Summary Statistics

	Std. Source Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS
Linear	1.11	0.9463	0.9396	0.9027	17.98
Quadratic	0.94	0.9668	0.9574	0.8854	21.18
<u>Cubic</u>	<u>0.23</u>	<u>0.9983</u>	<u>0.9974</u>	<u>0.9958</u>	<u>0.77</u>
Quartic	0.25	0.9983	0.9970	0.9889	2.06
Fifth	0.28	0.9983	0.9963	0.9439	10.36

Suggested

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

Sixth 0.29 0.9986 0.9958 0.2835 132.45

and the "Predicted R-Squared".

## Response 5 Impact Strength

### ANOVA for Response Surface Cubic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	
Model	184.55	3	61.52	1167.40	< 0.0001	significant
<i>A-% Ni</i>	53.62	1	53.62	1017.61	< 0.0001	
$A^2$	3.80	1	3.80	72.07	0.0001	
$A^3$	5.82	1	5.82	110.37	< 0.0001	
Residual	0.32	6	0.053			
Cor Total	184.87	9				

The Model F-value of 1167.40 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A,  $A^2$ ,  $A^3$  are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.23	R-Squared	0.9983
Mean	31.97	Adj R-Squared	0.9974
C.V. %	0.72	Pred R-Squared	0.9958
PRESS	0.77	Adeq Precision	79.891
-2 Log Likelihood	-6.16	BIC	3.05
		AICc	9.84

The "Pred R-Squared" of 0.9958 is in reasonable agreement with the "Adj R-Squared" of 0.9974; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 79.891 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error		95% CI		VIF
	Estimate	df	Error	Low	High		
Intercept	31.27	1	0.11	31.00	31.54		

A-% Ni	-9.41	1	0.30	-10.14	-8.69	6.73
A <sup>2</sup>	1.72	1	0.20	1.22	2.21	1.00
A <sup>3</sup>	3.95	1	0.38	3.03	4.88	6.73

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Impact Strength} = & \\ & +31.27 \\ & -9.41 * A \\ & +1.72 * A^2 \\ & +3.95 * A^3 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Impact Strength} = & \\ & +38.11867 \\ & +1.82630 * \% \text{ Ni} \\ & -2.52473 * \% \text{ Ni}^2 \\ & +0.34715 * \% \text{ Ni}^3 \end{aligned}$$

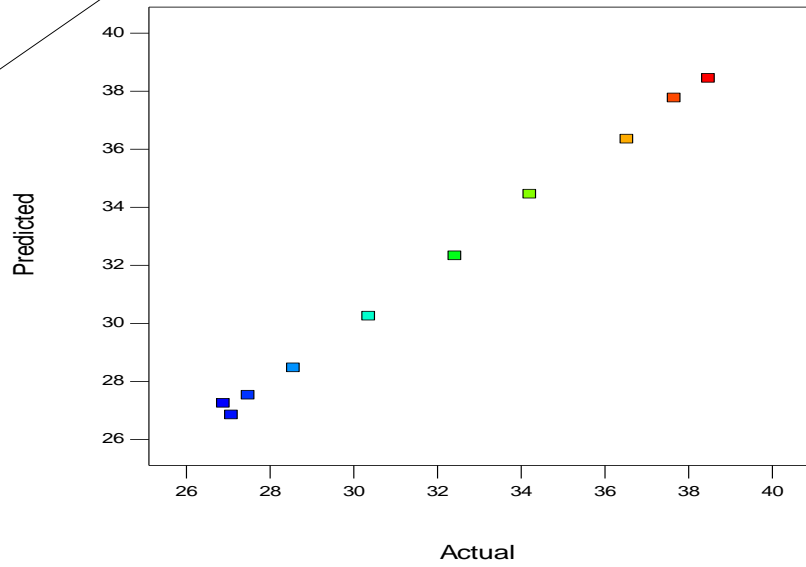
The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Impact Strength

Color points by value of  
Impact Strength:



Predicted vs. Actual

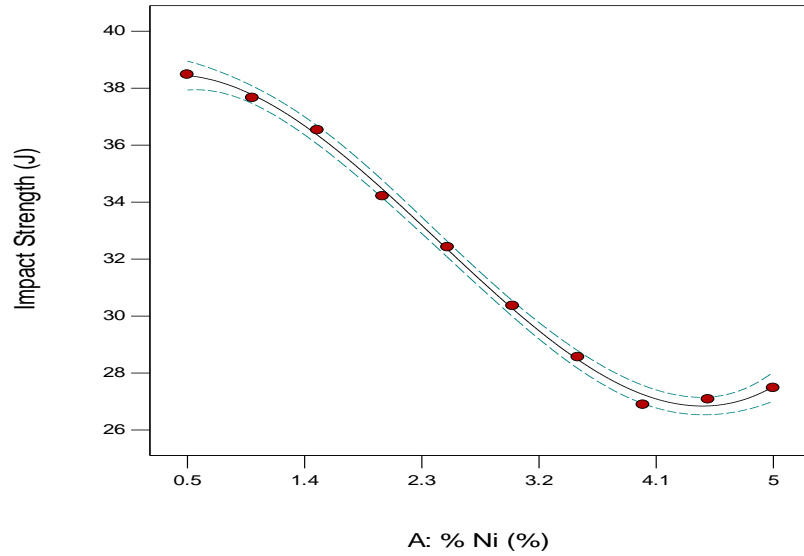


Design-Expert® Software  
Factor Coding: Actual  
Impact Strength (J)

● Design Points  
--- 95% CI Bands

X1 = A: % Ni

One Factor



**Response 6      Resistivity Transform:    None**

**Summary (detailed tables shown below)**

	Sequential Lack of Fit	Adjusted	Predicted
Source	p-value	R-Squared	R-Squared
Linear	< 0.0001	0.9127	0.8386

Suggested

Quadratic	0.1376		0.9288	0.7819
Cubic	0.0414		0.9607	0.7884
<u>Quartic</u>	<u>0.0367</u>		<u>0.9819</u>	<u>0.7630</u> <u>Suggested</u>
Fifth	0.0545		0.9919	0.7937
Sixth	0.2398		0.9937	-0.2127

**Sequential Model Sum of Squares [Type I]**

	Sum of	Mean	F	p-value	
Source	Squares	df	Square	Value	Prob > F
Mean vs Total	760.04	1	760.04		
<u>Linear vs Mean</u>	<u>32.00</u>	<u>1</u>	<u>32.00</u>	<u>95.07</u>	<u>0.0001</u> ≤ <u>Suggested</u>
Quadratic vs Linear	0.77	1	0.77	2.81	0.1376
Cubic vs Quadratic	1.01	1	1.01	6.69	0.0414
<u>Quartic vs Cubic</u>	<u>0.56</u>	<u>1</u>	<u>0.56</u>	<u>8.01</u>	<u>0.0367</u> <u>Suggested</u>
Fifth vs Quartic	0.23	1	0.23	7.25	0.0545
Sixth vs Fifth	0.052	1	0.052	2.14	0.2398
Residual	0.072	3	0.024		
Total	794.73	10	79.47		

"Sequential Model Sum of Squares [Type I]":  
Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

	Std.	Adjusted	Predicted		
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS
<u>Linear</u>	<u>0.58</u>	<u>0.9224</u>	<u>0.9127</u>	<u>0.8386</u>	<u>5.60</u> <u>Suggested</u>
Quadratic	0.52	0.9446	0.9288	0.7819	7.57
Cubic	0.39	0.9738	0.9607	0.7884	7.34
<u>Quartic</u>	<u>0.26</u>	<u>0.9899</u>	<u>0.9819</u>	<u>0.7630</u>	<u>8.22</u> <u>Suggested</u>
Fifth	0.18	0.9964	0.9919	0.7937	7.16

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

Sixth 0.16 0.9979 0.9937 -0.2127 42.07

and the "Predicted R-Squared".

## Response 6 Resistivity

### ANOVA for Response Surface Quartic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	34.34	4	8.59	122.91	< 0.0001	significant
A-% Ni	1.57	1	1.57	22.42	0.0052	
A <sup>2</sup>	0.24	1	0.24	3.45	0.1226	
A <sup>3</sup>	1.01	1	1.01	14.50	0.0125	
A <sup>4</sup>	0.56	1	0.56	8.01	0.0367	
Residual	0.35	5	0.070			
Cor Total	34.69	9				

The Model F-value of 122.91 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>3</sup>, A<sup>4</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.26	R-Squared	0.9899
Mean	8.72	Adj R-Squared	0.9819
C.V. %	3.03	Pred R-Squared	0.7630
PRESS	8.22	Adeq Precision	34.877
-2 Log Likelihood	-5.17	BIC	6.35
		AICc	19.83

The "Pred R-Squared" of 0.7630 is not as close to the "Adj R-Squared" of 0.9819 as one might normally expect; i.e. the difference is more than 0.2. This may indicate a large block

effector a possible problem with your model and/or data. Things to consider are model reduction, response transformation, outliers, etc. All empirical models should be tested by doing confirmation runs. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 34.877 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	8.68	1	0.16	8.27	9.10	
A-% Ni	1.61	1	0.34	0.74	2.48	6.73
A <sup>2</sup>	-1.64	1	0.89	-3.92	0.63	14.47
A <sup>3</sup>	1.65	1	0.43	0.54	2.76	6.73
A <sup>4</sup>	2.39	1	0.84	0.22	4.56	14.47

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Resistivity} = & \\ & +8.68 \\ & +1.61 * A \\ & -1.64 * A^2 \\ & +1.65 * A^3 \\ & +2.39 * A^4 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Resistivity} = & \\ & +6.57917 \\ & -1.96585 * \% \text{ Ni} \\ & +2.70960 * \% \text{ Ni}^2 \\ & -0.88051 * \% \text{ Ni}^3 \\ & +0.093217 * \% \text{ Ni}^4 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

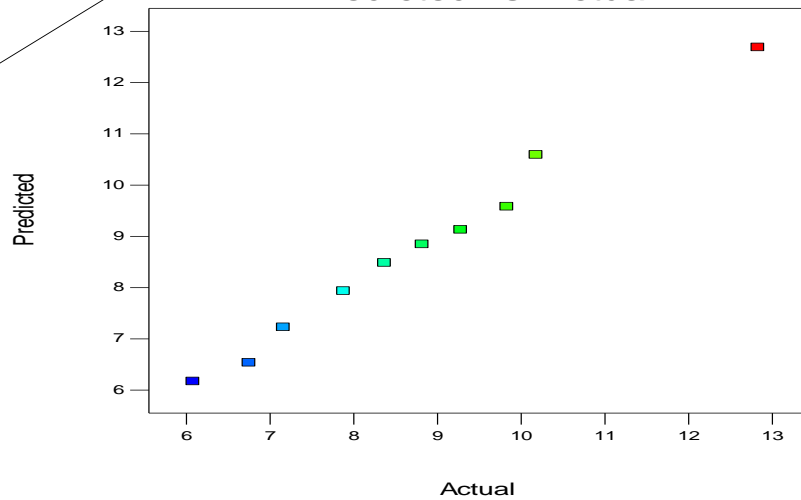


Design-Expert® Software  
Resistivity

Color points by value of  
Resistivity:



Predicted vs. Actual



**Response 7      Conductivity Transform:    None**

**Summary (detailed tables shown below)**

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	< 0.0001		0.8897	0.8432
<u>Quadratic</u>	<u>0.0040</u>		<u>0.9644</u>	<u>0.9387</u> <u>Suggested</u>
Cubic	0.2224		0.9682	0.9157
Quartic	0.0950		0.9793	0.9294
Fifth	0.6273		0.9758	0.6172
Sixth	0.2193		0.9821	0.6624

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F	p-value	Prob > F
Mean vs Total	329.59	1	329.59			
Linear vs Mean	29.69	1	29.69	73.61	< 0.0001	
<u>Quadratic vs Linear</u>	<u>2.31</u>	<u>1</u>	<u>2.31</u>	<u>17.76</u>	<u>0.0040</u>	

Suggested

Cubic vs Quadratic	0.22	1	0.22	1.85	0.2224
Quartic vs Cubic	0.32	1	0.32	4.22	0.0950
Fifth vs Quartic	0.024	1	0.024	0.28	0.6273
Sixth vs Fifth	0.16	1	0.16	2.40	0.2193
Residual	0.20	3	0.066		

"Sequential Model Sum of Squares [Type I]":  
Select the highest order polynomial where the

Total 362.51 10 36.25

additional terms are significant and the model is not aliased.

### Model Summary Statistics

	Std.		Adjusted	Predicted		
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS	
Linear	0.64	0.9020	0.8897	0.8432	5.16	
<u>Quadratic</u>	<u>0.36</u>	<u>0.9723</u>	<u>0.9644</u>	<u>0.9387</u>	<u>2.02</u>	<u>Suggested</u>
Cubic	0.34	0.9788	0.9682	0.9157	2.78	
Quartic	0.27	0.9885	0.9793	0.9294	2.32	
Fifth	0.30	0.9893	0.9758	0.6172	12.60	
Sixth	0.26	0.9940	0.9821	0.6624	11.11	

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

and the "Predicted R-Squared".

### Response 7 Conductivity

#### ANOVA for Response Surface Quadratic model

Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
--------	----------------	----	-------------	---------	------------------

Model	32.01	2	16.00	122.77	< 0.0001	significant
A-% Ni	29.69	1	29.69	227.79	< 0.0001	
A <sup>2</sup>	2.31	1	2.31	17.76	0.0040	
Residual	0.91	7	0.13			
Cor Total	32.92	9				

The Model F-value of 122.77 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.36	R-Squared	0.9723
Mean	5.74	Adj R-Squared	0.9644
C.V. %	6.29	Pred R-Squared	0.9387
PRESS	2.02	Adeq Precision	27.304
-2 Log Likelihood	4.44	BIC	11.35
		AICc	14.44

The "Pred R-Squared" of 0.9387 is in reasonable agreement with the "Adj R-Squared" of 0.9644; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 27.304 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard 95% CI 95% CI		
	Estimate	df	Error	Low	High VIF
Intercept	5.19	1	0.17	4.79	5.60
A-% Ni	-2.70	1	0.18	-3.12	-2.28 1.00
A <sup>2</sup>	1.34	1	0.32	0.59	2.09 1.00

#### Final Equation in Terms of Coded Factors:

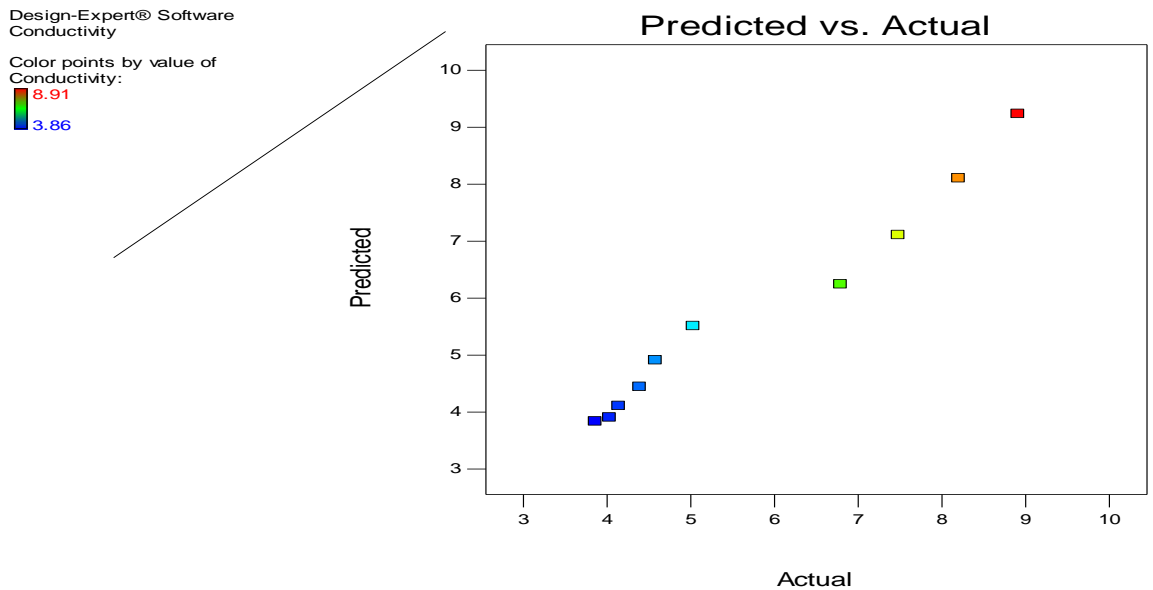
$$\begin{aligned} \text{Conductivity} = & \\ & +5.19 \\ & -2.70 * A \\ & +1.34 * A^2 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Conductivity} = & \\ & +10.49733 \\ & -2.65655 * \% \text{ Ni} \\ & +0.26485 * \% \text{ Ni}^2 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.



Factor Name	Level Low	Level High	Level Std. Dev.	Coding		
A	% Ni	2.66	0.50	5.00	0.000	Actual

Response	Predicted		Predicted Observed	Std Dev	CI for Mean		99% of Population	
	Mean	Median <sup>1</sup>			SE	95% CI	95% CI	95% TI

				Mean	low	high	low	high
Yield Strength	345.63	345.63	-	18.8738	9.05956	323.462	367.798	456.354
UTS	478.477	478.477	-	11.6858	5.60929	464.751	492.202	547.032
Hardness	235.973	235.973	-	12.264	5.88684	221.569	250.378	307.921
Elongation	22.4969	22.4969	-	0.270016	0.19675	21.8708	23.1231	24.9909
Impact Strength	31.6317	31.6317	-	0.229557	0.110189	31.3621	31.9013	32.9784
Resistivity	8.61948	8.61948	-	0.264292	0.160378	8.20721	9.03174	10.3649
Conductivity	5.30057	5.30057	-	0.361053	0.172527	4.89261	5.70853	7.2988

### Confirmation Report

Two-sided Confidence = 95% n = 1

Factor	Name	Level Low	Level High	Level Std. Dev.	Coding
A	% Ni	2.66	0.50	5.00	0.000 Actual

Response	Mean	Median <sup>1</sup>	Observed	Std Dev	n	SE Pred	95% PI low	Data Mean	95% PI high
Yield Strength	345.63	345.63	-	18.8738	1	20.94	294.40		396.86
UTS	478.477	478.477	-	11.6858	1	12.96	446.76		510.19
Hardness	235.973	235.973	-	12.264	1	13.60	202.69		269.26
Elongation	22.4969	22.4969	-	0.270016	1	0.33	21.43		23.56
Impact Strength	31.6317	31.6317	-	0.229557	1	0.25	31.01		32.25
Resistivity	8.61948	8.61948	-	0.264292	1	0.31	7.82		9.41
Conductivity	5.30057	5.30057	-	0.361053	1	0.40	4.35		6.25

## Design Expert Analysis for Vanadium

Response 1 Yield Strength Transform: None

Summary (detailed tables shown below)

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.0001		0.8454	0.7442
Quadratic	0.0244		0.9185	0.8094
<u>Cubic</u>	<u>0.0156</u>		<u>0.9668</u>	<u>0.9498</u> Suggested
Quartic	0.3141		0.9681	0.8616
Fifth	0.7826		0.9610	-0.4217
Sixth	0.0625		0.9863	-1.4100

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F	P-value
Mean vs Total	1.180E+006	1	1.180E+006		
Linear vs Mean	63315.28	1	63315.28	50.20	0.0001
Quadratic vs Linear	5434.92	1	5434.92	8.17	0.0244
<u>Cubic vs Quadratic</u>	<u>3027.90</u>	<u>1</u>	<u>3027.90</u>	<u>11.17</u>	<u>0.0156</u>
Quartic vs Cubic	325.60	1	325.60	1.25	0.3141
Fifth vs Quartic	27.70	1	27.70	0.087	0.7826
Sixth vs Fifth	938.44	1	938.44	8.41	0.0625
Residual	334.66	3	111.55		

Suggested

Total 1.253E+006 10 1.253E+005

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

Source	Std. Dev.	Adjusted R-Squared	Predicted R-Squared	Adjusted R-Squared	PRESS
Linear	35.51	0.8626	0.8454	0.7442	18776.56
Quadratic	25.79	0.9366	0.9185	0.8094	13990.01
<u>Cubic</u>	<u>16.46</u>	<u>0.9778</u>	<u>0.9668</u>	<u>0.9498</u>	<u>3685.29</u>
Quartic	16.13	0.9823	0.9681	0.8616	10162.71
Fifth	17.84	0.9827	0.9610	-0.4217	1.044E+005
Sixth	10.56	0.9954	0.9863	-1.4100	1.769E+005

Suggested

"Model Summary Statistics": Focus on the model maximizing

the "Adjusted R-Squared"  
and the "Predicted R-Squared".

**Response 1 Yield Strength**

**ANOVA for Response Surface Cubic model**

**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value	
Model	71778.10	3	23926.03	88.27	< 0.0001	significant
A-% V	21830.77	1	21830.77	80.54	0.0001	
A <sup>2</sup>	5434.92	1	5434.92	20.05	0.0042	
A <sup>3</sup>	3027.90	1	3027.90	11.17	0.0156	
Residual	1626.40	6	271.07			
Cor Total	73404.50	9				

The Model F-value of 88.27 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	16.46	R-Squared	0.9778
Mean	343.50	Adj R-Squared	0.9668
C.V. %	4.79	Pred R-Squared	0.9498
PRESS	3685.29	Adeq Precision	22.538
-2 Log Likelihood	79.29	BIC	88.50
		AICc	95.29

The "Pred R-Squared" of 0.9498 is in reasonable agreement with the "Adj R-Squared" of 0.9668; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 22.538 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard 95% CI			VIF
	Estimate	df	Error	Low	High	
Intercept	369.97	1	7.88	350.69	389.24	
A-% V	189.94	1	21.16	138.15	241.72	6.73
A <sup>2</sup>	-64.97	1	14.51	-100.47	-29.47	1.00
A <sup>3</sup>	-90.22	1	26.99	-156.28	-24.17	6.73

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Yield Strength} = & \\ & +369.97 \\ & +189.94 * A \\ & -64.97 * A^2 \\ & -90.22 * A^3 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Yield Strength} = & \\ & +205.50000 \\ & -24.70280 * \% V \\ & +52.51282 * \% V^2 \\ & -7.92075 * \% V^3 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

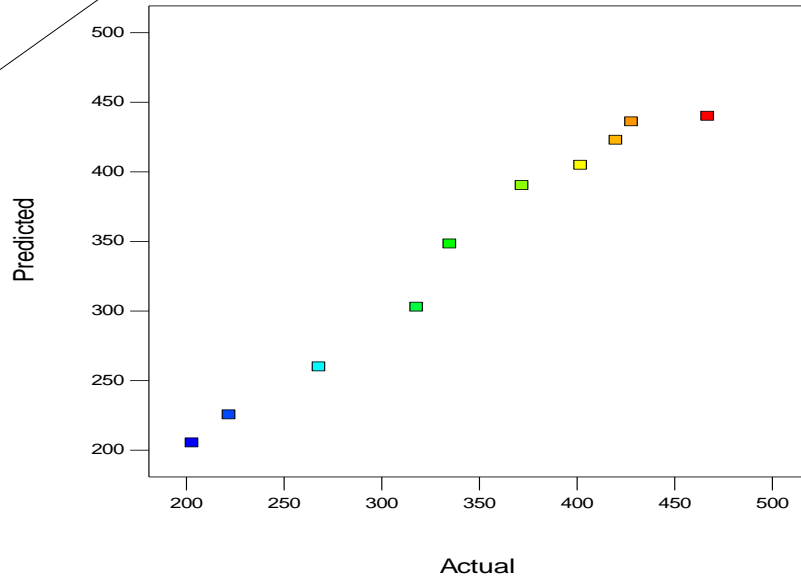


Design-Expert® Software  
Yield Strength

Color points by value of  
Yield Strength:



Predicted vs. Actual

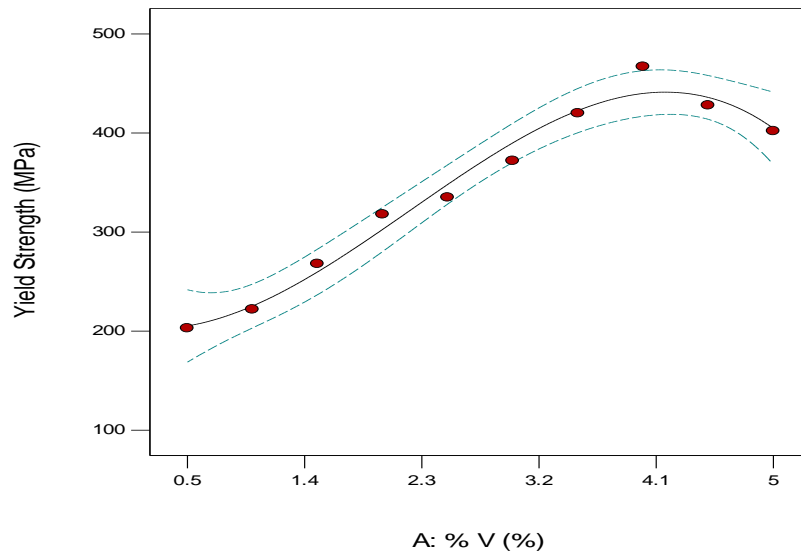


Design-Expert® Software  
Factor Coding: Actual  
Yield Strength (MPa)

● Design Points  
--- 95% CI Bands

X1 = A: % V

One Factor



**Response 2      UTS      Transform:   None**

**Summary (detailed tables shown below)**

	<b>Sequential Lack of Fit</b>	<b>Adjusted</b>	<b>Predicted</b>
<b>Source</b>	<b>p-value</b>	<b>R-Squared</b>	<b>R-Squared</b>
Linear	0.0002	0.8269	0.7176

Quadratic	0.0702	0.8801	0.7068
<u>Cubic</u>	<u>0.0037</u>	<u>0.9691</u>	<u>0.9430 Suggested</u>
Quartic	0.4160	0.9679	0.9379
Fifth	0.9301	0.9600	0.6470
Sixth	0.8418	0.9475	-5.0161

### Sequential Model Sum of Squares [Type I]

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob > F
Mean vs Total	2.201E+006	1	2.201E+006		
Linear vs Mean	30470.43	1	30470.43	43.98	0.0002
Quadratic vs Linear	2184.61	1	2184.61	4.55	0.0702
<u>Cubic vs Quadratic</u>	<u>2615.29</u>	<u>1</u>	<u>2615.29</u>	<u>21.13</u>	<u>0.0037</u>
Quartic vs Cubic	100.83	1	100.83	0.79	0.4160
Fifth vs Quartic	1.40	1	1.40	8.721E-003	0.9301
Sixth vs Fifth	9.94	1	9.94	0.047	0.8418
Residual	630.41	3	210.14		

Suggested

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 2.237E+006 10 2.237E+005

additional terms are significant and the model is not aliased.

### Model Summary Statistics

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS
Linear	26.32	0.8461	0.8269	0.7176	10169.92
Quadratic	21.90	0.9068	0.8801	0.7068	10559.84

Cubic	11.12	0.9794	0.9691	0.9430	2052.45	<u>Suggested</u>
Quartic	11.33	0.9822	0.9679	0.9379	2235.37	
Fifth	12.65	0.9822	0.9600	0.6470	12711.28	
Sixth	14.50	0.9825	0.9475	-5.0161	2.167E+005	"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

## Response 2 UTS

### ANOVA for Response Surface Cubic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value	
Model	35270.33	3	11756.78	95.00	< 0.0001	significant
A-% V	13102.05	1	13102.05	105.86	< 0.0001	
A <sup>2</sup>	2184.61	1	2184.61	17.65	0.0057	
A <sup>3</sup>	2615.29	1	2615.29	21.13	0.0037	
Residual	742.57	6	123.76			
Cor Total	36012.90	9				

The Model F-value of 95.00 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	11.12	R-Squared	0.9794
Mean	469.10	Adj R-Squared	0.9691
C.V. %	2.37	Pred R-Squared	0.9430
PRESS	2052.45	Adeq Precision	22.618
-2 Log Likelihood	71.45	BIC	80.66

AICc 87.45

The "Pred R-Squared" of 0.9430 is in reasonable agreement with the "Adj R-Squared" of 0.9691; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 22.618 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error	95% CI		VIF
	Estimate	df		Low	High	
Intercept	485.88	1	5.32	472.86	498.91	
A-% V	147.14	1	14.30	112.15	182.14	6.73
A <sup>2</sup>	-41.19	1	9.80	-65.18	-17.20	1.00
A <sup>3</sup>	-83.85	1	18.24	-128.48	-39.22	6.73

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned}
 \text{UTS} = & \\
 & +485.88 \\
 & +147.14 * A \\
 & -41.19 * A^2 \\
 & -83.85 * A^3
 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

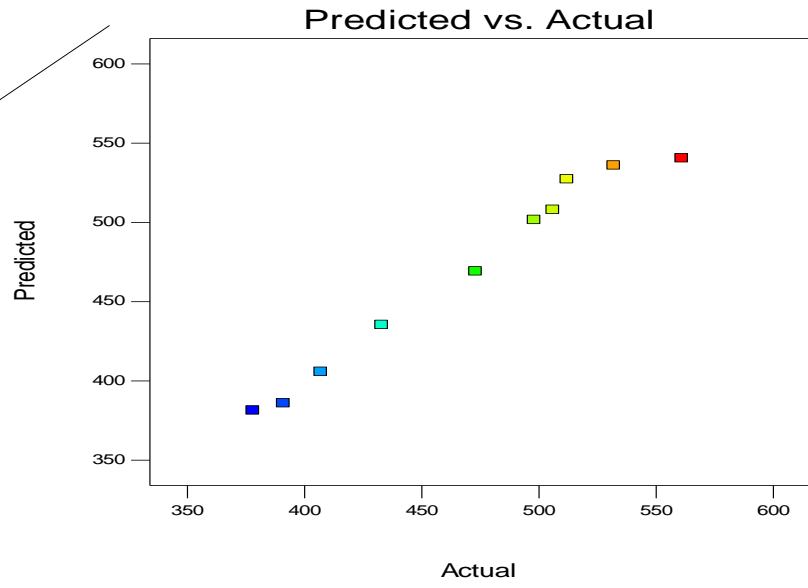
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned}
 \text{UTS} = & \\
 & +397.60000 \\
 & -56.86247 * \% V \\
 & +52.59441 * \% V^2 \\
 & -7.36131 * \% V^3
 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

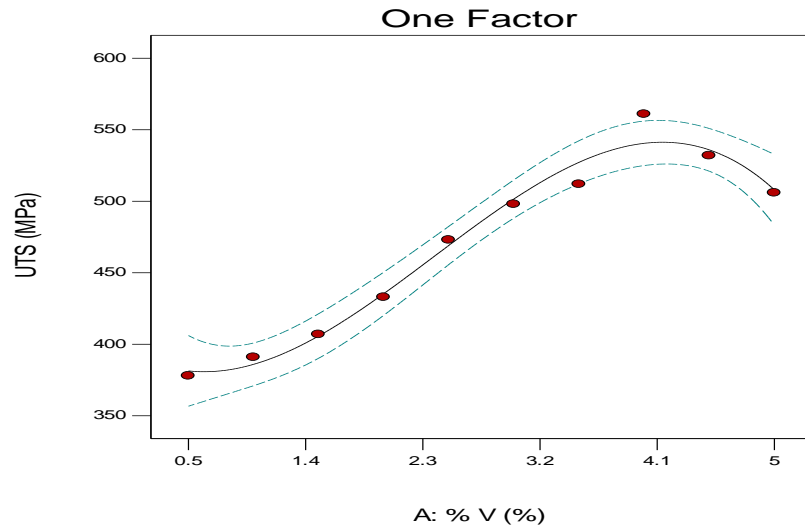
Design-Expert® Software  
UTS

Color points by value of  
UTS:  
561  
378



Design-Expert® Software  
Factor Coding: Actual  
UTS (MPa)

X1 = A: % V



Response 3                      Hardness   Transform:   None

Summary (detailed tables shown below)

	Sequential Lack of Fit	Adjusted	Predicted
Source	p-value	R-Squared	R-Squared
Linear	< 0.0001	0.8725	0.7945
Quadratic	0.0401	0.9235	0.8025
<u>Cubic</u>	<u>&lt; 0.0001</u>	<u>0.9944</u>	<u>0.9886</u> <u>Suggested</u>
Quartic	0.5919	0.9937	0.9597
Fifth	0.3435	0.9939	0.9299
Sixth	0.6145	0.9927	0.5396

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F	P-value
					Prob > F
Mean vs Total	6.121E+005	1	6.121E+005		
Linear vs Mean	70635.10	1	70635.10	62.61	< 0.0001
Quadratic vs Linear	4284.12	1	4284.12	6.33	0.0401
<u>Cubic vs Quadratic</u>	<u>4445.57</u>	<u>1</u>	<u>4445.57</u>	<u>90.23</u>	<u>0.0001</u> <u>Suggested</u>
Quartic vs Cubic	18.18	1	18.18	0.33	0.5919
Fifth vs Quartic	62.05	1	62.05	1.15	0.3435
Sixth vs Fifth	20.39	1	20.39	0.31	0.6145
Residual	194.99	3	65.00		

Total 6.917E+005 10 69172.80

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

	Std.	Adjusted		Predicted		
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS	
Linear	33.59	0.8867	0.8725	0.7945	16366.72	
Quadratic	26.03	0.9405	0.9235	0.8025	15731.02	
<u>Cubic</u>	<u>7.02</u>	<u>0.9963</u>	<u>0.9944</u>	<u>0.9886</u>	<u>906.71</u>	<u>Suggested</u>
Quartic	7.45	0.9965	0.9937	0.9597	3209.18	
Fifth	7.34	0.9973	0.9939	0.9299	5582.21	
Sixth	8.06	0.9976	0.9927	0.5396	36676.15	"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

**Response 3 Hardness**

**ANOVA for Response Surface Cubic model**

**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Model	79364.79	3	26454.93	536.96	< 0.0001	significant
A-% V	26880.80	1	26880.80	545.60	< 0.0001	
A <sup>2</sup>	4284.12	1	4284.12	86.96	< 0.0001	
A <sup>3</sup>	4445.57	1	4445.57	90.23	< 0.0001	
Residual	295.61	6	49.27			
Cor Total	79660.40	9				

The Model F-value of 536.96 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	7.02	R-Squared	0.9963
Mean	247.40	Adj R-Squared	0.9944
C.V. %	2.84	Pred R-Squared	0.9886
PRESS	906.71	Adeq Precision	53.987
-2 Log Likelihood	62.24	BIC	71.45
		AICc	78.24

The "Pred R-Squared" of 0.9886 is in reasonable agreement with the "Adj R-Squared" of 0.9944; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 53.987 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error		95% CI		VIF
	Estimate	df	Low	High	Low	High	
Intercept	270.90	1	3.36	262.68	279.12		
A-% V	210.76	1	9.02	188.68	232.84	6.73	
A <sup>2</sup>	-57.68	1	6.19	-72.82	-42.55	1.00	
A <sup>3</sup>	-109.32	1	11.51	-137.48	-81.16	6.73	

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Hardness} = & \\ & +270.90 \\ & +210.76 * A \\ & -57.68 * A^2 \\ & -109.32 * A^3 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

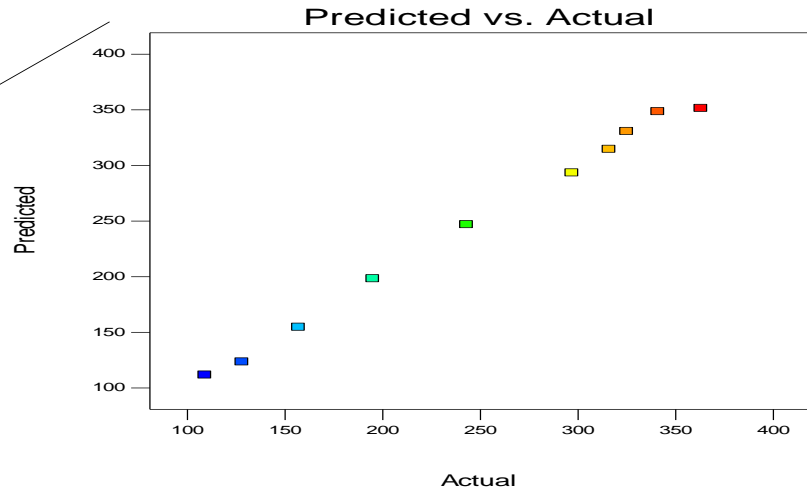
$$\begin{aligned} \text{Hardness} = & \\ & +126.73333 \\ & -61.40482 * \% V \\ & +67.78555 * \% V^2 \\ & -9.59751 * \% V^3 \end{aligned}$$



The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.

Design-Expert® Software  
Hardness

Color points by value of Hardness:

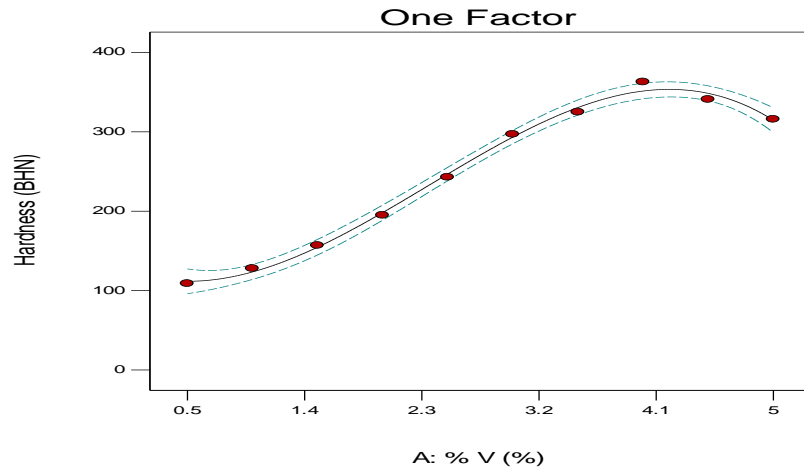


Design-Expert® Software

Factor Coding: Actual  
Hardness (BHN)

● Design Points  
--- 95% CI Bands

X1 = A: % V



**Response 4                      Elongation Transform:    None**

**Summary (detailed tables shown below)**

	<b>Sequential Lack of Fit</b>	<b>Adjusted</b>	<b>Predicted</b>
<b>Source</b>	<b>p-value</b>	<b>R-Squared</b>	<b>R-Squared</b>
Linear	< 0.0001	0.9095	0.8670
Quadratic	0.9480	0.8967	0.7666
Cubic	0.1203	0.9220	0.6998
<u>Quartic</u>	<u>0.0100</u>	<u>0.9780</u>	<u>0.8845 Suggested</u>
Fifth	0.4588	0.9764	0.0817
Sixth	0.0716	0.9910	-0.2352

**Sequential Model Sum of Squares [Type I]**

<b>Source</b>	<b>Sum of Squares</b>	<b>df</b>	<b>Mean Square</b>	<b>F Value</b>	<b>P-value Prob &gt; F</b>
Mean vs Total	4420.51	1	4420.51		
Linear vs Mean	91.46	1	91.46	91.48	< 0.0001
Quadratic vs Linear	5.219E-003	1	5.219E-003	4.571E-003	0.9480
Cubic vs Quadratic	2.82	1	2.82	3.27	0.1203
<u>Quartic vs Cubic</u>	<u>3.95</u>	<u>1</u>	<u>3.95</u>	<u>16.26</u>	<u>0.0100</u> <b>Suggested</b>
Fifth vs Quartic	0.17	1	0.17	0.67	0.4588
Sixth vs Fifth	0.74	1	0.74	7.49	0.0716
Residual	0.30	3	0.099		

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

Total 4519.97 10 452.00

**Model Summary Statistics**

	Std.		Adjusted	Predicted	
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS
Linear	1.00	0.9196	0.9095	0.8670	13.22
Quadratic	1.07	0.9196	0.8967	0.7666	23.21
Cubic	0.93	0.9480	0.9220	0.6998	29.86
<u>Quartic</u>	<u>0.49</u>	<u>0.9878</u>	<u>0.9780</u>	<u>0.8845</u>	<u>11.49</u>
Fifth	0.51	0.9895	0.9764	0.0817	91.33
Sixth	0.32	0.9970	0.9910	-0.2352	122.85

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

Suggested

**Response 4 Elongation**

**ANOVA for Response Surface Quartic model**  
**Analysis of variance table [Partial sum of squares - Type III]**

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	98.24	4	24.56	101.00	< 0.0001	significant
A-% V	27.42	1	27.42	112.74	0.0001	
A <sup>2</sup>	3.75	1	3.75	15.44	0.0111	
A <sup>3</sup>	2.82	1	2.82	11.61	0.0191	
A <sup>4</sup>	3.95	1	3.95	16.26	0.0100	
Residual	1.22	5	0.24			
Cor Total	99.46	9				

The Model F-value of 101.00 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup>, A<sup>3</sup>, A<sup>4</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.49	R-Squared	0.9878
Mean	21.03	Adj R-Squared	0.9780
C.V. %	2.35	Pred R-Squared	0.8845
PRESS	11.49	Adeq Precision	26.894
-2 Log Likelihood	7.31	BIC	18.82
		AICc	32.31

The "Pred R-Squared" of 0.8845 is in reasonable agreement with the "Adj R-Squared" of 0.9780; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 26.894 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard Error		95% CI		VIF
	Estimate	df	Error	Low	High		
Intercept	21.80	1	0.30	21.03	22.57		
A-% V	-6.73	1	0.63	-8.36	-5.10	6.73	
A <sup>2</sup>	-6.50	1	1.65	-10.74	-2.25	14.47	
A <sup>3</sup>	2.75	1	0.81	0.68	4.83	6.73	
A <sup>4</sup>	6.35	1	1.58	2.30	10.40	14.47	

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Elongation} = & \\ & +21.80 \\ & -6.73 * A \\ & -6.50 * A^2 \\ & +2.75 * A^3 \\ & +6.35 * A^4 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

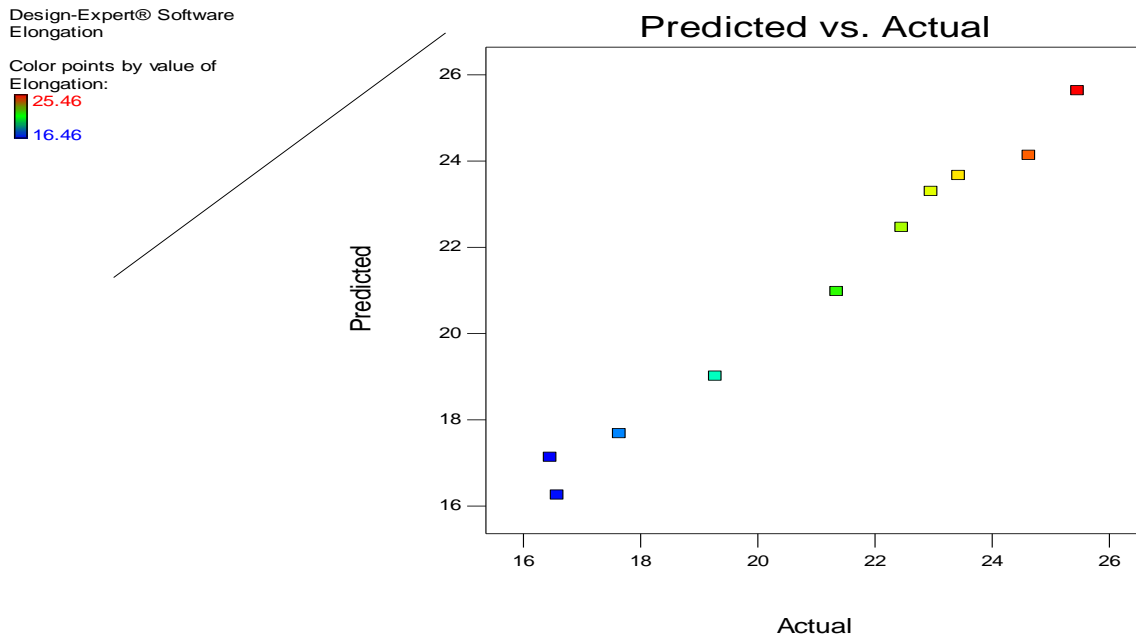
**Final Equation in Terms of Actual Factors:**

$$\begin{aligned} \text{Elongation} = & \\ & +29.47083 \\ & -11.07068 * \% V \\ & +7.97048 * \% V^2 \end{aligned}$$

$$-2.48510 * \% V^3$$

$$+0.24790 * \% V^4$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.



**Response 5                      Impact Strength Transform:    None**

**Summary (detailed tables shown below)**

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	< 0.0001		0.9419	0.9005
<u>Quadratic</u>	<u>0.0095</u>		<u>0.9762</u>	<u>0.9511</u> <u>Suggested</u>
Cubic	0.0791		0.9841	0.9570
Quartic	0.1459		0.9880	0.9789
Fifth	0.7540		0.9854	0.8221
Sixth	0.6627		0.9819	-2.3682

**Sequential Model Sum of Squares [Type I]**

	Sum of Squares	df	Mean Square	F Value	p-value	
Mean vs Total	9732.53	1	9732.53			
Linear vs Mean	124.08	1	124.08	147.01	< 0.0001	
<u>Quadratic vs Linear</u>	<u>4.33</u>	<u>1</u>	<u>4.33</u>	<u>12.49</u>	<u>0.0095</u>	<u>Suggested</u>
Cubic vs Quadratic	1.03	1	1.03	4.46	0.0791	
Quartic vs Cubic	0.52	1	0.52	2.96	0.1459	
Fifth vs Quartic	0.024	1	0.024	0.11	0.7540	
Sixth vs Fifth	0.061	1	0.061	0.23	0.6627	
Residual	0.79	3	0.26			

Total 9863.36 10 986.34

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

additional terms are significant and the model is not aliased.

**Model Summary Statistics**

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS	
Linear	0.92	0.9484	0.9419	0.9005	13.01	
<u>Quadratic</u>	<u>0.59</u>	<u>0.9815</u>	<u>0.9762</u>	<u>0.9511</u>	<u>6.39</u>	<u>Suggested</u>
Cubic	0.48	0.9894	0.9841	0.9570	5.62	
Quartic	0.42	0.9933	0.9880	0.9789	2.76	
Fifth	0.46	0.9935	0.9854	0.8221	23.27	
Sixth	0.51	0.9940	0.9819	-2.3682	440.66	

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

and the "Predicted R-Squared".

### Response 5 Impact Strength

#### ANOVA for Response Surface Quadratic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	128.40	2	64.20	185.33	< 0.0001	significant
A-% V	124.08	1	124.08	358.17	< 0.0001	
A <sup>2</sup>	4.33	1	4.33	12.49	0.0095	
Residual	2.42	7	0.35			
Cor Total	130.83	9				

The Model F-value of 185.33 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.59	R-Squared	0.9815
Mean	31.20	Adj R-Squared	0.9762
C.V. %	1.89	Pred R-Squared	0.9511
PRESS	6.39	Adeq Precision	34.237
-2 Log Likelihood	14.21	BIC	21.12
		AICc	24.21

The "Pred R-Squared" of 0.9511 is in reasonable agreement with the "Adj R-Squared" of 0.9762; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 34.237 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient Estimate	df	Standard Error	95% CI Low	95% CI High	VIF
Intercept	30.45	1	0.28	29.78	31.12	

A-% V	-5.52	1	0.29	-6.21	-4.83	1.00
A <sup>2</sup>	1.83	1	0.52	0.61	3.06	1.00

**Final Equation in Terms of Coded Factors:**

$$\begin{aligned} \text{Impact Strength} = & \\ & +30.45 \\ & -5.52 * A \\ & +1.83 * A^2 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

**Final Equation in Terms of Actual Factors:**

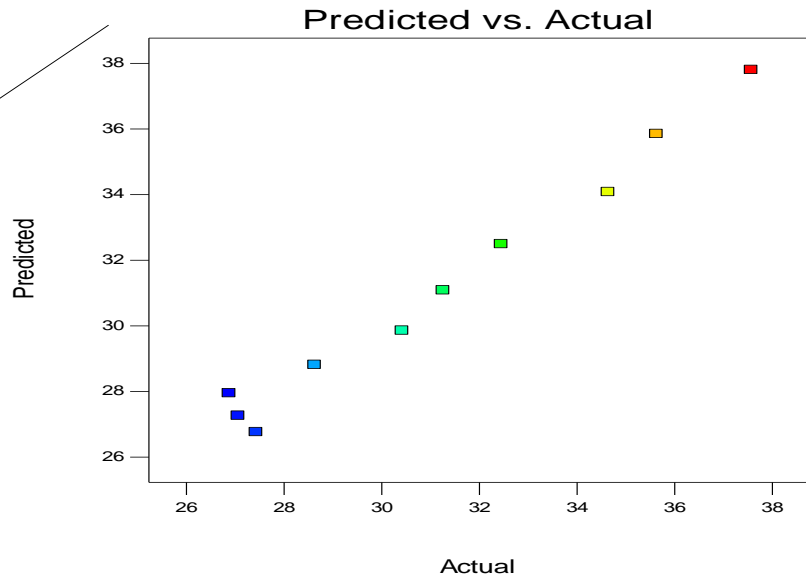
$$\begin{aligned} \text{Impact Strength} = & \\ & +39.93367 \\ & -4.44439 * \% V \\ & +0.36212 * \% V^2 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.



Design-Expert® Software  
Impact Strength

Color points by value of  
Impact Strength:



**Response 6      Resistivity Transform:    None**

**Summary (detailed tables shown below)**

Source	p-value	Adjusted R-Squared	Predicted R-Squared
Linear	0.2151	0.0828	-0.4183
<u>Quadratic</u>	<u>0.0003</u>	<u>0.8605</u>	<u>0.7581</u> <u>Suggested</u>
Cubic	0.0745	0.9083	0.6924
Quartic	0.1960	0.9238	0.4432
Fifth	0.0771	0.9603	0.8322
Sixth	0.9027	0.9474	-2.6766

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F	P-value
Mean vs Total	636.33	1	636.33		
Linear vs Mean	2.45	1	2.45	1.81	0.2151
<u>Quadratic vs Linear</u>	<u>9.36</u>	<u>1</u>	<u>9.36</u>	<u>45.58</u>	<u>0.0003</u>
Cubic vs	0.63	1	0.63	4.65	0.0745

Suggested

Quadratic					
Quartic vs Cubic	0.25	1	0.25	2.23	0.1960
Fifth vs Quartic	0.33	1	0.33	5.60	0.0771
Sixth vs Fifth	1.367E-003	1	1.367E-003	0.018	0.9027
Residual	0.23	3	0.077		

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 649.57 10 64.96

additional terms are significant and the model is not aliased.

### Model Summary Statistics

	Std.	Adjusted		Predicted	
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS
Linear	1.16	0.1847	0.0828	-0.4183	18.78
<u>Quadratic</u>	<u>0.45</u>	<u>0.8915</u>	<u>0.8605</u>	<u>0.7581</u>	<u>3.20</u>
Cubic	0.37	0.9388	0.9083	0.6924	4.07
Quartic	0.33	0.9577	0.9238	0.4432	7.37
Fifth	0.24	0.9824	0.9603	0.8322	2.22
Sixth	0.28	0.9825	0.9474	-2.6766	48.69

Suggested

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared"

and the "Predicted R-Squared".

## Response 6 Resistivity

### ANOVA for Response Surface Quadratic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	11.81	2	5.90	28.75	0.0004	Significant
A-% V	2.45	1	2.45	11.91	0.0107	
A <sup>2</sup>	9.36	1	9.36	45.58	0.0003	
Residual	1.44	7	0.21			
Cor Total	13.24	9				

The Model F-value of 28.75 implies the model is significant. There is only a 0.04% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.45	R-Squared	0.8915
Mean	7.98	Adj R-Squared	0.8605
C.V. %	5.68	Pred R-Squared	0.7581
PRESS	3.20	Adeq Precision	14.198
-2 Log Likelihood	8.98	BIC	15.89
		AICc	18.98

The "Pred R-Squared" of 0.7581 is in reasonable agreement with the "Adj R-Squared" of 0.8605; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 14.198 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard 95% CI 95% CI		
	Estimate	df	Error	Low	High VIF
Intercept	9.08	1	0.22	8.56	9.59
A-% V	0.77	1	0.22	0.24	1.31 1.00
A <sup>2</sup>	-2.70	1	0.40	-3.64	-1.75 1.00

### Final Equation in Terms of Coded Factors:

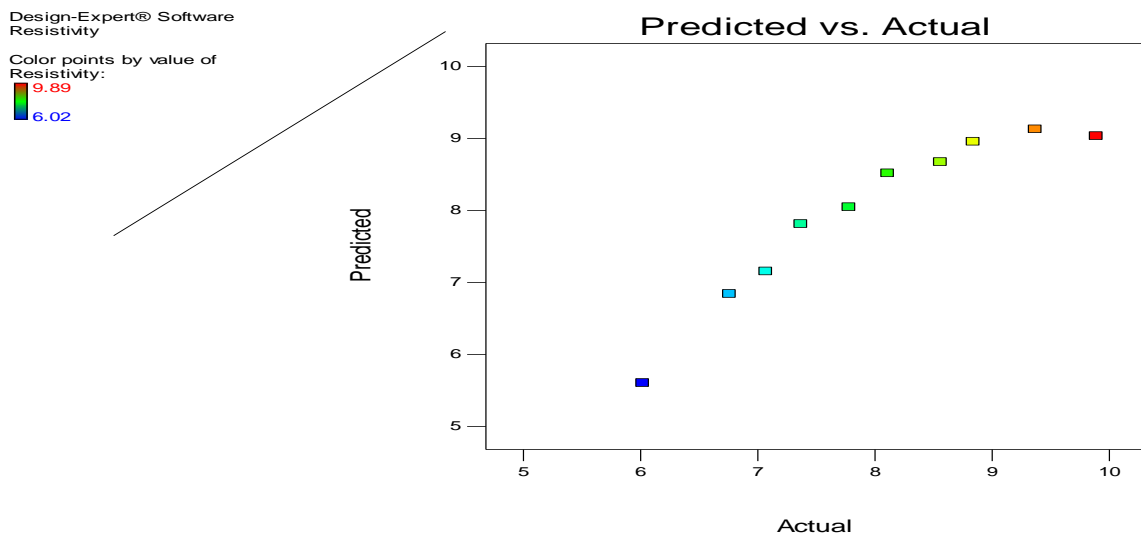
$$\begin{aligned} \text{Resistivity} = & \\ & +9.08 \\ & +0.77 * A \\ & -2.70 * A^2 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

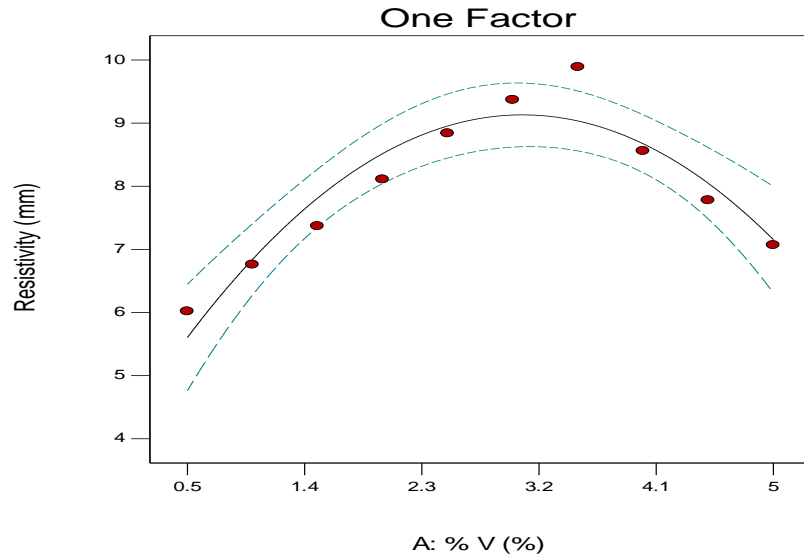
### Final Equation in Terms of Actual Factors:

$$\begin{aligned} \text{Resistivity} = & \\ & +4.10083 \\ & +3.27353 * \% V \\ & -0.53258 * \% V^2 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.



Design-Expert® Software  
 Factor Coding: Actual  
 Resistivity (mm)  
 ● Design Points  
 --- 95% CI Bands  
 X1 = A: % V



**Response 7**      **Conductivity Transform: None**

**Summary (detailed tables shown below)**

Source	Sequential p-value	Lack of Fit p-value	Adjusted R-Squared	Predicted R-Squared
Linear	< 0.0001		0.9374	0.9161
<u>Quadratic</u>	<u>0.0221</u>		<u>0.9678</u>	<u>0.9398</u> <b>Suggested</b>
Cubic	0.3527		0.9679	0.9061
Quartic	0.0764		0.9807	0.9524
Fifth	0.7827		0.9764	0.7590
Sixth	0.3632		0.9772	0.6519

**Sequential Model Sum of Squares [Type I]**

Source	Sum of Squares	df	Mean Square	F Value	P-value	Prob > F
Mean vs Total	319.23	1	319.23			
Linear vs Mean	32.84	1	32.84	135.88	< 0.0001	
<u>Quadratic vs Linear</u>	<u>1.06</u>	<u>1</u>	<u>1.06</u>	<u>8.56</u>	<u>0.0221</u>	<b>Suggested</b>
Cubic vs	0.13	1	0.13	1.01	0.3527	

Quadratic					
Quartic vs Cubic	0.37	1	0.37	4.96	0.0764
Fifth vs Quartic	7.949E-003	1	7.949E-003	0.087	0.7827
Sixth vs Fifth	0.10	1	0.10	1.14	0.3632
Residual	0.26	3	0.088		

"Sequential Model Sum of Squares [Type I]": Select the highest order polynomial where the

Total 354.00 10 35.40

additional terms are significant and the model is not aliased.

### Model Summary Statistics

	Std.		Adjusted	Predicted		
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS	
Linear	0.49	0.9444	0.9374	0.9161	2.92	
<u>Quadratic</u>	<u>0.35</u>	<u>0.9750</u>	<u>0.9678</u>	<u>0.9398</u>	<u>2.09</u>	<u>Suggested</u>
Cubic	0.35	0.9786	0.9679	0.9061	3.27	
Quartic	0.27	0.9893	0.9807	0.9524	1.66	
Fifth	0.30	0.9895	0.9764	0.7590	8.38	
Sixth	0.30	0.9924	0.9772	0.6519	12.10	

"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

## Response 7 Conductivity

### ANOVA for Response Surface Quadratic model

#### Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	33.90	2	16.95	136.46	< 0.0001	Significant
A-% V	32.84	1	32.84	264.35	< 0.0001	
A <sup>2</sup>	1.06	1	1.06	8.56	0.0221	
Residual	0.87	7	0.12			
Cor Total	34.77	9				

The Model F-value of 136.46 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, A<sup>2</sup> are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Std. Dev.	0.35	R-Squared	0.9750
Mean	5.65	Adj R-Squared	0.9678
C.V. %	6.24	Pred R-Squared	0.9398
PRESS	2.09	Adeq Precision	29.413
-2 Log Likelihood	3.96	BIC	10.86
		AICc	13.96

The "Pred R-Squared" of 0.9398 is in reasonable agreement with the "Adj R-Squared" of 0.9678; i.e. the difference is less than 0.2. "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 29.413 indicates an adequate signal. This model can be used to navigate the design space.

Factor	Coefficient		Standard 95% CI 95% CI		
	Estimate	df	Error	Low	High VIF
Intercept	5.28	1	0.17	4.88	5.68
A-% V	-2.84	1	0.17	-3.25	-2.43 1.00
A <sup>2</sup>	0.91	1	0.31	0.17	1.64 1.00

### Final Equation in Terms of Coded Factors:

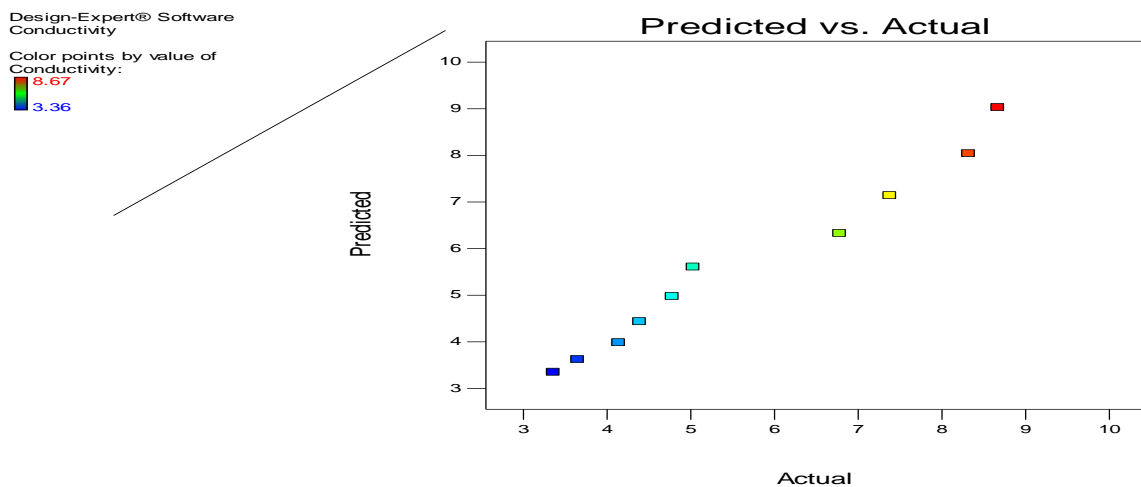
$$\begin{aligned} \text{Conductivity} = & \\ & +5.28 \\ & -2.84 * A \\ & +0.91 * A^2 \end{aligned}$$

The equation in terms of coded factors can be used to make predictions about the response for given levels of each factor. By default, the high levels of the factors are coded as +1 and the low levels of the factors are coded as -1. The coded equation is useful for identifying the relative impact of the factors by comparing the factor coefficients.

### Final Equation in Terms of Actual Factors:

$$\begin{aligned} \text{Conductivity} = & \\ & +10.10750 \\ & -2.24932 * \% V \\ & +0.17955 * \% V^2 \end{aligned}$$

The equation in terms of actual factors can be used to make predictions about the response for given levels of each factor. Here, the levels should be specified in the original units for each factor. This equation should not be used to determine the relative impact of each factor because the coefficients are scaled to accommodate the units of each factor and the intercept is not at the center of the design space.





**Factor Name Level Low Level High Level Std. Dev. Coding**

A % V 2.72 0.50 5.00 0.000 Actual

Response	Predicted Predicted		Observed	Std Dev	SE Mean	CI for Mean		99% of Population	
	Mean	Median <sup>1</sup>				95% CI low	95% CI high	95% TI low	95% TI high
Yield Strength	367.172	367.172	-	16.4641	7.88088	347.888	386.456	270.627	463.717
UTS	483.717	483.717	-	11.1248	5.32512	470.686	496.747	418.481	548.952
Hardness	267.8	267.8	-	7.01911	3.35984	259.578	276.021	226.64	308.959
Elongation	21.8959	21.8959	-	0.493118	0.299951	21.1249	22.667	18.6378	25.1541
Impact Strength	30.5314	30.5314	-	0.588572	0.281546	29.8656	31.1971	27.2734	33.7894
Resistivity	9.06351	9.06351	-	0.45314	0.216761	8.55095	9.57606	6.55518	11.5718
Conductivity	5.32148	5.32148	-	0.352456	0.168599	4.92281	5.72016	3.37049	7.27248

**Confirmation Report**

Two-sided Confidence = 95% n = 1

**Factor Name Level Low Level High Level Std. Dev. Coding**

A % V 2.72 0.50 5.00 0.000 Actual

Response	Predicted Predicted		Observed	Std Dev	n	SE Pred	95% PI low	Data Mean	95% PI high
	Mean	Median <sup>1</sup>							
Yield Strength	367.172	367.172	-	16.4641	1	18.25	322.51		411.84
UTS	483.717	483.717	-	11.1248	1	12.33	453.54		513.90
Hardness	267.8	267.8	-	7.01911	1	7.78	248.76		286.84
Elongation	21.8959	21.8959	-	0.493118	1	0.58	20.41		23.38
Impact Strength	30.5314	30.5314	-	0.588572	1	0.65	28.99		32.07
Resistivity	9.06351	9.06351	-	0.45314	1	0.50	7.88		10.25
Conductivity	5.32148	5.32148	-	0.352456	1	0.39	4.40		6.25

**APPENDIX 11**  
**PICTURES FROM EXPERIMENTAL PROCEDURE**



Figure (a) represents the electronic weighing balance



Figure (b) represents the process of measuring out copper using electronic weighing balance.



Figure (c) represents the process of measuring out aluminum using electronic weighing balance.



Figure (d) represents the process of measuring out carbide forming elements using electronic weighing balance.



Figure (e) represents the samples for mechanical properties test.



Figure (f) represents the samples for microstructural examination.

